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Unsteady Non-Uniform Flow of Molten Metals in Rectangular Open Channels

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Abstract

In the metallurgical industries, it is very important to characterize the flow of molten metals in open channels given that they are transported through these devices to different plant sections. However, unlike the flow of water which has been studied since ancient times, the flow of molten metals in open channels has received little attention. The unsteady non-uniform flow of blast furnace molten pig iron in a rectangular open channel is analyzed in this work by numerical solution of the Saint-Venant equations. The influence of mesh size on the convergence of molten metal height is studied to determine the proper mesh and time step sizes. A sinusoidal inflow pulse is imposed at the entrance of the channel in order to analyze the propagation of the resulting wave. The influence of the angle of inclination of the channel and the roughness coefficient of the walls on the amplitude and the dynamic behavior of the height of the molten metal are analyzed. Phase portraits of the channel state variables are constructed and interpreted. Numerical simulations show that as the angle of inclination of the channel increases, the amplitude of the formed wave decreases. From 10 degrees onwards, the peak of the wave descends even below the initial height. On the other hand, the roughness coefficient affects the molten pig iron height profiles in an inverse way than the angle of inclination. The amplitude of the formed wave increases as the roughness coefficient increases.

Keywords

Molten Metals Flow, Non-Uniform Flow, Open Channels, Pig Iron Transport, Saint-Venant Equations, Sinusoidal Pulse, Unsteady Flow

1. Introduction

Unlike the flow of water in open channels, which has been studied since ancient times, the flow of molten metals in open channels has received little treatment in the literature. The few published studies on the subject are in the context of Lorentz force flowmetry, in which molten metals flowing in open channels are exposed to electromagnetic fields in order to measure their volumetric flow rate, e.g. [1] [2] [3] [4] [5]. However, in the metallurgical industries, it is of paramount importance to characterize the flow of molten metals as they are economically transported through open channels to different plant sections. In the aluminum industry, molten aluminum is transported from the melting furnaces to degassing units to remove dissolved hydrogen, in order to avoid the formation of porosities and blowholes in the finished pieces. In the steel industry, molten pig iron is transported through open channels from the crucible of the blast furnace to the torpedo ladles in order to feed the basic oxygen converters, where the pig iron is converted into crude steel by removing impurities by oxidation with supersonic oxygen jets.

Open channels for transporting molten metals are totally different from channels used for transporting water. The latter have considerable lengths (of the order of kilometers) and very small slopes (0.005 m/m) [6] [7], while the former are characterized by short lengths (of the order of 20 m) and relatively high angles of inclination (five degrees of inclination or more). In addition, the channels that transport molten metals are made of refractory materials that withstand high temperatures, and are subject to relatively frequent maintenance due to the wear of the walls and the adhesion of solidified metal and slag. Molten metals cannot remain exposed to the atmosphere for a long time in an open channel since they re-oxidized and freeze easily, which explains their short length. Due to their short length, stable and uniform flow conditions are rarely achieved in open channels for the transport of molten metals. In these channels, the flow is frequently turbulent and transient in nature both in time and space.

In a recent work [8] the flow of molten aluminum in a rectangular open channel is studied. Three types of flow were considered, namely uniform flow, steady gradually varied flow, and unsteady non-uniform flow. The uniform flow is one in which the temporal and spatial derivatives of the variables contained in the Saint-Venant equations are equal to zero. For uniform flow, the above author used Manning's equation with a suggested roughness coefficient of 0.1, which gives extremely high values of volumetric flow rate when applied to blast furnace pig iron. For unsteady non-uniform flow, the Saint Venant equations were used and numerically solved using commercial software. In this case, the author applies an inadequate boundary condition in the downstream of the channel, which originates a distorted and inaccurate profile of the wave of the molten aluminum. For steady gradually varied flow the author solves the Saint Venant equations by considering null the time derivatives of the height and the volumetric flow rate of the molten aluminum. Again, the boundary condition applied in the downstream of the channel is inexact and leads to an incorrect profile of the height of the aluminum throughout the length of the channel.

Previously, the present authors analyzed the uniform flow of pig iron in a rectangular channel [9]. In this work, the unsteady non-uniform flow of blast furnace pig iron in a rectangular open channel is analyzed by numerical solution of the Saint-Venant equations, employing an explicit backwards finite difference method in space [10]. The influence of mesh size on the convergence of metal height values is studied to determine the most appropriate mesh and time step sizes. As in [8] and [11], a sinusoidal inflow pulse is imposed at the entrance of the channel in order to analyze the propagation of the resulting wave. The influence of the angle of inclination of the channel and the roughness coefficient of the walls on the amplitude and the dynamic behavior of the height of the molten metal are numerically studied. As a novelty, from the time evolution curves of the height and the volumetric flow rate, a phase portrait of the channel is constructed. Finally, the effects of the amplitude and the period of the sinusoidal inflow step on the dynamic response of the height of the molten metal in the channel are studied.

2. Mathematical Model

The equations that describe the one-dimensional unsteady flow of a liquid in an open channel can be obtained by means of mass and momentum balances, considering that the liquid is incompressible, that the vertical pressure distribution is hydrostatic, and that the density is constant in time [8] [10] [11]. The Saint-Venant equations describe the temporal and spatial changes in height and volumetric flow rate of a liquid flowing in an open channel. These equations consist of a continuity equation and a momentum equation, and can be expressed in various ways depending on the variables used, for example:

Continuity equation:

$$b\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial Q}{\partial t}+\frac{1}{b}\frac{\partial}{\partial x}\left(\frac{Q^2}{h}\right)+gbh\left(\frac{\partial h}{\partial x}+S_f-S_0\right)=0 \tag{2}$$

In the above equations $h(x, t)$ and $Q(x, t)$ are the height of the liquid and the volumetric flow rate, respectively; x and t are the coordinates in space and time, respectively. Also b is the width of the channel, g is the gravitational acceleration, S_f is the friction slope, and S_0 is the channel slope. S_f and S_0 can be determined from the following expressions:

$$S_f\left(x,t\right)=\frac{n^2Q^2P^{4/3}}{A^{10/3}} \tag{3}$$

$$S_0=\tan\left(\alpha\right) \tag{4}$$

where

n is the Manning roughness coefficient;

$P=2h+b$ is the wetted perimeter;

$A = bh$ is the flow area;

α (rad) is the inclination angle of the channel.

An algebraic manipulation of the momentum equation leads to the following expression [10]:

$$\frac{\partial Q}{\partial t} + \alpha \frac{\partial Q}{\partial x} + \beta = 0 \tag{5}$$

in which

$$\alpha = 2\left(\frac{Q}{bh}\right) + \frac{gh - \left(\frac{Q}{bh}\right)^2}{\left(\frac{Q}{bh}\right)\left(\frac{5}{3} - \frac{4}{3}R_h\right)} \tag{6}$$

$$\beta = gbh\left(S_f - S_0\right) \tag{7}$$

$$R_h = \frac{A}{P} = \frac{bh}{b + 2h} \tag{8}$$

Equations (1) and (5) are the Saint-Venant equations used in the present work. They require an initial condition and a boundary condition for each of the dependent variables h and Q.

Initial conditions:

$$h(x,0) = h_0 \tag{9}$$

$$Q(x,0) = Q_0 \tag{10}$$

Boundary conditions:

$$h(0,t) = h_0 \tag{11}$$

A sinusoidal inflow pulse is imposed at the upstream of the channel on the volumetric flow rate in order to analyze the propagation of the resulting wave:

$$Q(0,t) = Q_0 + \eta Q_0 \sin\left(\frac{2\pi t}{T}\right), t \le t_p \tag{12}$$

$$Q(0,t) = Q_0, t > t_p \tag{13}$$

where T = pulse period, t_p = pulse duration time, ηQ_0 = pulse amplitude.

The values of h_0 and Q_0 were taken from a problem presented and solved in [12], in which pig iron is transported to a torpedo ladle through a rectangular channel with a length of 20 m and an inclination of 5 degrees. The channel is 0.2 m wide, the height of the metal is kept at 0.15 m, and the roughness of the walls is 5×10^{-4} m. The calculated value of the volumetric flow rate is 0.038 m^3/s, which requires a value of Manning's roughness coefficient n = 0.036 $m^{-1/3}/s$. So it turns out that b = 0.2 m, h_0 = 0.15 m, and Q_0 = 0.038 m^3/s. The above values of b, h_0 and Q_0 yield a mean velocity u_m of 1.27 m/s of the pig iron in the channel. These data will be used in the subsequent numerical simulations.

3. Numerical Solution

The Saint-Venant Equations (1) and (5) were numerically solved employing an explicit backwards finite difference method for the space coordinate, as suggested in [10]. Explicit central and forward finite difference schemes were tested in this work but the numerical solutions obtained were unstable. For the time coordinate, a forward finite difference scheme was used. The discretized form of the momentum equation is as follows:

$$Q_i^{n+1} = Q_i^n - \left(\frac{\Delta t}{\Delta x}\right)\alpha_i^n\left(Q_i^n - Q_{i-1}^n\right) - \Delta t \beta_i^n \tag{14}$$

and the discretized form of the continuity equation is as follows:

$$h_i^{n+1} = h_i^n - \left(\frac{\Delta t}{b\Delta x}\right)\left(Q_i^{n+1} - Q_{i-1}^{n+1}\right) \tag{15}$$

To ensure the stability and convergence of the numerical solutions of the Saint-Venant equations, the following conditions must be met:

$$\lim_{\substack{x\to\infty \\ t\to\infty}} h(x,t) = h_0 \tag{16}$$

and

$$\lim_{\substack{x\to\infty \\ t\to\infty}} Q(x,t) = Q_0 \tag{17}$$

Channels for transporting molten metals such as aluminum and pig iron are relatively short in length, on the order of 20 m. It is to be expected that a channel of this length will not satisfy uniform flow conditions. In the numerical simulations, it was sought that the stability and convergence conditions given by Equations (16) and (17) were guaranteed. A channel length of 500 m was used, and these results were later employed to analyze smaller metallurgical channel lengths. Three numbers of nodes were tested in the numerical simulations, namely 1000, 2000 and 3000, which applied over a length of 500 m produced Δx values of 0.5, 0.25 and 0.166 m, respectively. Values of Δt above 1×10^{-3} s caused numerical instability, while values below 1×10^{-5} s markedly increased the processing time. Then it was decided to use a value of Δt of 1×10^{-4} s. **Figure 1** shows the results of the convergence tests using n = 0.036 $m^{-1/3}s$, b = 0.2 m, h_0 = 0.15 m, Q_0 = 0.038 m^3/s. The difference between the results for the height of the pig iron for Δx = 0.25 and Δx = 0.166 is not significant, so it was decided to use N = 2000 and Δx = 0.25 in the subsequent numerical simulations so as not to increase unnecessarily the computation time.

4. Results and Comments

Numerical simulations were carried out using the following values of parameters: n = 0.036 $m^{-1/3}s$, b = 0.2 m, h_0 = 0.15 m, Q_0 = 0.038 m^3/s, g = 9.81 m/s^2, N = 2001 nodes, Δx = 0.25 m, $\Delta t = 1 \times 10^{-4}$ s. As in [8] for the sinusoidal inflow pulse, a period T of 10 s and a pulse duration t_p of 5 s were used, which caused a half-wave pulse in the volumetric flow rate. The amplitude used was $0.75Q_0$. The times considered were 10, 20, 30, 60, 120 and 180 s.

Figure 2 shows the time evolution of the pig iron height for the times considered

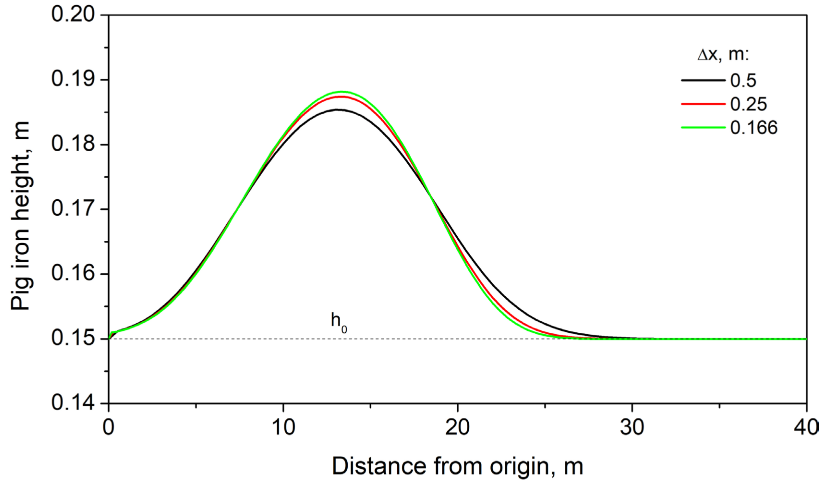

Figure 1. Convergence test results.

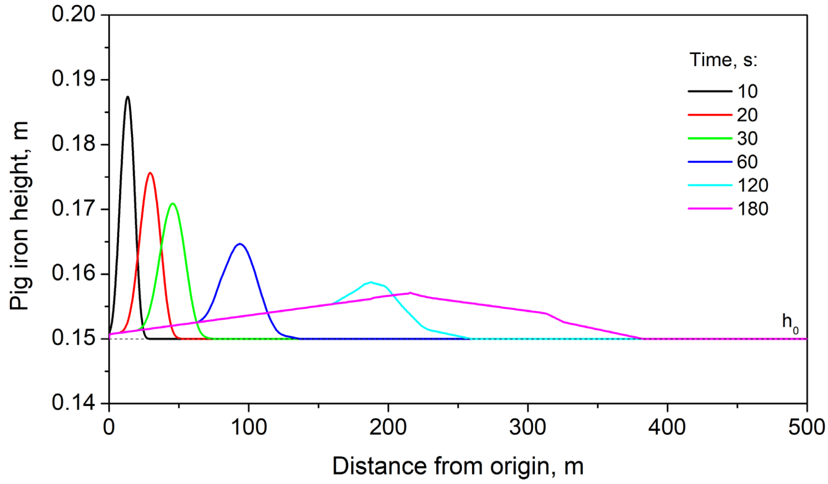

Figure 2. Time evolution of the pig iron height in the 500 m long channel for several times after the start of the sinusoidal inflow pulse in the channel.

above, after the sinusoidal inflow pulse in the channel started. It can be seen that after 10 s the height of the pig iron reaches its initial value h_0 for a channel length of 500 m. The resulting wave is observed to move along the channel and its amplitude decreases with time. If the wave is moving forward at the mean velocity of the metal in the channel, i.e. 1.27 m/s, it will reach the discharge end of the 500 m channel in about 395 s. As discussed above, a length of 500 m for an open channel carrying molten metals is extremely long. In the problem proposed in [12] it is mentioned that an industrial channel for the transport of pig iron has a length of 20 m. According to **Figure 3**, a 20 m long channel will reach a more or less uniform flow after about 30 s from the start of the sinusoidal input pulse considering a period of 10 s, duration of 5 s, and amplitude of 00285 m^3/s.

A phase portrait of the dependent variables h and Q is shown in **Figure 4**. Closed trajectories that start at the equilibrium point (h_0, Q_0) are moving away from this point due to the sinusoidal inflow pulse, and then return to it to restore uniform flow in the channel. The magnitude of the departure from the

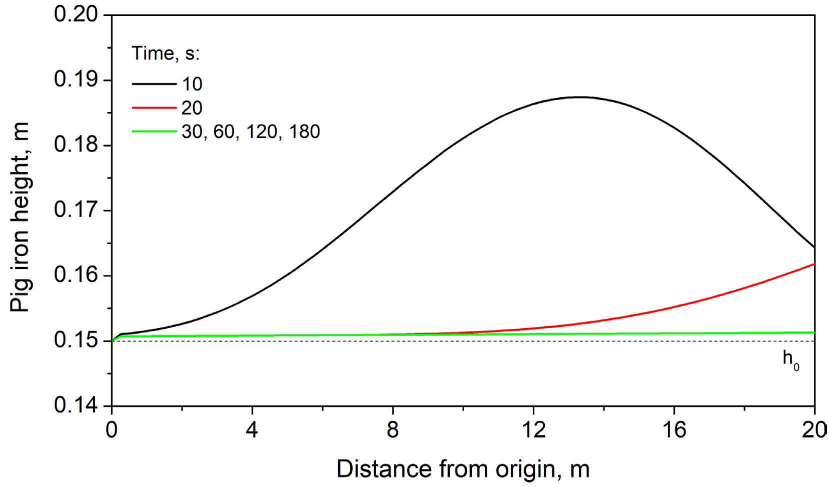

Figure 3. Time evolution of the pig iron height in a 20 m long channel after the sinusoidal inflow pulse in the channel started.

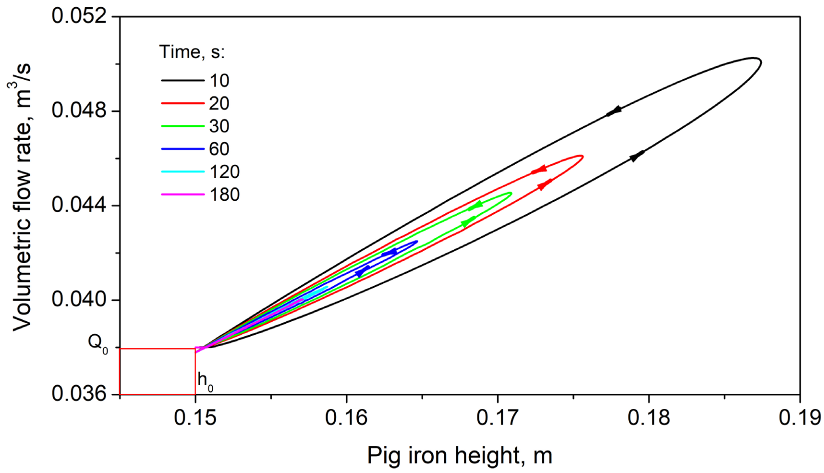

Figure 4. Phase portrait for the 500 m long channel.

equilibrium point (h_0, Q_0) depends on the time considered. The phase portrait of a 20 m channel shows open and long trajectories for short times, as shown in **Figure 5**, which means that the equilibrium point has not been reached. For long times the trajectories are closed but short since they eventually converge to that point.

The influence of the channel inclination angle on the pig iron height profiles can be seen in **Figure 6**, constructed using a roughness coefficient value of n = 0.036 $m^{-1/3}$ s. All the height profiles have been obtained when 10 s have elapsed since the start of the sinusoidal inflow pulse. It can be appreciated that as the angle of inclination of the channel increases, the amplitude of the formed wave decreases. From 10 degrees onwards, the peak of the wave descends below the initial height h_0 due to the predominance of gravitational forces over inertial ones. Besides, **Figure 7** shows the influence of the roughness coefficient n on the height profile of the molten metal in the channel. In this figure the angle of inclination α of the channel was kept constant at 5 degrees. The roughness coefficient

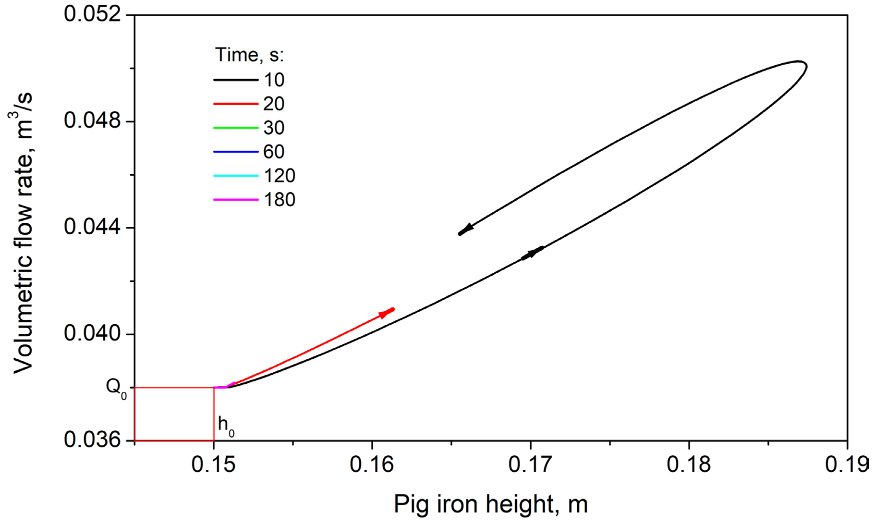

Figure 5. Phase portrait for a channel of 20 m length.

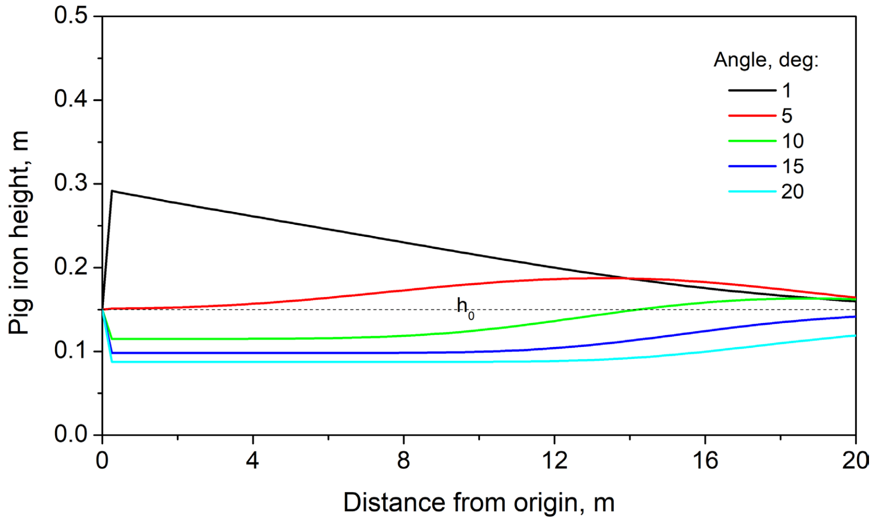

Figure 6. The influence of the channel inclination angle on the pig iron height profiles.

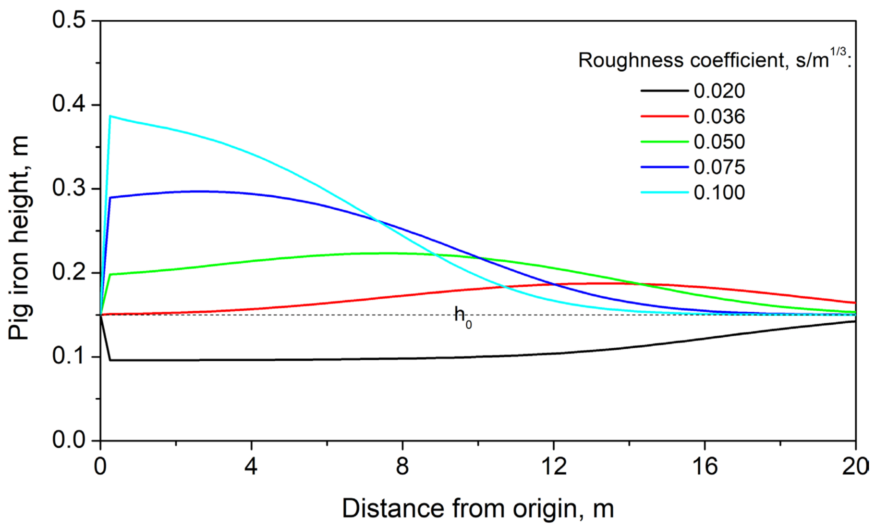

Figure 7. The influence of the roughness coefficient on the pig iron height profiles.

affects the height profiles in an inverse way than the angle of inclination. The amplitude of the wave increases as the roughness coefficient increases, as can be seen in **Figure 7**. The friction stresses slow down the downward movement of the molten metal in the channel, allowing a greater accumulation of the pig iron at the entrance of the channel, thus increasing the amplitude of the traveling wave.

Numerical simulations were carried out to study the effect of the period, amplitude and duration time of the sinusoidal pulse on the height profile of the pig iron in the channel. Changes in pig iron height for different positions along the channel length were monitored. Distances from the origin of the channel of 5, 10, 15 and 20 m were considered for a monitoring time of 30 s, assuming a slope angle of 5 degrees and a roughness coefficient of 0.036. **Figure 8** shows the changes in pig iron height with time for a sinusoidal pulse period $T = 10$ s, an amplitude $\eta Q_0 = 0.0285$ m^3/s, and a pulse duration $t_p = 5$ s. It can be seen that the amplitude of the wave decreases as the distance from the origin of the channel increases. It can also be noted that for all the distances considered, the heights of the pig iron converge to their initial value in a time of 30 s.

The effect of the period and duration of the sinusoidal inflow pulse can be observed in **Figure 9**. With respect to the results shown in **Figure 8**, the pulse period was reduced from 10 to 2 s, and the duration time from 5 to 1 s. Comparing both figures, it can be seen that although the amplitude of the pulse is the same, the amplitude of the resulting wave is smaller when the period and duration of the pulse are decreased. However, the response time of the system in which the effect of the pulse is appreciated is the same in both cases, around 1.8 s, regardless of the duration time of the pulse. Finally, the amplitude of the sinusoidal inflow pulse was decreased from 0.0285 to 0.019 m^3/s, whilst the period and duration time were kept constant at 10 s and 5 s, respectively. These results are shown in **Figure 10**. The following observations emerge from looking at **Figure 10**: 1) Amplitudes of the resulting waves are smaller by lowering the amplitude

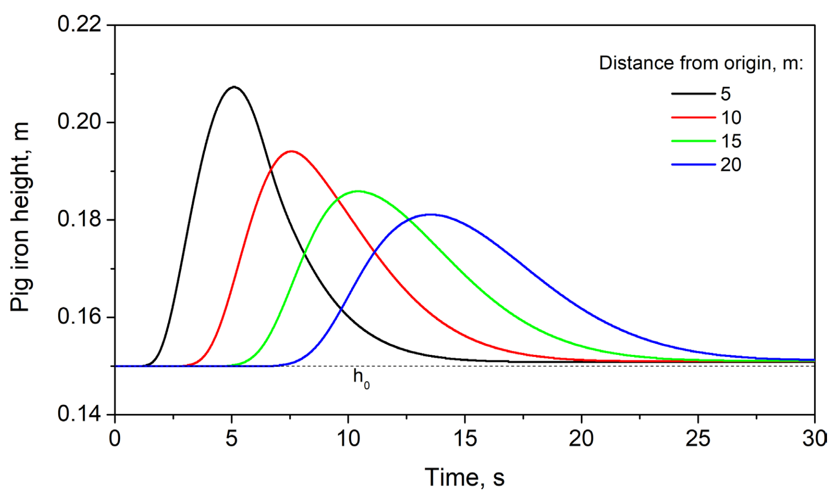

Figure 8. Changes in the pig iron height with time for a sinusoidal inflow pulse with period $T = 10$ s, amplitude $\eta Q_0 = 0.0285$ m^3/s, and pulse duration $t_p = 5$ s.

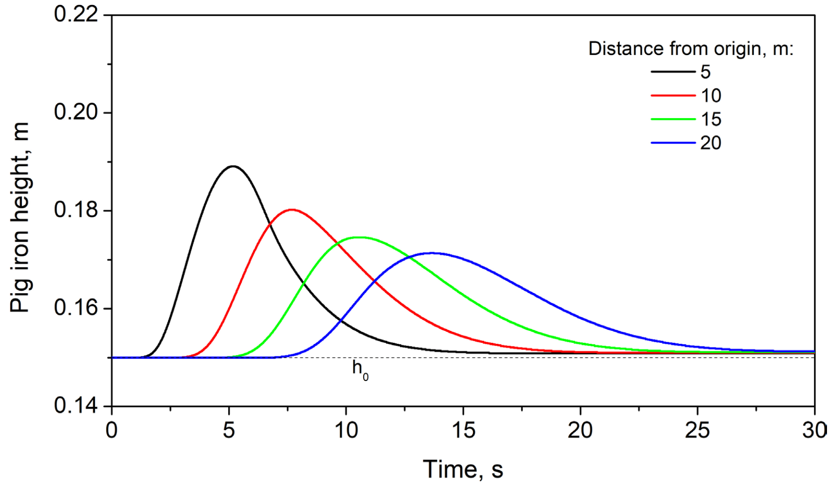

Figure 9. Changes in the pig iron height with time for a sinusoidal inflow pulse with period $T = 2$ s, amplitude $\eta Q_0 = 0.0285$ m^3/s, and pulse duration $t_p = 1$ s.

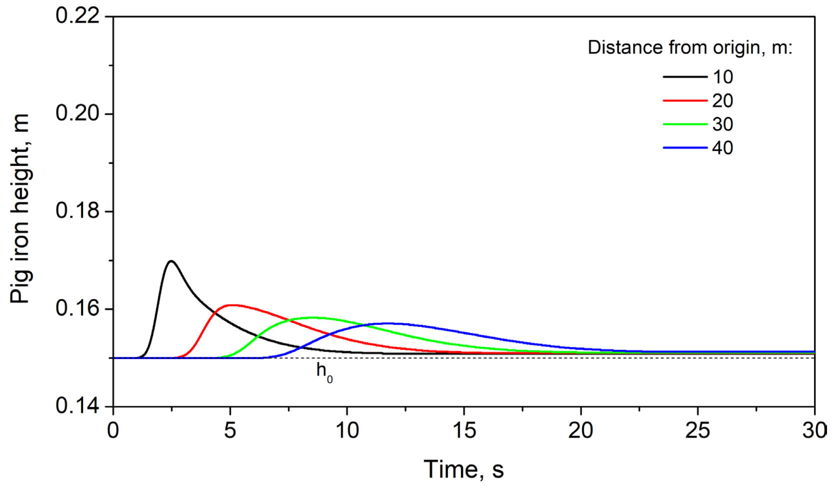

Figure 10. Changes in the pig iron height with time for a sinusoidal inflow pulse with period $T = 10$ s, amplitude $\eta Q_0 = 0.019$ m^3/s, and pulse duration $t_p = 5$ s.

of the sinusoidal inflow pulse for all positions considered. 2) Response time of the system to the input pulse decreases slightly from 1.8 to 1.6 s. 3) Convergence time to the initial height decreases from 30 to 25 s.

5. Conclusions

The transient non-uniform flow of molten metal in a rectangular open channel was numerically studied. To analyze the non-steady behavior of the pig iron flow, a sinusoidal inflow pulse of the volumetric flow rate was imposed. From the results of the numerical simulations the following conclusions can be drawn:

1) As the angle of inclination of the channel increases, the amplitude of the formed wave decreases. From 10 degrees onwards, the peak of the wave descends even below the initial height due to the predominance of gravitational forces over inertial and frictional ones.

2) The roughness coefficient affects the height profiles in an inverse way than

the angle of inclination. The amplitude of the formed wave increases as the roughness coefficient increases. The friction stresses slow down the downward movement of the molten metal in the channel, allowing a greater accumulation of the pig iron at the entrance of the channel, thus increasing the amplitude of the traveling wave.

3) The amplitude of the formed wave depends on the characteristics of the sinusoidal inflow pulse, namely its amplitude, its period and the duration of the pulse. For the same pulse amplitude, a smaller pulse period reduces the amplitude of the formed wave.

4) In channels of short length, such as those used for the transport of molten metals, obtaining a uniform flow takes longer than for long channels.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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