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On the Effects of Driven Element L/D Ratio and Length in VHF-SHF Yagi-Uda Arrays

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Abstract

While the Yagi-Uda array has been studied for decades, one issue appears to have received less attention than perhaps it should, namely, the effects on performance of the array's driven element length and its length-to-diameter ratio. This paper looks at that question. It shows that decreasing the L/D ratio increases impedance bandwidth, but it may shift the IBW band sufficiently far from the design frequency that other parameters such as gain and front-to-back ratio probably are adversely affected. It also shows that array performance is not relatively independent of element diameters. This paper also investigates the effect of lengthening the driven element, which can substantially improve IBW. Several iterations of a 3-element prototype and improved arrays are modeled with the Method of Moments and discussed in detail. A five step design procedure is recommended and applied to a Genetic Algorithm-optimized 3-element Yagi at 146 MHz. This array exhibits excellent performance in terms of gain, front-to-back ratio, and especially impedance bandwidth (nearly 14% for voltage standing wave ratio ≤ 2:1 with two frequencies at which 50 Ω is almost perfectly matched). While the analysis and recommended design steps are applied to cylindrical array elements, which commonly are aluminum tubing for stand-alone VHF-SHF Yagis, they can be applied to other element geometries as well using equivalent cylindrical radii, for example, Printed Circuit Board traces for planar arrays. One consequence of lengthening the driven element while reducing its L/D ratio is that some reactance is introduced at the array feedpoint which must be tuned out, and two approaches for doing so are suggested.

Keywords

Yagi, Array, Driven Element, Impedance Bandwidth

1. Introduction

The Yagi-Uda array ("Yagi") is a truly remarkable antenna. It has provided robust performance across a wide range of parameters ever since its introduction nearly a century ago [1] [2], and its performance has been studied for decades. Yet one design issue appears to remain unresolved: the optimum diameter and length for the array's Driven Element (DE). In particular, how the DE geometry affects a Yagi's impedance bandwidth (IBW) and other performance measures such as gain (G), front-to-back ratio (FBR), and half-power beamwidth (HPBW) seems not to be well settled. Gain and FBR refer to the maximum *E-plane* values. IBW is defined here as the range of frequencies for which the voltage standing wave ratio (VSWR) is less than or equal to 2:1, with caveats as discussed later on. Which DE diameter ostensibly is better, smaller or larger, depends to a large degree on the source of information. For example, one authoritative reference on antenna design states that a Yagi's element diameters do not significantly affect electrical characteristics [3@p.225]. According to this highly respected source, the array's IBW, G, FBR, and HPBW are *not* materially affected. Yet another well-respected source states that using elements with fairly large diameters reduces impedance change with frequency thereby improving IBW [4@p.11-15].

Whether or not "thin" or "fat" DE's are better appears to remain an open question, and likewise for its length. It is not clear that lengthening or shortening the DE can improve a Yagi's performance, yet this issue merits investigation.. This paper looks at these questions using a three-element Yagi as an example. As it turns out, both assertions about DE diameters above are correct, up to a point. The analysis shows that for the 3-element array a lower driven element length-to-diameter ratio (L/D ratio) and a somewhat longer length are generally better than a large L/D ratio with no change in length, but there are tradeoffs. Re-sizing DE diameter and length can improve IBW and with simple capacitor matching provide an essentially perfect match to the feed system, and this modification has no appreciable effect on G, FBR, or beamwidth, which can be good or bad, depending on how important those parameters are in a particular application. The results reported here suggest that the conclusions about DE L/D ratio and length should hold for a Yagi of any length, and, while the idea of using a "fat" DE does not depend on operating frequency, doing so obviously is most attractive at VHF/UHF/SHF (ITU Bands 8, 9, 10) where the array elements are smaller than at lower frequencies.

This paper is organized as follows: Section 1 discusses what issues are being addressed, DE diameter and length, and why. Section 2 provides a brief literature review. Section 3 discusses Yagi array structure and provides the analytical framework for this research. Section 4 discusses the research results. Section 4.1 explains the analytical methods while Sections 4.2 - 4.6 discuss the results for Yagis versions Ø through 4, respectively. In these sections data are displayed in tabular form and discussed in detail. In Section 4.7 the NEC-4 data are displayed graphically with two plots for each of the Yagi versions Ø through 4, inclusive. Section

4.8 compares the Proto and Improved Yagis and suggests a 5-step design procedure for improving IBW. Section 5 applies the results and methodology developed in this paper to the design of an improved Genetic Algorithm-optimized short boom 3-element array for the 2-meter band (146 MHz). Section 6 discusses the conclusions reached and the implications of this research

2. Literature Review

Because the Yagi-Uda array has existed for nearly a century, the engineering literature studying its characteristics, design considerations, different configurations and applications is voluminous. There are literally hundreds, perhaps thousands, of papers and books dealing with just about every aspect of the Yagi-Uda array, except, evidently, as pointed out in Section 1, the driven element length and its length-to-diameter ratio. The Yagi literature is so extensive that at any attempt at a comprehensive in-depth review is both beyond the scope of this paper and unnecessary because the existing literature is silent on the topic of this paper. Nonetheless, there are some key references on Yagi-Uda arrays that are generally relevant. The book by Stutzman and Thiele [3] is not only a classic on antenna theory and design, it provides the very basis for the research reported here, a well-designed 3-element array with published performance data. But it asserts array characteristics that conflict with statements made in [4]. Ramo *et al.* [5] suggest that a Yagi's drive point impedance should be increased because it usually is quite low and consequently more difficult to match to the feed system characteristic impedance. One of the earliest comprehensive works on Yagi design is [6], which was long considered the definitive source on Yagi arrays because it includes extensive experimental data. Additional analysis of Yagi configurations and design approaches appear in [7] [8] and [9], yet none of these books examines the specific issue of driven element length or length-to-diameter ratio.

3. Methodology

Typical Yagi geometry is shown in **Figure 1**. The array comprises parallel dipole elements spaced along its axis ("boom"). The radio frequency (RF) source excites the driven element (DE) which usually is a center-fed dipole (CFD) as shown (length L, diameter D). Other DE configurations are sometimes used, for example, a folded dipole, but they are not considered here. DE is flanked by several parasitic elements, on one side a generally longer element, R, that acts as a reflector, and on the other a group of generally shorter elements that act as directors (D1…DN). While this paper discusses Yagis fabricated using cylindrical elements, the analysis is equally applicable to arrays made from planar elements, for example PC board (PCB) traces, by using equivalent radii. For example, the effective diameter of the cylindrical conductor corresponding to a PC board trace of width w is simply $0.5w$ ([7], §9.4.5). A table of effective radii for a variety of conductor shapes is available in that reference.

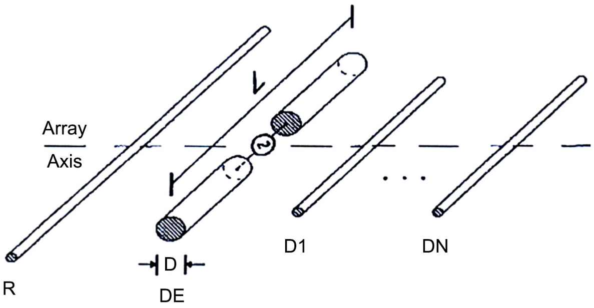


Figure 1. Yagi array structure.

In most Yagi designs all elements are the same diameter, and they are electrically "thin". *Thin* elements have their length-to-diameter ratio $L/D \gg 1$, whereas *fat* elements have $L/D \sim 1$. Same length thin and fat elements have different current distributions. In free space the current along an isolated very thin element is nearly sinusoidal, but this approximation becomes progressively worse with increasing element diameter, that is, as the element becomes fatter, and how good the approximation is depends a great deal on the L/D ratio.

An element's self-impedance is determined by its free-space current distribution, which, as pointed out, varies considerably with the L/D ratio. A Yagi's *input impedance*, Z_{in}, which is the complex sum of the radiation resistance and the input reactance, is determined by DE's self-impedance and its mutual impedance with every other element in the array, but the DE influence is dominant. The objective here is to improve the Yagi's performance, especially with respect to IBW, by adjusting the DE self-impedance to improve the match to the feed system characteristic impedance, Z_0, which typically is 50 Ω purely resistive.

The conjecture that changing DE's length and L/D ratio can significantly alter a Yagi's input impedance rests on the following observations:

1) In well-designed Yagis, the radiation resistance is usually lower than 50 Ω, often by quite a bit, but not always.

2) In general, VSWR increases more quickly with a Yagi's input (feedpoint) reactance when its radiation resistance is less than Z_0.

3) The radiation resistance of a center-fed dipole (CFD) passes through a maximum as its L/D ratio decreases.

Low input resistance: For the first observation, that Yagis exhibit low input resistance is apparent from the plethora of published Yagi design and measurement data (e.g. [3]-[9]). The two other observations are discussed in detail below.

VSWR variation with antenna input reactance: For an antenna with input impedance $Z_{in} = R_{in} + jX_{in}$, $j = \sqrt{-1}$, the *normalized* impedance is $Z_N = Z_{in}/Z_0 = R_N + jX_N$ where $R_N = R_{in}/Z_0$, $X_N = X_{in}/Z_0$ and Z_0 is assumed to be real (purely resistive). R_{in} is the radiation resistance, and in most antenna systems the feed system characteristic impedance, Z_0, is $50 + j0$ Ω.

The reflection coefficient, ρ, and VSWR are given by ([9], §1-10)

$$\rho = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_N - 1}{Z_N + 1}$$

$$\text{VSWR} = \frac{1 + |\rho|}{1 - |\rho|}$$

$$\Rightarrow \text{VSWR} = \frac{\sqrt{|R_N + 1|^2 + X_N^2} + \sqrt{|R_N - 1|^2 + X_N^2}}{\sqrt{|R_N + 1|^2 + X_N^2} - \sqrt{|R_N - 1|^2 + X_N^2}}$$

Figure 2 plots VSWR//50 Ω vs. X_N parametric in the normalized resistance R_N ("//" means "relative to"). In a 50 Ω system an actual feedpoint resistance of 25 Ω, for example, corresponds to $R_N = 0.5$ (top curve in the figure). Following that curve, an input reactance of 100 Ω ($X_N = 2$) corresponds to a VSWR of 10.4:1. Note, importantly, that the VSWR formula shows that only the magnitude of the reactance is important, not its sign (inductive or capacitive).

It is evident from **Figure 2** that VSWR is more sensitive to increases in X_N when R_N is low. For example, with $R_N = 0.5$, increasing X_N from 1 to 2.5 causes VSWR to increase from 4.3 to 14.9 (10.6 points), but when $R_N = 2$ the same change increases VSWR by only 2.8 points, from 2.6 to 5.4. Because of their generally low input resistance, Yagis tend to exhibit a similar VSWR sensitivity. It consequently is reasonable to expect that a DE with higher radiation resistance will improve the array's performance by reducing its VSWR sensitivity to the input reactance. Therefore the initial design objective in selecting a better DE is *increasing its radiation resistance* when $R_{in} < Z_0$. For arrays with $R_{in} > Z_0$ the objective is to *reduce* R_{in} to a value as close as possible to Z_0. The analysis in this paper focuses on increasing R_{in} because for most Yagis $R_{in} < Z_0$. However, the techniques and CFD data presented here, with obvious modifications, can be used just as effectively when $R_{in} > Z_0$.

<u>How CFD radiation resistance changes with L/D ratio</u>: The self-impedance of an isolated free-space CFD is plotted as a function of diameter in **Figure 3**, parametric in its length (dimensions in wavelengths, λ). The radiation resistance, R_{in}, appears in **Figure 3(a)** and the reactance, X_{in}, in **Figure 3(b)**. The different curves are for dipole lengths as annotated. CFD diameter varies from zero ($L/D = \infty$) to 0.1λ with element lengths $L = 0.450\lambda$, 0.453λ, 0.475λ and 0.500λ. For all four lengths the resistance reaches a maximum in the vicinity of ~ 0.065λ diameter. The maximum resistance for a fat dipole is considerably higher than it is for a thin one. For example, for the half-wave CFD ($L = 0.500\lambda$) the input resistance is less than 80 Ω for a very thin element ($D \sim 0.001\lambda$), but it is about 113 Ω for a fat one ($D \sim 0.065\lambda$). This characteristic is important in trying to increase the Yagi's feedpoint resistance. At a given length a fat element exhibits a higher radiation resistance than a thin one, often by quite a bit.

Turning to **Figure 3(b)**, the half-wave CFD is inductive ($X_{in} > 0$) for all diameters up to about 0.085λ where it passes through resonance ($X_{in} = 0$) to become capacitive. The other elements all start out capacitive, cross through resonance to become inductive, then cross a second resonance point to become

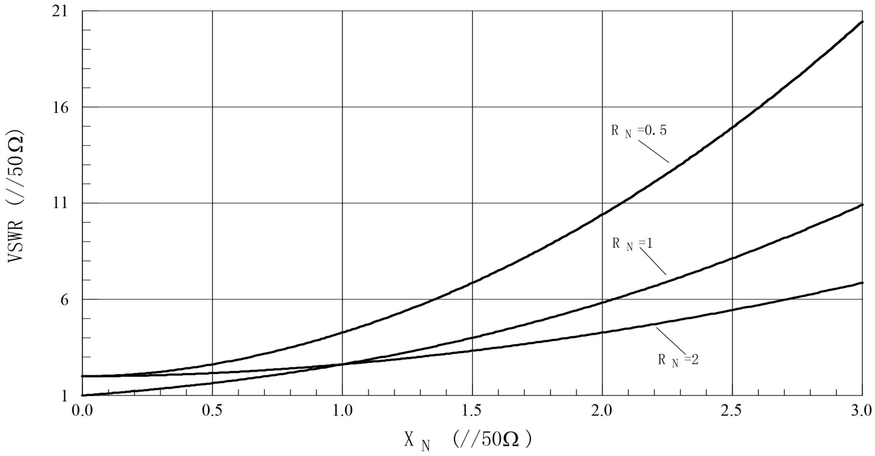
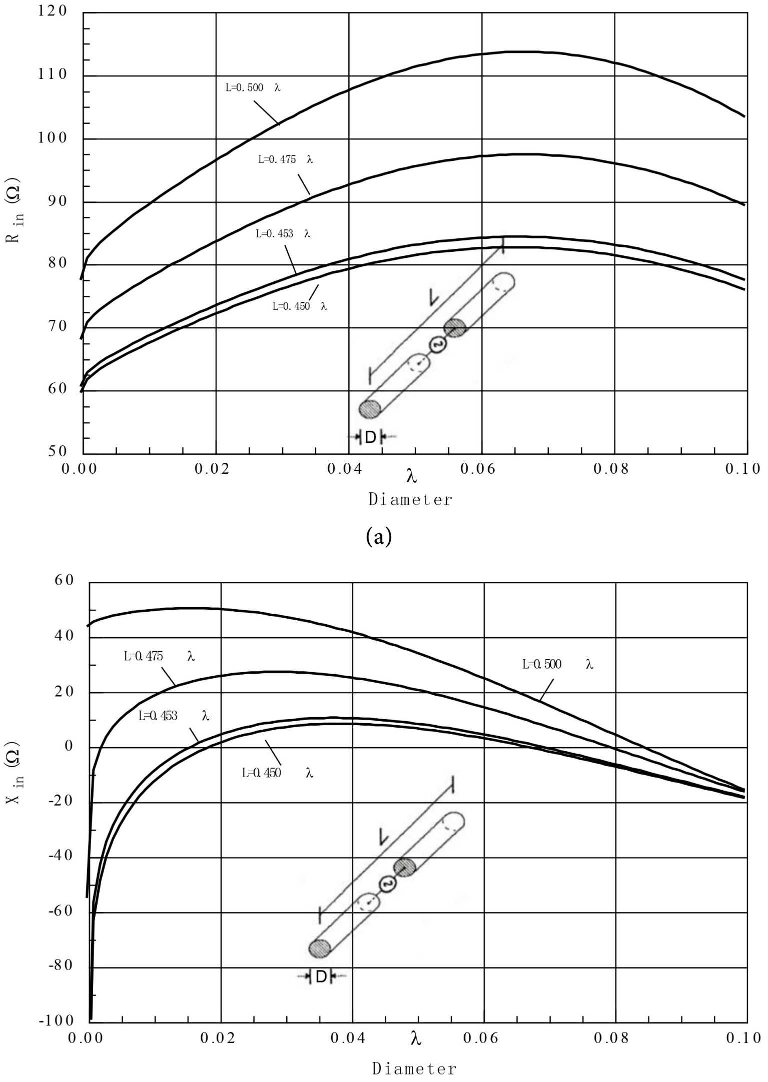


Figure 2. VSWR vs. Normalized Reactance. [parametric in R_N].



(a)

(b)

Figure 3. (a) CFD Radiation Resistance. [parametric in dipole length]. (b) CFD Input Reactance. [parametric in dipole length].

capacitive again. It is significant that near the diameter corresponding to maximum R_{in}, ~0.065λ, both of the shorter CFD's, 0.453λ and 0.450λ, are nearly resonant, but their radiation resistances are much lower than those of the longer non-resonant (inductive) elements with lengths 0.475λ and 0.500λ. It is evident that how the reactance changes with diameter is important in determining the DE's resonant frequency. While resonating DE is not the primary objective, which instead is increasing R_{in}, as will be seen it is an important consideration.

4. Research Results

4.1. Analysis

This paper employs a well-designed 3-element Yagi array as the starting point, referred to as Version Ø, which is modified by "fattening" DE (decreasing L/D ratio) and by lengthening it. The effects of doing so are then investigated. There are four modified arrays (Versions 1 though 4, inclusive, referred to as "improved" arrays). The Version Ø "prototype" or "proto" array is described in Table 5-4 in [3], the first table entry. It was chosen as the prototype because, in addition to detailed array dimensions, the table includes performance data which can be compared directly to the data calculated in this study. In [3] the performance of Version Ø was computed using a Method of Moments (MoM) code described in Chapter 7, a program which in some respects is perhaps a bit long in the tooth today because Stutzman and Thiele's seminal textbook was first published in 1981. By contrast, all modeling described in this paper was done with the Numerical Electromagnetics Code Version 4.2 [10], which is a state-of-the-art MoM code that has evolved and improved over several decades' time and that is widely considered to be the "gold standard" for MoM modeling of wire antennas. There is a more recent version, NEC-5 [11] [12], but it does not provide any advantage for modeling Yagis.

The NEC-4 computed array data are presented in two ways: 1) a table summarizing results, and 2) plots of the array's Gain, FBR, Z_{in} and VSWR//50 Ω. Discussion of the results, version by version, and the corresponding data tables appear in the sections that follow, 4.2 through 4.6 for Versions Ø through 4, respectively. All of the graphical data are presented in Section 4.7. The proto array and each of the improved arrays has associated with it two plots, the first, Gain and FBR, and the second, Z_{in} and VSWR. Note that the data in [3] were computed *only* at the Yagi design frequency, F_0, whereas the NEC-4 data for this study were calculated for the relative frequency range $F_L \le F/F_0 \le 1.10$ where the lower relative frequency F_L is 0.8 or 0.9 depending on the desired plot resolution. Thus, the NEC-4 data cover a wide band of frequencies around the design frequency, whereas the data in [3] are only at one frequency.

In order to compare these data, each table comprises two sections, left and right, to reflect that dichotomy. The left side of the table shows the NEC-4 computed *best values* of key parameters in the calculated frequency band and the relative frequency, F/F_0, at which they occur. The right side of the table compares the prototype data from [3] directly to NEC-4's computed performance *at the design frequency* F_0. The NEC data are on the left of the separator * / * and [3]'s data on the right, "fmt" means "format," and "n/a" "not applicable." Note that the proto VSWR was computed from the Z_{in} data listed in [3] not from NEC's data, but the other parameters in the right table section were computed by NEC-4.

Also included in each table is the NEC-4 AGT (Average Gain Test), which measures the fidelity of the NEC-4 model with respect to computing Z_{in}. For simple structures in free space, such as those considered here, its value should be very close to 1, which indicates an accurate impedance calculation. For large, complex structures an AGT within about ten percent [0.9 - 1.1] corresponds to an acceptable level of accuracy for the impedance calculation. AGT thus reflects how accurately NEC has modeled an antenna's source current distribution. The range of AGT values over the calculated frequency band is shown in the table's left section, whereas AGT at the design frequency F_0 is on the right.

NEC-4 models solid cylindrical wires, not the hollow metal tubes usually used to fabricate Yagis. However, NEC does take into account skin depth ([13], ch. 4) which accommodates thin-wall tubes. In the case of aluminum with a typical conductivity of 2.5×10^7 S/m the skin depth at 50 MHz is ≈0.025 mm and even less at higher frequencies, which is far less than typical wall thickness.

4.2. Yagi Version 0, Prototype Array

The prototype antenna comprises a 0.479λ reflector, 0.453λ CFD DE, and a 0.451λ director (λ is the wavelength). All elements are 0.005λ diameter, so that $90.2 \le L/D \le 95.8$. DE is quite thin with $L/D = 90.6$. The three elements are uniformly spaced 0.25λ along the boom. Results for this array appear in **Table 1**.

Table 1. Yagi Version 0, Prototype Array.

Ver. 0, PROTOTYPE Yagi, Parameter's *Best* Value (NEC-4) @ Relative Frequency F/F_0			Ver. 0, Proto NEC-4 Data compared to [3] Data @ Design Freq, $F/F_0 = 1$: fmt NEC/[3]
Parameter	--	F/F_0	
Gain	9.5 dBi	0.9892	9.34/9.4 dBi
FBR	6.34 dB	0.9784	5.53/5.6 dB
VSWR	2.44	0.9916	2.58/2.49
Z_{in} @ VSWR	$20.7 + j5\ \Omega$	0.9916	$22.4 + j17.9\ \Omega/22.3 + j15\ \Omega$
IBW	0%		n/a
HPBW	*E* plane		54˚/66˚
	H plane		70˚/84˚
AGT	0.98496 - 0.98685		0.98505

From the table's left side, the prototype's maximum gain of 9.5 dBi occurs at $0.9892F_0$, about 1% below the design frequency. Similarly, the maximum FBR of 6.34 dB occurs at $0.9784F_0$. The minimum VSWR of 2.44 occurs at $0.9916F_0$, not at F_0 itself. This behavior is quite common in Yagis, the best performance for some parameter does not occur at the design frequency itself, F_0, but often slightly away from it.

Turning to the right side of the table, NEC-4 calculates the array's maximum gain at F_0 to be 9.34 dBi while [3] lists it as 9.4 dBi. Generally the agreement between NEC-4 and [3] is quite good except for the Half Power Beam Widths (HPBW) where NEC-4's *E-plane* and *H-plane* values are narrower by 12˚ and 14˚, respectively (the *E-plane* contains the array's boom/elements, and the *H-plane* is perpendicular). These differences are likely due to improvements to NEC-4's MoM algorithms and to coding improvements that enhance the accuracy of NEC-4's calculations compared to the MoM algorithm that was used in the early 1980s.

Prototype Array Gain & FBR: **Figure 5** in Section 4.7.1 plots the prototype array's performance as a function of relative frequency from $100(1-F_L)\%$ below the design frequency to 10% above. **Figure 5(a)** plots *E-plane* maximum gain and FBR. The curves are similar in shape with maxima occurring at the relative frequencies listed in **Table 1**. Both maxima occur at a value of F/F_0 somewhat below 1.00, not at F_0 itself.

Proto Array Z_{in} & VSWR: The prototype's Z_{in} and VSWR//50 Ω are plotted in **Figure 5(b)**. X_{in} increases monotonically from ~100 Ω capacitive ($X_{in} < 0$), passes through resonance near $F/F_0 \approx 0.985$, then increases to around +100 Ω inductive near $F/F_0 \approx 1.055$ after which it begins to plateau at around +115 Ω. The variation in R_{in} is quite different. It is very flat at around 25 Ω up to $F/F_0 \approx 1.01$ after which it increases monotonically to about 130 Ω at $F/F_0 \approx 1.10$. As to VSWR, its minimum is 2.44 at $F/F_0 = 0.9916$. It decreases quickly to this point and increases slowly thereafter, reaching a plateau around 4.8:1 above $F/F_0 \approx 1.05$. Without a matching network or impedance transformer this Yagi simply has poor VSWR performance. Applying the 2:1 standard results in IBW of *0%* because *nowhere* does VSWR fall to or below 2. The question is whether or not this Yagi's performance can be improved by increasing DE diameter (lower L/D ratio) and tweaking its length so as to increase its radiation resistance. While increasing DE self-impedance is not a new idea, just how to do it is not well-settled in the literature. Some sources suggest using a folded dipole DE because Z_{22} (the DE self-impedance) is then increased by a factor of four [5@p.642]. Of course, even doing so does not guaranty a good match to Z_0, even though it may be better, but the effect on IBW and other performance parameters is unclear. What the author believes to be new in this work is using *both* the driven element diameter *and* its length to achieve a higher radiation resistance based on the analysis and discussion in Section 3 or, in cases where $R_{in} > Z_0$, reducing it.

4.3. Yagi Version 1, Improved Array/"Fat" DE

Version 1 of the improved Yagi is created by using a fatter driven element in the proto array, and that is the *only change* at this point. All other dimensions remain the same. The DE diameter is increased to 0.065λ, yielding $L/D = 6.97$ compared to the prototype's value of 90.6. If this antenna were built to operate at 299.8 MHz where the wavelength is 1 meter, the new DE diameter is 6.5 cm (2.56 inches). For practical purposes a 2.5 inch diameter cylinder suffices, and that happens to be a standard metal tube size (usually aluminum in UHF Yagis). If a 0.065λ diameter tube is too large to be practical, than a "cage dipole" structure might suffice as an alternative.

The improved array's performance is summarized in **Table 2** and plotted in **Figure 6**. IBW is much better with the fatter DE, but it is in a band whose location relative to F_0 may be problematic because it is fairly far away from maximum gain and FBR, which cannot be corrected simply by scaling F_0 to a new design frequency because the separation between the IBW band and gain and FBR will remain the same.

Improved Yagi Ver. 1—Gain & FBR: Careful examination of the gain and FBR plots in **Figure 5(a)** (proto) and **Figure 6(a)** (improved) in Sections 4.7.1 and 4.7.2 reveals that the curves are nearly the same. There is no appreciable change in gain or FBR as a result of increasing the DE diameter from a quite thin 0.005λ to a much fatter 0.065λ. Comparing **Table 1** and **Table 2**, HPBW are essentially the same for the Proto and Version 1 arrays. At the design frequency F_0, the improved array's gain is 9.41 dBi and its FBR 5.42 dB compared to the prototype's values of 9.4 dBi and 5.6 dB, respectively. Although at this point it is speculative because an exhaustive investigation has not been done, it does seem reasonable to conclude that *substituting a "fat" DE for a "thin" one in a Yagi antenna will not materially affect the values of either its maximum gain or its front-to-back ratio or their locations in relative frequency, or the half-power beam widths.* This conjecture is further supported by subsequent results.

Table 2. Yagi Version 1, Improved Array, "Fat" DE.

IMPROVED Yagi Ver. 1, Parameter's *Best* NEC-4 Value @ Relative Frequency F/F_0			IMPROVED Ver. 1, NEC-4 Data compared to [3] Data @ Design Freq, $F/F_0 = 1$: fmt NEC/[3]
Parameter	--	F/F_0	
Gain	9.57 dBi	0.9895	9.41/9.4 dBi
FBR	6.24 dB	0.9775	5.42/5.6 dB
VSWR	1.66	0.9170	5.14/2.49
Z_{in} @ VSWR	$30.3 - j2\ \Omega$	0.9170	$40.2 + j81.2\ \Omega/22.3 + j15\ \Omega$
IBW	0.8745 - 0.9465 (7.2%)		n/a
HPBW	*E*-plane		53°/66°
	H-plane		71°/84°
AGT	0.9966 - 1.00780		0.99811

Improved Yagi Ver 1.—Z_{in} & VSWR: An entirely different picture emerges when the proto and improved arrays are compared for Z_{in} and VSWR (**Figure 5(b)** to **Figure 6(b)**). The improved Yagi's feedpoint reactance X_{in} increases monotonically to a maximum of about +140 Ω near $F/F_0 \approx 1.05$. DE is inductive throughout the range $0.92 \le F/F_0 \le 1$ except for a region below $\approx 0.92 F/F_0$ where it is moderately capacitive with $\text{Min}(X_{in}) \approx -50\ \Omega$. At F_0 the input reactance is $X_{in} = +j81.2\ \Omega$, which is important to know because it determines how this Yagi might be fed.

The radiation resistance R_{in} is fairly flat at around 30 Ω up to $F/F_0 \approx 0.975$ where it begins to increase dramatically to about 240 Ω at $F/F_0 \approx 1.10$. At F_0 its value is 40.2 Ω, which is close to the target value of 50 Ω, but not quite there. If R_{in} were precisely 50 Ω then a perfect match to a 50 Ω feed could be obtained by simply tuning out any reactance.

Also plotted in **Figure 6(b)** is the improved Yagi's VSWR. It exhibits less variability than the prototype array's, and, importantly, over the modeled range $0.80 \le F/F_0 \le 1.10$ it falls below 2:1 for $F/F_0 \sim [0.8745 - 0.9465]$. By contrast, the prototype array's VSWR never falls below 2:1 so that its IBW is 0%. Even at this point the improved array's IBW is 7.2% with no effort to create an even better match to the 50 Ω feed. This result is a direct consequence of substituting a "fat" DE for the "thin" one.

4.4. Yagi Version 2, Improved Array/"Fat" DE, Cap Loading

Version 2 of the improved Yagi is created by tuning out the feedpoint inductance in Version 1. At F_0 in Version 1 $Z_{in} = 40.2 + j81.2\ \Omega$, so the array may be further modified by adding negative (capacitive) reactance that offsets the +81.22 Ω inductive reactance. This is accomplished by loading the feed point with a series capacitor of 6.539 pF, which, in the NEC-4 model, is added at the DE feed segment using a NEC "LD" card. In practice, one half of the capacitance must be added in series to each of the DE's arms in order to maintain the array's electrical balance.

The effects of adding capacitive loading is shown in **Table 3** and in **Figure 7** (Section 4.7.3) The added capacitive reactance balances out DE's inductive reactance resulting from its being too short (recall that up to now DE's length has been fixed by the proto array dimensions in [3]).

Comparing the gain and FBR figures in **Tables 1-3** it is apparent that using the fat DE with or without capacitive loading does not materially affect array gain or FBR or HPBW. Only IBW is affected. By tuning out DE's inductance the VSWR at F_0 is reduced from 5.13:1 to 1.25, which is a very good, but not perfect, match to the feed system. The VSWR is so much lower because DE is essentially resonant at F_0 with a fairly high radiation resistance ($Z_{in} = 40.2 + j0.008\ \Omega$). In Version 1 without cap loading the IBW is 7.24%, but the IBW band was shifted more than 9% below F_0 with minimum VSWR of 1.66:1. In Version 2 with capacitor loading IBW is narrower at 3.3%, but it is centered essentially at F_0 with a minimum VSWR of 1.245:1.

Table 3. Yagi Version 2, Improved Array, "Fat" DE, Cap Loading (6.539 pF).

IMPROVED Yagi Ver. 2, with CAP LOADING, Parameter's *Best* NEC-4 Value @ Relative Frequency F/F_0			IMPROVED Array Ver. 2, with CAP LOADING, NEC-4 Data compared to [3] Data @ Design Freq, $F/F_0 = 1$: fmt NEC/[3]
Parameter	--	F/F_0	
Gain	9.57 dBi	0.9895	9.41/9.4 dBi
FBR	6.24 dB	0.9775	5.42/5.6 dB
VSWR	1.22	1.0020	1.25/2.49
Z_{in} @ VSWR	$41.8 + j3.7\ \Omega$	1.0020	$40.2 + j0.008\ \Omega/22.3 + j15\ \Omega$
IBW	0.9860 - 1.0190 (3.3%)		n/a
HPBW	*E*-plane		53˚/66˚
	H-plane		71˚/84˚
AGT	0.99766 - 1.00780		0.99811

Thus, capacitor loading results in a significantly better VSWR but a narrower IBW without materially affecting G, FBR or HPBW. Of course, simple as it is, the capacitors added to each of DE's arms do constitute a basic matching network, and the network's frequency response affects the impedance bandwidth. In this case the cap loading causes X_{in} to vary more quickly than without it which causes IBW to shrink. Of course, no "matching" at all would be ideal, but it seems that adding two capacitors to the DE arms is an acceptable compromise because of its simplicity and, as discussed in Section 5.1, how easily it can be implemented.

4.5. Yagi Version 3, Improved Array/"Fat," Stretched DE

The prototype array's driven element is a 0.005λ diameter, 0.453λ long CFD. So far the improved arrays Versions 1 and 2 have increased its diameter to 0.065λ while maintaining its length. As a result, Version 1 achieves an IBW of 7.2% compared to the proto's value of 0%. However, Version 1's IBW is in a band well below the design frequency F_0, and at F_0 the VSWR is quite high at 5.13:1. This issue was addressed in Version 2 by tuning out DE's inductive reactance at F_0. That results in lowering VSWR to 1.25:1 in a band essentially centered on F_0, but IBW is narrower at 3.2% because the capacitor causes X_{in} to vary more quickly than without it

While improved Version 2 has much better VSWR than Version 1, it may be possible to do better still if R_{in} can be raised from 40.2 Ω to a value closer to 50 Ω. This can be done by slightly lengthening the driven element ("stretching" it) to take advantage of how a CFD's radiation resistance varies with its length (see Figure 3(a) and Figure 4). The reason for believing a longer DE will increase the radiation resistance is apparent from Figure 4 that plots Z_{in} for a free space CFD as a function length for two dipole diameters. The 0.065λ diameter is what has been used so far, and the 0.025λ diameter CFD will be used in another array design discussed in Section 5.

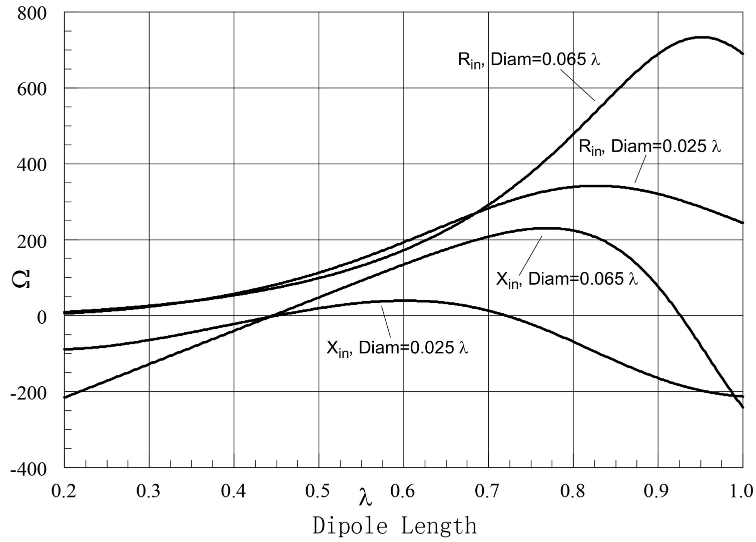


Figure 4. Free Space CFD Z_{in} vs. Length.

The 0.065λ element reaches a maximum above 700 Ω For at $L \approx 0.95\lambda$ whereas the 0.025λ element peaks near $L \approx 0.84\lambda$ at approximately 350 Ω. For both diameter dipoles R_{in} is 100 Ω at about a half wavelength. This value of radiation resistance can be used to advantage to better match the Yagi's input impedance to the feed system because it is relatively high. With the target value of 50 Ω, it is reasonable to expect that a free space CFD's R_{in} around 100 Ω can be substantially lowered because the effect of the array's parasitic elements is to do precisely that. They act as resistances in parallel with the free space CFD's R_{in} thereby lowering it. If R_{in} is increased sufficiently by lengthening DE and then lowered enough by the array's parasitic effect, it may be possible to obtain a much better match. A simple, quick approach for determining what DE's new length should be is to use NEC-4. After a few runs it was determined that increasing the driven element length from 0.453λ to 0.477λ increases R_{in} to 49.5 Ω which potentially provides an almost perfect match to Z_0.

Table 4 and **Figure 8** show how this Yagi performs with the stretched DE. As in previous cases, the gain and FBR are essentially unaffected. And, as before, IBW is large (11.2%), but its band is shifted well below the design frequency, in this case more than 10% below F_0 with $F_{\mathrm{minVSWR}} = 0.8945$ and Min(VSWR) = 1.37. At the design frequency VSWR is quite high at 6.537 because even though R_{in} is almost exactly 50 Ω (49.5), Z_{in} is highly inductive at $+j107.7$ Ω.

4.6. Yagi Version 4, Improved Array/"Fat," Stretched DE, Cap Loading

Without capacitive loading, the lengthened DE in Version 3 has $Z_{in} = 49.5 + j107.7$ Ω resulting in a high VSWR at F_0. Just as in Version 2, adding series capacitance at the DE feedpoint can be used to tune out the inductive reactance and resonate DE. Inserting 4.929pF achieves that end to give $Z_{in} = 49.5 + j0.035$ Ω with VSWR = 1.01, essentially a perfect match to the 50 Ω feed.

Table 4. Yagi Version 3, Improved Array, Stretched, "Fat" DE.

IMPROVED Yagi with LONG DE, Parameter's *Best* NEC-4 Value @ Relative Frequency F/F_0			IMPROVED Array with "STRETCHED" DE, NEC-4 Data compared to [3] Data @ Design Freq, $F/F_0 = 1$: fmt NEC/[3]
Parameter	--	F/F_0	
Gain	9.57 dBi	0.9900	9.44/9.4 dBi
FBR	6.17 dB	0.9780	5.39/5.6 dB
VSWR	1.36	0.8945	6.54/2.49
Z_{in} @ VSWR	$36.9 - j1.2\ \Omega$	0.8945	$49.5 + j107.7\ \Omega/22.3 + j15\ \Omega$
IBW	0.8255 - 0.9375 (11.2%)		n/a
HPBW	*E*-plane		53˚/66˚
	H-plane		71˚/84˚
AGT	0.99706 - 1.00766		0.99775

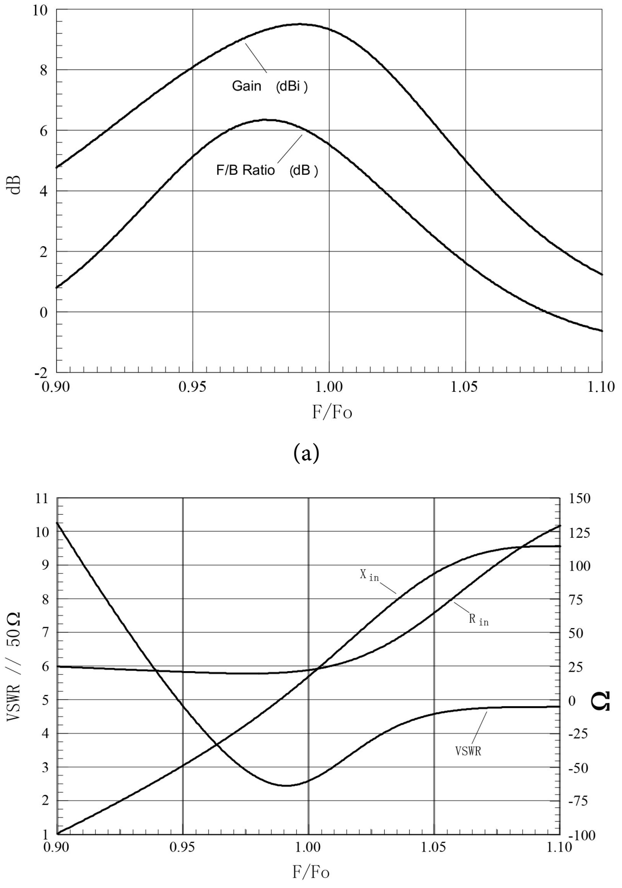
The performance of Yagi Version 4 is shown in **Table 5** and **Figure 9** (Section 4.7.5). IBW now is centered on F_0 with maximum gain and FBR within about 1% to 2% of F_0. However, as seen previously, an undesirable effect of adding capacitance is to narrow IBW because of how X_{in} varies with frequency. In this case it was reduced from 11.2% to 3.1%. Nevertheless, these results show that two simple modifications to a Yagi's driven element when $R_{in} < Z_0$, viz., making it "fatter" and longer, can dramatically improve performance by centering the IBW band at F_0 while maintaining G, FBR and HPBW and by eliminating the need for an external matching network.

Table 5. Yagi Version 4, Improved Array, Long, "Fat" DE, Cap Loaded, 4.929pF.

IMPROVED Yagi with LONG DE, CAP LOADING, Parameter's *Best* NEC-4 Value @ Relative Frequency F/F_0			IMPROVED Array with CAP LOADED LONG DE, NEC-4 Data compared to [3] Data @ Design Freq, $F/F_0 = 1$: fmt NEC/[3]
Parameter	--	F/F_0	
Gain	9.57 dBi	0.9896	9.44/9.4 dBi
FBR	6.17 dB	0.9772	5.39/5.6 dB
VSWR	1.01	1.0000	1.01/2.49
Z_{in} @ VSWR	$49.5 + j0.035\ \Omega$	1.0000	$49.5 + j0.035/22.3 + j15\ \Omega$
IBW	0.9848 - 1.0156 (3.08%)		n/a
HPBW	*E*-plane		53˚/66˚
	H-plane		71˚/84˚
AGT	0.99706 - 1.00766		0.99775

4.7. Plots: Gain, FBR, Z_{in}, and VSWR, Yagi Versions Ø to 4

4.7.1. Prototype Array Version Ø (Figure 5)

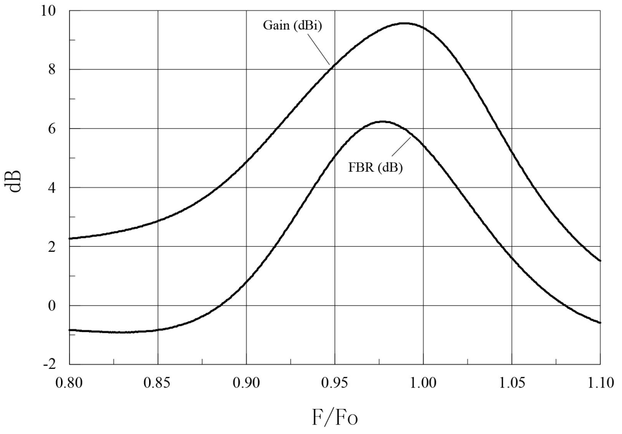


(a)

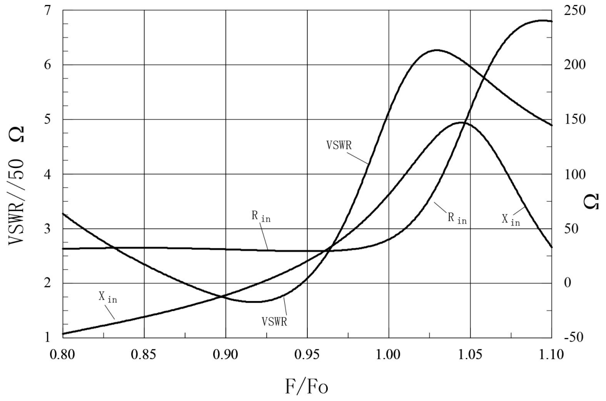
(b)

Figure 5. (a) Ver Ø, Proto 3-El Yagi, G & FBR. [R-0.479λ, DE-0.453λ, D1-0.451λ, S-0.25λ, diam all elmts 0.005λ]. (b) Ver Ø, Proto 3-El Yagi, Z_{in} & VSWR. [R-0.479λ, DE-0.453λ, D1-0.451λ, S-0.25λ, diam all elmts 0.005λ].

4.7.2. Improved Array Version 1 (Figure 6)



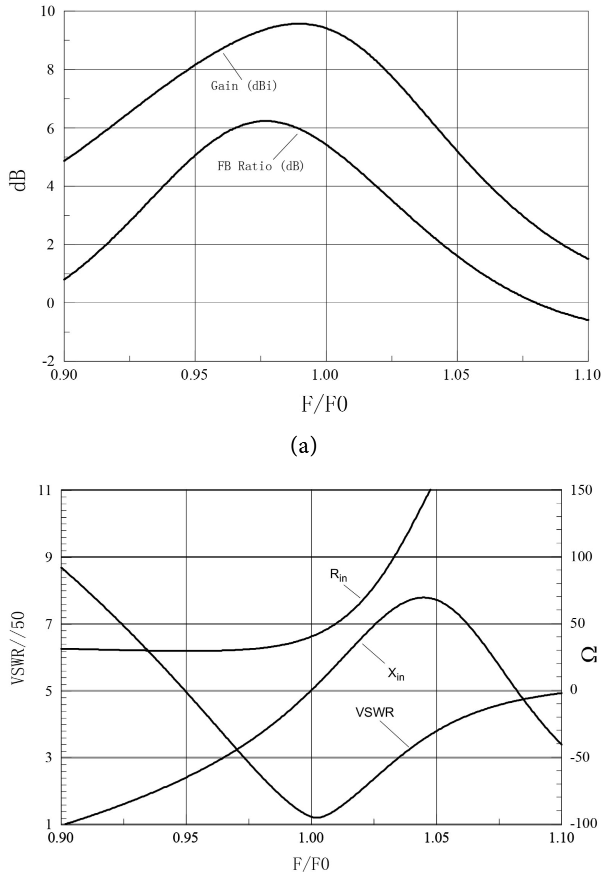
(a)



(b)

Figure 6. (a) Ver. 1, Improved 3-El Yagi, "Fat" DE, G & FBR. [R-0.479λ, DE-0.453λ, D1-0.451λ, S-0.25λ, diams: R/D1-0.005λ, DE-0.065λ]. (b) Ver. 1, Improved 3-El Yagi, "Fat" DE, Z_{in} & VSWR. [R-0.479λ, DE-0.453λ, D1-0.451λ, S-0.25λ, diams: R/D1-0.005λ, DE-0.065λ].

4.7.3. Improved Array Version 2 (Figure 7)

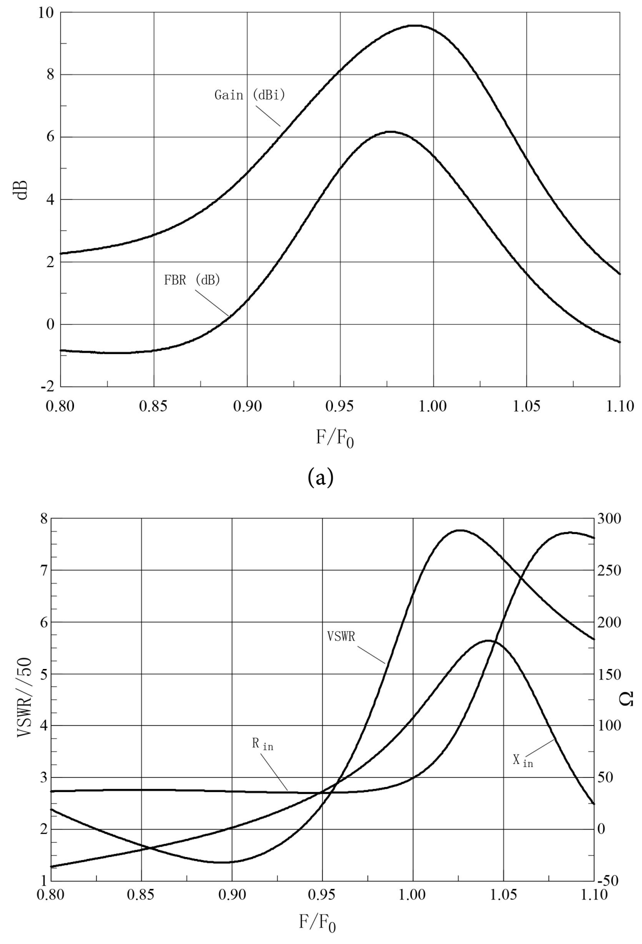


(a)

(b)

Figure 7. (a) Ver. 2, Improved 3-El Yagi, "Fat" DE, Cap Loading, G & FBR. [R-0.479λ, DE-0.453λ, D1-0.451λ, S-0.25λ, R/D1-0.005λ diam, DE-0.065λ, 6.539pF Cap]. (b) Ver. 2, Improved 3-El Yagi, "Fat" DE, Cap Loading, Z_{in}&VSWR. [R-0.479λ, DE-0.453λ, D1-0.451λ, S-0.25λ, R/D1-0.005λ diam, DE-0.065λ, 6.539pF Cap].

4.7.4. Improved Array Version 3 (Figure 8)

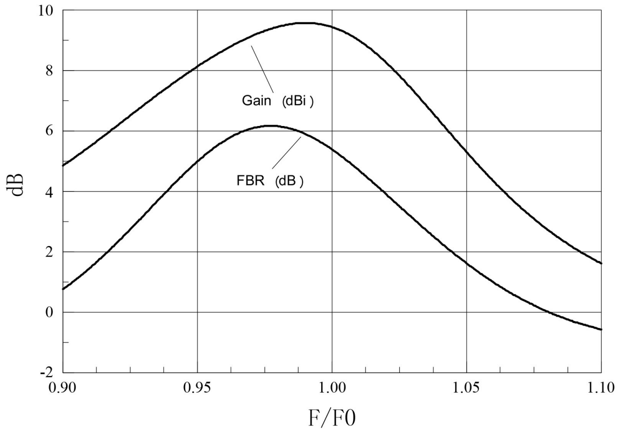


(a)

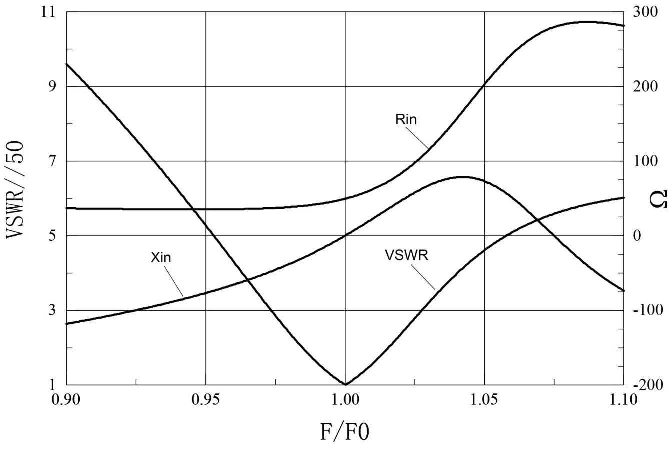
(b)

Figure 8. (a) Ver. 3, Improved 3-El Yagi, "Fat", Long DE, G&FBR. [R-0.479λ, DE-0.477λ, D1-0.451λ, S-0.25λ, diams: R/D1-0.005λ, DE-0.065λ]. (b) Ver. 3, Improved 3-El Yagi, "Fat", Long DE, Z_{in}&VSWR. [R-0.479λ, DE-0.477λ, D1-0.451λ, S-0.25λ, diams: R/D1-0.005λ, DE-0.065λ].

4.7.5. Improved Array Version 4 (Figure 9)



(a)



(b)

Figure 9. (a) Ver. 4, Improved 3-El Yagi, "Fat", Long DE, Cap, G & FBR. [R-0.479λ, DE-0.477λ, D1-0.451λ, S-0.25λ, diams:R/D1-0.005λ, DE-0.065λ, 4.929pF cap]. (b) Ver. 4, Improved 3-El Yagi, "Fat", Long DE, Cap, Z_{in} & VSWR. [R-0.479λ, DE-0.477λ, D1-0.451λ, S-0.25λ, diams: R/D1-0.005λ, DE-0.065λ, 4.929pF cap].

4.8. Comparison of Proto/Improved Yagis & Design Suggestions

Table 6 compares results for the five Yagis discussed thus far in this paper. The investigation began with a well-designed prototype array from recognized source that was modified through four successive iterations. In each modified version the driven element is "fat." In Versions 3 and 4 it also is "stretched." Impedance Bandwidth is defined using a VSWR//50 Ω ≤ 2:1 standard, which is common in the industry, and, surprisingly, the proto array has IBW of zero because nowhere does its VSWR fall to or below 2:1. The objective of this work therefore is to enlarge IBW while maintaining or improving other performance parameters.

It is evident from Table 6 that re-sizing the Yagi's driven element can produce significantly better results. In particular, Version 4 achieves an essentially perfect match to the feed system (VSWR = 1.01) with a 3.1% IBW and very good gain and FBR. Fattening and lengthening DE increases the radiation resistance to ≈50Ω. Similar improvements in systems that are not 50 Ω should be possible by properly re-sizing the driven element. In the test cases considered here, the downside to this approach is introducing what can be a substantial amount of inductive reactance in Z_{in}. However, this reactance can be tuned out by adding an appropriate series capacitance to the feedpoint. By resonating DE VSWR is determined only by the ratio of R_{in} to Z_0. IBW is narrowed because of how X_{in} changes with frequency. Nevertheless, the final result is likely to be in line with the specs of typical well-designed Yagis whose IBW is on the order of 2% [7@p515].

An even better example of how important a Yagi's driven element length and L/D ratio can be will be discussed in the next section in which a 3-element genetic algorithm (GA)-optimized array exhibits even better performance than array Version 4.

Table 6. Performance Comparison of Yagi Versions.

Array # / Table #	Design Parameters	Impedance BW (VSWR ≤ 2:1)		**@ F_0** [Design Frequency]				
		IBW (%)	F_{minVSWR}	VSWR	Z_{in} (Ω)	G (dBi)	FBR (dB)	HPBW (˚)
Ver 0/0	**PROTO**, All Elmnts. 0.005λ	0	n/a	2.59	22.4 + j17.9	9.34	5.53	54/70
Ver 1/1	DE 0.065λ × 0.453λ NO CAP	7.2	0.9170	5.14	40.2 + j81.2	9.41	5.42	53/71
Ver 2/2	DE 0.065λ × 0.453λ LOADED DE 6.539 pF CAP	3.3	1.0020	1.25	40.2 + j0.008	9.41	5.42	53/71
Ver 3/3	DE 0.065λ × **0.477λ** STRETCHED DE, NO CAP	11.2	0.8945	6.54	49.5 + j107.7	9.44	5.39	53/71
Ver 4/4	DE 0.065λ × **0.477λ** STRETCHED DE 4.929pF CAP	3.1	1.0000	1.01	49.5 + j0.035	9.44	5.39	53/71

These results suggest a five step design procedure to improve a Yagi's IBW without adversely affecting HPBW or maximum gain and FBR (value and location in relative fequency). For $R_{in} < Z_0$, increasing R_{in}:

1) Lower DE L/D ratio as much as practicably possible using the analysis in Section 3 and the data in Figure 4.

2) Use a program such as NEC-4 to determine a stretched DE length that brings R_{in} as close to as possible to Z_0.

3) Tune out feedpoint reactance at F_0 (resonate DE) so that VSWR depends only on the ratio of R_{in} to Z_0 which should be very close to a perfect match if the DE length was chosen properly.

4) Examine IBW and confirm that HPBW and the values and locations in relative frequency of max gain and FBR have not been adversely affected.

5) For $R_{in} > Z_0$, reducing R_{in}: Although not common, in some Yagis $R_{in} > Z_0$, so the objective is to reduce the radiation resistance, not increase it. Figure 3(a) and Figure 4 provide data that can serve as a starting point. Depending on the DE length, Figure 3(a) suggests that it may not be necessary or desirable to fatten DE because thin elements exhibit low radiation resistances. However, the driven element length should be shortened instead of lengthened because doing so reduces R_{in} even further. The specific required length for a given DE diameter perhaps is most easily determined using NEC-4 or a similar program, just as NEC-4 was used for the 3-element arrays in this paper. Then follow steps 3 and 4.

This 5-step procedure should apply to any Yagi regardless of the number of elements because VSWR depends only ratio of R_{in} to Z_0 when the driven element has been resonated.

5. Design of a 3-Element 146 MHz Array

5.1. GA-Optimized 3-Element Yagi-Uda Array

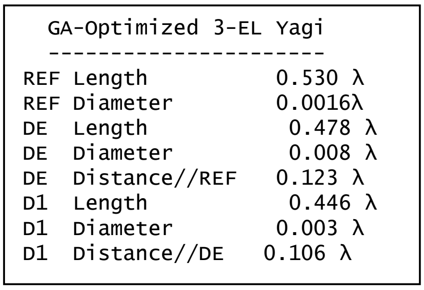
In this section the array design techniques discussed in Sections 1 through 4 are applied to an optimized 3-element Yagi to further illustrate how useful they can be in achieving even better array performance. In this example the improvement in IBW is very substantial while other performance measures are not adversely affected

The starting point is the array described in ([14], not peer reviewed) which was optimized for all eight design parameters using a binary-coded GA: three elements, length and diameter for each, and two boom positions (DE and D1 with REF at the origin). The optimized values for these parameters appear in **Figure 10**. This Yagi is quite short with a boom length of only 0.229λ, just over a quarter wave at the design frequency F_0, which in this case is 146 MHz. As shown in **Figure 11**, its performance is excellent by all the measures considered here. DE is nearly resonant without any tuning yielding a VSWR of 1.49:1. Array gain is lower than the values for the designs in **Table 6**, but consistent with the antenna's shorter boom. This antenna exhibits a remarkably high FBR of 54 dB. Its IBW is just over 9%, from $F/F_0 = 0.924$ to 1.0156, which is quite good.

The bottom line is that this indeed is a very good Yagi design. But there is a problem, a practical one. The optimized element diameters do not translate to readily available standard sizes, so this array cannot be easily fabricated using standard size aluminum tubes. And if a standard size were used, say, 1/2-inch diameter tubing for all three elements, then performance suffers. In that case, VSWR increases to 2.7, IBW shrinks to 2.1%, and FBR drops considerably to 19.3 dB. Only the gain increases, by about half a decibel. It will be interesting to see if the techniques discussed in this paper, which is an expansion of previously published work ([15], not peer reviewed), can reverse some or all of the negative effects of using 1/2-inch standard diameter elements.

5.2. Improved 146 MHz Array

Because of the limitations described above, the GA-optimized array was modified using the 5-step procedure suggested in Section 4.8, modeled using standard 1/2-inch diameter tubing for the reflector and director elements, and a stretched 2-inch diameter driven element (0.0247λ diameter at 146 MHz). The DE was lengthened from the optimized value of 0.478λ to 0.585λ, which has the effect of increasing Z_{in} to $50 + j(76)\ \Omega$. The inductive reactance can be tuned out by inserting 14.29 pF capacitance at the feedpoint, and the results are shown in **Figure 12**, which also includes the NEC-4 input file for this design.



```
   GA-Optimized 3-EL Yagi
   ----------------------
REF Length          0.530 λ
REF Diameter        0.0016λ
DE  Length           0.478 λ
DE  Diameter         0.008 λ
DE  Distance//REF   0.123 λ
D1  Length           0.446 λ
D1  Diameter        0.003 λ
D1  Distance//DE   0.106 λ
```

Figure 10. GA-Optimized Array Dimensions. [non-standard element diameters].

```
At Design FrequencyF0 = 146MHZ:
------------------------------------
VSWR = 1.49
Zin  = 33.87 + j(-3.04) ohms
Gain =  7.03dBi
FBR  = 54.02dB
AGT  = 0.99415
------------------------------------
Min VSWR  = 1.312 @ F/F0  = 0.9756
IBW       = 0.924-1.0156 (9.16%)
Max Gain  = 8 dBi @ F/F0=1.05
Max FBR   = 55.04 dB @ F/F0=1.0004
AGT Range: 0.99304-0.99685
```

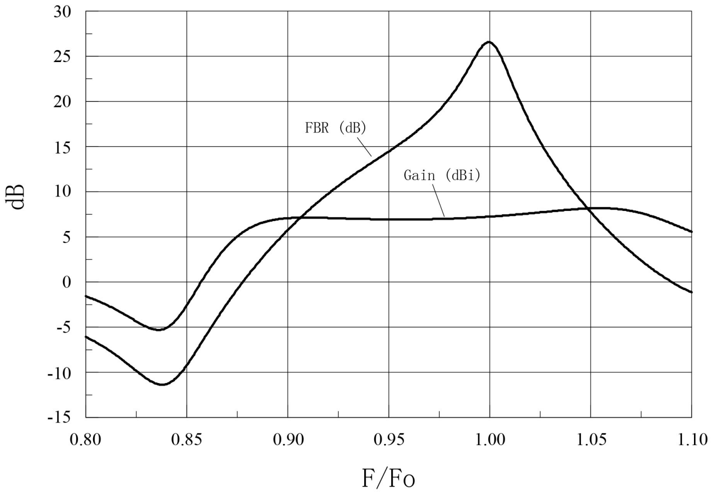
Figure 11. NEC-4 Computed Performance. GA-Optimized Yagi, Non-standard Elemen.

```
At Design FrequencyF0= 146MHZ:
--------------------------------
VSWR = 1
Zin = 50.08 + j(0) ohms
Gain= 7.25dBi
FBR  = 26.56dB
AGT  =0.98939
--------------------------------
Min VSWR = 1.002 @ F/F0=1
IBW  = 0.881-1.02 (13.9%) [VSWR=<2:1]
Max Gain =8.18 dBi @ F/F0=1.057
Max FBR  = 26.57 dB @ F/F0= 0.9995
AGT Range:  0.98522-1.0064
=========================================
NEC INPUT FILE
--------------
CM
CM Driven Element Loaded with 14.2942 pF Capacitance.
CM
CM DIMENSIONS THIS RUN:
CM ===================
CM NOTE: BRACKETED [..] DIMENSIONS IN WAVELENGTHS.
CM REF Length        = 42.85in [0.53]
CM REF Diameter      = 0.50in   [0.0062]
CM REF Boom Position = 0.in    [0.]
CM DE Length         = 47.29in [0.585]
CM DE Diameter       = 2in     [0.0247]
CM DE Boom Position  = 9.94in  [0.123]
CM D1 Length         = 36.06in [0.446]
CM D1 Diameter       = 0.50in   [0.0062]
CM D1 Boom Position  = 18.51in [0.229]
CM
CM NOTE: ALL COORDINATES BELOW ARE IN METERS.
CE
GW1 9 0.       -0.54415  0. 0.        0.54415  0. 0.00635
GW2 9 0.252568 -0.600618 0. 0.252568  0.600618 0. 0.0254
GW3 9 0.470227 -0.457907 0. 0.470227  0.457907 0. 0.00635
GE
LD 0 2 5 5 0. 0. 14.2942E-12
EX 0 2 5 1 1. 0 0 0
FR 0 0 0 0 146 0
RP 0 19 19 1001 0. 0. 5. 10. 100000.
XQ
```

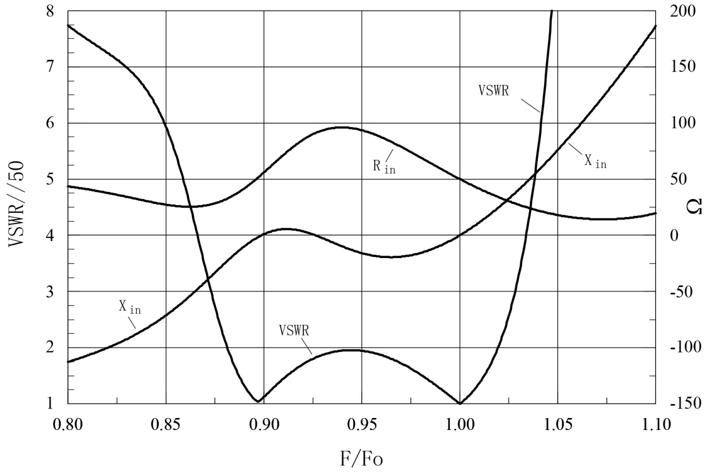
Figure 12. Performance of GA-Optimized Array with "Fat," Stretched DE & Cap loading.

The VSWR is reduced to essentially a perfect match at F_0, and IBW is increased to almost 14%. Gain is slightly higher at 7.25 dBi, and FBR has recovered somewhat to 26.6 dB. Nonetheless, this FBR value is far below what the optimized array's FBR would be if it were built using only the optimized diameters for each element. This creates a clear trade-off: use only custom size tubing to preserve FBR, or use standard size tubing even though FBR is lower. The decision depends on how important FBR is in the intended application.

Figure 13 plots the array's performance: Gain and FBR in **Figure 13(a)**; Z_{in} and VSWR in **Figure 13(b)**. Unlike the Gain/FBR curves in Yagi Versions 1 through 4, which were similarly shaped with a more or less constant offset, the curves in this case are quite dissimilar. The gain is fairly flat over a large portion of the modeled frequency range while FBR has a very pronounced peak at the design frequency. The Z_{in} and VSWR curves also are quite different from the previous arrays' curves, especially VSWR. It has two minima that are close to a perfect match, one at the design frequency and the second about 11% below F_0, In between VSWR increases to a maximum just below 2, but it never exceeds 2:1 which accounts for the large IBW (13.9%, 128.6 - 148.9 MHz).



(a)



(b)

Figure 13. (a) GA-3-El Yagi, "Fat," Stretched DE, Cap Loaded (14.29 pF), Gain & FBR. (b) GA-3-El Yagi, "Fat," Stretched DE, Cap Loaded (14.29 pF), Z_{in} & VSWR.

As to the radiation resistance, it has a value of almost exactly 50 Ω at F_0 where DE resonates due to the added feedpoint capacitance. The peak in R_{in} more or less coincides with the VSWR peak near 2:1, and in fact it is the R_{in} value near 100 Ω that primarily determines VSWR because X_{in} is only moderately negative in that frequency range.

One question that comes up is how to add the required feedpoint capacitance (for $X_{in} > 0$) or inductance (for $X_{in} < 0$). Of course, the simplest approach is to use an off-the shelf capacitor or inductor with the required value and power handling capability, but there is a strong likelihood that such a component is not available OTS, so the only choice might be to fabricate one that is built into the Yagi's feedpoint. As between the two, capacitor or inductor, it seems that the inductor should be the easier, probably an air-core coil wound with a sufficiently heavy gauge, insulated wire with one-half of the total required inductance connected to each half of the driven element to preserve electrical balance.

The capacitor, however, may be more problematic. Yet either of two very simple approaches might work well. A "parallel-plate" capacitor can be created by using the DE's outer surface as one of the plates and a separating insulator under a top plate which is connected to the Yagi's feed cable. One half of the required 14.29 pF capacitance must be added in series with each of the driven element arms. An estimate of required dimensions can be obtained using the capacitance formula for flat parallel plates, $C = \varepsilon_0\varepsilon_r LW/T$ Farad where L, W and T are the top plate length and width and the insulator thickness, respectively (all in meters), ε_0 is the permittivity of free space (8.854×10^{-12} F/m), and ε_r the insulation dielectric constant. If the top plate were a 1-inch square patch of, say, copper with a 1/8-inch thick insulator with dielectric constant of 4 between it and the DE's surface, the capacitance is 7.2 pF, just about half the total as required. Of course, trimming would be necessary, as would be the selection of an appropriate high dielectric strength insulator, maybe a rubber material such as neoprene. Another similar, and even simpler, approach might be to lay an insulated wire on DE's surface parallel to its axis. The required length could be estimated using the capacitance formula for a single wire over a ground plane as long as its height restriction is met with the shortened wavelength in the dielectric insulation, viz., $C = 2\pi\varepsilon_0\varepsilon_r/\ln(2h/a)$ Farad/meter (of wire length), where h is the wire center distance from the DE surface and a the wire radius ([16], §2.2). Either of these approaches could constitute an easily implemented feed that resonates the driven element at the design frequency. And either one eliminates the need for structural matching such as a gamma or hairpin match or for a complex external matching circuit.

6. Conclusions and Implications

This paper investigated the effects of re-sizing the driven element in two Yagi arrays. DE length and L/D ratio can have a major impact on an array's performance and should be treated as important design parameters. The results of this research suggest a 5-step design approach that increases IBW without materially affecting HPBW or the values and locations in relative frequency of maximum Gain and FBR. Although applied to systems with a 50 Ω characteristic impedance in this paper, the suggested design approach should be useful regardless of the characteristic impedance.

The suggested approach can bring the array's radiation resistance R_{in} as close as possible to the feed system characteristic impedance Z_0. Resonating DE then centers the IBW band on the design frequency F_0 with VSWR ≈ 1. While the arrays in this paper have $R_{in} < Z_0$, the analysis and methodology are equally applicable to Yagis with $R_{in} > Z_0$ with slight modifications. And while this paper uses 3-element arrays as examples, its analysis and techniques are applicable to Yagi-Uda arrays with any number of elements.

Reactance of the opposite sign must be introduced at the feedpoint in order to resonate DE at F_0. The arrays discussed in this paper required capacitive reactance, and two simple methods for introducing it are proposed. These methods should be useful in resonating the driven element in any Yagi-Uda array.

The Yagis in this paper use cylindrical elements, but there is no reason why the suggested design approach cannot be applied to other element geometries as well using an effective cylindrical radius, for example, planar PCB Yagis, presumably with similar results, possibly providing better performance, and eliminating a complex matching network for feeding the array. The discussion cites a reference that provides a table of effective cylindrical radii for several other conductor shapes that also might be useful in fabricating different types of Yagi-Uda arrays.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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