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Estimation of the Power of the Anomalous Microwave Emission

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Abstract

Context and Background: The product of the electromagnetic (EM) wave's power P times its period τ, *i.e.* $P\tau$, is the amount of energy conserved in EM wave's absorption in matter. Whether $P\tau$ is the amount of energy conserved in the emission of EM waves from matter is not assessed. **Motivation:** In this research, we perform a computational study to explore the ability of $P\tau$ to represent the amount of energy conserved in EM wave's emission from matter. **Hypothesis:** Since the magnitude of the power P of emitted EM waves computed through Larmor's formula for a rotating dipole is excessively small, we alternatively hypothesize that $P\tau$ and the law of conservation of energy can lead to a realistic estimation of P. **Methods:** We estimate the power P_{AME} of the anomalous microwave emission (AME), a well-characterized radiation generated in the interstellar medium (ISM) by spinning dust grains, and one possible source of contamination of the cosmic microwave background (CMB). For our estimation of P_{AME}, we assume the AME to be generated in a molecular cloud mostly populated by spinning silicate nanoparticles (SSNs) or polycyclic aromatic hydrocarbon (PAH) spinning dust grains. Indeed, SSNs and PAHs are listed among the most probable sources of AME, and their characteristics are well-known. We discriminate between realistic and non-realistic values of P_{AME} based upon the magnitude of two parameters that depend on P_{AME}: the significant distance z, and the time of photon production T. The parameter z is the space interval from the spinning dust grain within which the spinning dust grain's electric field is effective. **Results:** Using the information available for AME, SSNs and PAHs, we estimate the power P_{AME} using both Larmor's formula and $P\tau$. We compare and comment the results obtained for z and T. **Conclusions:** Our study highlights the effectiveness of $P\tau$ over Larmor's formula in providing a realistic value of P_{AME}. This finding might have consequences in quantum technology of single photon detection and production.

Keywords

Anomalous Microwave Emission, Spinning Dust, Power of Emitted Radiation

1. Introduction

Is $P\tau$, *i.e.* the product of light's power P times its period τ, the amount of energy conserved in the emission of electromagnetic (EM) waves from matter? This outstanding question is not addressed in the current scientific literature. In this research, we computationally tackle this problem. Before outlining our strategy, we summarize the status of this field of research. Recently, Boone *et al.* [1] found that $P\tau$ is the amount of energy conserved in the absorption of EM waves in matter. To date, this finding is quantitatively proven in the absorption of visible and infrared (IR) light by capacitors [1] [2], by thin films in IR spectra [3], by the retina in vertebrate's vision [4], by molecules in photoredox catalysis reactions [5], in photo-thermoelectric devices [5], and by neurons under two-photon excitation [5]. On the contrary, only some indirect evidence that $P\tau$ could work in emission of EM waves exists, and is offered by the radiation from free electrons [6] [7]. From the computational and the experimental points of view, one of the difficulties in studying EM wave's emission lies in finding sources of EM waves with a well-determined emitted power and frequency ν (or period $\tau = \nu^{-1}$). In response to this difficulty, in our computational study we investigate the anomalous microwave emission (AME) radiated by spinning silicate nanoparticles (SSNs) or polycyclic aromatic hydrocarbon (PAH) spinning dust grains in the interstellar medium (ISM). With the available information, we determine the emitted power P_{AME} and the period τ_{AME}. On one hand, estimating τ_{AME} is straightforward, since $\tau_{\mathrm{AME}} = 1/\nu_{\mathrm{AME}}$, and ν_{AME} is well-known to peak in intensity in the frequency interval between 10 and 100 GHz [8] [9]. On the other hand, there are two routes to estimating P_{AME}: one is through the classical electrodynamics Larmor's formula; another is through $P\tau$ and the law of conservation of energy. We determine P_{AME} from both routes, and discriminate whether the obtained values of P_{AME} are realistic or not through the values of two P_{AME}-dependent parameters. These two parameters are: z, the significant distance from a spinning dust grain within which its electric field is effective, and T, the time of photon production. Discussing the values of z and T will enable us to draw conclusions on the ability of $P_{\mathrm{AME}}\tau_{\mathrm{AME}}$ to represent the amount of energy conserved in the emission of AME from spinning dust grains.

Before diving into the details of our investigation, let us outline an overview on the AME. Generally, the production of AME is ascribed to the dipole moment of the spinning dust grains in the ISM [10] [11]. The value of the dipole moment, the shape, size, composition, and the density of the AME are well-known. Indeed, the AME is one of the sources of contamination of the cosmic microwave background (CMB). Thus, the AME needs to be subtracted from the CMB signal in order to enable the extraction of information of cosmological relevance from the CMB [12]. For this reason, in the past twenty years the AME received large attention from many research groups.

Since long time, the PAH spinning dust grains are considered the main contributors to the generation of the AME [9] [11]. More recently, however, the lack of correlation between AME and PAH spinning dust grains [13] prompted the research community to consider also other sources. Among them, SSNs [14], magnetic grains [15], and diamond nanoparticles [16] were proposed. In this research, we focus first on SSNs as spinning dust grains generating the AME, and utilize the information on SSN's size, shape, composition, density, and electric dipole available from Hoang and Lazarian [15]. The amount of SSNs in the ISM is unknown and might be even very small, however the abundant information on SSNs will easily enable us sketching a rough prediction on P_{AME} and the parameters z and T. We then compute P_{AME} and the parameters z and T also for PAH spinning dust grains and compare the results with those obtained for the SSNs.

The AME was first discovered in the late 1990s [17] [18] [19], and further characterized, through direct observations made possible by the Planck satellite. The AME was clearly identified in the Perseus and ρ Ophiuchi molecular clouds, and in two compact radiation-emitting dust regions known as AME-G173.6 + 2.8 and AME-G107.1 + 5.2 [8]. Additional sources were described in a later publication [9]. Hoang *et al.* [20] investigated other sources of AME, e.g. those in Herbig A- and B-type stars, and in T-Tauri young variable stars, which are located near molecular clouds. The existence of a relationship between AME and spinning dust grains in the ISM was first proposed by Draine & Lazarian [10] [11]. More information on AME can be found in numerous publications, e.g. [21]-[30].

As mentioned above, there is general consensus among the authors regarding the frequency range of ν_{AME}, which is in the 10 to 100 GHz interval. Some authors further highlight that the magnitude of ν_{AME} is of the same order of the rotational frequency ω_{rot} of the spinning dust grains that generate the AME [31] [32] [33]. Specifically:

$$\nu_{\mathrm{AME}} \approx \left(\frac{5}{3}\right)^{\frac{1}{4}} \omega_{\mathrm{rot}} . \tag{1}$$

Equation (1) signifies that the rotational frequency ω_{rot} is in the GHz range. It is very likely that such high rotational frequency is enabled by the small size of the spinning dust grains, whose radius is typically of the order of nm, or a fraction of it [11] [14].

Almost all authors report that Larmor's formula offers a good estimation of P_{AME}, the power of the AME emitted by spinning dust grains. Various authors [10] [11] [14] [16] [20] [33] [34] report Larmor's formula in Gaussian units as:

$$P = \frac{2}{3}\frac{\mu^2 \omega_{\mathrm{rot}}^4}{c^3} , \tag{2}$$

where μ is the electric dipole moment in units of D (Debye, corresponding to 3.33564×10^{-30} C·m) of the spinning dust grains, and c is the speed of light in vacuum approximated to 3×10^{8} m·s^{-1}. In the available literature, however, actual values of P_{AME} are extremely difficult to find, and the few reported values are very small: e.g. $P_{\text{AME}} \sim 10^{-47}$ W per grain [15]. This lack of information on P_{AME} prompts us to compute the value of P_{AME} from Larmor's formula expressed with Standard International (S.I.) units. The small values we obtained stimulate us to follow an alternative route to compute P_{AME} by applying $P\tau$ and the law of conservation of energy.

To apply $P\tau$ and the law of conservation of energy to AME's emission, we first identify the types of energy of the spinning dust grains that are transformed into the EM wave's energy. We hypothesize that the spinning dust grain's electrical and mechanical energies fulfill such task. The electrical energy is due to the electric dipole moment μ of the spinning dust grain, while the mechanical energy involves its moment of inertia I. In order to significantly and effectively contribute to the conserved energy $P\tau$, the spinning dust grain's electrical and mechanical energies are required to be of the same order of magnitude of $P\tau$. To compute the electrical and mechanical energies, we utilize the known information on the spinning dust grain's size, shape, composition, density, and electric dipole.

2. Methods

2.1. $P\tau$ and the Law of Conservation of Energy in the Interaction between EM Waves and Matter

It is well established that the power P radiated by a rotating dipole is given by Larmor's formula, which we will discuss in the next section. On the other hand, it is less straightforward to establish how much is the energy conserved when energy is transferred to a radiated field or EM wave by an accelerating charge, or a spinning dipole. Is this energy equal to the energy $h\nu$ of a photon, where h is Planck's constant (6.63×10^{-34} J·s)? Or, is this energy equal to that of n photons? If so, what is the relationship between the emitted power P and the number n of photons of the emitted EM wave at constant frequency ν? These questions prompted us to investigate whether it is possible to use the law of conservation of energy to respond. To this end, we assume that an accelerating charge, a spinning dipole or, in general, matter capable of emitting EM waves, have electrical energy (E_{el}), thermal energy (E_{th}), and mechanical energy (E_{mec}). These energies contribute to the generation of the radiated field or EM wave. Vice-versa, the energy E_{EM} of the radiated field depends on them. Thus, for the law of conservation of energy, $E_{\text{el}} + E_{\text{th}} + E_{\text{mec}} = E_{\text{EM}}$, assuming that the energy dissipated during the energy transfer process between the radiating object and the EM wave is negligible. We can easily give an analytical expression to E_{el}, E_{th}, and E_{mec} and estimate their magnitude. However, it is less straightforward to establish an analytical expression and estimate for the magnitude of E_{EM}. Imagining that the radiating object emits at a constant frequency ν, we have various options for E_{EM}: $h\nu$, $nh\nu$, or $P\Delta t$. Here Δt is an arbitrary time interval. However, using the law of conservation of energy in absorption phenomena, where the power of the EM waves absorbed by a system is known, it was found that $\Delta t = \tau = \nu^{-1}$, where τ is the period of the EM wave and ν its frequency. The equation $E_{\text{EM}} = P\tau$ captures the order of magnitude of the variables affected by the absorption process, and justifies the power-dependent response of the absorption [1] [2] [3] [4] [5]. Here, we hypothesize that the approach used in absorption phenomena is true also in emission phenomena. Thus, we assume $E_{\text{EM}} = P\tau$, and test the validity of this choice. We appreciate the fact that $P\tau$ changes continuously because, in agreement with Draine and Lazarian [10] [11] we want to describe the phenomenon of EM wave emission by charges and dipoles in the realm of classical physics.

2.2. Larmor's Formula

Larmor's formula in Equation (2) is expressed in Gaussian units. In order to compute P_{AME} in S.I. units, *i.e.* in Watts (W), we transform Larmor's formula as follows:

$$P_{\text{AME}} = \frac{2}{3}\frac{k\mu^2\omega_{\text{rot}}^4}{c^3}. \tag{3}$$

In Equation (3), the dipole moment μ in units of D (Debye) is multiplied by $k^{1/2}$, where k is Coulomb's constant (8.99 10^9 $\text{N}\cdot\text{m}^2\cdot\text{C}^{-2}$).

2.3. Characteristic Properties of the SSNs and PAH Spinning Dust Grains

In our calculations, for simplicity, we assume that the AME is generated by spherical SSNs with $MgSiO_3$ composition [14]. We consider the number of atoms N in one SSN to be $N = 7N_{\text{Si}} = 417.74a_{-7}^3$, where $a_{-7} = a/10^{-7}$ cm, and $a = 0.4$ nm is the radius of a spherical SSN [14]. With this choice of a, it turns out that $N = 27$. The dipole moment for such an SSN is $\mu = 12.2$ D, where $D = 3.33564 \times 10^{-30}$ C m, and the SSN's density is $\delta \approx 4$ g cm^{-3} [14]. We estimate the mass of each spherical SSN to be $m = \delta\frac{4}{3}\pi a^3 = 1.07 \times 10^{-24}\ \text{kg}$, and its moment of inertia I to be $I = \frac{2}{5}ma^2 = 6.85 \times 10^{-44}\ \text{kg}\cdot\text{m}^2$. The characteristic properties of the SSNs are summarized in **Table 1**. Those of cylindrical PAH spinning dust grains, taken from Draine and Lazarian [11], are collected in **Table 2**.

Table 1. Summary of the characteristics of a spinning silicate nanoparticle (SSN) taken from Ref. [14].

N	μ	δ	m	I	ω_{rot}
27	12.2 D	4 g·cm^{-3}	1.07×10^{-24} kg	6.85×10^{-44} kg·m^2	30 GHz

N is the average number of atoms in one SSN, μ is the SSN's dipole moment in units of D (Debye), δ its density, m its mass, I its moment of inertia, and ω_{rot} its rotational frequency. For our calculations, we assume spherical SSNs with radius a = 0.4 nm.

Table 2. Summary of the characteristics of polycyclic aromatic hydrocarbon (PAH) spinning dust grains taken from Draine & Lazarian [11].

N	μ	δ	m	I	ω_{rot}
25.43	2 D	2 g·cm^{-3}	0.59×10^{-24} kg	3.80×10^{-44} k·gm^2	30 GHz

N is the average number of atoms in one PAH spinning dust grain, μ is a PAH's average dipole moment in units of D (Debye), δ its density, m its mass, I its moment of inertia, and ω_{rot} its rotational frequency. For our calculations, we assume cylindrical PAH spinning dust grain with radius $a = 0.36$ nm and height $h = 2a$.

3. Results

3.1. Power of the Emitted AME Estimated from Larmor's Formula

We assume for simplicity that, for the SSNs, $\omega_{\text{rot}} = \nu_{\text{AME}} = 30\ \text{GHz}$. By multiplying the dipole moment μ by the number N of atoms in the SSNs, we obtain $\mu_{\text{effective}} = N\mu$. By replacing μ with $\mu_{\text{effective}} = N\mu$ in Equation (3), we estimate $P_{\text{AME}} = 0.22 \times 10^{-27}$ W. Reasoning in the same way, for PAH spinning dust grains we obtain $P_{\text{AME}} = 5.24 \times 10^{-30}$ W.

3.2. Power of the Emitted AME Estimated from $P\tau$ and the Law of Conservation of Energy

An SSN is charged and gives rise to an electric dipole [10] [11], which rotates at a rate of $\omega_{\text{rot}} \approx 30$ GHz, as assumed in Section 3.1. We hypothesize that the energy $P_{\text{AME}}\tau_{\text{AME}}$ of the AME, where $\tau_{\text{AME}} = \frac{1}{\nu_{\text{AME}}}$, is generated by the spinning of the SSNs with rotational kinetic energy $E_{\text{rot}} = \frac{1}{2} I \omega_{\text{rot}}^2$ plus an electric energy contribution E_{dipole}, due to the dipole $\mu_{\text{effective}} = N\mu$ of the SSN, defined in Section 3.1, such that:

$$E_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0} N^2 \mu^2 \frac{1}{z^3}. \tag{4}$$

Thus $E_{\text{rot}} + E_{\text{dipole}} = P_{\text{AME}}\tau_{\text{AME}}$, in agreement with our discussion in Section 2.1, or:

$$\frac{1}{2} I \omega_{\text{rot}}^2 + \frac{1}{4\pi\varepsilon_0} N^2 \mu^2 \frac{1}{z^3} = P_{\text{AME}}\tau_{\text{AME}}. \tag{5}$$

In Equation (4) and Equation (5), z is the significant distance from the SSN within which the SSN's electric field is effective. The constant ε_0 is the dielectric permittivity in vacuum (8.854×10^{-12} Fm^{-1}). In Equation (5), $P_{\text{AME}}\tau_{\text{AME}}$ is the amount of energy conserved in the emission of EM waves from the SSN, in agreement with our discussion in Section 2.1. To be significant, both E_{rot} and E_{dipole} are required to be of the same order of magnitude of $P_{\text{AME}}\tau_{\text{AME}}$. This constraint enables us estimating the magnitude of both P_{AME} and z. We estimate the power P_{AME} as $P_{\text{AME}} \approx E_{\text{rot}} / \tau_{\text{AME}}$. Since $\tau_{\text{AME}} = \frac{1}{\nu_{\text{AME}}}$, and $\omega_{\text{rot}} = \nu_{\text{AME}} = 30\ \text{GHz}$, it turns out that $P_{\text{AME}} = 0.93$ pW. Reasoning in the same way, for PAH spinning dust grains we obtain $P_{\text{AME}} = 0.51$ pW. We estimate z in Section 3.3.

3.3. Significant Distance z from Power P_{AME} Derived through Larmor's Formula and $P\tau$

Using Larmor's formula, the power P_{AME} emitted by the SSNs is $P_{\text{AME}} = 0.22 \times 10^{-27}$ W, as discussed in Section 3.1. In this case, $P_{\text{AME}}\tau_{\text{AME}} = 0.0068 \times 10^{-36}$ J and, using Equation (4) expressed as $z^3 = \frac{1}{4\pi\varepsilon_0} N^2 \mu^2 \left(E_{\text{dipole}}\right)^{-1}$ and $P_{\text{AME}}\tau_{\text{AME}} \approx E_{\text{dipole}}$, we obtain $z = 1.17$ cm. Reasoning in the same way, for PAH spinning dust grains we obtain $z = 1.14$ cm. On the other hand, using $P\tau$ and the law of conservation of energy, for SSNs we estimate $P_{\text{AME}} = 0.93$ pW. Then, $E_{\text{dipole}} \approx P_{\text{AME}}\tau_{\text{AME}} = 0.031 \times 10^{-21}\,\text{J}$, and $z = 70.5$ nm. Reasoning in the same way, for PAH spinning dust grains we obtain $z = 24.88$ nm.

3.4. Time of Photon Production T Obtained with Larmor's Formula and $P\tau$

For SSNs, with $P_{\text{AME}} = 0.22 \times 10^{-27}$ W from Larmor's formula, and 0.93 pW from $P\tau$ and the law of conservation of energy, we estimate the time of production T of photons with energy $h\nu_{\text{AME}} = 19.89 \times 10^{-24}$ J as $T = h\nu_{\text{AME}}/P_{\text{AME}}$. Here, h is Planck's constant (6.63×10^{-34} J·s). It turns out that $T = 1.05$ days with P_{AME} from Larmor's formula, and $T = 21.4$ ps with P_{AME} from $P\tau$. Analogously, for PAH spinning dust grains, we obtain $T = 1.44$ months with P_{AME} from Larmor's formula, and $T = 39.0$ ps with P_{AME} from $P\tau$.

4. Discussion

So far, we estimated the power P_{AME} and the parameters z and T of the AME produced by SSNs and PAH spinning dust grains. Our results are summarized in Table 3 and Table 4. We now discuss the consequences of the different values of P_{AME}, z and T obtained from Larmor's formula and from $P\tau$ and the law of conservation of energy.

4.1. The Power P_{AME}

As reported in Section 3.1 and 3.2, and in Table 3, for SSNs we estimate $P_{\text{AME}} = 0.22 \times 10^{-27}$ W from Larmor's formula, and $P_{\text{AME}} = 0.93$ pW from $P\tau$ and the law of conservation of energy. These two values are 15 orders of magnitude apart. For the PAH spinning dust grains the difference is 18 orders of magnitude. This difference requires some comment. Larmor's formula in Equation (3) is a widely known and well-established result of classical electrodynamics. It depends on two constants: Coulomb's constant $k = 8.99 \times 10^9$ N·m^2·C^{-2}, and the speed of light in vacuum, $c = 3 \times 10^8$ m·s^{-1}. The rotational frequency ω_{rot} and the dipole moment μ are the two variables in Larmor's formula. Both these variables are properties of the spinning dust grain, which are fixed. Indeed, the rotational frequency ω_{rot} is required to be of a magnitude comparable to that of the frequency of the emitted AME. Moreover, the dipole moment μ varies in a small range of values (see e.g. [11]), and its unit in the S.I. system is defined by the conversion

Table 3. Summary of the characteristics of the AME produced by spherical spinning silicate nanoparticles (SSNs).

	P_{AME} (W)	z (m)	T
Larmor	0.22×10^{-27}	1.17×10^{-2}	1.05 days
$P\tau$	0.93×10^{-12}	70.5×10^{-9}	21.4 ps

We list the power P_{AME} of the anomalous microwave emission (AME) emitted by the SSNs. We estimate P_{AME} with Larmor's formula and using $P\tau$ and the law of conservation of energy. We also report the parameters z and T. The parameter z is the significant distance from the SSN's dipole within which the SSN's electric field is effective. The parameter T is the time of production of photons with energy $h\nu_{\mathrm{AME}} = 19.89 \times 10^{-24}$ J.

Table 4. Summary of the characteristics of the AME produced by cylindrical polycyclic aromatic hydrocarbon (PAH) spinning dust grains.

	P_{AME} (W)	z (m)	T
Larmor	5.24×10^{-30}	1.14×10^{-2}	1.44 months
$P\tau$	0.51×10^{-12}	24.88×10^{-9}	39.0 ps

We list the power P_{AME} of the anomalous microwave emission (AME) emitted by the PAH spinning dust grains. We estimate P_{AME} with Larmor's formula and using $P\tau$ and the law of conservation of energy. We also report the parameters z and T. The parameter z is the significant distance from the PAH dipole within which the PAH's electric field is effective. The parameter T is the time of production of photons with energy $h\nu_{\mathrm{AME}}$=19.89×10^{-24} J.

factor $D = 3.33564 \times 10^{-30}$ C·m. These constraints on ω_{rot} and μ impose a low degree of freedom to the value of P_{AME}. On the other hand, Equation (5) for $P\tau$ and the law of conservation of energy depends on the properties of the spinning dust grain: the rotational frequency ω_{rot}, the dipole moment μ, but also the mass and the geometry (size and shape) of the spinning dust grain. Thus, the power P_{AME} can be estimated with a larger degree of freedom (mass, size, shape) with $P\tau$ than with Larmor's formula.

4.2. The Significant Distance z

We recall that z is the significant distance from the spinning dust grain within which the spinning dust grain's electric field is effective. The significant distance z of $z = 1.17$ cm for SSNs obtained, e.g., from P_{AME} calculated through Larmor's formula implies that the electric field around the SSN is effective within a large radius. In this case, in dense molecular clouds populated by SSNs, the electric field of one SSN may couple with the electric fields of a large number of neighboring SSNs, slowing down their frequency of rotation ω_{rot}. This hypothesis is realistic because the typical number density of molecules in a molecular cloud is about 3×10^{8} m^{-3} [35]. Therefore, approximately 2000 molecules may dwell in a sphere of radius 1.17 cm, and thus also hundreds of SSNs (assuming that molecular clouds are predominantly populated by SSNs). In these conditions, the interaction among the electric fields of the SSNs could lead to decreasing their frequency of rotation ω_{rot}, thus interrupting the production of AME at 30 GHz. On the other hand, a value of $z = 70.5$ nm, obtained from P_{AME} estimated through $P\tau$ and the law of conservation of energy, implies that the electric field around the SSN is effective within a short radius. In this case, the coupling of the electric fields of neighboring SSNs even in a dense molecular cloud is negligible, thus minimizing the probability of reducing the frequency of rotation ω_{rot} of the surrounding SSNs and interrupting AME's production. Indeed, the typical number density of molecules in a molecular cloud of about 3×10^8 m^{-3} [35] implies that only 0.440×10^{-12} molecules (notice the negative exponent!) may dwell in a sphere of radius 70.5 nm, minimizing the probability to find SSNs at a close distance, of the order of nm, one from the other. This statement follows the assumption that molecular clouds are predominantly populated by SSNs. Reasoning in the same way, we reach similar conclusions for PAH spinning dust grains.

4.3. The Time of Photon Production T

For SSNs, with $P_{\text{AME}} = 0.22 \times 10^{-27}$ W calculated through Larmor's formula, we estimate a time of photon production of $T = 1.05$ days. Considering that the period of AME photons is $\tau_{\text{AME}} = 33$ ps, $T = 1.05$ days corresponds to 2.7×10^{15} periods, if the photon would be able to keep a coherent phase over all this time. In addition, given that photons move at the speed of 3×10^8 ms^{-1}, a time $T = 1.05$ days signifies that the "forming" photon travels away from its origin's point by about 27×10^9 km. What do 2.7×10^{15} periods mean for a "forming" photon? What does it mean such a delocalization from the photon's origin point? What happens to the "forming" photon if it interferes with other photons or "forming" photons? What would happen to its coherence? On the other hand, with $P_{\text{AME}} =$ 0.93 pW estimated through $P\tau$ and the law of conservation of energy, we obtain a time of photon production $T = 21.4$ ps, which is of the same order of magnitude of $\tau_{\text{AME}} = 33$ ps. The T magnitude of 21.4 ps signifies that the "forming" photon travels away from its origin point by about 6.5 mm, in a hypothetical spherical volume in which approximately 350 other "forming" photons might be found. Thus, in the interior of the hypothetical spherical volume, the photons experience a favorable environment where to develop their characteristic frequency. Similar considerations can be applied to PAH spinning dust grains.

4.4. Comparison with Polycyclic Aromatic Hydrocarbon (PAH) Spinning Dust Grains

In Table 2 we collect the information on N, μ, δ, m, I, ω_{rot}, and the radius a for cylindrical PAH spinning dust grains, which are listed among the possible sources of AME [9] [11]. For our calculations, we assume cylindrical PAH spinning dust grain with radius $a = 0.36$ nm and height $h = 2a$. We then compute P_{AME} with Larmor's formula and with $P\tau$ and the law of conservation of energy. In addition, we estimate the parameters z and T, and report the results in Table 4. The P_{AME} from Larmor's formula is three orders of magnitude smaller for PAH spinning dust grains than it is for SSNs, while the time of photon production T for PAH spinning dust grains reaches 1.44 months. The significant distance z for PAH spinning dust grains is of the same order of magnitude as that for the SSNs. We ascribe the large difference between the values of P_{AME} and T obtained for PAH spinning dust grains and SSNs to the difference in magnitude of their dipole moment μ. On the other hand, the P_{AME} from $P\tau$ and the law of conservation of energy is of the same order of magnitude of that of SSNs, so are also z and T. These findings suggest that small variations of composition, size and shape of the spinning dust grains do not affect significantly the characteristics of the AME generated by spinning dust grains.

5. Conclusion

The purpose of this research is to find the energy conserved in the emission of electromagnetic (EM) waves from matter. We tackle this problem through a computational investigation in which we estimate the power P_{AME} of the anomalous microwave emission (AME) generated by spinning dust grains in the interstellar medium (ISM). We first compute P_{AME} through Larmor's formula obtaining $P_{\text{AME}} \leq 10^{-27}$ W. In this case, the estimated significant distance z of the electric field around the spinning dust grains suggests couplings between the spinning dust grains that might slow down their rotations. Moreover, the obtained time of photon production T suggests that the "forming" photons could suffer severe interferences while forming. We then compute P_{AME} from $P\tau$, *i.e.* the product of the EM wave's power P times its period τ. In this case, we estimate P_{AME}~pW. Such P_{AME} gives rise to a value of z small enough to prevent coupling with the electric fields of neighboring grains. Moreover, with P_{AME}~pW, the magnitude of T is small enough to offer to the "forming" photons a favorable environment where to develop their characteristic frequency. These results are independent of the composition of the spinning dust grains and validate the hypothesis that $P\tau$ is the amount of energy conserved in the interaction between EM waves and matter in emission phenomena. Our results may impact a vast range of light-matter interaction phenomena. We envision consequences, for example, for the quantum technology of single photon detection and production, which needs further investigation.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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