

Upward Transition Probabilities $B(E2) \uparrow$ Properties Study of Even-Even $^{104-114}\text{Ru}$ Nuclei

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Abstract

In this research work, the upward transition probabilities for the transition levels, $0^+ \rightarrow 2^+$, $2^+ \rightarrow 4^+$, $4^+ \rightarrow 6^+$ and $6^+ \rightarrow 8^+$ levels of even-even neutron rich $^{104-114}\text{Ru}$ isotopes have been calculated by using the Global Best Fit (GBF) method. In addition, the associated parameters such as, Quadrupole moment and Deformation parameter of even-even $^{104-114}\text{Ru}$ have been calculated. The dependency of these nuclear parameters shows the nuclear magic number tendency.

Keywords

Upward Transition Probability, Quadrupole Moment, Deformation Parameter, Global Best Fit (GBF) Method

1. Introduction

In nuclear physics, the most important part of interest is the shape of nucleus. The ground state shape of nucleus is spherical and it may deviate from this shape, which is closely related to nuclear “magic numbers”. We will consider nuclear “magic numbers” and their evolution along the nuclear chart. In stable nuclei, large gaps exist between nuclear shells when the proton or neutron number is equal to 2, 8, 20, 28, 50, 82 and 126 [1] [2]. These gaps result in large transition energy values between the ground and first excited states, relatively low quadrupole moment and small neutron capture cross sections. The “magic numbers” and their values are not preserved; they evolve for unstable nuclei due to nuclear structure effects. Therefore, nuclear properties of the first excited 2^+ states in even-even nuclei provide important information on the evolution of nuclear properties and shell model studies. The electric quadrupole reduced transition probabilities are important for the nuclear structural information. In

recent years, the electric quadrupole reduced transition probabilities of even-even neutron rich nuclei $^{102-112}\text{Pd}$ [3] [4], $^{104-112}\text{Cd}$ [5] and $^{100-102}\text{Ru}$ [6] have been studied using Interacting Boson Model-1 (IBM-1). This model (IBM-1) was developed by Iachello and Arima and the associated quadrupole moment, deformation parameter was studied [3] [4]. All these nuclei studied the downward reduced transition probabilities using the IBM-1. In 1999, excitation energies, $E2$ transition probabilities, quadrupole excitation properties for even-even $^{104-114}\text{Ru}$ and $^{106-110}\text{Pd}$ nuclei have been studied [7]. After this study, Skyrme force SLy4 for even-even $^{94-110}\text{Ru}$ nuclei has been investigated using the Hartree-Fock-Bogoliubov (HFB) method [8] and the triaxial shapes properties for the even-even neutron-rich $^{106-108}\text{Mo}$ and $^{108-112}\text{Ru}$ nuclei have been also investigated [9]. Therefore, we have focused the even-even $^{104-114}\text{Ru}$ isotopes' properties for the transition levels $0^+ \rightarrow 2^+$, $2^+ \rightarrow 4^+$, $4^+ \rightarrow 6^+$ and $6^+ \rightarrow 8^+$. For this purpose, we have used the GBF model to investigate the basic information of the $^{104-114}\text{Ru}$ nuclei because this model describes three basic properties: mass and energy dependency with γ -ray transition probability; localization and emphasis for the anchor nucleus; regionalized by the magic number [10].

Using GBF method, we have calculated the upward transition probabilities, $B(E2) \uparrow$ of neutron-rich even-even $^{104-114}\text{Ru}$ nuclei. Associated parameters like quadrupole moment (Q_0) and deformation parameter (β_2) have also been estimated. The study also reveals the effects of the estimated parameters on the nuclear structure. This also reveals how the estimated parameters affect the structure of the nucleus. This method presents the near magic number $N = 50$ region for the ^{104}Ru nucleus. In this paper, energy and mass dependencies have been showed with the calculated $B(E2) \uparrow$ values. The relationship among $B(E2) \uparrow$, Q_0 and β_2 are also given in graphical. Finally, this paper is arranged as: GBF model has been described in 2.1 Section. $B(E2) \uparrow$, Q_0 and β_2 are discussed in Section 2.2, 2.3 and 2.4 respectively.

2. Theory

In this section we describe the procedure used to compute the electric quadrupole reduced transition probabilities and the corresponding electric quadrupole moment and deformation parameter. The procedure summary of this theory is described in the following flowchart (Figure 1):

2.1. Global Best Fit (GBF) Method

According to the Global Best Fit Method, a knowledge of the energy E (Kev) of the 2_1^+ state is all that is required to make a prediction for the corresponding mean life time for the γ -ray, τ_γ (in ps) and hence, the $B(E2) \uparrow$ ($e^2\text{b}^2$) value. Within the framework of the hydrodynamic model with irrotational flow, Bohr and Mottelson [11] have derived simple expressions for the τ_γ value is given by

$$\tau_\gamma \approx 0.6 \times 10^{14} E^{-4} Z^{-2} A^{1/3} \quad (1)$$

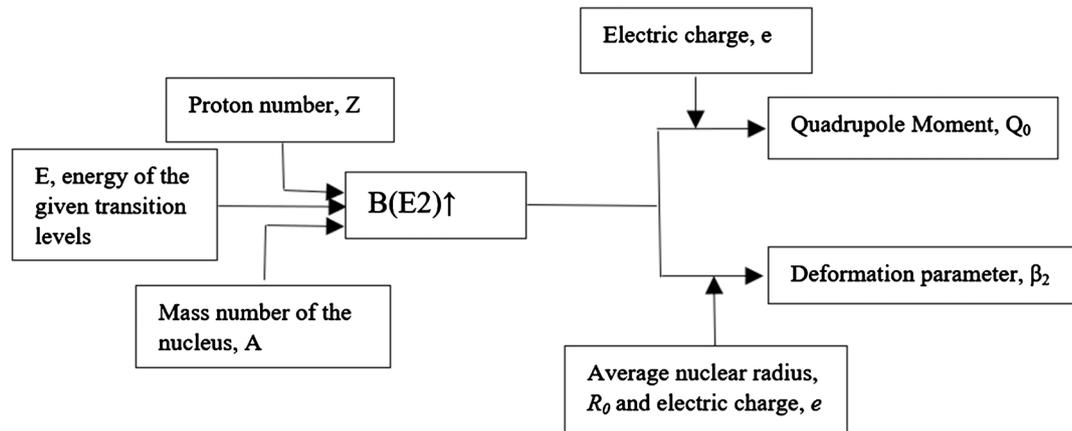


Figure 1. Flowchart of the given procedure.

For small harmonic vibrations of spherical nucleus, the τ_γ value is,

$$\tau_\gamma \approx 1.4 \times 10^{14} E^{-4} Z^{-2} A^{1/3} \quad (2)$$

For collective rotations of axially symmetric nuclei. The $E^{-4}Z^{-2}$ dependence in the above expressions was adopted by Grodzins [12] in his empirical fits (for all even-even nuclei), but he replaced $A^{1/3}$ with A . When the exponents of E and A were allowed to vary, we found earlier [10] [13] that the best global fit to the data [14] was obtained by

$$\tau_\gamma = 1.25 \times 10^{14} E^{-4} Z^{-2} A^{0.69} \quad (3)$$

Hence, τ_γ and $B(E2) \uparrow$ is related by the equation [7],

$$\tau_\gamma = 40.81 \times 10^{13} E^{-5} [B(E2) \uparrow / e^2 b^2]^{-1} \quad (4)$$

When converted to $B(E2) \uparrow$, this expression led to

$$B(E2) \uparrow = 3.26 E^{-1} Z^2 A^{-0.69} \quad (5)$$

We also showed that, the $1/E$ dependence is more important than the exact A dependence. If the exponent of A is fixed as $-2/3$ (instead of -0.69), the revised best fit to the data was found [15] to be,

$$B(E2) \uparrow = 2.6 E^{-1} Z^2 A^{-2/3} \quad (6)$$

Here, $B(E2) \uparrow$ is the electric quadrupole transition probability, E is the excitation energy, Z is the atomic number and A is the mass number.

2.2. Electric Quadrupole Reduced Transition Probability

The upward transition probabilities $B(E2) \uparrow$ is the transition of a particle from lower energy state to higher energy state [10]. It can be calculated by using the following equation obtained from GBF method,

$$B(E2; l_i \rightarrow l_f) \uparrow = 2.6 E^{-1} Z^2 A^{-2/3} \quad (7)$$

Here, l_i is the lower energy state and l_f is the higher energy state and the subscript i and f indicate the initial and final respectively.

2.3. Electric Quadrupole Moment

The nuclear electricintrinsic quadrupole moment is a parameter which describes the nuclear charge distribution. A non-zero intrinsic quadrupole moment Q_0 indicates that the charge distribution is not spherically symmetric. By convention the value of Q_0 is taken to be positive ($Q_0 > 0$) if the ellipsoid is prolate and negative ($Q_0 < 0$) if it is oblate [16]. Intrinsic quadrupole moment, Q_0 is related to the electric quadrupole transition probabilities $B(E2) \uparrow$, calculated by the following equation [17]

$$Q_0 = \left[\frac{16\pi B(E2) \uparrow}{5e^2} \right]^{1/2} \quad (8)$$

Here, Q_0 measured in barn (b) unit.

2.4. Deformation Parameter

Deformation Parameter is the parameter which measures the elongation of the axially symmetric shape of a deformed nucleus from its spherical shape. Deformation parameter is denoted by β_2 which is, related to $B(E2) \uparrow$, calculated by the equation [17]

$$\beta_2 = \left(4\pi/3 ZR_0^2 \right) \left[B(E2) \uparrow / e^2 \right]^{1/2} \quad (9)$$

Here, R_0 is the average radius nuclear which can be obtained from the following equation,

$$R_0^2 = 0.0144A^{2/3} \text{ barn (b)}. \quad (10)$$

3. Results and Discussion

The values of the E , R_0 , $B(E2) \uparrow$, β_2 and Q_0 for the even-even $^{104-114}\text{Ru}$ nuclei are given in **Table 1**. For the even-even $^{104-114}\text{Ru}$ nuclei, E has been obtained from the references [18]-[23] and other values $B(E2) \uparrow$, Q_0 , β_2 and R_0 , have been obtained using the Equations (7)-(10) respectively. Using these values, **Figures 2-6** have been drawn in below where their relations and behaviors have been discussed.

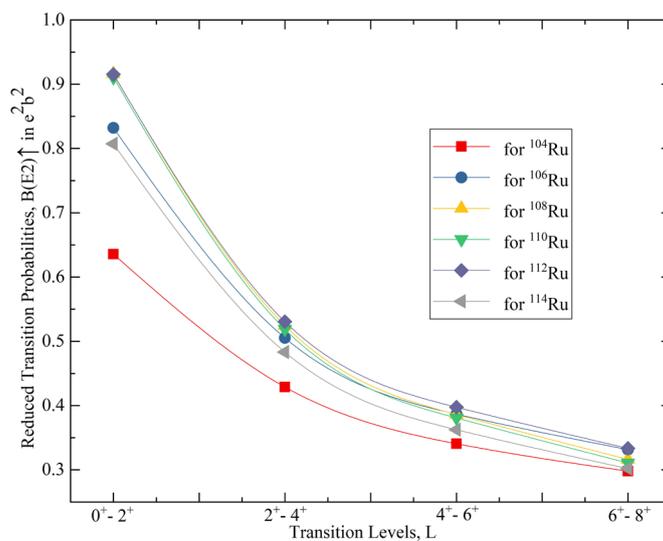
Electric quadrupole transition probabilities $B(E2) \uparrow$ is drawn as a function of transition levels for even-even $^{104-114}\text{Ru}$ nuclei in **Figure 2**. This shows the decrease of $B(E2) \uparrow$ with increasing the transition levels for each nucleus. For the $0^+ - 2^+$ transition level, transition probability is higher than that of the other transition levels for each nucleus. In this level, transition probability for the ^{104}Ru is the lowest than the other nuclei.

Deformation parameter is drawn as a function of $B(E2) \uparrow$ in **Figure 3**. This figure shows the transition probability increasing with the increase of deformation parameter. For $^{104-114}\text{Ru}$ nuclei, deformation parameter change follows the almost linear relationship with respect to the transition levels.

Quadrupole moment variations with the change of $B(E2) \uparrow$ are shown in **Figure 4**. **Figure 4** shows that quadrupole moments increase linearly with the

Table 1. The electric quadrupole transition probabilities, deformation parameters and quadrupole moments of $^{104-114}\text{Ru}$ isotope.

Nuclei	Transition Level, $I_i \rightarrow I_f$	Energy, E in KeV [18]-[23]	Average Radius, R_0^2 (b)	Upward Transition, $B(E2) \uparrow$ (e^2b^2)	Deformation parameter, β_2	Quadrupole Moment, Q_0 (b)
^{104}Ru	$0^+ - 2^+$	358.02	0.31845	0.6357	0.2383	2.5279
	$2^+ - 4^+$	530.46		0.4290	0.1958	2.0767
	$4^+ - 6^+$	668.12		0.3407	0.1744	1.8507
	$6^+ - 8^+$	764		0.2979	0.1631	1.7305
^{106}Ru	$0^+ - 2^+$	270.07	0.32252	0.8321	0.2692	2.8922
	$2^+ - 4^+$	444.63		0.5054	0.2098	2.254
	$4^+ - 6^+$	581.1		0.3867	0.1835	1.9716
^{108}Ru	$0^+ - 2^+$	242.24	0.32657	0.9162	0.279	3.0349
	$2^+ - 4^+$	422.96		0.5247	0.2111	2.2967
	$4^+ - 6^+$	574.8		0.3861	0.1811	1.9701
	$6^+ - 8^+$	701.6		0.3163	0.1639	1.7831
^{110}Ru	$0^+ - 2^+$	240.73	0.33059	0.9107	0.2748	3.0257
	$2^+ - 4^+$	422.62		0.5188	0.2074	2.2837
	$4^+ - 6^+$	575.75		0.3808	0.1777	1.9565
	$6^+ - 8^+$	705.4		0.3108	0.1605	1.7676
^{112}Ru	$0^+ - 2^+$	236.66	0.33458	0.9153	0.2722	3.0334
	$2^+ - 4^+$	408.24		0.5305	0.2072	2.3093
	$4^+ - 6^+$	545		0.3974	0.1793	1.9987
	$6^+ - 8^+$	649.5		0.3335	0.1643	1.831
^{114}Ru	$0^+ - 2^+$	265.19	0.33855	0.8073	0.2526	2.8488
	$2^+ - 4^+$	443.01		0.4832	0.1954	2.204
	$4^+ - 6^+$	590.6		0.3625	0.1693	1.9089
	$6^+ - 8^+$	709.1		0.3019	0.1545	1.7421

**Figure 2.** The change of reduced transition probabilities with transition levels.

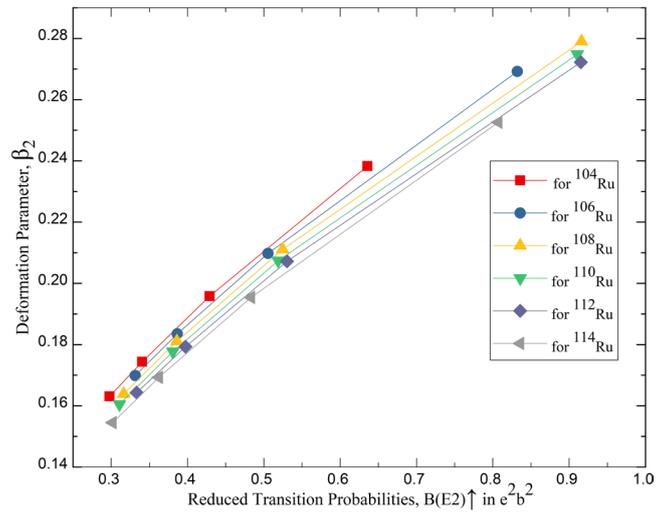


Figure 3. Deformation parameter change with respect to the reduced transition probabilities.

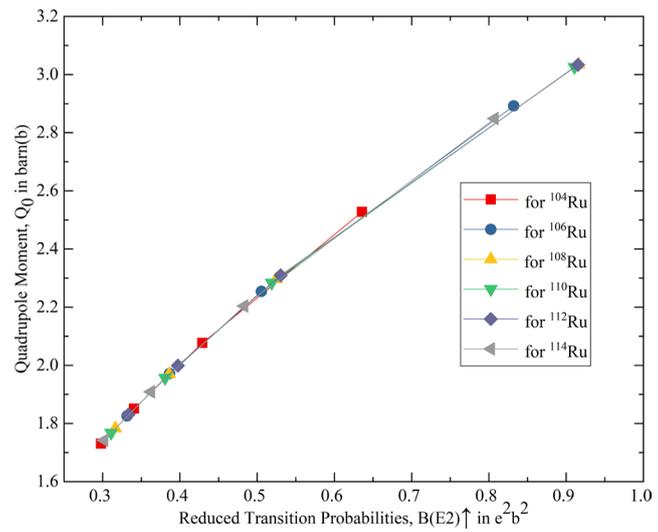


Figure 4. Quadrupole moment variations with the change of reduced transition probabilities.

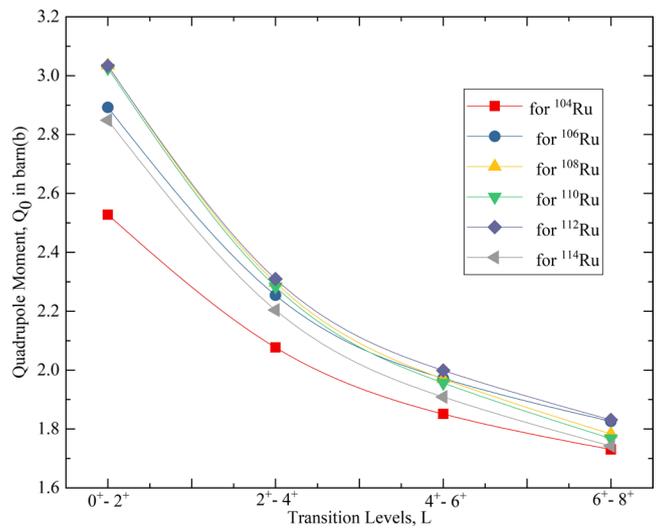


Figure 5. Quadrupole moment change with respect to the transition levels.

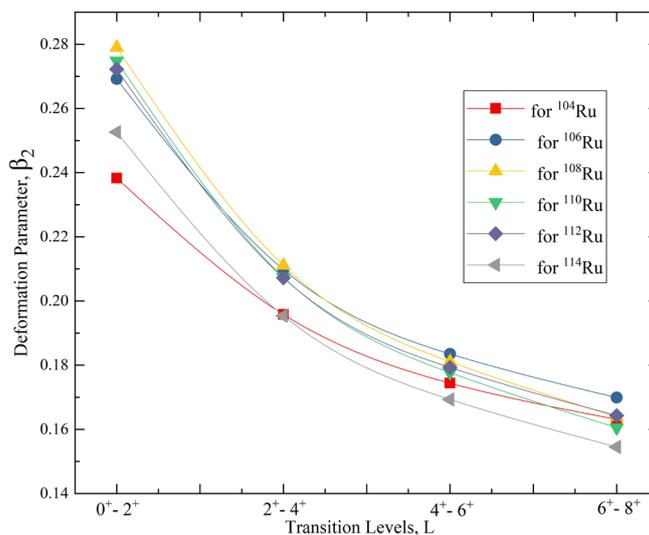


Figure 6. The change of deformation parameter against the transition levels.

transition probabilities increasing. In the figure, we see that quadrupole moment is lower for the isotope ¹⁰⁴Ru.

Quadrupole moments are represented with the variation of transition levels for even-even ¹⁰⁴⁻¹¹⁴Ru nuclei in **Figure 5**. In this figure, we see the quadrupole moments decrease with increasing the transition levels for each nucleus and it reaches in the range 1.8 - 3.0 barn (b).

Figure 6 shows the deformation parameters variation with respect to the transition levels for these even-even nuclei. From this figure, we can conclude that at first transition level 0⁺ - 2⁺ the deformation of the nucleus shape will be maximum for each nucleus, gradually the deformations decrease for upper transition levels.

4. Conclusion

It is seen from the data and corresponding graphs, when the transition levels of any nuclei increased the electric quadrupole moment and reduced transition probabilities $B(E2) \uparrow$ of the given nuclei are decreased. The deformation parameters also decrease with increasing transition levels. It concludes from the data and corresponding graph, the transition probabilities, quadrupole moment, and deformation parameters have comparatively lower values for the isotopes which have neutron number close to magic number 50.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

- [1] Goepfert-Mayer, M. (1950) Nuclear Configurations in the Spin-Orbit Coupling Model. I. Empirical Evidence. *Physical Review*, **78**, 16.

- <https://doi.org/10.1103/PhysRev.78.16>
- [2] Sorlin, O. and Porquet, M.-G. (2008) Nuclear Magic Numbers: New Features far from Stability. *Progress in Particle and Nuclear Physics*, **61**, 602. <https://doi.org/10.1016/j.ppnp.2008.05.001>
- [3] Hossain, I., Saeed, M.A., Ghani, N.N.A.M.B., Sa'adeh, H., Hussein, M. and Abdullah, H.Y. (2014) Electromagnetic Reduced Transition Properties of the Ground State Band of Even-Even 102-106Pd Isotopes by Means of Interacting Boson Model-I. *Indian Journal of Physics*, **88**, 5-9. <https://doi.org/10.1007/s12648-013-0374-5>
- [4] Hossain, I., Abdullah, H.Y., Ahmad, I.M. and Saeed, M.A. (2014) B(E2) Value of Even-Even ¹⁰⁸⁻¹¹²Pd Isotopes by Interacting Boson Model-I. *Chinese Physics C*, **38**, Article ID: 024103. <https://doi.org/10.1088/1674-1137/38/2/024103>
- [5] Abdullah, H.Y., Hossain, I., Ahmed, I.M., Ahmed, S.T., Kasmin, M.K., Saeed, M.A. and Ibrahim, N. (2013) Electromagnetic Reduced Transition Properties of Even-Even ¹⁰⁴⁻¹¹²Cd Isotopes. *Indian Journal of Physics*, **87**, 571-574. <https://doi.org/10.1007/s12648-013-0257-9>
- [6] Hossain, I., Ahmed, I.M., Sharrad, F.I., Abdullah, H.Y., Salman, A.D. and Al-Dahan, N. (2015) Yrast States and B(E2) Values of Even 100-102Ru Isotopes Using Interacting Boson Model (IBM-1). *Chiang Mai Journal of Science*, **42**, 996-1004. <https://www.researchgate.net/publication/273684750>
- [7] Zajac, K., Przbniak, L., Pomorski, K., Rohoziiiski, S.G. and Srebrny, J. (1999) The Low-Lying Quadrupole Collective Excitations of Ru and Pd Isotopes. *Nuclear Physics A*, **653**, 71-87. [https://doi.org/10.1016/S0375-9474\(99\)00161-X](https://doi.org/10.1016/S0375-9474(99)00161-X)
- [8] Bayram, T. (2013) A Microscopic Analysis on Shape of Ruthenium Isotopes. *Romanian Journal of Physics*, **58**, 931-938.
- [9] Zhang, C.L., Bhat, G.H., Nazarewicz, W., Sheikh, J.A. and Shi, Y. (2015) Theoretical Study of Triaxial Shapes of Neutron-Rich Mo and Ru Nuclei. *Physical Review C*, **92**, Article ID: 034307. <https://doi.org/10.1103/PhysRevC.92.034307>
- [10] Raman, S., Nestor Jr., C.W., Kahane, S. and Bhatt, K.H. (1989) Predictions of B(E2; 0⁺ → 2⁺) Values for Even-Even Nuclei. *Atomic Data and Nuclear Data Tables*, **42**, 1-54. [https://doi.org/10.1016/0092-640X\(89\)90031-4](https://doi.org/10.1016/0092-640X(89)90031-4)
- [11] Raman, S., Nestor Jr., C.W. and Tikkanen, P. (2001) Transition Probability from the Ground to the First-Excited 2⁺ State of Even-Even Nuclides. *Atomic Data and Nuclear Data Tables*, **78**, 1-128. <https://doi.org/10.1006/adnd.2001.0858>
- [12] Grodzins, L. (1962) The Uniform Behavior of Electric Quadrupole Transition Probabilities from First 2⁺ States in Even-Even Nuclei. *Physics Letters*, **2**, 88. [https://doi.org/10.1016/0031-9163\(62\)90162-2](https://doi.org/10.1016/0031-9163(62)90162-2)
- [13] Raman, S., Nestor Jr., C.W. and Bhatt, K.H. (1988) Systematics of B(E2; 0⁺ → 2⁺) Values for Even-Even Nuclei. *Physical Review C*, **37**, 805. <https://doi.org/10.1103/PhysRevC.37.805>
- [14] Raman, S., Malarkey, C.H., Milner, W.T., Nestor Jr., C.W. and Stelson, P.H. (1987) Transition Probability, B(E2) ↑, from the Ground to the First-Excited 2⁺ State of Even-Even Nuclides. *Atomic Data and Nuclear Data Tables*, **36**, 1-96. [https://doi.org/10.1016/0092-640X\(87\)90016-7](https://doi.org/10.1016/0092-640X(87)90016-7)
- [15] Raman, S., Nestor Jr., C.W., Kahane, S. and Bhatt, K.H. (1991) Low-Lying Collective Quadrupole and Octupole Strengths in Even-Even Nuclei. *Physical Review C*, **43**, 556. <https://doi.org/10.1103/PhysRevC.43.556>
- [16] Basdevant, J.-L., Rich, J. and Spiro, M. (2005) *Fundamentals in Nuclear Physics*. Springer, Berlin, 58-60.

- [17] Jarallah, N.T. and Hussain, J.H. (2018) Determination of the Shape for ($_{54}\text{Xe}$ and $_{82}\text{Pb}$) Nuclei from Deformation Parameters (β , δ). *Iraqi Journal of Science*, **57**, 2015-2016.
- [18] Blachot, J. (2007) Nuclear Data Sheets for A = 104. *Nuclear Data Sheets*, **108**, 2062. <https://doi.org/10.1016/j.nds.2007.09.001>
- [19] De Frenne, D. and Negret, A. (2008) Nuclear Data Sheets for A = 106. *Nuclear Data Sheets*, **109**, 974. <https://doi.org/10.1016/j.nds.2008.03.002>
- [20] Blachot, J. (1997) Nuclear Data Sheets for A = 108. *Nuclear Data Sheets*, **81**, 599-752. <https://doi.org/10.1006/ndsh.1997.0016>
- [21] Gurdal, G. and Kondev, F.G. (2012) Nuclear Data Sheets for A = 110. *Nuclear Data Sheets*, **113**, 1338. <https://doi.org/10.1016/j.nds.2012.05.002>
- [22] De Frenne, D. and Jacobs, E. (1996) Nuclear Data Sheets for A = 112. *Nuclear Data Sheets*, **79**, 646. <https://doi.org/10.1006/ndsh.1996.0015>
- [23] Blachot, J. (2012) Nuclear Data Sheets for A = 114. *Nuclear Data Sheets*, **113**, 524. <https://doi.org/10.1016/j.nds.2012.02.002>

Acronyms

Acronyms' Name

Acronyms' Symbol

Upward transition probabilities	$B(E2) \uparrow$
Global Best Fit	GBF
Quadrupole moment	Q_0
Deformation parameter	β_2
Interacting Boson Model-1	IBM-1
Mean life time for the γ -ray	τ_γ
Energy	E
Proton number	Z
Nuclear mass number	A
Initial energy level	I_i
Final energy level	I_f
Electric charge	e
Nuclear average radius	R_0