World Journal of Mechanics, 2024, 14, 1-7
https://www.scirp.org/journal/wjm
ISSN Online: 2160-0503
ISSN Print: 2160-049X

# Improved Units of Measure in Rotational Mechanics

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**How to cite this paper:** Petti, R.J. (2024) Improved Units of Measure in Rotational Mechanics. *World Journal of Mechanics*, **14**, 1-7.
https://doi.org/10.4236/wjm.2024.141001

**Received:** January 1, 2024
**Accepted:** January 28, 2024
**Published:** January 31, 2024



## Abstract

The SI system of units in rotational mechanics yields correct numerical results, but it produces physically incorrect units of measure in many cases. SI units also violate the principle of general covariance—the general rule for defining continuous coordinates and units in mathematics and mathematical physics. After 30+ years of wrestling with these problems, the ultimate authority on units of measure has declared that Newton–meter and Joule are not equivalent in rotational mechanics, as they are in the rest of physics. This article proposes a simple modification to SI units called "Nonstandard International units" ("NI units") until a better name is agreed upon. NI units yield correct numerical results and physically correct units of measure, and they satisfy the principle of general covariance. The main obstacle to the adoption of NI units is the consensus among users that the radius of rotation should have the unit meter because the radius can be measured with a ruler. NI units assigned to radius should have units meter/radian because the radius is a conversion factor between angular size and circumferential length, as in arclength = $r\theta$. To manage the social consensus behind SI units, the author recommends retaining SI units as they are, and informing users who want correct units that NI units solve the technical problems of SI units.

## Keywords

Rotational Mechanics, Angular Unit, Torque, Moment of Inertia, Angular Momentum, General Covariance

## 1. Introduction

Rotational mechanics is a centuries-old field of classical physics. Isaac Newton wrote about rotational mechanics in *Principia Mathematica*. Newton derived Kepler's second law from his mechanics and law of gravitation [1]. In 1746 Daniel

Bernoulli and Leonard Euler proved the conservation of angular momentum [2].

The SI system of units for rotational mechanics always provides correct numerical results [3], but it assigns physically incorrect units of measure to key variables. This problem has been known for decades. The Finite Element Method is now the accepted approach to solving continuum and some discrete component problems in all areas of engineering [4]. However, it is particularly challenging to assign appropriate units of measure in rotational models.

These issues have led *Bureau International des Poids et Mesures* (BIPM) to declare that Newton-meter and Joule are not interchangeable in rotational mechanics, though they are interchangeable in the rest of physics.

This work compares SI units with a modified system of units called "Nonstandard International units" ("NI units") until a better name is agreed upon. NI units satisfy all requirements of the principle of general covariance, which is the basic rule for defining differentiable coordinates and associated units of measure in differential geometry and mathematical physics. SI units violate this principle in all cases where SI units differ from NI units.

The conversions between SI and NI units are very simple:

1) From NI units to SI units: remove unit rad everywhere it appears in NI units.

2) From SI units to NI units: assign to angular position $\theta$ unit rad, and apply the rules of general covariance to assign units to other variables; for example, angular velocity $\omega = \lim_{\Delta t \to 0} \Delta\theta/\Delta t$ must have units rad/s.

It appears that the user community accepted SI units for rotational mechanics for three reasons.

1) Most users are convinced that the radius of rotation "$r$" must be assigned the unit meter because it can be measured with a ruler. Insistence on this condition is the root of all the problems. NI units assign to r the units m/rad.

2) SI units yield correct numerical results, as do NI units.

3) Most users valued conditions (a) and (b) more than the consistency of physical units and the principle of general covariance.

## 2. SI Units and NI Units for Rotational Mechanics

**Table 1** displays SI units and NI units in rotational mechanics.

SI units for rotational mechanics have four basic variables: angle (no units); radius (meter); time (second), mass (kg).

NI units have four basic variables: angle (radian), circumference (meter); time (second); mass (kg).

Alternatively NI units have basic variables angle (radian), radius (m/rad); time (second); mass (kg).

- Units for other variables are determined by assigned to units from each variable in the defining equation.
- SI units violate this rule for circumference and arc length.

Very few articles have been published in refereed journals that challenge the

Table 1. SI units and NI units for rotational mechanics.

| quantity | symbol | relation to other variables | SI units | type | NI units | type |
|---|---|---|---|---|---|---|
| **Kinematics** | | | | | | |
| Circumference | $C$ | $C = 2\pi r$ | m | | m | base unit |
| Circular arc length | $Arc$ | $Arc := r\theta$ | m | | m | |
| Radius of rotation | $r$ | $R = C/(2\pi)$ | m | base unit | m/rad | |
| Angle | $\theta$ | $\theta$ = angle = $Arc/r$ | 1 | base unit | rad | base unit |
| Angular velocity | $\omega$ | $\omega := \mathrm{d}\theta/\mathrm{d}t = 2\pi f$ | 1/s | | rad/s | |
| Angular acceleration | $A$ | $\alpha := \mathrm{d}\omega/\mathrm{d}t$ | $1/s^2$ | | $rad/s^2$ | |
| 2π | | Convert cycle to radian | 1 | | rad/cycle | |
| Frequency | $f$ | $f := \omega/2\pi$ | 1/s | | cycle/s | |
| Period | $T$ | $T = 1/f$ | S | | s/cycle | |
| **Mechanics** | | | | | | |
| Mass density (2-D) | $\rho$ | $\rho$:= dmass/dArea | $kg/m^2$ | base unit | $kg/m^2$ | base unit |
| Moment of inertia | $I$ | $I := \int \rho r^2 \mathrm{dArea}$ | $Kg/m^2$ | | kg $m^2/rad^2$ | |
| Angular momentum | $L$ | $I\omega$ | J s = kg $m^2$/s | | J s/rad = kg $m^2$/s/rad | |
| Torque (rotational force) | $\tau$ | $\tau := \mathrm{d}L/\mathrm{d}t$, or $\tau := -\mathrm{d}KE/\mathrm{d}\theta$ | N m<br>N m/rad | | J/rad = N m/rad<br>J/rad= N m/rad | |
| Kinetic energy | $KE$ | $KE := \int \tau \mathrm{d}\theta = \frac{1}{2} I\omega^2$ | J | | J = N m | |
| Potential energy ($k$ = elastic constant) | $PE$ | $PE := \int \tau \mathrm{d}\theta = \frac{1}{2} k\theta^2$ | J | | J = N m | |

correctness of SI units. *Metrologia* has published a few, the best of which is [5]. Quincy recognizes that angle is not a ratio of lengths, nor is it dimensionless. He gets bogged down with radians and degrees in the same formulas. The article does not address the critical issue that radius r must have units m/rad, nor does it benefit from the comprehensive principle of general covariance.

## 3. Comparison of SI Units and NI Units

### 3.1. Angular Position

The SI system of units requires that angular position $\theta$ be measured in radians. However, it assigns to $\theta$ no units, which itself is a violation of the principle of general covariance. SI units remove the unit radian from all variables and equations in rotational mechanics. If a user chooses to use degrees or cycles as the angular unit, the variables and equations do not indicate where to insert conversion factors 2 π rad/cycle or 180/π degrees/rad. The inability to use other established units for angle violates the principle of general covariance.

The NI system of units measures angular position $\theta$ in radians, and assigns the unit radian to $\theta$. In NI units, the variables and equations contain information that enables a user to change to another angle unit such as degree or cycle. As a consequence, NI units changes units assigned to angular velocity "$\omega$", torque "$\tau$", angular momentum "$L$", radius "$r$", and other variables as shown in **Table 1**.

### 3.2. Torque

SI units assign to torque τ units N–m. BIPM asserts that, (a) in rotational mechanics, J and N-m are not interchangeable units, as they are in the rest of physics; (b) N-m and J both have units of energy, but only N-m should be used to describe torque.

> "The SI unit [for torque] is Newton-metre. Even though torque has the same dimension as energy, the Joule is never used for expressing torque" [6].
>
> *"Torque units*. The Newton-meter, the SI unit of torque, is the magnitude… The Joule should never be used as a synonym for this unit…" [7] [8].

This assignment violates the principle of general covariance.

The experience of winding up a spring-powered device demonstrates that the force applied to the mechanism is proportional to the angle through which the spring is turned. The SI units for torque violate the fact that torque should have units Δenergy/Δangle. SI units also assign physically incorrect units to angular momentum $\boldsymbol{L}$.

For at least the past three decades, most references on rotational mechanics do not quote units for torque [9].

NI units assign to $\tau$ units J/rad. NI units enable the two classical definitions of torque to agree on units, which are units of rotational force.

$$\tau = \mathrm{d}\boldsymbol{L}/\mathrm{d}t\left(\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2/\mathrm{rad}\sim\mathrm{J}/\mathrm{rad}\right),\quad \tau = -\mathrm{d}E/\mathrm{d}\theta\left(\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2/\mathrm{rad}\sim\mathrm{J}/\mathrm{rad}\right) \tag{1}$$

### 3.3. Radius of Rotation and Circumference

SI units assign to radius of rotation r the unit meter. The fact that r can be measured with a simple ruler adds to confidence in this assignment. SI units assign no units to $\theta$, though $\theta$ must still be measure in radians, in order to preserve assignment of unit meter to $r$. This violates the principle of general covariance.

NI units assigns to circumference $C$ the unit meter and to radius r the units m/rad. NI units treat $r$ as a conversion factor between $C$ measured in radians and $C$ measured in meters. The assignment to $r$ of units m/rad generates more resistance to NI units than any other feature of NI units.

## 4. The Principle of General Covariance

In the twentieth century, differential geometry and general relativity came together to solve the most difficult challenges in coordinate systems in mathematical physics. General relativity generally has no orthonormal coordinates and no path-independent parallel translation; the geometrical length of a line segment generally is not the difference in coordinates, and the shape of space time itself changes continuously in dynamic situations.

The solution to these challenges is the principle of general covariance [10] [11] which requires that the coordinates, equations, tensor fields, and units of measure in mathematical physics are invariant under arbitrary differentiable coordinate transformations. A coordinate transformation is essentially the Jacobian of a differentiable map of one coordinate system to a second coordinate system. The transformation usually consists of multiplication by a conversion factor for each pair of similar units (or at most an inhomogeneous linear transformation, as for Kelvins and degrees Celsius). This principle meets the needs of the rest of physics, including Newtonian mechanics, general relativity, and quantum field theory.

Here are example applications of general covariance.

- If coordinate "x" has unit meter, and coordinate "t" has unit second, then dx/dt must have units m/s.
- If a coordinate system requires that θ be measured in radians, then the variable θ must have the unit radian.
- If the definition of a derived variable includes θ, then units for the derived variable must include the unit for θ.

Assignments of SI units to variables angle, torque, and angular momentum violate the principle of general covariance. NI units observe the principle of general covariance everywhere.

The article blames this violation on the presence of curvature in the base manifold. The author has never seen or imagined a case where violation of the principle of general covariance is not cause by incorrect in incomplete application of the principle.

## 5. Summary

SI units yield correct numerical results in rotational mechanics. However, these units do not yield physically correct units of measure. Three problems remain:

- Requiring angular position θ to be measured in radians but assigning no units to $\theta$ violates the principle of general covariance.
- Users cannot change the angular unit in rotational mechanics, because all angular units have been removed from the variables and equations. The variables and equations do not indicate where to put the conversion factors.
- Assigning to torque τ units of energy, either N-m or J, contradicts the practical result that "τ" has physical meaning Δenergy/Δangle, or J/rad. To avoid equating torque with "ordinary energy," BIPM asserts that J and N-m are interchangeable everywhere in physics, except in rotational mechanics. This assignment is a glaring violation of the principle of general covariance.

SI units violate the principle of general covariance in numerous places.

NI units solve all the problems with units of measure in rotational mechanics. The key changes are:

- Angular position θ has unit rad, and θ should be measured in radians. These

units enable the equations of rotational mechanics to provide the information needed to convert any angular unit to any other angular unit.
- Radius “r” should have units m/rad. Many metrologists resist this assignment because the radius can be measured with a ruler, so it appears that “r” must have the unit meter. NI units treat radius r as a conversion factor between circumference measured in meters and circumference measured in radians. Circumference can be measured with a ruler without violating the principle of general covariance.

One might ask why the engineering and physics communities settled for units with these problems. The author believes this occurred because:
- A strong consensus among users preferred that radius has the unit meter. This condition is the root of the problems.
- SI units deliver correct numerical results.
- Most of the community cares more about the two conditions above than about consistency among units of measure or the principle of general covariance.

The SI system of units is probably the most reasonable system that assigns to radius the unit meter.

## 6. Recommendations

The main factor favoring SI units for rotational mechanics is that SI units are more familiar to users. In particular, users are more familiar with radius as a basic variable with unit meters.

The main factors favoring NI units for rotational mechanics are:
- NI units observe the principle of general covariance, SI units do not.
- The NI units for angle, torque and other variables match the physical meanings of these variables. SI units do not.
- NI units retain the equivalence of J and N-m. According to BIPM, SI units must invalidate the equivalence of J and N-m.
- Users can change the unit assigned to the angle because locations of angular units in the equations indicate where to include conversion factors for different angular units.

The author’s recommendations are:
- Users who want physically correct units should use NI units. This solves the technical problems with SI units.
- Users who don’t need physical units—or cannot accept that radius has units m/rad—can continue to use SI units.

## Acknowledgments

The author acknowledges Ishaan Khurana of Brandeis University for advice about presentation in early drafts of this article.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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