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Optimal Resource Allocation on Physical and Human Capital: Theoretical Modelling and Empirical Case Study of the United States

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Abstract

This study expands the theoretical framework proposed by (Mankiw et al., 1992) to explore optimal resource allocation between physical capital and human capital accumulation. Using United States data spanning from 1950 to 2019, it empirically examines the long-term cointegration relationships among per-capita final output, physical capital, and human capital. The empirical estimates, derived from a theory-based model, indicate that sustaining economic optimality in the US necessitates allocating approximately 25.13% to developing human capital and 26.23% to accumulating physical capital from real GDP. These allocations underscore the crucial roles of both physical and human capital in production, highlighting the equal importance of human capital in shaping economic output. Furthermore, the analysis reveals short-term interdependencies between these capitals and their immediate responses to output fluctuations. These insights into short-term dynamics provide essential implications for policymaking, enabling informed decisions on resource allocation and strategic economic planning.

Keywords

Human Capital, Endogenous Growth, Optimal Growth Path, Cointegration, United States

1. Introduction

In numerous endogenous growth theories, the sustained expansion of per-capita output hinges on the pivotal concept of non-diminishing returns to reproducible factor inputs within the production process. This foundational mechanism encompasses a spectrum of elements, notably the notion of broadly defined aggregate capital stock, the spillover effects generated by public capital investment, the allocation of resources toward research and development, and perhaps most crucially, the integration of human capital into the production framework.

The distinct impact of human capital, set apart from its physical counterpart, on propelling growth has been a focus of attention among endogenous-growth theorists dating back to seminal works such as those by (Frankel, 1962) and (Uzawa, 1965). An array of studies has delved into how human capital engenders spillover effects in these models. For instance, (Griliches, 1979) and (Romer, 1986) argued that while individual firms possess specific knowledge, collective industry knowledge offers external benefits, although it remains exogenous concerning an individual firm's profit-maximizing strategy. (Lucas, 1988) contributed by delineating how human capital serves as a conduit for creating and disseminating knowledge across firms. He outlined a model wherein the knowledge amassed by skilled workers could be accumulated within the education sector, using a fraction of the current knowledge level—an aspect that, though derived in theory, remained contingent on multiple exogenous parameters, precluding the assurance of its steady-state value falling between 0 and 1, and challenging empirical estimations across different nations within a two-sector model.

Lucas' (Lucas, 1988) two-sector model finds its counterpart in the single-sector system investigated by (Mankiw et al., 1992). Their framework elucidates the accumulation of both human and physical capital through the sacrifice of current consumption. In this paradigm, final output is apportioned to physical and human capital investments, and consumption by fractions denoted by s_K, s_H, and $(1-s_K-s_H)$ respectively—parameters that remained exogenous. Their subsequent empirical findings revealed that approximately 80% of variations in per-capita income across nations could be explained through regressions consistent with their theoretical framework. Over the following decades, the empirical insights from (Mankiw et al., 1992) have served as a pivotal reference for both theoretical and empirical studies on the role of human capital in the process of economic growth. Currently, there has been a surge in the quantity and quality of global human capital, enabling sustained growth in many countries. However, it has also been noted that there is a continual decrease in the relative price of skilled labor compared to unskilled workers, a phenomenon elaborated by (Jones & Romer, 2010) in their reinterpretation of Kaldor's stylized growth facts.

This paper endeavors to depart from the assumption of constant and exogenous s_K and s_H posited by (Mankiw et al., 1992), aiming to establish a robust micro-foundation within this continuum of literature. We envisage a scenario where a representative household-worker optimally allocates resources toward consumption, physical capital accumulation, and human capital development, seeking to maximize lifetime utility. Furthermore, we extend the deterministic growth model of (Mankiw et al., 1992) into a stochastic discrete-time version, encompassing both exogenous and endogenous growth factors. This approach equips us with theory-consistent regressions, enabling estimation of factor input shares and the optimal resource allocation fraction between physical and human capital. Policy implications are presented in the empirical segment, exemplified through a case study centered on the United States.

While prior literature extensively studies the role of human capital in growth, this paper introduces a significant innovation. Departing from static assumptions, we develop a dynamic model for optimal resource allocation. Our framework involves a representative household-worker optimizing resource allocation for consumption, physical, and human capital, enhancing the deterministic model into a stochastic version. This innovation aims to provide a nuanced understanding of how resource allocation influences economic growth. By offering a more dynamic and theoretically robust model, this research contributes to a deeper comprehension of optimal resource allocation's impact on growth dynamics.

The subsequent sections of this paper unfold as follows: Section 2 introduces our extension of the model proposed by (Mankiw et al., 1992), aiming to derive optimal resource allocation within a stochastic framework. This extension serves as the foundation for anchoring the time-series aspects of the cointegration growth-accounting framework. Section 3 provides a comprehensive overview of the aggregate data related to our system variables, all obtained from the United States. Within this section, empirical analyses, including cointegration tests, Granger causality examinations, and assessments of short-run dynamics through impulse response analyses, are conducted. These analyses are guided by the long-run parameter estimates derived from the structural model. Finally, Section 4 encapsulates the conclusions drawn from the findings presented throughout this paper.

2. One-Sector Stochastic Growth Model with Physical and Human Capital

2.1. Theory

At any time *t*, let Y_t represent the aggregate output, K_t denote the aggregate physical capital stock, H_t indicate the aggregate human capital stock, L_t signify the total number of employed workers (typically assumed to mirror an economy's population level), and A represent the constant level of total factor productivity (TFP). The conventional labor-augmented production technology adheres to the Cobb-Douglas form.

$$Y_t = AK_t^{\alpha} H_t^{\beta} \left[(1+x)^t L_t \right]^{1-(\alpha+\beta)} \epsilon_t^P , \tag{1}$$

In Equation (1), we expand upon the production function proposed by (Mankiw et al., 1992). In addition to accounting for constant returns to scale, we introduce an exogenous Harrod-neutral rate of technological progress, denoted as *x*, along with multiplicative stochastic shocks, ϵ_t^P, intended to model potential short-run business cycles. The exogenous parameters α, β, and x are not constrained, as their empirical estimates contribute to understanding the dynamics of endogenous and/or exogenous growth within an economy. These parameter configurations will be discussed later. Through division of both sides of Equation (1), it can be reformulated in per-capita terms:

$$y_t = A(1+x)^{[1-(\alpha+\beta)]t} k_t^{\alpha} h_t^{\beta} \epsilon_t^{P}. \quad (2)$$

Here, the lowercase variables, y, k, and h, denote per-capita output, physical capital, and human capital, respectively. Detrending Equation (2) enables the representation of all variables in the context of per efficiency-unit of labor input in production.

$$\hat{y}_t = A\hat{k}_t^{\alpha}\hat{h}_t^{\beta}\epsilon_t^{P}, \text{ where } \hat{y}_t = \frac{Y_t}{(1+x)^t L_t}; \; \hat{k}_t = \frac{K_t}{(1+x)^t L_t}; \; \hat{h}_t = \frac{H_t}{(1+x)^t L_t}. \quad (3)$$

For the sake of mathematical simplification, we presume complete depreciation for both types of capital stock in each period. However, the trajectories of their dynamics and the characteristics of steady-state values remain unaffected even if the depreciation rates are less than 100%.[1] Hence, the investment flow at time t translates into the capital stock at time $t+1$. Output, at each time point, is allocated to consumption, as well as the investment in physical capital and human capital. An endogenous variable s_t signifies the fraction allocated to accumulate physical capital, with $(1-s_t)$ representing the fraction allocated to human capital. Assuming a log-utility function for consumption per efficiency unit of labor ($\hat{c}_t$), the optimization problem for the household-worker's lifetime utility is formulated by the value function as follows.

$$V(\hat{k}_0, \hat{h}_0) = \max_{\{\hat{c}_t, s_t, \hat{k}_{t+1}, \hat{h}_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0\left(\sum_{t=0}^{\infty}\delta^t \ln \hat{c}_t\right); \text{ discount factor } \delta \in (0,1), \quad (4)$$

subject to

$$\hat{k}_{t+1} = s_t\hat{y}_t - \hat{c}_t = s_t A\hat{k}_t^{\alpha}\hat{h}_t^{\beta}\epsilon_t^{P} - \hat{c}_t; \quad (5)$$

$$\hat{h}_{t+1} = (1-s_t)\hat{y}_t = (1-s_t)A\hat{k}_t^{\alpha}\hat{h}_t^{\beta}\epsilon_t^{P}; \quad (6)$$

$$\hat{k}_0, \; \hat{h}_0 \text{ are given; and} \quad (7)$$

$$\hat{c}_t, \hat{k}_{t+1}, \hat{h}_{t+1} \geq 0, \quad s_t \in [0,1], \quad (8)$$

for all $t \in \mathbb{N}$.

To address the intertemporal utility maximization problem of the household-worker, considering constraints (5) to (8), we apply the (Bellman, 1957) principle of optimality, transforming Equation (4) into the subsequent limiting value function:

$$V(\hat{k}_t, \hat{h}_t, \epsilon_t^{P}) = \max_{\{\hat{c}_t, s_t, \hat{k}_{t+1}, \hat{h}_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \ln \hat{c}_t + \delta\mathbb{E}_0\left[V(\hat{k}_{t+1}, \hat{h}_{t+1}, \epsilon_{t+1}^{P} \mid \hat{k}_t, \hat{h}_t, \epsilon_t^{P})\right], \quad (9)$$

subject to constraints (5) to (8). A conjecture of the aforementioned value func-

[1]Capital accumulation with a 100% depreciation rate, while unrealistic, has been commonly adopted in prior theoretical studies, like (Glomm & Ravikumar, 1994), primarily for the sake of achieving mathematically solvable derivations.

tion assumes the following form:

$$V\left(\hat{k}_t, \hat{h}_t, \epsilon_t^P\right) = B_0 + B_1 \ln \hat{k}_t + B_2 \ln \hat{h}_t + B_3 \ln \epsilon_t^P, \tag{10}$$

where B_0, B_1, B_2, and B_3 represent combinations of parameters within the system, to be verified subsequently. The optimality conditions for maximizing the right-hand side of the combined Equations (9) and (10) involve obtaining the following two first-order derivatives with respect to $\hat{c}_t$ and s_t:

$$\frac{1}{\hat{c}_t} = \frac{\delta B_1}{\hat{k}_{t+1}} \quad \text{and} \tag{11}$$

$$\frac{\hat{k}_{t+1}}{\hat{h}_{t+1}} = \frac{B_1}{B_2}. \tag{12}$$

Equation (11) delineates the household-worker's optimal intertemporal allocation of resources between current consumption and future physical capital stock. Equation (12) implies a constant and time-invariant ratio between physical capital and human capital for optimality. By substituting Equations (11) and (12) into Equations (5) and (6), the resulting expressions are:

$$\hat{c}_t = \left(\frac{1}{1+\delta B_1}\right) s_t A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P, \tag{13}$$

$$s_t = \frac{1+\delta B_1}{1+\delta B_1+\delta B_2}, \tag{14}$$

$$\hat{k}_{t+1} = \left(\frac{\delta B_1}{1+\delta B_1}\right) s_t A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P, \text{ and} \tag{15}$$

$$\hat{h}_{t+1} = \left(\frac{\delta B_2}{1+\delta B_1}\right) s_t A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P. \tag{16}$$

Substituting Equations (13) to (16) into the right-hand side of the combined Equations (9) and (10) facilitates the verification of

$$B_1 = \frac{\alpha}{1-\delta(\alpha+\beta)} \quad \text{and} \quad B_2 = \frac{\beta}{1-\delta(\alpha+\beta)}. \tag{17}$$

B_1 and B_2 serve as crucial determinants in resolving the optimal sequence of $\left\{\hat{c}_t, s_t, \hat{k}_{t+1}, \hat{h}_{t+1}\right\}_{t=0}^{\infty}$ for the representative household-worker. Derived from B_1, Equation (13) reveals the optimal consumption trajectory as:

$$\hat{c}_t = \left(1-\delta\alpha-\delta\beta\right) A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P. \tag{18}$$

Here, the marginal propensity to consume equates to one minus the summation of one-period discounted shares of physical capital and human capital within the production process, $(\delta\alpha+\delta\beta)$. These discounted shares, $\delta\alpha$ and $\delta\beta$, correspondingly denote the optimal fractions of output allocated to physical capital and human capital investment. This relationship is evident when substituting B_1 and B_2 into Equations (15) and (16), as follows.

$$\hat{k}_{t+1} = \delta\alpha A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P \quad \text{and} \tag{19}$$

$$\hat{h}_{t+1} = \delta \beta A \hat{k}_t^{\alpha} \hat{h}_t^{\beta} \epsilon_t^P . \tag{20}$$

The optimal investment fraction, s_t, is deduced from equations (6) and (20), or obtained by substituting B_1 and B_2 into Equation (14) as:

$$s_t = 1 - \delta \beta . \tag{21}$$

Hence, the optimal s_t equals one minus the one-period discounted share attributed to human capital in output. As β approaches zero, s_t tends towards one, indicating that the household-worker lacks the motivation to invest in human capital. In this case, the model simplifies to a conventional Ramsey planning economy, where the only accumulable factor in the system is physical capital. Conversely, a greater share of human capital in output signifies that accumulating human capital proves more productive than focusing on physical capital accumulation.

The model described above encompasses four distinct scenarios within an economy, contingent upon the interplay of parameters, as illustrated in Table 1.

Assuming $\epsilon_t^P = 1$ and $0 < \alpha + \beta < 1$ for scenarios (a) and (b), the steady-state values of $\hat{k}^*$ and $\hat{h}^*$ from Equations (19) and (20) are both unique and stable, expressed as:

$$\hat{k}^* = \phi^{\frac{1-\beta}{1-(\alpha+\beta)}} \text{ and} \tag{22}$$

$$\hat{h}^* = (\delta \beta A)^{\frac{1}{1-\beta}} \phi^{\frac{\alpha}{1-(\alpha+\beta)}}, \text{ where } \phi = (\delta \alpha A)(\delta \beta A)^{\frac{\beta}{1-\beta}} > 0 . \tag{23}$$

In scenarios (a) and (b), when the reproducible factors, $\hat{k}_t$ and $\hat{h}_t$, exhibit diminishing returns to scale, sustained growth in per-capita output occurs in the long run solely if the exogenous rate of technological progress, x, maintains a positive value. Conversely, scenarios (c) and (d), characterized by constant returns to reproducible factors in output $(\alpha + \beta = 1)$, render the steady-state values of $\hat{k}^*$ and $\hat{h}^*$ nonexistent. In such instances, the long-run growth rates denoted by γ for per-capita variables along the balanced growth path remain identical, perpetual and non-explosive:

$$\gamma_y = \gamma_k = \gamma_h = \gamma_c = x + \delta A \alpha^{\alpha} (1-\alpha)^{1-\alpha} - 1, \text{ where } x \geq 0 . \tag{24}$$

2.2. Simulation and Calibration

Utilizing the earlier theoretical findings, it becomes feasible to simulate short-term and long-term transitions of the system's variables. Equations (18) to (20) delineate the resolutions to the household-worker's intertemporal utility maximization

Table 1. Growth scenario and corresponding parameters in the system.

Parametric combination	Growth Scenario
(a) $\alpha + \beta < 1$ and $x = 0$	No perpetual growth, steady state exists
(b) $\alpha + \beta < 1$ and $x > 0$	Pure exogenous growth
(c) $\alpha + \beta = 1$ and $x = 0$	Strict endogenous growth
(d) $\alpha + \beta = 1$ and $x > 0$	Co-existence of endogenous and exogenous growth

issue. In conjunction with Equations (3) and (21), we can represent the precise log-linear equilibrium conditions in a state-space format as:

$$\begin{bmatrix} 1 \\ \ln \hat{k}_{t+1} \\ \ln \hat{h}_{t+1} \\ \ln \epsilon_t^P \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \ln(\delta\alpha A) & \alpha & \beta & 1 \\ \ln(\delta\beta A) & \alpha & \beta & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \ln \hat{k}_t \\ \ln \hat{h}_t \\ \ln \epsilon_t^P \end{bmatrix} \text{ and } \quad (25)$$

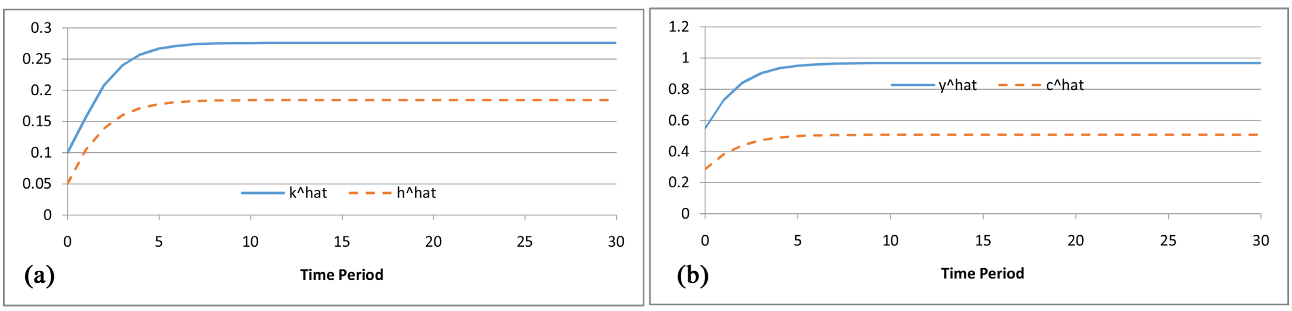
$$\begin{bmatrix} \ln \hat{c}_t \\ \ln \hat{y}_t \end{bmatrix} = \begin{bmatrix} \ln\left[(1-\delta\alpha-\delta\beta)A\right] & \alpha & \beta & 1 \\ \ln A & \alpha & \beta & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \ln \hat{k}_t \\ \ln \hat{h}_t \\ \ln \epsilon_t^P \end{bmatrix}. \quad (26)$$

To calibrate the system outlined above, all parameters are assumed to fall within reasonable ranges. Specifically, for scenarios (a) and (b) in Table 1, we set $A = 2$, $\alpha = 0.3$, and $\beta = 0.2$. With the household-worker's subjective discount rate defined as $\delta = 0.95$, the optimal investment fraction is calculated as $s_t = (1-\delta\beta) = 0.81$ across all time periods. This implies that, at each point in time, the distribution of output should allocate 52.5% to consumption, 28.5% to physical capital accumulation, and 19% to human capital accumulation. Concurrently, we assume $\epsilon_t^P = 1$ for a deterministic case, intending to introduce this shock later during our theoretical impulse-response analysis. Figure 1 illustrates the optimal trajectories of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$ based on the provided initial values of $\hat{k}_0 = 0.1$ and $\hat{h}_0 = 0.05$.

Continuing, to illustrate scenarios (c) and (d) as detailed in Table 1, we alter the share of human capital in output from $\beta = 0.2$ to $\beta = 0.7$ to ensure $(\alpha + \beta) = 1$. Under these conditions, the steady-state equilibrium depicted in Figure 1 becomes nonexistent, and the growth rate of the system, as derived from Equation (24), can be expressed as:

$$\gamma_y = \gamma_k = \gamma_h = \gamma_c = x + \delta A \alpha^{\alpha} (1-\alpha)^{1-\alpha} - 1 = \begin{cases} 3.15\% & \text{for } x = 0 \ (\text{scenario}(c)) \\ 5.15\% & \text{for } x = 0.02 \ (\text{scenario}(d)) \end{cases}$$

Next, incorporating a stochastic component $\epsilon_t^P \sim^{iid} N(1, \sigma_P^2)$ into the model



Note: Using our calibrated parameters, the steady-state values of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$ are unique and are determined to be 0.28, 0.18, 0.97, and 0.51, respectively.

Figure 1. Optimal transitions of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$, in the case of scenarios (a) and (b)[2].

enables us to conduct a theoretical impulse-response analysis spanning from the short to the long run. Scenario (b) serves as an illustrative example for this purpose. Assuming a one-percentage increase in TFP, the percentage responses of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$ are depicted in Figure 2.

In Figure 2, $\hat{y}_t$ and $\hat{c}_t$ exhibit contemporaneous and positive responses to the one-percentage-point shock, whereas the responses of $\hat{k}_t$ and $\hat{h}_t$ occur with a lag of one period. Across all variables, the initial positive responses gradually diminish, settling within 6 to 7 time periods.

Furthermore, this positive standard shock introduces fluctuations in the long-run growth rate. In the case of scenario (b), assuming the exogenous rate of technological progress is $x = 0.02$, it also represents the long-run rate of growth in per-capita output. Following a one-percentage increase in TFP at time T, Figure 3 illustrates that this growth rate contemporaneously rises from 2% to 3%, but at time $T + 1$, it decreases to 1.5%, aligning with the reduction observed in the positive response from $\hat{y}_t$ shown in Figure 2. The long-run growth rate of 2% is then restored after 6 to 7 time periods.

2.3. Theory-Implied Cointegration Equations

In the preceding subsection, although our simulation results appear reasonable, it is crucial to note that the system parameters, α, β, and x, should not be arbitrarily assumed but instead properly estimated. In our theoretical model, we provide theory-consistent cointegrating spaces for the per-capita variables y_t, k_t, and h_t. By log-linearizing Equations (19), (20), and (3), and applying a lag operator, L, we obtain:

$$\left[1-(\alpha+\beta)L\right](\ln k_t - xt) = \lambda_1 + L\ln \epsilon_t^P ; \quad (27)$$

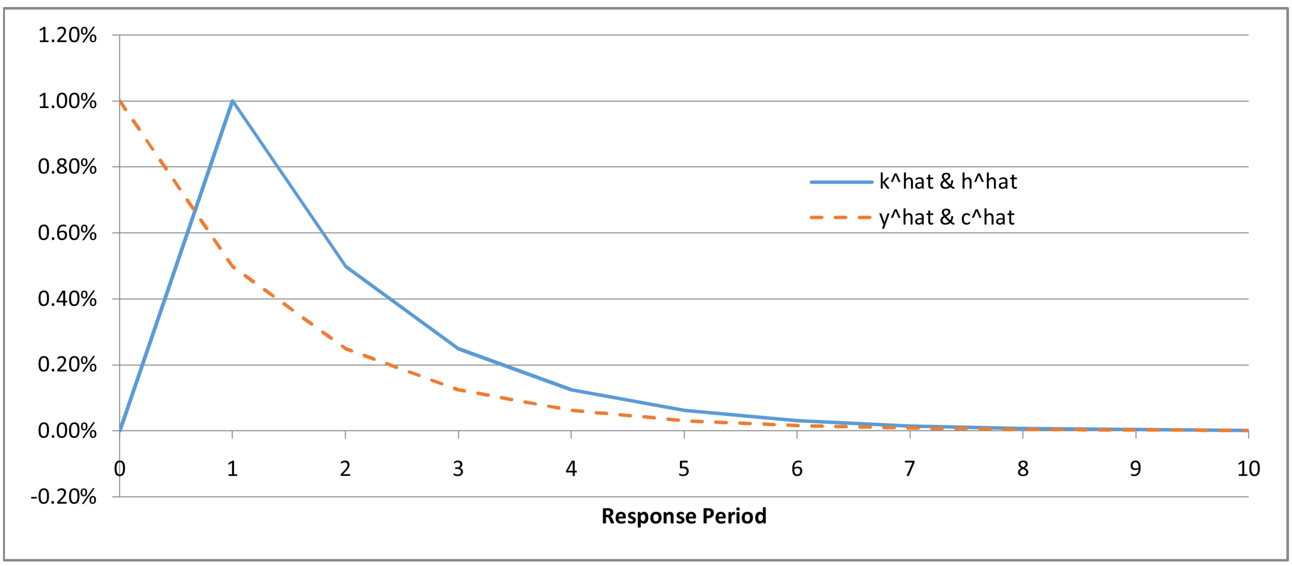


Figure 2. The percentage responses of $\hat{k}_t$, $\hat{h}_t$, $\hat{y}_t$, and $\hat{c}_t$ to 1% increase in TFP, in the case of scenario (b).

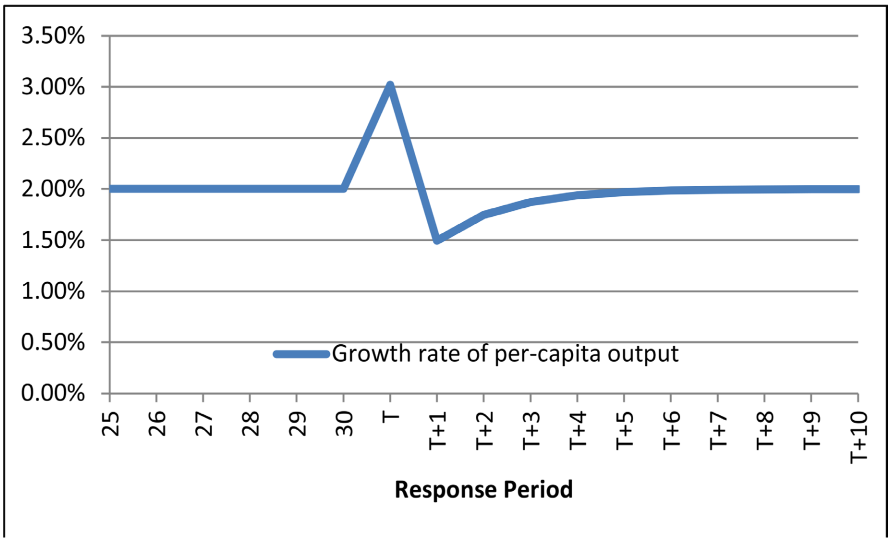


Figure 3. The response of the long-run growth rate in the case of scenario (b).

$$\left[1-(\alpha+\beta)L\right](\ln h_t - xt) = \lambda_2 + L\ln\epsilon_t^P; \quad (28)$$

$$\left[1-(\alpha+\beta)L\right](\ln y_t - xt) = \lambda_3 + \ln\epsilon_t^P. \quad (29)$$

Here λ_1, λ_2, and λ_3 represent constant combinations of parameters, and $\ln\epsilon_t^P$ is an *i.i.d.* shock with a zero mean and constant variance.

From a time-series perspective, $\ln y_t$, $\ln k_t$, and $\ln h_t$ are potentially I(1) variables, allowing for a maximum of two linearly independent cointegrating vectors. Assuming that $\alpha+\beta=1$ to ensure non-explosive and perpetual growth, Equations (27) to (29) result in the cointegrating space as:

$$\ln y_t - \ln k_t = \kappa_1 + \ln\epsilon_t^P \text{ and} \quad (30)$$

$$\ln y_t - \ln h_t = \kappa_2 + \ln\epsilon_t^P \text{, where } \kappa_1 \text{ and } \kappa_2 \text{ are constant.} \quad (31)$$

If the existence of a cointegrating space with two cointegrating vectors is rejected, then at most one cointegrating equation can be found. By multiplying Equation (30) on both sides by α and Equation (31) on both sides by $(1-\alpha)$, and then summing them up, we obtain:

$$\ln y_t - \alpha\ln k_t - (1-\alpha)\ln h_t = \kappa_3 + \ln\epsilon_t^P \text{, where } \kappa_3 \text{ is constant.} \quad (32)$$

If we relax the constraint that $\alpha+\beta=1$, the unrestricted cointegrating equation can be derived from Equation (2), resulting in:

$$\ln y_t - \alpha\ln k_t - \beta\ln h_t - \left[1-(\alpha+\beta)\right]xt - \ln A = \ln\epsilon_t^P. \quad (33)$$

Equation (32) emerges as a constrained version of Equation (33) when assuming $\alpha+\beta=1$. The validity of this hypothesis, in conjunction with the impact of the exogenous rate of technological progress, *x*, will be subjected to empirical testing in the subsequent section of this paper.

3. Empirical Estimation

3.1. Data and Descriptive Statistics

The data source for all variables in the theoretical model is directly available from Penn World Table (PWT) version 10.01, updated in early 2023, encompassing a substantial time span from 1950 to 2019 for the United States and most advanced countries. This dataset stands as the most current and comprehensive worldwide macroeconomic dataset constructed by (Feenstra et al., 2015). Within this dataset, we observe real GDP denoted as Y_t, total population represented as L_t, real aggregate physical capital stock indicated by K_t, and the calculated index for aggregate human capital stock, noted as H_t.[2] Both real GDP and aggregate physical capital stock are quantified at constant 2017 national prices, expressed in million 2017 US dollars. The division of Y_t, K_t, and H_t by the total population allows us to observe the upward trajectory of the logarithmic lowercase per-capita variables: $\ln y_t$, $\ln k_t$, and $\ln h_t$, depicted in **Figure 4**.

Throughout our sample period, both the physical capital stock per capita (k_t) and real GDP per capita (y_t) exhibited consistent growth, with average annual rates of 2.03% and 1.65%, respectively. In contrast, the expansion of human capital stock per capita, h_t, experienced a distinct slowdown starting in 1980. Specifically, the average annual growth rate of h_t stood at 1.24% from 1950 to 1980 but notably declined to 0.48% between 1981 and 2019. This trend concurs with the assertion made by (Carnevale & Rose, 2011) that the United States has

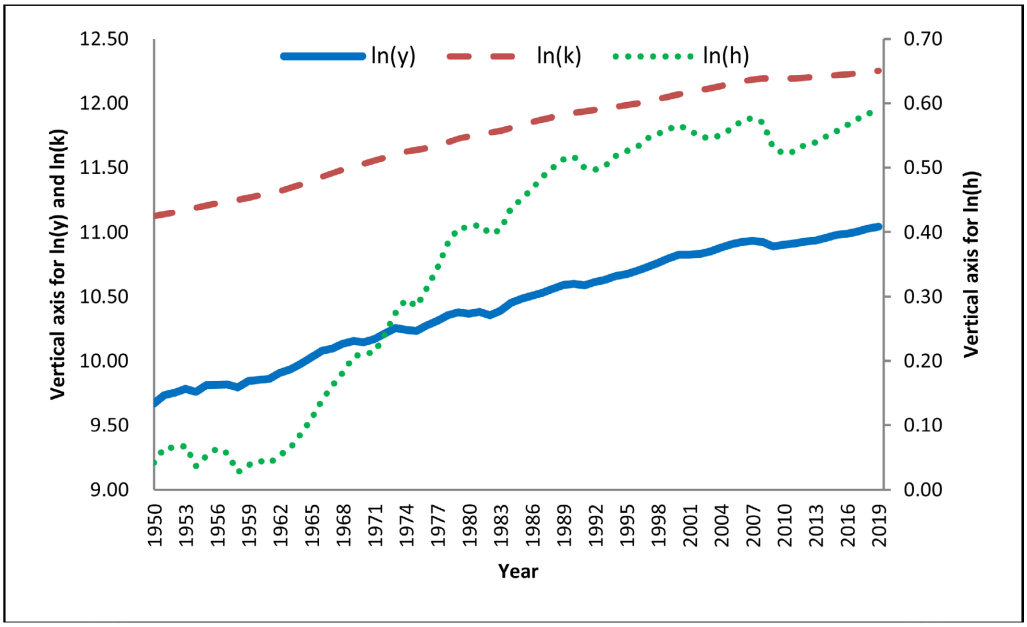


Figure 4. Historical Trends of $\ln y_t$, $\ln k_t$, and $\ln h_t$ in the United States from 1950 to 2019.

[2]The aggregate human capital stock is calculated, as per PWT (8.1) documentation, by multiplying the number of workers in an economy (*emp*, in millions) by the economy's human capital per worker (*hc*). The *hc* represents an index of human capital that takes into account both years of schooling (Barro & Lee, 2013) and returns to education (Psacharopoulos, 1994).

witnessed a deficiency in producing college-educated workers since 1980.

3.2. Stationarity of Variables

In general, following logarithmic transformation, most macroeconomic variables tend to exhibit stochastic trends in the long run. To assess the stationarity of $\ln y_t$, $\ln k_t$, and $\ln h_t$, **Figure 4** suggests that both intercepts and time trends should be accounted for when conducting conventional unit root tests for these variables in their original levels. However, when examining first differences, considering only intercepts is adequate, unless a significant residual trending component persists, as observed in the case of $\Delta \ln k_t$ in our sample. **Table 2** provides a summary of the outcomes derived from the Augmented Dickey Fuller (ADF) unit root tests (Said & Dickey, 1984).

Table 2 indicates, at a 5% significance level, that $\ln y_t$, $\ln k_t$, and $\ln h_t$ are all identified as nonstationary variables. This result suggests that their long-run cointegrating relationships can be examined through balanced regressions. While incorporating these integrated variables into an ad hoc single aggregate production function may introduce endogeneity bias, utilizing cointegration properties derived from a theory-consistent model helps avoid spurious regressions and simultaneity bias. To illustrate this, we conduct a Johansen cointegration test (Johansen, 1991) employing a vector autoregressive (VAR) model.

Table 2. ADF unit root tests for logarithmic variables in levels and first differences.

	Variables in Levels		
	$\ln y_t$	$\ln k_t$	$\ln h_t$
Sample Period	1950-2019	1950-2019	1950-2019
Specification	constant and trend	constant and trend	constant and trend
Leg length	1	2	1
ADF t-statistic	−1.3525	0.3731	−0.9789
5% critical value	−3.4773	−3.4783	−3.4773
	Variables in First Differences		
	$\Delta \ln y_t$	$\Delta \ln k_t$	$\Delta \ln h_t$
Sample Period	1951-2019	1951-2019	1951-2019
Specification	constant	constant and trend	constant
Leg length	0	1	0
ADF t-statistic	−7.4104	−3.7476	−5.3323
5% critical value	−2.9048	−3.4783	−2.9048

a. Lag lengths were optimally chosen using the Schwarz Information Criterion (SIC), with a maximum lag length of 10. The decision to include a constant and/or trend depends on their graphical representation and significance in the ADF test equations.

3.3. Long-Run Cointegration Relationships

When multiple non-stationary variables share common stochastic trends, cointegration relationships can be observed through specific stationary and linear combinations, and this can be empirically tested using a VAR system encompassing these variables. In this process, it is imperative to select appropriate lag lengths by employing criteria such as the Schwarz Information Criterion, Akaike Information Criterion, and Hannan-Quinn Information Criterion. Subsequently, during the execution of Johansen cointegration tests, the VAR model with lag p, or VAR(p), undergoes transformation into a Vector Error Correction (VEC) model with lag $p - 1$, denoted as VEC ($p - 1$). The Johansen cointegration test outcomes within the VEC (1) model are summarized in Table 3.

According to Table 3, at a standard significance level of 5%, both the trace and maximum eigenvalue test statistics indicate the presence of two cointegration vectors (CVs) encompassing $\ln y_t$, $\ln k_t$, and $\ln h_t$, incorporating a time trend. However, only one CV is identified at a 1% significance level based on both test statistics. The prospect of one or two CVs aligns with the theoretical model's predictions. Following the transformation of the VAR model into a VEC model, two cointegration equations are derived for $\ln y_t$, $\ln k_t$, and $\ln h_t$, incorporating a time trend.

$$\ln y_t = 0.8599 + \underset{(0.1179)}{0.7871^{***}} \ln k_t + \underset{(0.0022)}{0.0086^{***}} t + \ln \hat{\epsilon}_t^P \quad \text{and} \tag{34}$$

$$\ln h_t = 15.9079 + \underset{(0.2538)}{1.4411^{***}} \ln k_t + \underset{(0.0048)}{0.0195^{***}} t + \ln \hat{\epsilon}_t^P, \tag{35}$$

where standard errors are in parentheses, and $\hat{\epsilon}_t^P$ represents the regression residual. The significance level of 1% is denoted by ***. The first CV represents a conventional production function in the Solow–Swan model, where labor and physical capital serve as the sole factor inputs under constant returns to scale. The estimated significant share of physical capital at 0.7871 signifies the United States as a conventionally capital-intensive economy. Additionally, this economy demonstrates exogenous growth, with an annual technological progress rate of 0.86% as the Harrod-neutral rate. This rate is inferred from the estimated coefficient derived from Equation (34), illustrating the time trend's impact.

Upon reducing the significance level to 1% in the Johansen cointegration tests,

Table 3. Johansen Cointegration Tests for $\ln y_t$, $\ln k_t$, and $\ln h_t$, a Linear Time Trend is Included.

Null hypothesis	Trace test statistic	Maximum eigenvalue test statistic	Conclusion
No CV	50.9652***	24.7706***	Reject the null
At most one CV	26.1946**	20.1138**	Reject the null
At most two CVs	6.0808	6.0808	Do not reject the null

a. Significance levels of 5% and 1% are denoted by ** and ***, respectively.

as indicated by both the trace and maximum eigenvalue test statistics in Table 3, the analysis identifies the presence of only one cointegration vector, characterized by Equation (36):

$$\ln y_t = 6.5657 + \underset{(0.1409)}{0.2761^{**}} \ln k_t + \underset{(0.1016)}{0.2645^{***}} \ln h_t + \underset{(0.0026)}{0.0147^{***}} t + \ln \hat{\epsilon}_t^P , \quad (36)$$

where standard errors are represented in parentheses, and the significance levels of 5% and 1% are denoted by ** and ***, respectively. The major estimates in Equation (36) exhibit statistical significance and fall within reasonable ranges.

Notably, the estimated shares of physical and human capital in the production process, denoted as α and β, respectively, approximate at 0.2761 and 0.2645, indicating their nearly equivalent magnitudes. This parity underscores the comparable importance of human capital stock alongside physical capital stock in shaping the trajectory of the US economy across a 70-year span. In addition, the estimated coefficient of the time trend suggests a consistent and exogenously driven technological growth rate of approximately 1.47% in the United States. This finding implies a stable trend indicating sustained external influences fostering technological advancements within the economy over the specified period.

Furthermore, drawing from the earlier theoretical section that derived the optimal investment share in human capital as $\delta\beta$, assuming a discount rate of $\delta = 0.95$ alongside the aforementioned estimated α and β, computations suggest a recommended allocation of approximately 25.13% and 26.23% of real GDP towards developing human capital and accumulating physical capital, respectively, to attain long-term economic optimality in the United States.

In our theoretical model, we categorize four scenarios in Table 1 based on the estimates of the coefficients. Table 4 provides a summary of the test results derived from formal log-likelihood ratio (LR) tests for the VEC model.

The first test evaluates the null hypothesis of constant returns to per-capita variables in Equation (33). With a reasonable level of significance, the rejection of $\alpha + \beta = 1$ in favor of $\alpha + \beta < 1$ could be justified, suggesting the potential presence of exogenous growth in the United States. The second test centers on the null hypothesis of a zero Harrod-neutral rate of technological progress ($x = 0$), and this hypothesis is rejected as well. These two outcomes are jointly

Table 4. Hypothesis tests differentiate endogenous and exogenous growth scenarios.

Cointegrating equation: $\ln y_t = \beta_0 + \beta_1 \ln k_t + \beta_2 \ln h_t + \beta_3 t + \ln \epsilon_t^P$ (Equation (33))				
Null hypothesis	Model implication	LR test statistic	Critical value	Conclusion
(1) $\beta_1 + \beta_2 = 1$	$\alpha + \beta = 1$	32.0027	$\chi^2_{0.05}(1) = 3.84$	Reject the null
(2) $\beta_3 = 0$	$x = 0$	39.5572	$\chi^2_{0.05}(1) = 3.84$	Reject the null
(3) $\beta_1 + \beta_2 = 1$ and $\beta_3 = 0$	$\alpha + \beta = 1$ and $x = 0$	39.5771	$\chi^2_{0.05}(2) = 5.99$	Reject the null

a. The calculation of LR test statistics is based on the determinants of residual covariance of restricted and unrestricted VEC models.

scrutinized and supported in the third test, affirming that, over the past 70 years, the United States has experienced periods of pure exogenous growth. During this phase, significant contributions from both human and physical capital were observed, even though these capital stocks exhibited diminishing returns.

3.4. Granger Causality Tests and Impulse Response Analysis

As our VEC model fulfills the stability condition owing to the stationarity of $\ln y_t$, $\ln k_t$, and $\ln h_t$ after first differencing, it allows us to conduct Granger causality tests. These tests aid in evaluating the predictability of one variable concerning another. The outcomes of the block homogeneity Granger causality tests are summarized in Table 5.

In a VAR or VEC system, a variable is considered relatively endogenous if it is Granger-caused by other variables without exhibiting reverse predictability. The findings from Table 5 indicate that the lagged effects of $\Delta \ln h_t$ and $\Delta \ln k_t$ on $\Delta \ln y_t$ lack significance, as evidenced by the associated test statistics—2.4176, 3.6241, and 4.3135—all failing to surpass the respective critical values. However, $\Delta \ln y_t$ significantly Granger causes both $\Delta \ln h_t$ and $\Delta \ln k_t$, supported by respective test statistics of 6.6776 and 15.3013, both surpassing the corresponding critical values. Moreover, $\Delta \ln h_t$ displays uni-directional Granger causality towards $\Delta \ln k_t$, with a causation strength of 6.3031 from $\Delta \ln h_t$ to $\Delta \ln k_t$, while the reverse causation from $\Delta \ln k_t$ to $\Delta \ln h_t$ is comparatively weaker at 2.5184. These Granger causality tests elucidate that real GDP per capita appears relatively exogenous compared to both physical capital per capita and human capital per capita. Among these variables, physical capital per capita emerges as the most endogenous within the system. These lagged effects provide insights into

Table 5. Granger Causality Tests for First-differenced $\ln y_t$, $\ln k_t$, and $\ln h_t$.

Null Hypothesis	χ^2 Statistic	Critical Value	Conclusion
$\Delta \ln k_t$ does not Granger cause $\Delta \ln y_t$	2.4176	$\chi^2_{0.05}(1) = 3.84$	Do not reject the null
$\Delta \ln h_t$ does not Granger cause $\Delta \ln y_t$	3.6241	$\chi^2_{0.05}(1) = 3.84$	Do not reject the null
$\Delta \ln k_t$ and $\Delta \ln h_t$ do not Granger cause $\Delta \ln y_t$	4.3135	$\chi^2_{0.05}(2) = 5.99$	Do not reject the null
$\Delta \ln y_t$ does not Granger cause $\Delta \ln k_t$	15.3013	$\chi^2_{0.05}(1) = 3.84$	Reject the null
$\Delta \ln h_t$ does not Granger cause $\Delta \ln k_t$	6.3031	$\chi^2_{0.05}(1) = 3.84$	Reject the null
$\Delta \ln y_t$ and $\Delta \ln h_t$ do not Granger cause $\Delta \ln k_t$	15.3367	$\chi^2_{0.05}(2) = 5.99$	Reject the null
$\Delta \ln y_t$ does not Granger cause $\Delta \ln h_t$	6.6776	$\chi^2_{0.05}(1) = 3.84$	Reject the null
$\Delta \ln k_t$ does not Granger cause $\Delta \ln h_t$	2.5184	$\chi^2_{0.05}(1) = 3.84$	Do not reject the null
$\Delta \ln y_t$ and $\Delta \ln k_t$ do not Granger cause $\Delta \ln h_t$	9.1806	$\chi^2_{0.05}(2) = 5.99$	Reject the null

a. The above Granger causality tests are performed at 5% level of significance.

the transmission channels among the variables. However, it is crucial to emphasize that these outcomes do not challenge our previous statements regarding the endogenous growth of per-capita output linked with per-capita physical and human capital inputs. The estimations primarily focus on establishing the long-term cointegration relationship among $\ln y_t$, $\ln k_t$, and $\ln h_t$, rather than on their short-term causal interactions.

These short-run Granger causal relationships enable us to conduct generalized impulse response analyses,[3] considering the test results within our VEC system order $\{\ln y_t, \ln h_t, \ln k_t\}$. While the impulse response analysis comprises nine interactions among the three variables, we choose to present two meaningful and intuitive figures. Firstly, **Figure 5** illustrates the responses of the two factor inputs, $\ln k_t$ and $\ln h_t$, to each other's shocks.

Figure 5 illustrates the dynamic relationship between two key components of economic growth: physical capital and human capital. The depicted responses highlight their reactions to sudden changes, or impulses, in each other.

The observed negative reaction of $\ln k_t$ to a positive shock in $\ln h_t$ suggests an interesting dynamic. When there is an unexpected increase in human capital, initially, there is a slight negative response in the increase of physical capital. This reaction might imply a shift in focus or resource allocation away from building more physical infrastructure immediately after a surge in human capital. It hints that in the short term, an emphasis on improving human skills or knowledge might temporarily reduce the immediate drive to invest in additional physical resources.

On the other hand, the positive response of $\ln h_t$ to a positive shock in $\ln k_t$ indicates a different dynamic. An unforeseen increase in physical capital seems

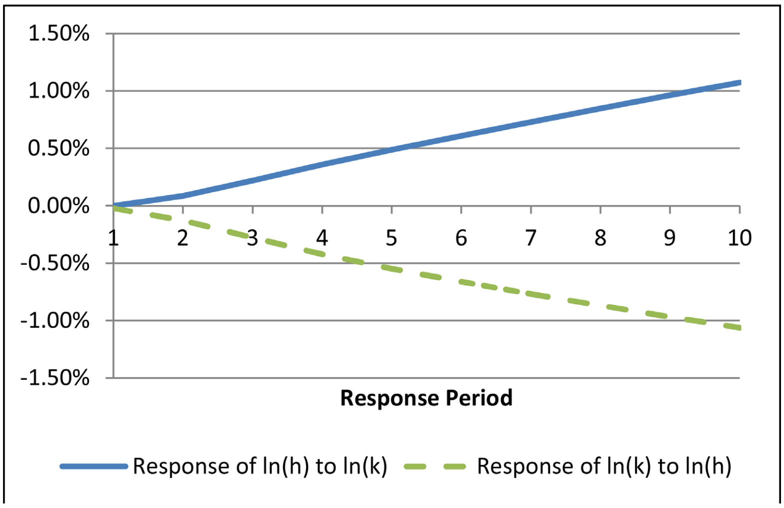


Figure 5. Mutual Impulse Responses between $\ln k_t$ and $\ln h_t$.

[3]To mitigate potential issues such as autocorrelated errors and causal-ordering problems inherent in conventional impulse responses, we employ the generalized impulse response functions proposed by (Koop et al., 1996) and (Pesaran & Shin, 1998). These methods are chosen to circumvent these specific challenges.

to stimulate the growth or enhancement of human skills or knowledge. This positive reaction suggests that improvements or expansions in physical infrastructure might create an environment conducive to the development of human capabilities. It implies that a well-equipped or technologically advanced infrastructure might foster an environment conducive to nurturing and improving human capital over time.

Secondly, a transient surge in output per capita, as depicted in Figure 6, gives rise to sustained elevations in the growth rates of both physical and human capital per capita.

Figure 6 portrays the aftermath of a sudden boost in output per capita on the growth rates of physical and human capital. It reveals an interesting pattern in their responses over time. Initially, when there is a positive jolt to output per capita, human capital experiences a significantly higher growth spurt compared to physical capital. This suggests that an immediate increase in productivity initially prompts a rapid surge in the growth of skills, education, or knowledge among the workforce, outpacing the initial growth in physical capital. However, as time progresses, the growth rate of physical capital catches up and begins to escalate steadily, indicating a more prolonged and continuous increase in the growth of infrastructure, machinery, or tangible assets. Meanwhile, the growth rate of human capital, after its initial rapid ascent, starts to diminish over time, ultimately plateauing or declining back towards normalcy. This pattern implies that while an initial productivity surge triggers a sharp rise in human capital growth, the growth rate of physical capital gradually accelerates and catches up, eventually surpassing or sustaining its growth, while the growth rate of human capital subsides or returns to baseline levels over extended periods.

4. Concluding Remarks

This study expands upon the theoretical framework established by (Mankiw,

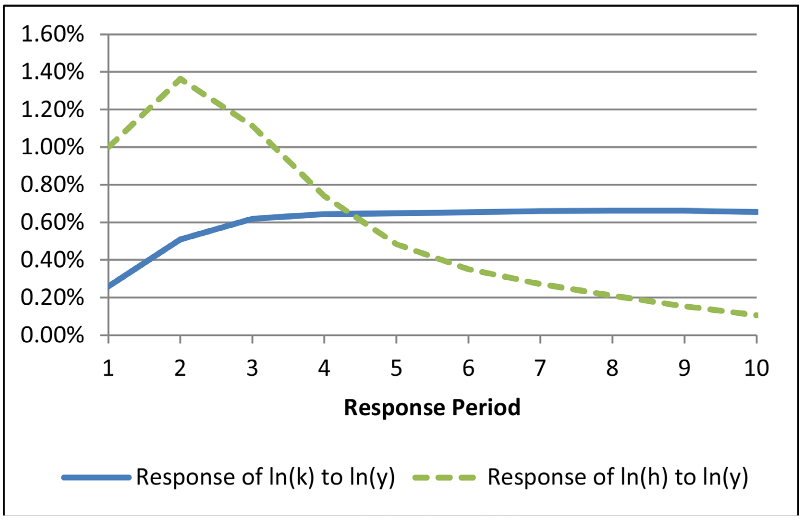


Figure 6. Responses of $\ln k_t$ to $\ln y_t$ and $\ln h_t$ to $\ln y_t$.

Romer & Weil, 1992) to explore the optimality problem, specifically delving into optimal resource allocation among consumption, physical capital, and human capital accumulation. During their era, data quality and quantity posed challenges, potentially leading to endogeneity bias that might have affected their estimated shares of physical and human capital inputs. However, in the present context, time-series data has become abundant, and corresponding econometric estimation techniques have matured. Therefore, the theoretical extensions made by (Mankiw et al., 1992) in this paper are substantiated through empirical analysis utilizing United States data. Over the period spanning 1950 to 2019, empirical evidence showcases long-run cointegration relationships among final output, physical capital, and human capital when measured per capita. The empirical estimates derived from the theory-consistent model indicate that allocating approximately 25.13% and 26.23% of real GDP to developing human capital and accumulating physical capital, respectively, would foster sustained economic optimality in the United States. The estimated shares of physical and human capital in the production process not only underscore their respective contributions to final output but also offer valuable insights for policymakers regarding optimal capital investment allocation. These findings furnish policymakers with crucial information when making decisions concerning resource allocation, thereby guiding strategic planning and policy formulation for sustainable economic development.

Moreover, the examination of per-capita variables reveals crucial insights into their short-term interdependencies and responses to varying impulse. These dynamics provide valuable glimpses into the immediate reactions of physical and human capital to changing economic conditions. Notably, the contrasting responses observed suggest an intricate relationship between these variables. In the short term, fluctuations in output per capita are observed to instigate sustained co-movement in the growth rates of both physical and human capital per capita, underscoring their reciprocal influences. Understanding these short-term interactions offers a practical understanding of immediate economic dynamics, aiding in more informed policy responses and resource allocation strategies.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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