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Criteria for Submergence of Ice Blocks in Front of Ice Cover

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Abstract

During winter, ice jams develop when floating ice blocks accumulate in rivers. Ice jams can dramatically decrease in the capacity of flow in a river and can cause ice flooding due to increase in water level. Submergence of floating ice blocks in front of ice cover is critical for the development of an ice jam. In this study, the effect of the rotation angle of ice blocks on the submergence of ice block was assessed. The impacts of both the drag force caused by the flow and the hydraulic pressure force on the rotation of ice block were studied. Considering both the maximum moment for anti-overturn of an ice block, and the associated rotation angle θ_1, equations for describing the criteria for ice block entrainment in front of ice cover have been derived. On the basis of the theorem for moment equilibrium, relating the moment acting on a horizontal ice block with the maximum anti-overturn moment of an ice block, the criteria for assessing the overturn-and-submergence of an ice block have been proposed. To verify results using the derived equations for calculating the critical flow velocity for ice block submergence in front of ice cover, data was collected from flume experiments in the laboratory. Experiments have been conducted using different sizes of ice block under different flow conditions in a flume which is 26.68 m long, 0.40 m wide, and 0.6 m deep. Model ice blocks were made of polypropylene and have nearly the same as the mass density of the natural ice. Using proposed method for assessing ice block submergence in front of ice cover, calculated critical flow velocities agree well with those of experiments.

Keywords

Critical Velocity, Ice Block, Maximum Anti-Overturn Moment, Kinetic Energy

1. Introduction

In winter, ice pans may form in northern rivers. Depending on velocity of flowing water, floating ice pans or ice blocks can be entrained and submerged in front of the ice cover. Throughout winter period, the submerged accumulate under ice cover. As a result, an ice jam forms and the thickness of ice jam will be increased during the accumulation process. Under such a condition, the cross section under the ice jam for passing flow decreases, which significantly increases water levels upstream of the ice-jammed section compared to under open channel flow conditions. Upstream of the ice jam section, flooding may be caused by a dramatic increase in water levels. An example of ice flooding occurred in 1982 in the Hequ Reach of the Yellow River [1].

River ice jams in northern rivers can also affect the safety of hydraulic structures such as pump stations and hydropower stations, and may cause loss of life. Therefore, studying the mechanism of ice jam development in northern rivers, such as critical conditions for the ice block submergence in front of ice cover, is important.

Research work regarding critical conditions for ice block submergence in front of ice cover has been carried out by some researchers. For example, a formula was derived to determine the critical flow velocity (V_c) for the entrainment of ice blocks in front of an ice cover [2],

$$V_c = \left(0.035 gL\right)^{1/2} \tag{1}$$

where, V_c is the critical flow velocity; g is the gravitational acceleration; L is the length of the ice block.

Kivisild stated that the critical flow Froude number for the entrainment of ice blocks in front of an ice cover was 0.08 [3]. To address the critical condition of the submergence of a single ice block in front of ice cover, laboratory experiments were conducted [4]. From the one-dimensional hydrodynamics analysis, using a moment equilibrium method, the following formula was derived to determine the critical flow Froude number (Fr) for ice block submergence in front of ice cover,

$$Fr = \sqrt{\frac{2\left(t/h\right)\left(1-\rho_i/\rho_w\right)}{C_s + \left[1/\left(1-t/h\right)\right] - \left(1+\beta\right)}} \tag{2}$$

where, t is the ice block thickness; h is the upstream water depth of the block; ρ_i is the mass density of ice block; ρ_w is the mass density of water; C_s is the approaching flow velocity coefficient, C_s > 1.0; β is the velocity head coefficient under the ice cover.

Using experimental data collected by other researchers [4], an equation was developed to determine the critical flow Froude number for the entrainment of the ice block in front of an ice cover [5]:

$$\frac{V_c}{\left[gt\left(1-\rho_i/\rho_w\right)\right]^{1/2}} = \frac{2\left(1-t/h\right)}{\left[5-3\left(1-t/h\right)^2\right]^{1/2}} \tag{3}$$

The effects of flow Froude number on the formation of ice jam and ice accumulation have been also studied by researchers [6]. Using field measurements of ice block submergence in front of an ice cover in the Hequ Reach of the Yellow River as well as laboratory experiments, it is found that the critical flow Froude number for the entrainment of the ice block in front of an ice cover depends not only on the ratio of the thickness of the ice block (t) to the flowing water depth under ice cover (h), but also on the ratio of the ice block thickness to the ice block length (L). They derived the following equation to determine the critical flow Froude number for the entrainment of ice block in front of an ice cover:

$$F_c = 0.271\left(\frac{t}{h}\right)^{-0.101}\left(\frac{t}{L}\right)^{-0.088} \tag{4}$$

Wang et al used laboratory experiments to study the critical condition for the entrainment of ice blocks in front of an ice cover [7] [8] [9] [10]. Based on the data obtained, the authors found that the critical ice Froude number is related to the thickness, length, and width of the ice blocks. Based on their experiments, they derived the following equation for the critical ice Froude number for the entrainment of ice blocks in front of an ice cover,

$$\frac{V_c}{\sqrt{gt}} = 0.1034\left(\frac{t}{L}\right)^{-0.113}\left(\frac{b}{L}\right)^{0.260}\left(\frac{t}{h}\right)^{-0.492} \tag{5}$$

To assess the influence of the shape of ice block leading edge on its stability, experiments were conducted and the pressure difference distribution of the bottom surface of an ice block was measured with different leading edge shapes as well as the stability of rectangular ice block using the moment balance method [11].

Based on the experimental data [11], simulated numerical simulation was conducted to assess the distribution of the hydraulic pressure force on the bottom of ice blocks with both rectangular and semi-circular leading edge shapes [12]. On the basis Equation 3, the criteria was proposed to determine if an ice block can be entrained and submerged in front of ice cover [12],

$$\frac{V_c}{\left[gt\left(1-\rho_i/\rho_w\right)\right]^{1/2}} = k\frac{2\left(1-t/h\right)}{\left[5-3\left(1-t/h\right)^2\right]^{1/2}} \tag{6}$$

The coefficient k ranges from 1.15 to 1.35. The coefficient k takes the minimum value if the frontal cross section of the ice block has a quasi-rectangular shape [12].

Considering the drag force and the Venturi pressure effect, the critical Froude number for submergence of an ice block in front of ice cover was studied using the method of the equilibrium of moments [13]. They stated that the longer the ice block, the higher the flow velocity is needed for the entrainment of the ice block in front of the ice cover. Also, the critical flow velocity for the entrainment of ice blocks decreases with the increase in the ratio of the ice block thickness to flow depth.

To date, research results regarding the criteria for entrainment of ice block in front of ice cover are either limited to field measurements in a specific river or to laboratory experiments. On the other side, some researchers studied the criteria for entrainment of ice block on the basis of theory of mechanics. Results based on theory of mechanics are relatively more comprehend. However, the rotation angle of the ice block has not been considered in these studies when the maximum anti-overturn moment is generated. The overturned entrainment process of ice block is shown in **Figure 1**.

As shown in **Figure 1**, when the angle between ice block and ice cover increased from 0 to θ_1, the moment for anti-overturn of an ice block increases approximately linearly [14]. When the angle between the ice block and the ice cover changes from θ_1 to θ_2, the moment for anti-overturn of an ice block first increases slowly until a maximum is reached. Following the maximum, the moment for anti-overturn of an ice block decreased gradually. This means that the maximum moment for anti-overturn of an ice block occurs when the angle between ice block and ice cover is $\theta_1 < \theta < \theta_2$, but very close to θ_1. The maximum moment for anti-overturn of an ice block (R_{max}) could be calculated by Equation (7). After that, as the angle between ice block and ice cover increases from θ_2 to π/2, the moment for anti-overturn of an ice block decreases.

$$R_{max} = \left(\rho_w - \rho_i\right)\frac{gtbL^2}{2} \tag{7}$$

The moment for anti-overturn of an ice block changes with the rotation angle of the ice block. For an ice block to rotate, the moment acting on an ice block must be greater than the maximum moment for anti-overturn of an ice block. Therefore, to study the critical conditions for an ice block to be entrained and submerged in front of ice cover, the moment balance method should be followed. The moment analysis should be conducted provided that the ice block is rotated to an angle of θ_1 (the position for the maximum anti-overturn moment).

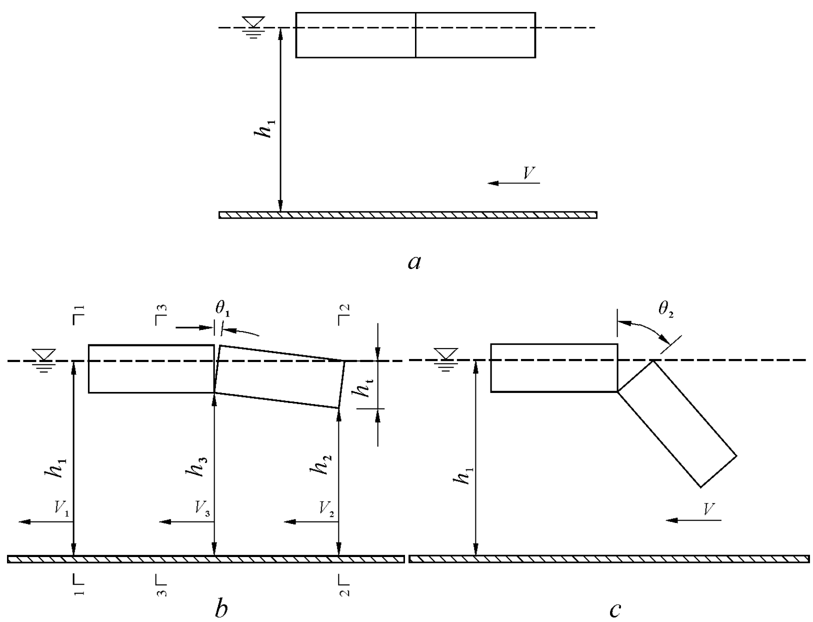

Figure 1. The "overturned" entrainment process of ice block in front of ice cover.

2. Analysis of Moment Acting on an Ice Block

When an ice block floats in the surface of flowing water, it is subjected to buoyancy force, gravitational force, drag force caused by flowing water, hydraulic pressure force, etc. The pressure acting on the ice block in front of the ice cover is the lowest force. The pressure force is mainly caused by the venturi effect and the interaction of the ice block and frontal ice cover. When an ice block approaches the front of an ice cover, it will be non-horizontal affected by flowing water in front of the ice cover. The pressure under ice block/cover gradually changes along the direction of flow, due to the decrease of the cross sectional area caused by the ice block/cover, which can be estimated using energy equation. As a consequence, the venturi effect occurs on the ice block. At the leading edge of the ice cover, the local flow velocity increases and accelerates in addition to the flow separation. Thus, the pressure at the leading edge of the ice block/cover reduces to a minimum, resulting in the maximum pressure difference.

As shown in **Figure 1(b)**, the rotation angle of an ice block to the position of (θ_1) can be calculated using following equation [15],

$$\theta_1 = \sin^{-1}\left[\frac{-(\rho_i/\rho_w)(L/t)+\sqrt{(L/t)^2+1-(\rho_i/\rho_w)^2}}{(L/t)^2+1}\right] \tag{8}$$

2.1. Moment Calculation

2.1.1. Drag Force Caused by the Flowing Water

$$F_1 = \frac{1}{2}\rho_w C_D A V_1^2 \tag{9}$$

where, C_D is the drag coefficient, V_1 is the average cross sectional velocity in front of ice cover (m/s), A is the cross sectional area of the ice block which is perpendicular to the flow direction (m^2). The drag force acting on an individual ice block can be further expressed as

$$F_1 = 1.42\left[\left(bt/L^2\right)^{1/3} - 0.26\right]\cdot\frac{1}{2}\rho_w V_1^2\cdot bt \tag{10}$$

where, b, t and L are the width, thickness and length of an individual ice block, respectively. Thus, the overturning moment M_1 generated by the drag force caused by the flowing water is:

$$M_1 = F_1\frac{t}{2} = \frac{1.42}{4}\rho_w V_1^2\left[\left(bt/L^2\right)^{1/3} - 0.26\right]bt^2 \tag{11}$$

2.1.2. Venturi Effect

When the study ice block is floating horizontally on water surface, the decrease of the cross section at the frontal edge of ice block is the same as that at the rear edge. Thus, the pressure decrease generated by the increase in flow velocity is constant in the longitudinal direction (flow direction) along the study ice block. Namely, the overturning moment for the rotating ice block cannot be generated, as shown in **Figure 1(a)**. However, when the ice block is rotated at an angle of θ_1, the force due to the reduction of pressure along the length of the ice block can be described as following,

$$F_2 = \frac{1}{2}\rho_w \left(V_2^2 - V_3^2\right) bL = \frac{1}{2}\rho_w V_1^2 \left(\alpha_1^2 - \alpha_2^2\right) bL \tag{12}$$

where, $\begin{cases} h_2 = h_1 - h_t = h_1 - t\cos\theta_1 \\ h_3 = h_2 + L\sin\theta_1 = h_1 - t\cos\theta_1 + L\sin\theta_1 \\ \alpha_1 = h_1/h_2; \alpha_2 = h_1/h_3 \end{cases}$.

In which, L is the length of the block (m); t is the thickness of the block (m); h_t is the projection length of the upstream submerged ice block in vertical direction (m). h_1, h_2 and h_3 are respectively the water depth (m) at sections 1-1, 2-2 and 3-3, as shown in Figure 1(b). Thus, the overturning moment M_2 generated from the venturi effect can be determined as following,

$$M_2 = F_2 \frac{L}{2}\cos\theta_1 = \frac{1}{4}\rho_w V_1^2 \left(\alpha_1^2 - \alpha_2^2\right) bL^2 \cos\theta_1 \tag{13}$$

2.1.3. Leading Edge Effect

At the leading edge of ice block, the force due to the pressure difference can be described as following,

$$F_3 = \Phi \rho_w V_2^2 bL = 0.35\Phi \rho_w V_1^2 \alpha_1^2 bL \tag{14}$$

where: Φ stands for a reduction factor. Note: Φ values for: when the length of submerged ice block $L < 0.5X_{50}$, then, $\Phi = 1$; when $0.5X_{50} < L < 3X_{50}$, then $\Phi = 0.5$; when $L > 3X_{50}$, then $\Phi = 0.2$. In the present study, $\Phi = 0.5$ [16]. Where, X_{50} is used to assess the impact of the length of submerged ice block on the process of ice block submergence. It is the location at which the pressure is midway between the initial and final pressure plateau values [11]. Thus, the overturning moment M_3 generated by the leading edge effect is,

$$M_3 = F_3 \frac{L}{2}\cos\theta_1 = 0.175\Phi \rho_w V_1^2 \alpha_1^2 bL^2 \cos\theta_1 \tag{15}$$

2.2. Maximum Anti-Overturn Moment

In present study, Equation (7) [14] was used to calculate the maximum anti-overturn moment for ice blocks submergence. When the ice block is in front of an ice cover is rotated at an angle of θ_1 to be submerged, the total anti-overturn moment M_T acting on the ice block should be,

$$R_{\max} \le M_T = M_1 + M_2 + M_3 \tag{16}$$

2.2.1. Calculation of the Critical Flow Velocity

The total anti-overturn moment M_T acting on ice block can be expressed as following,

$$M_T = \frac{1.42}{4}\rho_w V_1^2 \left[\left(\frac{bt}{L^2}\right)^{1/3} - 0.26\right] bt^2 + \frac{1}{4}\rho_w V_1^2 \left(\alpha_1^2 - \alpha_2^2\right) bL^2 \cos\theta_1 + \frac{0.35}{2}\Phi \rho_w V_1^2 \alpha_1^2 bL^2 \cos\theta_1 \tag{17}$$

The critical flow velocity can be obtained by simplification of above equation as following,

$$V_c = \sqrt{\frac{(\rho_w - \rho_i) gt}{0.71\left[\left(\frac{bt}{L^2}\right)^{1/3} - 0.26\right]\rho_w\left(\frac{t}{L}\right)^2 + 0.50\rho_w\left(\alpha_1^2 - \alpha_2^2\right)\cos\theta_1 + 0.35\Phi\rho_w\alpha_1^2\cos\theta_1}} \tag{18}$$

2.2.2. Verification of Calculation Results

To verify results using the derived equations for calculating the critical flow velocity for ice block submergence in front of ice cover, data was collected from flume experiments in the laboratory. As shown in Figure 2, the flume used in this study is 26.68 m long, 0.40 m wide, and 0.6 m deep. With different flow depths, the average approaching flow velocities were 0.05 m/s, 0.07 m/s, 0.09 m/s, 0.11 m/s, 0.12 m/s, 0.13 m/s, 0.14 m/s, 0.15 m/s, 0.16 m/s, 0.17 m/s, 0.18 m/s, 0.20 m/s and 0.22 m/s, respectively. Model ice blocks were made of polypro-py-lene with the mass density of 918 kg/m^3, which is nearly the same as the mass density of the natural ice at 917 kg/m^3. The model ice blocks had the following dimensions (length × width × height): 0.02 m × 0.02 m × 0.01 m, 0.03 m × 0.03 m × 0.01 m, 0.04 m × 0.04 m × 0.01 m and 0.06 m × 0.06 m × 0.01 m, respectively, the same sizes (the same dimensions) as those used by other researchers [17]. In total, 22 observation cross sections (CS) along the flume with an equal distance of 1.2 m were set up. In the downstream section from CS-20 to CS-21, one styrofoam plate was placed on the water surface as the model ice cover. Model ice blocks were placed into the flume in the upstream section between CS-4 and CS-5. The model ice cover in the downstream section from CS-20 to CS-21 initiated the formation of an ice cover by model ice blocks, and progress upstream, depending on hydraulic conditions, such as the flow velocity and the ice discharge.

The submergence process of ice blocks was classified into two states: "some ice blocks submergence" and "massive ice blocks submergence". The "some ice blocks submergence" refers to that a portion of ice blocks submerged in front of the ice cover, and normally accompanied with the "incomplete rotate" phenomenon, as shown in Figure 3(a). The "massive ice blocks submergence" refers to ice blocks that moved to the leading edge of the ice jam, and the submergence phenomenon of majority ice blocks was observed. In addition, the submergence process of each individual ice block was normally rotated 180˚, as shown in Figure 3(b). In such a case, only a small number of ice blocks were juxtaposed in front of ice cover, and the ice cover progressed upstream.

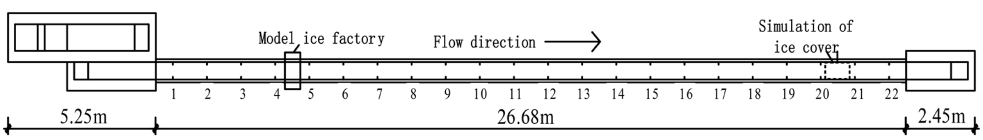

Figure 2. Setup of the laboratory flume for experiments.

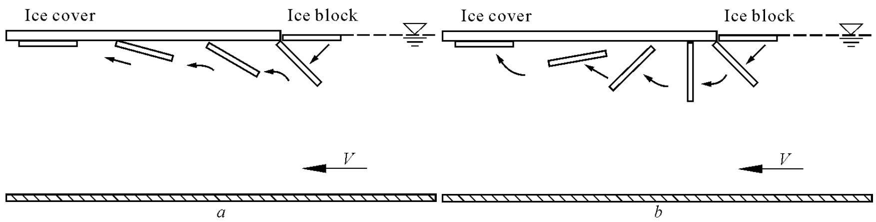

Figure 3. Ice block submergence in front of an ice cover.

Compared with the experimental results, as shown in **Figure 4**, the calculated critical velocity, using Equation (18), lies within the velocity range of "some ice blocks submergence". It should be noted that Equation 18 can only be used to calculate the critical flow velocity for ice blocks submergence when the ice block in front of the ice cover is rotated by an angle of θ_1. If the ice block in front of ice cover was not rotated at an angle of θ_1, the ice block may not be submerged even if the flow velocity exceeds the critical velocity. Under such a condition, the "massive ice blocks submergence" does not occur even if the flow velocity reaches the critical velocity, as shown in **Figure 4**.

3. Critical Velocity for "Massive Ice Blocks Submergence"

When flow velocity reaches the critical velocity for ice blocks in front of ice cover, changes of other factors such as the turbulence field may lead to an ice block to rotate an angle of θ_1, resulting in the "some ice blocks submergence" case. However, to obtain a formula for determining the critical velocity for the case of "massive ice blocks submergence" (instead of "some ice blocks submergence"), ice blocks should be considered floating horizontally on the surface of the water, see **Figure 1(a)**. This means that for flow with high velocity, the overturning moment generated by flow on a horizontal ice block can be greater than the maximum anti-overturn moment of an ice block.

Based on laboratory experiments, it is found that when an ice block began to rotate, the overturning moment acting on the ice block increased compared to that acting on a horizontal ice block [18]. When the rotation angle of ice block was θ_1 (critical position), the overturning moment increased by 25%. Namely, the overturning moment acting on ice block under the condition of the critical state (θ_1 position) was about 1.25 times of that acting on a horizontal ice block. Therefore, one can relate the overturning moment acting on the horizontal ice block to that of under the condition of the critical state (θ_1 position) as follows,

$$\frac{M_T}{1.25} = M \tag{19}$$

where, M is the overturning moment of a horizontal ice block floating on water surface in front of ice cover.

Using Equation (19), the overturning moment acting on ice blocks under the condition of the critical state (θ_1 position) can be converted into the overturning moment acting on ice blocks floating horizontally on the surface of the water. Following, the relationship between the overturning moment acting on ice

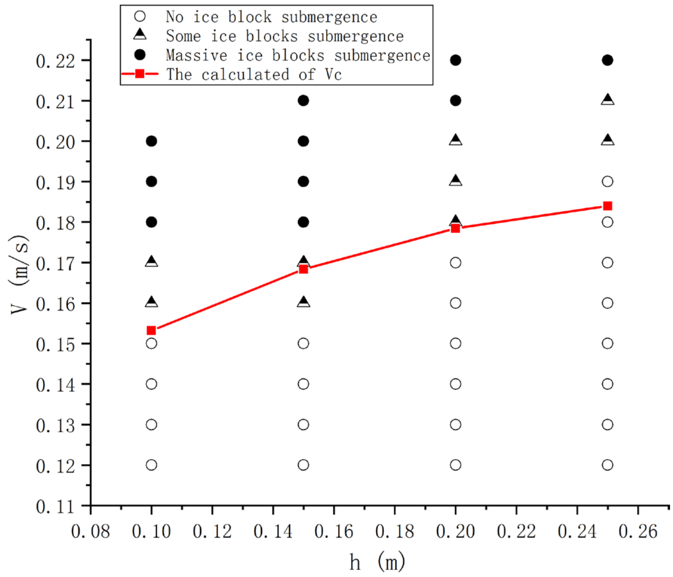

Figure 4. The relationship between the critical flow velocity and water depth (Ice block: $L \times W \times t = 0.06\ \text{m} \times 0.06\ \text{m} \times 0.01\ \text{m}$).

blocks floating horizontally on water surface and the maximum anti-overturning moment can be established. A higher critical flow velocity for submergence of a horizontal ice block (V_{c1}) can be obtained.

Regarding the overturning moment under the condition of the critical state (θ_1 position), the overturning moment acting on the horizontal ice block can be determined as follows,

$$M = 0.8\left\{0.355\rho_w V_1^2\left[\left(\frac{bt}{L^2}\right)^{1/3} - 0.26\right]bt^2 + 0.25\rho_w V_1^2\left(\alpha_1^2 - \alpha_2^2\right)bL^2\cos\theta_1 + 0.175\Phi\rho_w V_1^2\alpha_1^2 bL^2\cos\theta_1\right\} \tag{20}$$

The critical flow velocity for submergence of a horizontal ice block (V_{c1}) is obtained by:

$$V_{c1} = \sqrt{\frac{1.25\left(\rho_w - \rho_i\right)gt}{0.71\rho_w\left[\left(\frac{bt}{L^2}\right)^{1/3} - 0.26\right]\left(\frac{t}{L}\right)^2 + 0.5\rho_w\left(\alpha_1^2 - \alpha_2^2\right)\cos\theta_1 + 0.35\Phi\rho_w\alpha_1^2\cos\theta_1}} \tag{21}$$

Calculated results using Equation (21) are compared to those of laboratory experiments, as shown in **Figure 5**. One can see that the calculated critical velocity for submergence of a horizontal ice block can be used to determine whether the "massive ice blocks submergence" will present in front of ice cover.

4. Discussions

To date, the criteria for entrainment of an ice block in front of the ice cover did not consider the rotation angle of the ice block when the maximum anti-overturn moment was generated. During the period of the collision/submergence of ice blocks in front of ice cover, the incoming ice blocks will collide with ice blocks in front of ice cover. The kinetic energy of the incoming ice blocks will affect the rotation and submergence of ice blocks in front of ice cover, and provide a variety of possibilities for forming ice jam at the leading edge of ice cover. With respect to the rotation angle of ice block in front of the ice cover, the entrainment of an ice block can be classified as following 3 cases:

1) Case 1: If the rotation angle of ice block B is small ($\theta < \theta_1$), the incoming ice block C with velocity V collides with ice block B, and will provide a horizontal force where ice block B and ice block C come in contact. Above the center of mass of the ice block, the anti-overturning moment is generated. In other words, the kinetic energy of ice block C was transferred to ice block B as both the potential energy (anti-overturning) and kinetic energy of ice block B turned counterclockwise, as showed in Figure 6. In such this case, the collision between ice block C with ice block B will help ice block B to remain horizontal.

2) Case 2: If the rotation angle of ice block B is large ($\theta > \theta_1$), as shown in Figure 7, the incoming ice block C with velocity V collides with ice block B, and will cause two different types of collision resulting in an accumulation of ice blocks. As shown in Figure 8, the lower left corner of ice block B will rotate above the lower right corner of ice block A. Thus, ice block A will exert an upward force on ice block B. In such a case, ice block C can prevent ice block B from being submerged quickly.

3) Case 3: If the rotation angle of ice block B is large enough ($\theta > \theta_1$), another type of collision and accumulation of ice blocks will happen. As shown in Figure 9, the lower left corner of ice block B will be forced below the lower right corner of ice block A. In this case, ice block C can not provide the corresponding support to ice block B. If the ice block C collides with ice block B with a speed of V, then ice block B will be forced to submerge into the water by both the gravity force and impact of ice block C.

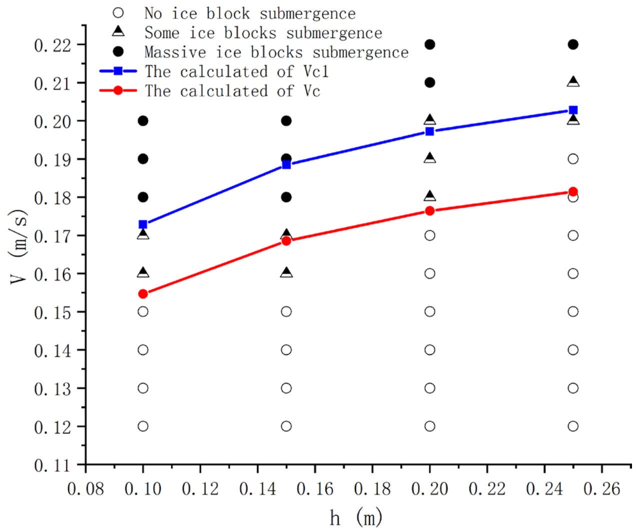

Figure 5. The relationship between the critical flow velocity and water depth (Ice block: $L \times W \times t = 0.04\ \text{m} \times 0.04\ \text{m} \times 0.01\ \text{m}$).

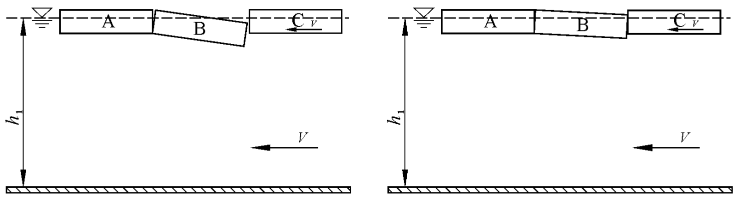

Figure 6. Ice blocks collision with a small angle rotation of ice block B.

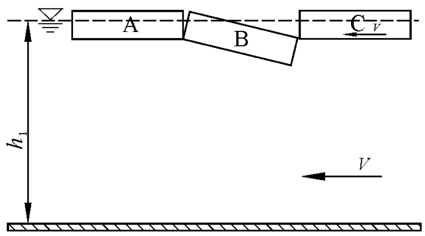

Figure 7. Ice block collision with a large angle rotation of ice block B.

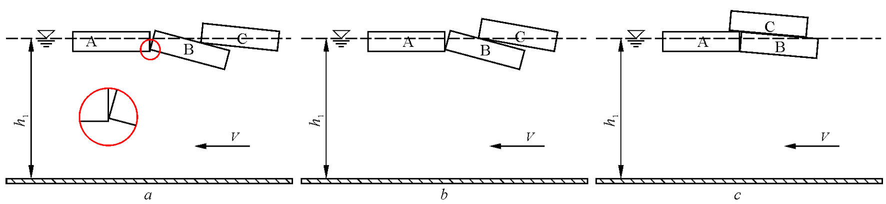

Figure 8. Ice block collision: the lower left corner of ice block B above the lower right corner of ice block A.

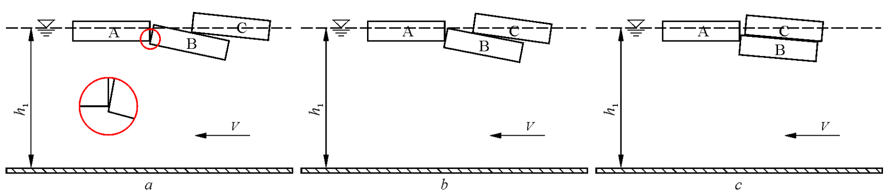

Figure 9. Ice block collision: the lower left corner of ice block B below the lower right corner of ice block A.

The fragmentation of the ice block was not taken into account in present study. For instance, if the edges of ice blocks are broken, ice block submergence in front of ice cover will be more complicated.

5. Conclusion

During winter, the submergence of floating ice blocks in front of ice cover is critical for the development of an ice jam. Ice jams can dramatically decrease in the capacity of flow in a river and can cause ice flooding. In the reported research, the criteria for entrainment of an ice block in front of the ice cover did not consider the rotation angle of the ice block when the maximum anti-overturn moment was generated. In present study, the effect of the rotation angle of ice blocks on the submergence process of ice block was assessed. Considering both the maximum moment for anti-overturn of an ice block, and the associated rotation angle θ_1, equations for describing the criteria for the entrainment of ice blocks in front of the ice cover have been derived. It was found that the formula for calculating the critical velocity can only reflect the "some ice blocks submergence" case. In the case of "massive ice block submergence", ice blocks in front of ice cover are considered floating horizontally on water surface. The overturning moment of the horizontally floated ice block is obtained based on the condition of the critical state (θ_1 position). Thereafter, the maximum anti-overturning moment of the ice block can be achieved. An equation for calculating the critical velocity V_{c1} reflects the "massive ice blocks submergence" has been derived. To verify results using the derived equations for calculating the critical flow velocity for ice block submergence in front of ice cover, flume experiments were carried out in the laboratory. Using the proposed method for calculating the critical flow velocity of an ice block in front of ice cover, the calculated critical flow velocities agree well with those of laboratory experiments.

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Notation

The following symbols are used in this paper:

A = the cross sectional area of the ice block which is perpendicular to the flow direction;

g = the gravitational acceleration;

b = the width of ice block;

C_s = the approaching flow velocity coefficient;

C_D = the drag coefficient;

h = the upstream water depth of the block;

h_t = the projection length of the upstream submerged ice block in vertical direction;

L = the length of the ice block;

R_{max} = the maximum moment for anti-overturn of an ice block;

t = the ice block thickness;

F_1 = the drag force caused by the flowing water;

F_2 = the force due to the reduction of pressure along the length of the ice block;

F_3 = the force due to the pressure difference;

M_1 = the overturning moment generated by the drag force;

M_2 = overturning moment generated from the venturi effect;

M_3 = the overturning moment generated by the leading edge effect;

M_T = the total anti-overturn moment acting on the ice block;

M = the overturning moment of a horizontal ice block floating on water surface;

V_1 = the average cross sectional velocity in front of ice cover (m/s);

V_c = the critical flow velocity;

V_{c1} = thecritical flow velocity for submergence of a horizontal ice block;

ρ_i = the mass density of ice block;

ρ_w = the mass density of water;

θ_1 = the position for the maximum anti-overturn moment;

θ = the angle between ice block and ice cover;

β = the velocity head coefficient under the ice cover;

Φ = the reduction factor.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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