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Reliability Assessment of a Fully Laterally Restrained Steel Floor Beam to Eurocode 3

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Abstract

Parametric uncertainties should always be considered when setting design criteria in order to ensure safe and cost effective design of engineered structures. This paper presents the results of the reliability assessment of a fully laterally restrained steel floor I-beam to Eurocode 3 design rules. The failure modes considered are bending, shear and deflection. These were solved to obtain reliability indices using first order reliability method coded in MATLAB environment. Parametric sensitivity analyses were carried out at varying values of the design parameters to show their relative contributions to the safety of the beam. It was seen that reliability indices generally decreased with an increase in load ratio, imposed load, beam span in bending, shear stress and deflection respectively. In addition, increasing the beam span beyond 10 m, load ratio above 1.4 and imposed load beyond 30 kN/m made the beam fail as these parameters gave negative reliability indices. For failure in deflection, reliability index rose with an increase in the radius of gyration and overall depth of the beam section accordingly. Furthermore, the reliability index surged as the thickness of the web increased when taking into account, shear failure. The results of the analysis showed that the steel beam is very safe in shear and at some load ratios and imposed loads for failure in bending and deflection respectively. The average values of reliability indices obtained for load ratios ranging from 1.0 to 1.4 fell from 3.017 to 3.457 for all failure mode studied. These values are within the recommended reliability indices by the Joint Committee on Structural Safety for structure with moderate failure consequences and beams in flexure.

Keywords

First-Order Reliability Method, Reliability-Indices

1. Introduction

Laterally restrained beams are beams that are laterally restrained by secondary steel members or concrete in order to prevent buckling of the compression flange. Engineering structures are always designed to have reasonable safety margins [1] [2] [3] [4] [5]. Consequently, there is a need to accurately determine the limit state in order to achieve an efficient design. According to Afolayan [6], the problems of civil engineering structures are non-deterministic. As a result, civil and structural engineers always find it difficult to estimate the actual load that acts on them. Thus the use of safety factors in civil engineering design may not provide a reliable and economical design of structures as there are no provisions to assist in evaluating whether the design is conservative or not [7] [8] [9].

Improper knowledge of the uncertainties of structural design parameters has caused the collapse of large span steel beams in the roof structure of a Church in Akwa Ibom State of Nigeria, killing many of the worshippers and damaging properties worth millions of dollars [10]. It has therefore become a task of paramount importance for both the civil and structural engineers to consider the variability in the resistance and load quantities when setting design criteria for engineered structures.

Structural deterioration and degradation of structural capacity are the common reason for safety assessment of structures. Engineering design must produce structures such that both the ultimate and serviceability limit states are not exceeded when they are being put to use. The limit states are functions of the structural capacity and load effects. Catering for the variability that occurs in the design parameters such as the strength and geometric properties of the engineering materials and structural action has always been the problem of the structural engineer. The structural engineer is always faced with the problem of how to cater to the variability that occurs in the strength and the geometric properties of the engineering materials.

The reliability of a structure is the ability of the structure to fulfill the purpose for which it was designed at some specific period of time [11]. According to Ditlevsen [12], the presence of uncertainties in both the structural resistance and loads is the reason why engineered structures exhibit some level of uncertainty under service loads. Engineered structures should therefore be designed to serve their intended purpose with a certain level of reliability. The duty of the structural engineer is to design and maintain the structure with a reasonable level of reliability. Structural reliability analysis deals with the rational treatment of parameter uncertainties. Therefore, probabilistic and statistics provide a framework for dealing with these uncertainties in order to produce structures that have an adequate level of reliability [13]-[18].

The current design procedures for laterally restrained steel beams are based on design codes such as Eurocode 3 [19]. The need for reliability analysis is due to the fact that the parameters used in code-based designs are uncertain. This could lead to unsafe, conservative or uneconomical design of structures. There-

fore, the present study demonstrates the applicability of the reliability method in structural engineering design of laterally restrained beams using simple procedures that require information on the uncertainties of the design parameters, to achieve some levels of reliability that may not have been properly considered in the conventional deterministic analyses.

In this paper, the reliability assessment of a fully laterally restrained I steel beam supporting an office floor is carried out with respect to the limited state of bending, shear and deflection. This was achieved using the Euro code 3 [19] design criteria and the first order reliability method (FORM). A MATLAB based program on FORM was written and implored to implement the reliability estimates.

2. Development of Yield Functions for Reliability Analysis

The yield functions were derived according to the design standards of Eurocode 3 [19] for the steel beam structure. **Figure 1** presents the floor plan of the office building under study and beam A-A is the beam of interest. This beam is an I-steel beam that is simply supported and subjected to uniformly permanent and variable loadings respectively as shown in **Figure 2**. **Figure 3** depicts the cross section of the beam.
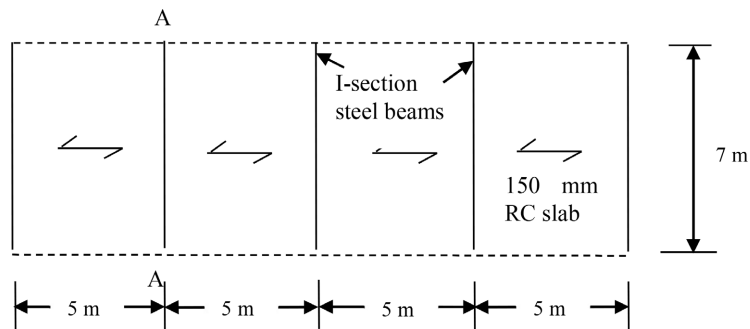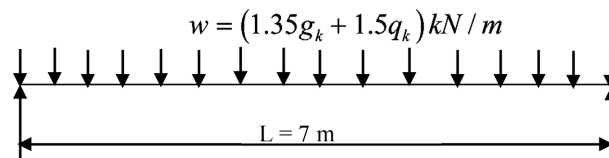

Figure 1. Floor plan of an office building.

$$w = \left(1.35g_k + 1.5q_k\right)kN/m$$

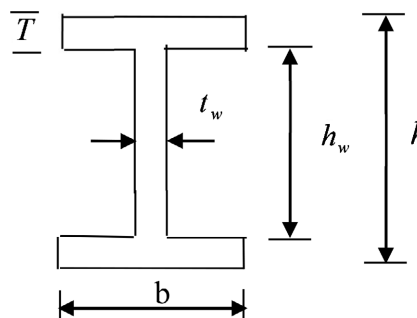**Figure 2.** I-section steel beam under permanent and uniform loading.

Figure 3. Beam cross section.

2.1. Bending Criterion

The design constraint in bending is an inequality problem given by:

$$M_{Ed} \leq M_{Pl,Rd}. \tag{1}$$

where, $M_{Pl,Rd}$ = design moment of resistance of the beam gross section; M_{Ed} = design moment due to applied load on the beam.

The maximum design bending moment is given by:

$$M_{Ed} = wL^2/8. \tag{2}$$

where, w is the uniformly distributed load due to permanent and variable actions on the beam.

The uniformly distributed load w in terms of load ratio α and characteristics variable load q_k is given by:

$$w = 1.5 * q_k (0.90\alpha + 1). \tag{3}$$

where, the load ratio α is given as:

$$\alpha = g_k/q_k. \tag{4}$$

Applying Equations (2)-(4), the maximum design bending moment is given as:

$$M_{app} = \left[1.5 * q_k (0.9\alpha + 1) L^2 \right]/8 \tag{5}$$

The design moment of resistance of the beam gross section is given by:

$$M_{Pl,Rd} = \left(W_{pl} * f_y \right)/\sqrt{3\gamma_m} \tag{6}$$

where, W_{pl} = plastic section modulus; f_y = yield strength of steel; γ_m = partial safety factor for material strength; q_k= characteristic variable action; α = load factor; L – beam span.

Applying Equations (3)-(5), the limit state function in bending is given by:

$$G(x)_1 = \left\{ \left(W_{pl} * f_y \right)/\sqrt{3\gamma_m} \right\} - \left\{ \left[1.5 * q_k * (0.9\alpha + 1) L^2 \right]/8 \right\} \tag{7}$$

2.2. Shear Criterion

The design constraint in shear is given by:

$$V_{Ed} \leq V_{Pl,Rd} \tag{8}$$

where,

V_{Ed} = design shear force on beam; $V_{Pl,Rd}$ = design plastic shear resistance of the beam.

According to Euro code 3 [19], the design plastic shear resistance of a universal beam section when shear occurs parallel to the web is given by:

$$V_{Pl,Rd} = \left(A_v * f_y \right)/\left(\sqrt{3}\gamma_m \right). \tag{9}$$

According to Arya [20],

$$A_V = 1.04 h t_w \tag{10}$$

where,

h = overall depth of the beam section; t_w = web thickness; A_V = shear area.

The design force on the beam is:

$$V = wL/2. \tag{11}$$

The design factor on the beam in terms of load factor, characteristic variable load and beam span is given by:

$$V = \left[1.5 * q_k (0.9\alpha + 1) L\right]/2 \tag{12}$$

Applying Equations (9), (10), and (12), the yield surface equation in shear is given by:

$$G(x)_2 = \left\{ \left(1.04 * h * t_w * f_y\right) / \sqrt{3}\gamma_m \right\} - \left\{ \left[1.5 * q_k (0.90a + 1) L\right]/2 \right\} \tag{13}$$

2.3. Deflection Criterion

The design constraint in deflection is given by:

$$\delta_{\max} \le \delta_{all} \tag{14}$$

where,

$\delta_{\max}$ = maximum deflection due to applied load on the beam; δ_{all} = limiting value of deflection of the beam.

The maximum deflection at mid span of the beam due to uniformly distributed load is given by:

$$\delta_{\max} = 5wL^4/384EI \tag{15}$$

where, E and I represent the modulus of elasticity and moment of inertia of the beam accordingly.

According to Arya [20], the limiting value of deflection of the beam is given by:

$$\delta_{all} = L/350 \tag{16}$$

Applying Equations (3), (14), (15) and (16), the limit state function in deflection is given by:

$$G(x)_3 = \left\{ L/350 \right\} - \left\{ \left[5 * 1.5 * q_k (0.90\alpha + 1) L^4\right]/384EI \right\} \tag{17}$$

From the theory of mechanics of materials, moment of inertia is given by:

$$I = Ar^2 \tag{18}$$

Applying Equation (18), the yield surface equation in deflection is given by:

$$G(x)_3 = \left\{ L/350 \right\} - \left\{ \left[5 * 1.5 * q_k (0.90\alpha + 1) L^4\right] / \left[384EAr^2\right] \right\} \tag{19}$$

where,

A = area of steel section; r = radius of gyration of the beam section.

3. Methodology

3.1. First Order Reliability Method

According to first order reliability method, the safety margin of a structural system

is expressed in terms of uncertain variables [1]. The vector $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ represents uncertain inputs variables that affect structural performance. The multivariate density function of the uncertain load and resistance quantities is given by:

$$F_X(X) = P\left(\bigcap_{i=1}^{n} \{X_i \leq x_i\} \right) \tag{20}$$

where, $F_x(X)$ denotes the multivariate density function of *x*.

The safety margin, $G(x)$ of a structure is dependent on the basic variables that affect the limit state considered. It is defined such that $G(x) > 0$ represents the safe state of a structure; $G(x) < 0$ represents the unsafe state of s structure and $G(x) = 0$ is the failure surface *i.e.*, it is the line that separates the safe state from the unsafe state. The safe state from the unsafe state of the structure.

The probability of failure of a structure is defined by Afolayan [1] as:

$$p_f = P[G(x) \leq 0] = \varphi(-\beta) \tag{21}$$

where, β = safety index and it represents the minimum distance from the origin to the failure surface *i.e.* when $G(x) = 0$. This condition can be depicted mathematically as follows:

$$\beta = \min\{\|x\|\} \text{ for } \{x : G(x) < 0\} \tag{22}$$

Using the first order reliability approach requires all variables in the failure function which are not normally distributed to be transformed into equivalent normally distributed variables. According to Ranganthan [5], the parameter of the equivalent normally distributed random variables can be estimated by imposing two conditions, *i.e.* the cumulative distribution functions and the probability density function of the actual variable. The equivalent normal variable should be equal at the design point, *i.e.* $x = (x_1^*, x_2^*, x_3^*, \cdots, x_n^*)$ on the failure surface.

Considering each statistical independent non variable individually and equating it with an equivalent normal variable at the design point yields:

$$\Phi\left\{ (x_1^* - \mu_{x_i}^N) \big/ \sigma_{x_i}^N \right\} = Fx_i(x_i^*) \tag{23}$$

where $\Phi()$ = cumulative distribution function of the standard normal variant at the design point; $\mu_{x_i}^N$ and $\sigma_{x_i}^N$ represents the mean and standard deviation of the equivalent normal variable at the design point respectively; $Fx_i(x_i^*)$ = cumulative distribution function of the original non normal variables.

Application of Equation (23) yields:

$$\mu_{x_i}^N = x_i^* - \Phi^{-1}\left[Fx_i(x_i^*) \right] \sigma_{x_i}^N. \tag{24}$$

Equating the probability distribution functions of the original variable and the equivalent normal variable at the design point yields gives:

$$\left\{ \varphi \big/ \sigma_{x_i}^N \right\} * \left\{ (x_i^* - \mu_{x_i}^N) \big/ \sigma_{x_i}^N \right\} = fx_i(x_i^*) \tag{25}$$

where $\varphi()$ and $fx_i(x_i^*)$ are the probability distribution function of the equivalent standard normal and the original non normal random variable respec-

tively.

Rearranging Equation (24) gives:

$$\Phi^{-1}\left[Fx_i\left(x_i^*\right)\right]\sigma_{x_i}^N = x_i^* - \mu_{x_i}^N \qquad (26)$$

Application of Equations (25) and (26) yields the standard deviation of the equivalent variables as:

$$\sigma_{x_i}^N = \varphi * \left\{\Phi^{-1}\left[Fx_i\left(x_i^*\right)\right]\Big/ fx_i\left(x_i^*\right)\right\}. \qquad (27)$$

3.2. Reliability Analysis

A four span reinforced concrete office floor, 150 mm thick is supported by 5 Nr 1 composite steel beams in S275 steel and subjected to an imposed load of 5 kN/m². The structural design of the beam A-A as shown in **Figure 1** was carried out according to the design criteria of Eurocode 3 [19] and 605 * 305 * 238 kg/m UB section was selected. This beam satisfied the design criteria of bending, shear and deflection respectively. The statistical properties of the basic random variables are given in **Table 1**.

4. Results and Discussion

The reliability indices corresponding to the failure mode of bending, shear and deflection were obtained using a written MATLAB program that is based on the

Table 1. Statistics of the basic random variables.

Basic variables	Statistics			Probability distribution
	Mean	Standard deviation	Coefficient of variation	
Thickness of flange f_t	31.4 mm	1.57 mm	0.50	Normal
Overall depth of beam, h	635.8 mm	19.074 mm	0.03	Normal
Radius of gyration of beam section, r	263 mm	7.89 mm	0.03	Normal
Beam span, L	7000 mm	350 mm	0.05	Gumbel
Load ratio, α	1	-	-	Fixed
Imposed load on beam, q_k	25 kN/m	6.25 kN/m	0.25	Gumbel
Plastic modulus of beam section, W_{pl}	7,486,000 mm³	374,300 mm³	0.05	Normal
Design strength of beam, f_y	265 N/mm²	18.55 N/mm²	0.07	Lognormal
Cross section area of beam, A	30,300 mm²	909 mm²	0.03	Normal
Partial safety factor for material strength, γ_m	1.05	0.1575	0.15	Lognormal
Thickness of web, t_w	18.4 mm	0.552 mm	0.03	Normal
Depth of web, h_w	573 mm	17.19 mm	0.03	Normal
Moment of inertia of beam section, I_x	2,094,710,000 mm⁴	104,735,500 mm⁴	0.05	Normal
Modulus of elasticity of steel, E	210000 N/mm²	6300 N/mm²	0.03	Lognormal

Source: [3] [5] [15].

first order reliability method. The graphs obtained from the results program are shown in **Figure 4** to **Figure 13**.

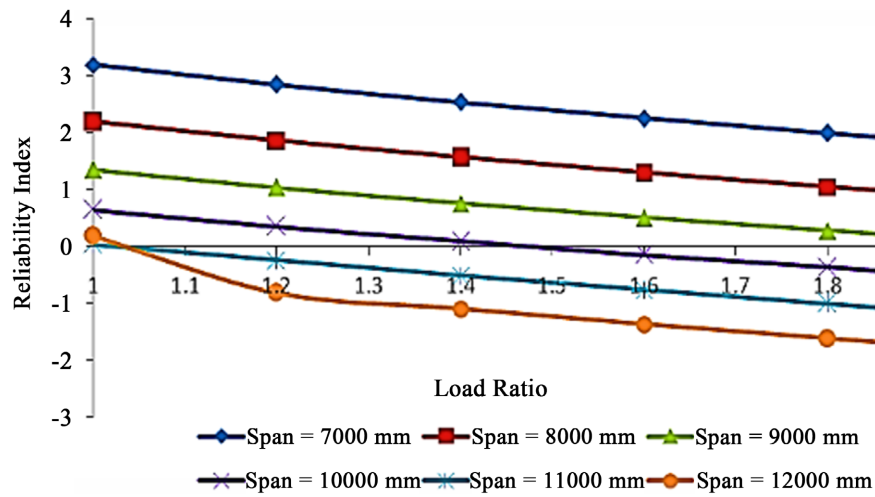From **Figure 4**, **Figure 6** and **Figure 10**, it can be seen that reliability indices

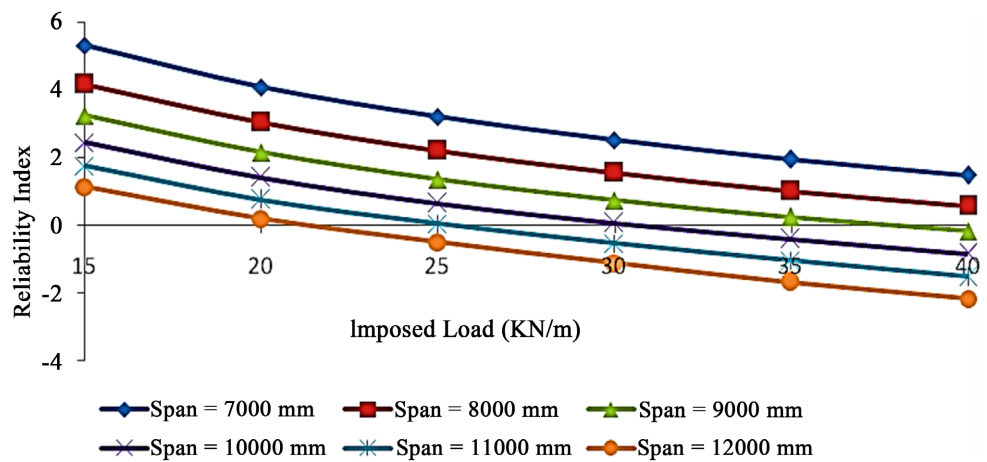**Figure 4.** Reliability index against load ratio for varying beam span (bending criterion).

Figure 5. Reliability index against imposed load ratio for varying beam span (bending criterion).
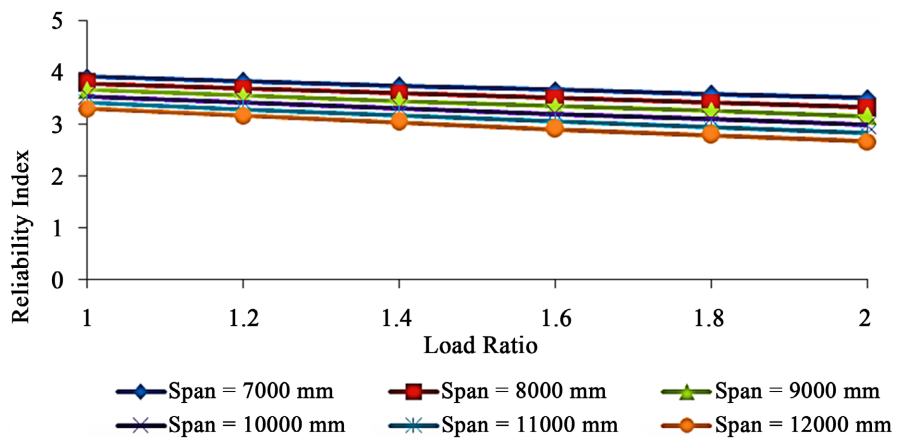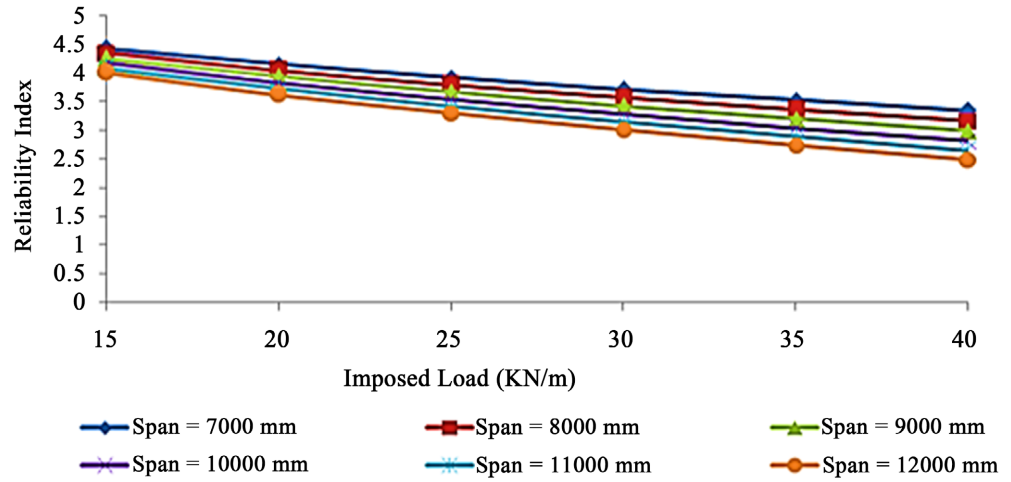

Figure 6. Reliability index against load ratio for varying beam span (shear criterion).

Figure 7. Reliability index against imposed load for varying beam span (shear criterion).
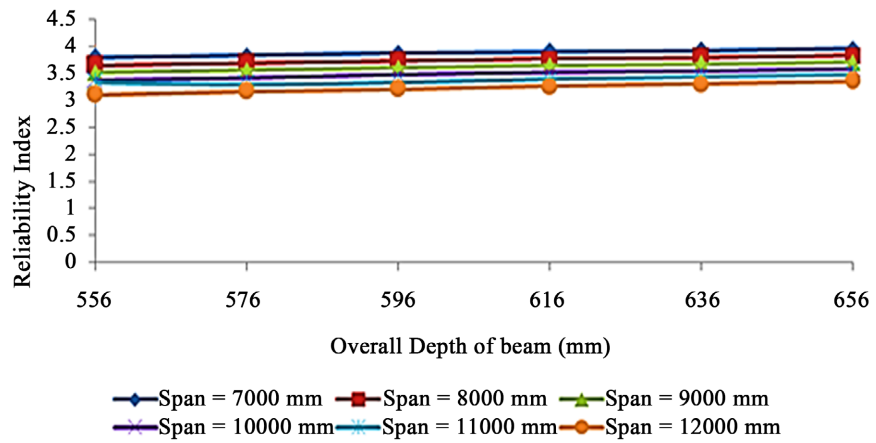

Figure 8. Reliability index against overall depth of beam for varying beam span (shear criterion).
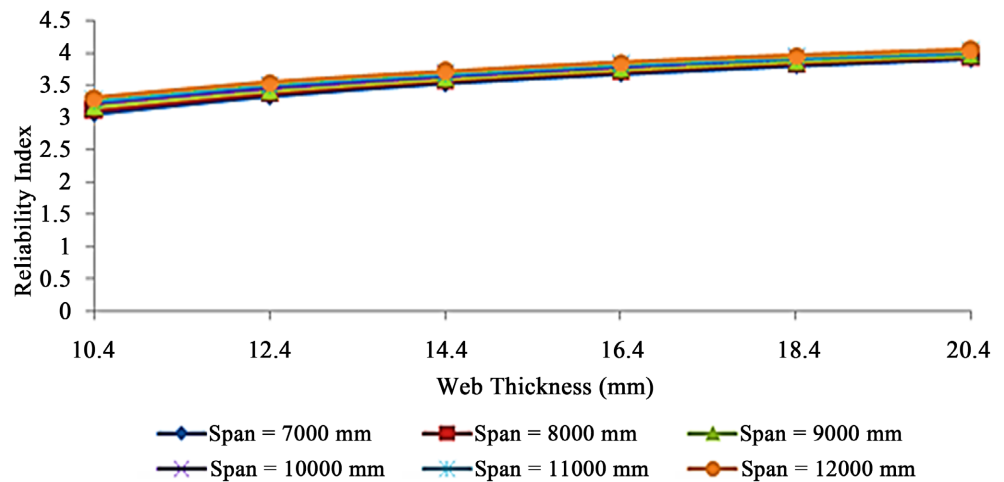

Figure 9. Reliability index against web thickness for varying beam span (shear criterion).

of the beam generally decreased as the beam span and load ratio increased for the limit state of bending shear and deflection respectively. This agrees with [14] and [16], that the load and resistance variables affect the reliability levels of

structures. This decrease in reliability level may be as a result of increase in applied bending moment with increase in span of the beam and load ratio. Increment in the beam span beyond 10 m and load ratio above 1.4 will cause failure of the beam as they gave negative values of safety indices. This also is in synergy with [13], that when the values of the reliability indices are negative, the structure is not safe at all.
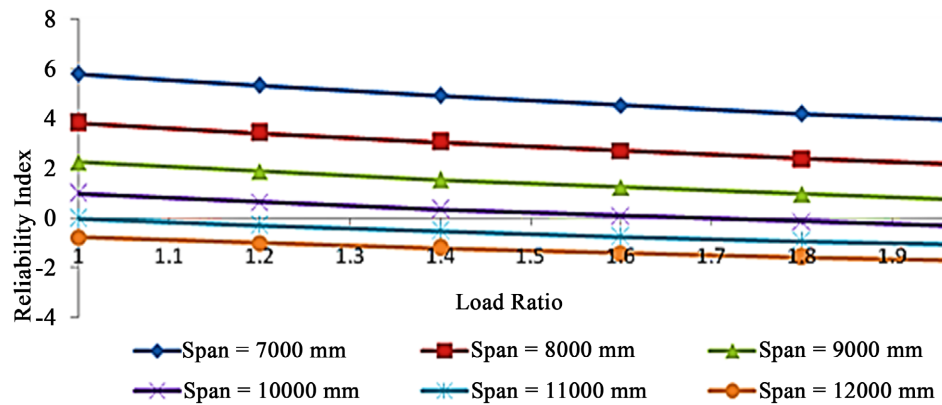

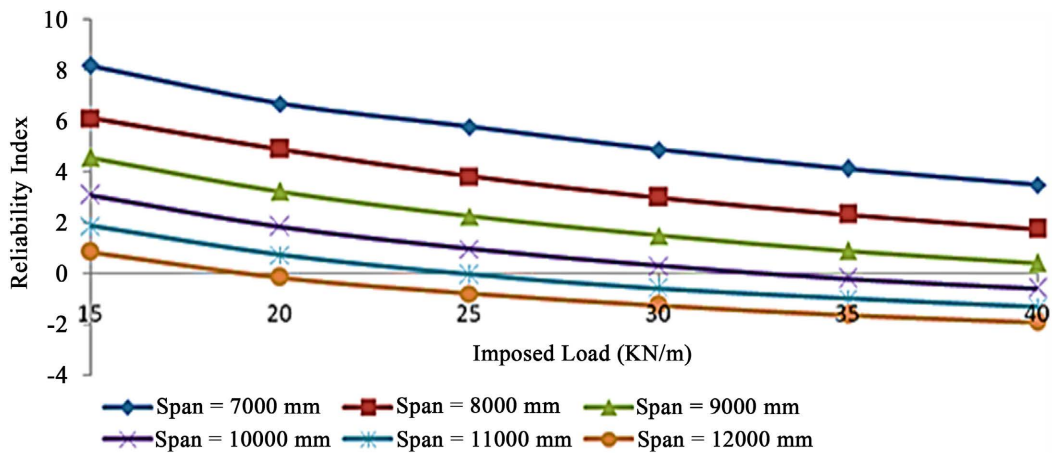**Figure 10.** Reliability index against load ratio for varying beam span (deflection criterion).

Figure 11. Reliability index against imposed load for varying beam span (deflection criterion).
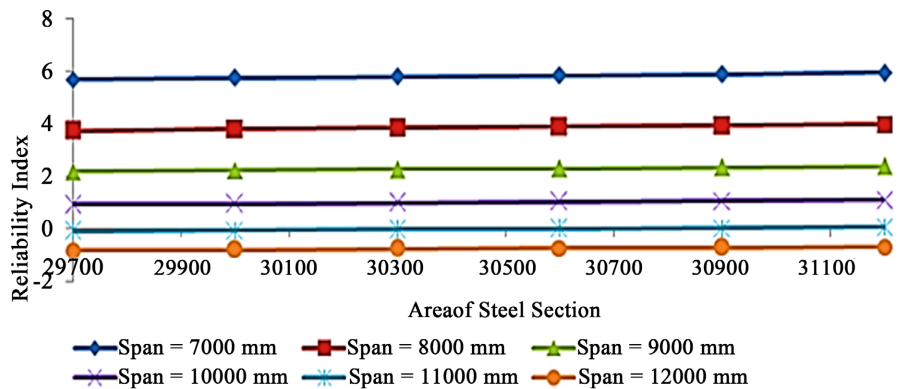

Figure 12. Reliability index against area of steel section for varying beam span (deflection criterion).
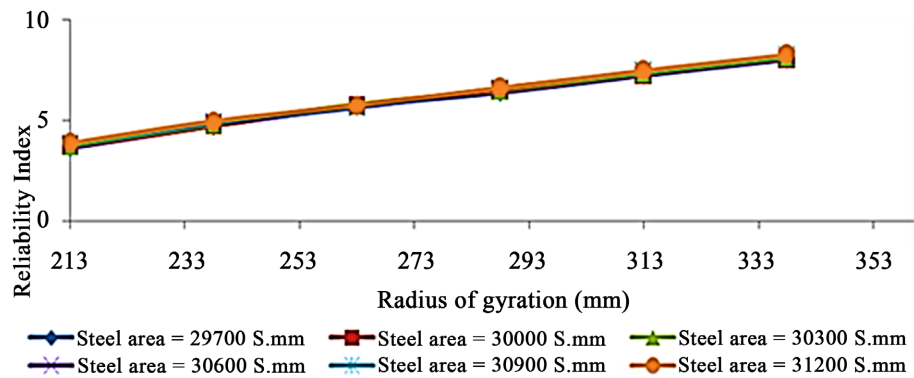
Figure 13. Reliability index against radius of gyration for varying beam span (deflection criterion)-11.

In view of **Figure 5**, **Figure 7** and **Figure 11**, it is apparent that the reliability index dropped as the beam span and imposed load heightened with respect to bending, shear and deflection limit states respectively. This is because, a rise in the value of the imposed load reduces the load bearing capacity of the beam due to elevation in applied bending moment. Raising the beam span above 10 m and the imposed load more than 30 kN/m will cause the beam to fail since this will lead to bleak values of reliability indices [21] [22]. Taking **Figure 8** into account, it is seen that reliability index expands with rising overall depth of beam section for failure in deflection. This may be attributed to the increase in shear area and overall depth of the beam section for failure in deflection.

It can be observed from **Figure 9** that the reliability index rises as the thickness of the web increases for failure in shear. This could be caused from increased shear area of beam with its corresponding rise in shear resistance. Considering **Figure 12** and **Figure 13**, the reliability index surges as the area and radius of gyration of the steel section increase. This trend may be due to the gain in moment of inertia of the steel section that arises when the area and radius of gyration of the section increase. As the moment of inertia and radius of gyration of the steel section improves, the resistance of the beam to failure in deflection also heightens leading to rises in reliability index.

The reliability indices obtained from this research work, for the three failure modes *i.e.*, bending, shear and deflection respectively, all fell within the range of 3.1 and 4.2 for structures with moderate consequences of failure. Comparing these values to the recommendations by the Loading and Safety Regulations for Structural Design, report, No. 36 [21] and the Joint Committee on Structural Safety [22], it can be seen that the beam is very safe in shear. It is also safe in bending and deflection but only at some load ratios and imposed loads.

5. Conclusions

The results of the reliability analysis of a simply supported composite I steel beam corresponding to failure in bending, shear and deflection according to the design criteria of Eurocode 3 [19], using first order reliability method (FORM)

have been presented in this study. The failure functions in bending, shear and deflection were developed. A MATLAB code on FORM was written and then used to implement the reliability estimations.

The outcome of these procedures revealed that the reliability indices generally decreased with an increase in load ratio and beam span for failure in bending, shear and deflection. Also, beam spans beyond 10m and imposed loads above 30kN/m led to failure of the beam. Furthermore, it was observed that the reliability index rose with elevation in the overall depth of beam section for failure in deflection.

With respect to shear failure, it was seen that as the thickness of the web heightened, the reliability index improved. In addition, increased area and radius of gyration of the steel section bettered the reliability index. The beam considered is very safe in shear. However, under bending and deflection, it is safe only at some load ratios and imposed loads.

Acknowledgements

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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