Theoretical Economics Letters, 2023, 13, 1253-1265
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Capital Market Power and Economic Growth in an Overlapping-Generations Model with Rational Expectations

Paul Ritschel

Faculty of Business Studies and Economics, University of Kaiserslautern-Landau, Kaiserslautern and Landau in der Pfalz, Germany
Email: paul.ritschel@wiwi.uni-kl.de

How to cite this paper: Ritschel, P. (2023). Capital Market Power and Economic Growth in an Overlapping-Generations Model with Rational Expectations. *Theoretical Economics Letters, 13,* 1253-1265.
https://doi.org/10.4236/tel.2023.135069

Received: July 23, 2023
Accepted: October 9, 2023
Published: October 12, 2023

Abstract

This paper investigates how market power of agents in the capital market affects economic growth and output fluctuations in an overlapping-generations model with endogenous capital accumulation. Agents interact strategically by anticipating the influence of their savings behavior on the equilibrium return on capital. We demonstrate that imperfect competition reduces economic growth because agents under-save relative to the competitive benchmark. Moreover, it is shown that there exists a uniquely determined Nash-equilibrium trajectory of the economy. However, this trajectory may be non-monotonic and thus differ qualitatively from the perfect-foresight trajectory in the case of perfect competition. Competitive limits can be recovered through population growth. These findings imply that competition in the capital market is an important driver of strong and smooth economic growth.

Keywords

Overlapping Generations, Market Power, Imperfect Competition, Economic Growth, Endogenous Fluctuations

1. Introduction

The two-period lived overlapping-generations (OLG) framework is a workhorse model in modern macroeconomics. It is simple and has a wide range of applications, especially in the context of economic growth and the business cycle. Traditionally, the model assumes competitive markets and rational expectations of agents, cf. Azariadis (1993) and De La Croix and Michel (2002).

Over the past four decades, however, a small canvas of literature has emerged that relaxes the competitive-markets assumption by investigating the relationship between market power, economic growth, and endogenous fluctuations in OLG models. While, to the best of our knowledge, market power in the *capital market* has not yet been addressed in this field, there are several contributions that analyze imperfect competition in other markets. Imperfect competition in the *labor market* is examined in, among others, Jacobsen (2000); Dos Santos Ferreira and Lloyd-Braga (2002); and Coimbra, Lloyd-Braga, and Modesto (2005). Using both one-sector and multi-sector OLG models, imperfectly competitive *output markets* are analyzed in Laitner (1982); Chatterjee and Cooper (1989); Chatterjee, Cooper, and Ravikumar (1993); Rivard (1994); Aloi, Dixon, and Lloyd-Braga (2000); as well as Kaas and Madden (2005). Finally, Dos Santos Ferreira and Lloyd-Braga (2005, 2008) examine departures from perfect competition in the *intermediate goods market*. The OLG model developed in Goenka, Kelly, and Spear (1998), which displays the full variety of complex dynamics, is probably the contribution most closely related to this paper as it also explores strategic interaction on the side of the agents (and not firms).

The motivation of the paper at hand is twofold. Firstly, it is motivated by the gap in the literature mentioned above. Secondly, by the insight that in overlapping-generations models with endogenous capital accumulation and rational expectations, agents should have some market power in the capital market. More specifically, the course of this paper will demonstrate that rational agents with self-fulfilling beliefs should exploit their knowledge of the production function to influence the return on savings and, therefore, no longer take prices as given.

The paper contributes to the literature by demonstrating that if agents interact strategically in the capital market, then: 1) there exists a uniquely determined Nash-equilibrium trajectory of the economy and, 2) there will generally be a reduction in the steady-state output level. In addition, there may result an adverse impact on the qualitative dynamics as it no longer needs to be monotonic. Although periodic business cycles and complex dynamics are ruled out, which is surprising given the findings in Goenka et al. (1998), it is shown that the model allows for initial fluctuations that render its dynamics qualitatively distinct from the dynamics of the competitive benchmark.

The remainder of this paper is organized as follows. The next section introduces the basic model and discusses all necessary assumptions. Section 3 presents the competitive benchmark and derives its perfect-foresight dynamics. In Section 4, we investigate how market power affects agents' savings behavior, economic growth, and the induced dynamics. Section 5 concludes. All proofs are provided in the appendix.

2. Model Prerequisites

Consider a simple overlapping-generations model with endogenous capital accumulation, one sector, and discrete time $t = 0,1,\cdots,\infty$. There are markets for capital, labor, and a physical good. At the beginning of each period $t \geq 0$, a new generation comprising $N_t \in \mathbb{N}_+$ identical two-period lived agents is born. In $t = 0$, there exists an initial old generation endowed with capital $K_0 > 0$. In principle, the evolution of the population over time may be specified by any sequence $\{N_t\}_{t=0}^{\infty}$. However, for technical reasons and simplicity of the exposition, we assume that there exists some $\gamma \in \mathbb{R}_+$ such that $\{N_t\}_{t=0}^{\infty}$ is governed by

$$N_{t+1} = (1+\gamma) N_t, \quad N_0 > 0.$$

For notational convenience, we define $n := (1+\gamma)^{-1}$. Agents are risk-averse and their preferences are represented by a life-cycle utility function $U : \mathbb{R}_+^2 \to \mathbb{R}$ defined by

$$U(c^1, c^2) := u(c^1) + v(c^2),$$

where $c^1, c^2 \geq 0$ denote youthful and old-age consumption, respectively. We impose the following assumptions on agents' preferences.

Assumption 1 (Preferences). The utility functions u and v are twice continuously differentiable, strictly increasing, strictly concave, and satisfy the Inada conditions

$$\lim_{c \to 0} u'(c), v'(c) = \infty \quad \text{and} \quad \lim_{c \to \infty} u'(c), v'(c) = 0.$$

In addition, the Arrow-Pratt coefficient of relative risk aversion satisfies

$$0 < \frac{-v''(c)c}{v'(c)} \leq 1 \quad \text{for all } c \geq 0.$$

The properties of the utility functions stated in Assumption 1 are standard in the literature on OLG models. The Inada conditions cause that agents will always choose strictly positive levels of both youthful and old-age consumption. Moreover, the strict concavity implies that youthful and old-age consumption are normal goods and, in addition, that the marginal rate of intertemporal substitution is strictly increasing in the amount saved and supplied to the capital market. Finally, the restriction on the Arrow-Pratt coefficient ensures that an agent's optimal savings are non-decreasing in the return on savings, implying that youthful and old-age consumption are also gross substitutes.

The physical good is produced from capital $K \geq 0$ and labor $N \geq 0$ by a large number of firms that engage in perfect competition. The production technology is neoclassical with constant returns to scale. We denote by $k := K/N$ the capital-labor ratio and by $f : \mathbb{R}_+ \to \mathbb{R}_+$ the production function of the representative firm such that $y = f(k)$ is GDP per capita. Capital and labor are paid their respective marginal products while the price of the good is normalized to one. The capital depreciation rate during production is $0 \leq \delta \leq 1$. We assume that the production technology satisfies the following properties.

Assumption 2 (Technology). The production function f is thrice continuously differentiable, strictly increasing, strictly concave, and satisfies the strong Inada conditions

$$\lim_{k \to 0} f'(k) = \infty \quad \text{and} \quad \lim_{k \to \infty} f'(k) = 0.$$

In addition, *f* satisfies

$$\varepsilon_1(k) := \frac{f''(k)k}{f'(k)} > -1 \quad \text{and} \quad \varepsilon_2(k) := \frac{f'''(k)k}{f''(k)} > -2 \quad \text{for all } k \geq 0.$$

As will become clear shortly, the two elasticity properties imposed on *f* in Assumption 2 facilitate the existence and uniqueness of a solution to the agent's savings decision problem in the case of market power. In particular, they imply that the capital income of the old generation is well behaved. Note that these properties are satisfied by most standard production functions in the literature.

To complete the model, we normalize that each young agent supplies one unit of labor inelastically to a perfectly competitive labor market. Old agents are retired and consume the proceeds of their savings.

3. The Competitive Benchmark

This section briefly describes the case of perfect competition in the capital market and the induced perfect-foresight dynamics.[1] In each period *t*, agents form a *rational expectation* $R_t^e > 0$ with respect to the return on savings $R_{t+1} > 0$ realized in $t+1$. Agents take this expectation as given, i.e., act as price takers. The objective of the young agent is to maximize his utility of lifetime consumption. Given the wage income w_t and the expectation R_t^e, his savings decision problem is

$$\max_{0 \leq s \leq w_t} u(w_t - s) + v(R_t^e s). \tag{1}$$

By Assumption 1, the *savings function* $s_t = s(w_t, R_t^e)$ of the agent is well defined by the first-order condition

$$\frac{u'(w_t - s_t)}{v'(R_t^e s_t)} = R_t^e. \tag{2}$$

In period *t*, there are N_t identical young agents with homogeneous expectations so that the capital-labor ratio k_{t+1} of the subsequent period $t+1$ becomes

$$k_{t+1} = \frac{N_t}{N_{t+1}} s_t = ns(w_t, R_t^e). \tag{3}$$

Since capital is paid its marginal product, the return on savings realized in $t+1$ is

$$R_{t+1} = 1 + f'(k_{t+1}) - \delta = 1 + f'(ns(w_t, R_t^e)) - \delta. \tag{4}$$

Observe from (4) that the expected return R_t^e generally feeds back into the realized, market-clearing return R_{t+1}. Therefore, a prerequisite for rational ex-

[1]The results presented in this section are well-known. For a detailed exposition, we refer to Chapter 1 in De La Croix and Michelle (2002).

pectations in this OLG model is that agents know the production function *f* because, otherwise, they cannot take into account the expectations-feedback effect. Indeed, a rational expectation R_t^e is determined by

$$R_t^e = 1 + f'\left(ns\left(w_t, R_t^e\right)\right) - \delta$$

and defines a perfect forecasting rule in the sense of Böhm and Wenzelburger (1999) that depends on the fundamentals of the economy. However, if an agent knows the production function, then he might as well incorporate the feedback effect in his decision problem and anticipate how his savings affect the return on savings in equilibrium, or, in other words, exert market power. The informational requirements are the same as for rational expectations. This insight motivates us to analyze strategic interaction in the capital market. Before we do so, however, it remains to derive the dynamics of the benchmark model.

The wage income w_t of a young agent in period *t* is determined by the marginal product of labor,

$$w_t = w\left(k_t\right) := f\left(k_t\right) - f'\left(k_t\right)k_t. \tag{5}$$

From Equations (3)-(5), we can conclude that the evolution of capital-labor ratios under *perfect foresight* is governed by the implicit difference equation

$$k_{t+1} = ns\left(w\left(k_t\right), 1 + f'\left(k_{t+1}\right) - \delta\right), \quad t \ge 0. \tag{6}$$

It is well-known that Assumptions 1 and 2 imply that the time-one map

$$k_{t+1} = G\left(k_t\right) \tag{7}$$

defined by (6) is a single-valued function that lives globally on $\mathbb{R}_{++}$ (De La Croix & Michel, 2002). The following lemma demonstrates that the perfect-foresight dynamics induced by *G* does not permit endogenous fluctuations.

Lemma 1. Let Assumptions 1 and 2 be satisfied. Then $G' > 0$ such that the perfect-foresight dynamics generated by (7) is monotonic.

Lemma 1 rules out business cycles and complex dynamics. The growth path $\left\{k_t\right\}_{t=0}^{\infty}$ (i.e., the sequence of all competitive rational-expectations equilibria) generated by *G* is either monotonically increasing or decreasing, depending on the initial capital stock k_0.

4. Strategic Interaction

The preceding discussion has revealed that agents with rational expectations may exploit their knowledge of the production function to exert market power in the capital market. The corresponding savings decision problem is as follows. Consider a young agent $i \in \left\{1, \cdots, N_t\right\}$ in period *t*. The agent anticipates that saving the amount s_t^i leads to the realized return on savings

$$R_{t+1} = 1 + f'\left(\left(s_t^i + s_t^{-i}\right)N_{t+1}^{-1}\right) - \delta,$$

where $s_t^{-i} \ge 0$ denotes aggregate savings of all other young agents except *i*. Since agents now face a situation of strategic interaction (there is Cournotian competition between N_t symmetric players), we use *Nash equilibrium in pure strategies* as a solution concept. Formally, a pure strategy of agent *i* is an amount of savings s_t^i and his strategy set is

$$\mathcal{S}_t^i := \left\{ s^i : 0 \le s^i \le w_t \right\}.$$

Given the wage income w_t, agent *i*'s best response to s_t^{-i} is determined by a solution to

$$\max_{s^i \in \mathcal{S}_t^i} u\left(w_t - s^i\right) + v\left(\left(1 + f'\left(\kappa_t\left(s^i\right)\right) - \delta\right) s^i\right), \tag{8}$$

where $\kappa_t\left(s^i\right) := \left(s^i + s_t^{-i}\right) N_{t+1}^{-1}$ for notational reasons. Existence and uniqueness of the agent's best response is established in the lemma below.

Lemma 2. Under the hypotheses of Assumptions 1 and 2, let $w_t > 0$ and $s_t^{-i} \ge 0$ be given. Then there exists a unique solution $0 < s_t^i < w_t$ to Problem (8), which is determined by

$$\frac{u'\left(w_t - s_t^i\right)}{v'\left(\left(1 + f'\left(\kappa_t\left(s_t^i\right)\right) - \delta\right) s_t^i\right)} = 1 + f'\left(\kappa_t\left(s_t^i\right)\right)\left[1 + \frac{s_t^i}{s_t^i + s_t^{-i}} \varepsilon_1\left(\kappa_t\left(s_t^i\right)\right)\right] - \delta. \tag{9}$$

Next, we can exploit that young agents are homogeneous, implying that[2]

$$s_t^{-i} = \left(N_t - 1\right) s_t^i \quad \text{and} \quad \kappa_t\left(s_t^i\right) = n s_t^i.$$

Consequently, (9) becomes

$$\frac{u'\left(w_t - s_t\right)}{v'\left(\left(1 + f'\left(n s_t\right) - \delta\right) s_t\right)} = 1 + f'\left(n s_t\right)\left[1 + \frac{\varepsilon_1\left(n s_t\right)}{N_t}\right] - \delta. \tag{10}$$

It follows from Lemma 2 that Condition (10) admits a unique solution $s_t = \mathfrak{s}_t\left(w_t\right)$ that stipulates the savings of a young agent in a symmetric Nash equilibrium in period *t*. The term $\varepsilon_1\left(n s_t\right)/N_t$ in (10) captures how individual savings s_t affect the equilibrium return on savings R_{t+1} or, in other words, measures the market power of a young agent in period *t*. His degree of market power is determined by the elasticity of the marginal product of capital ε_1 and by the number of competitors in the capital market N_t. We are now in the position to state a central result.

Proposition 3. Let Assumptions 1 and 2 be satisfied. Then the following holds.

(i) If expectations R_t^e are rational, then $\mathfrak{s}_t\left(w\right) < s\left(w, R_t^e\right)$ for all $w > 0$ and all $t \ge 0$. In particular, $\mathfrak{s}_t\left(w\right) \to s\left(w, R_t^e\right)$ if and only if $N_t \to \infty$.

(ii) Nash-equilibrium savings satisfy $\mathfrak{s}_t'\left(w\right) > 0$ for all $w > 0$ and all $t \ge 0$.

Proposition 3 demonstrates that an agent with market power saves "strategically" less than his counterpart with rational expectations in order to realize a higher return. The competitive savings level obtains in the limit $N_t \to \infty$ because the weight of each individual agent then becomes negligibly small relative to the size of the capital market. In this case, the savings function of a price taker

[2] The symmetry of agents also implies that we can omit the superscript *i*.

reobtains and the Nash equilibrium coincides with the competitive rational-expectations equilibrium. At this point, it is worthwhile considering a short example.

Example 1. *Consider the log-linear preferences* $u(c^1)=\ln(c^1)$ *and* $v(c^2)=\beta\ln(c^2)$, *where* $\beta>0$, *combined with the Cobb-Douglas production function* $f(k)=Ak^{\alpha}$, *where* $A>0$ *and* $0<\alpha<1$. *Let capital depreciate fully,* $\delta=1$, *and the population grow exponentially,* $N_t=(1+\gamma)^t N_0$. *For this parameterization, the savings functions compute*

$$s(w_t,R_t^e)=\frac{\beta}{1+\beta}w_t \quad \text{and} \quad \mathfrak{s}_t(w_t)=\frac{\beta-\beta(1-\alpha)N_t^{-1}}{1+\beta-\beta(1-\alpha)N_t^{-1}}w_t.$$

It is easily verified that $\mathfrak{s}_t(w_t)\to s(w_t,R_t^e)$ as $N_t\to\infty$.

The accumulation of capital is examined next. Observe from (10) and the fact that labor is paid its marginal product, $w_t=w(k_t)$, that a Nash equilibrium in period t is determined by the capital-labor ratio k_t alone. Accordingly, the economy's *Nash-equilibrium trajectory* $\{k_t\}_{t=0}^{\infty}$ is uniquely determined by

$$k_{t+1}=\mathcal{G}_t(k_t):=n\mathfrak{s}_t(w(k_t)), \quad t\geq 0. \tag{11}$$

The following corollary is an immediate implication of Proposition 3 (i).

Corollary 1. Let Assumptions 1 and 2 be satisfied. Then $\mathcal{G}_t(k)<G(k)$ for all $k>0$ and all $t\geq 0$. In particular, $\mathcal{G}_t(k)\to G(k)$ if and only if $N_t\to\infty$.

Corollary 1 reveals that market power has a curbing effect on capital accumulation because agents strategically withhold funds from the production sector. Against this background, it is interesting to analyze how the economy's long-run development is affected. Denote by $k^{\text{Nash}}>0$ and $k^{\text{RE}}>0$ the limits $\lim_{t\to\infty}k_t$ of the sequences $\{k_t\}_{t=0}^{\infty}$ that are recursively generated by (11) and (7), respectively.[3] We may then state the following key result.

Proposition 4. Let Assumptions 1 and 2 be satisfied. If the limit k^{Nash} exists, then $k^{\text{Nash}}\leq k^{\text{RE}}$, where equality holds if and only if $\lim_{t\to\infty}N_t=\infty$.

Proposition 4 demonstrates that imperfect competition in the capital market negatively affects long-run economic growth due to reduced savings of agents. The competitive limit is recovered if and only if the population grows infinitely large. In this case, agents' market power is gradually eliminated and the long-term development of the economy is unaffected.

The final objective of this paper is to investigate whether market power may adversely affect the qualitative dynamics of the economy. It turns out that for a stationary population profile, $\gamma=0$, the dynamics induced by (11) is monotonic.

Lemma 5. Let Assumptions 1 and 2 be satisfied and suppose that $N_t=N_0$ for all $t\geq 0$. Then $\mathcal{G}_t\equiv\mathcal{G}$ for all $t\geq 0$ and $\mathcal{G}'>0$ such that the dynamics generated by (11) is monotonic.

[3]The existence of $k^{\text{RE}}>0$ is well-known. It is implied by the monotonicity of the perfect-foresight dynamics (7) and the fact that the production function f does not permit sustained economic growth.

If the population size is constant, then the number of competing agents in the capital market is time-invariant and strategic interaction has no impact on the qualitative dynamics at all. Indeed, the dynamics (11) is then qualitatively indistinguishable from the perfect-foresight dynamics (7).

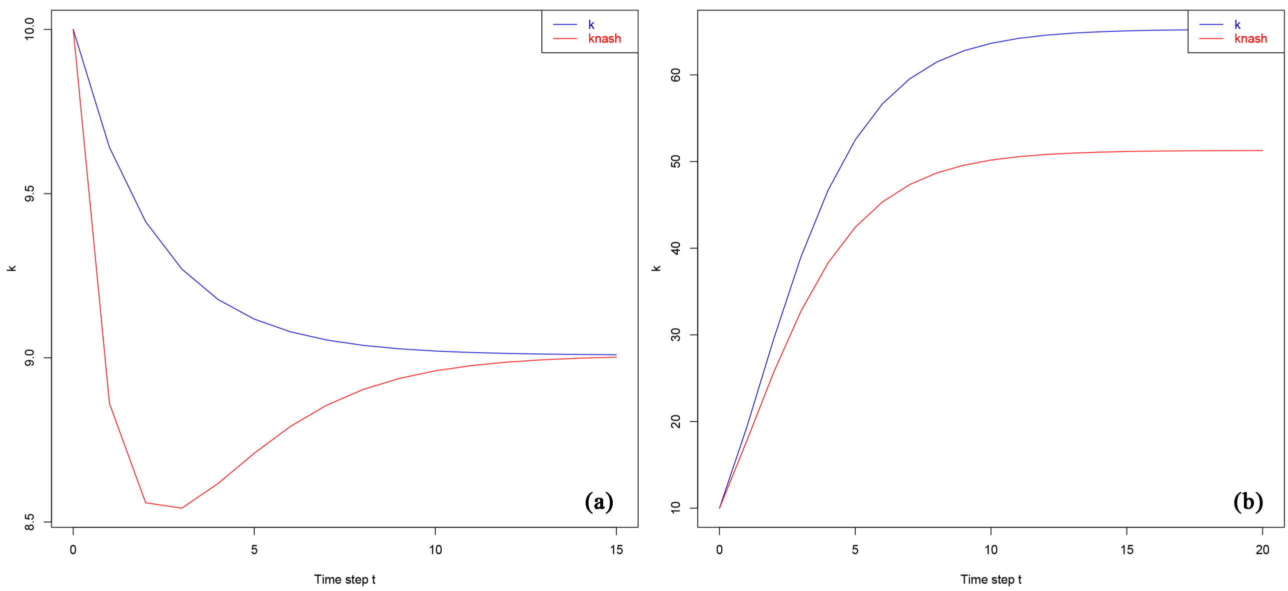
For a non-stationary population profile, however, the dynamics are *not* necessarily qualitatively equivalent. Indeed, the sequence of Nash equilibria $\{k_t\}_{t=0}^{\infty}$ generated by (11) may well be non-monotonic and output fluctuations can occur, as Example 2 below shows. Nevertheless, it should be emphasized that for $\gamma > 0$, the population growth eventually forces the competitive limit. Thus, initial fluctuations will sooner or later die out and the dynamics become monotonic. For this reason, the dynamical system (11) will never exhibit periodic equilibria or complex dynamics.

Example 2. *Conside the parameterization presented in Example* 1. *Figure* 1 *depicts the growth paths* $\{k_t\}_{t=0}^{\infty}$ *generated by* (7) *and* (11), *respectively.*

Clearly, Figure 1(a) *shows that the dynamics induced by* (11) *is non-monotonic whereas the perfect-foresight dynamics* (7) *is monotonic. Note that since* $N_\infty = \infty$, *the growth paths attain the same steady state* $k^{\text{Nash}} = k^{\text{RE}} > 0$. Figure 1(b), *on the contrary, shows that if* $N_\infty < \infty$, *then market power entails a reduction in long-run economic growth.*

5. Conclusion

Using a simple overlapping-generations model, this paper has demonstrated that market power in the capital market generally reduces real economic growth. The root cause is that agents strategically withhold funds from the productive sector



Imperfect Competition, Perfect Competition. $A = 37$, $\alpha = 0.65$, $\beta = 0.5$, $N_0 = 3$, $k_0 = 10$.

Figure 1. Qualitative dynamics depending on the form of competition in the capital market. (a) Different qualitative dynamics, $\gamma = 1$; (b) Reduction in economic growth, $\gamma = 0$.

in order to drive up the return on savings. Additionally, imperfect competition may adversely affect the qualitative dynamics of the economy as it no longer needs to be monotonic. These insights imply that maintaining competition and reducing barriers to entry in the capital market are crucial factors for strong and smooth economic growth. Future research should investigate this issue from an empirical perspective. Finally, analyzing if regulatory intervention can mitigate the adverse impact on economic growth could be a fruitful avenue for further research.

Acknowledgements

This paper has benefited from valuable comments and ideas of Jan Wenzelburger, Philipp Weinschenk, and Tom Rauber.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Aloi, M., Dixon, H. D., & Lloyd-Braga, T. (2000). Endogenous Fluctuations in an Open Economy with Increasing Returns to Scale. *Journal of Economic Dynamics and Control, 24,* 97-125. https://doi.org/10.1016/S0165-1889(98)00063-3

Azariadis, C. (1993). *Intertemporal Macroeconomics*. Wiley-Blackwell.

Böhm, V., & Wenzelburger, J. (1999). Expectations, Forecasting, and Perfect Foresight: A Dynamical Systems Approach. *Macroeconomic Dynamics, 3,* 167-186. https://doi.org/10.1017/S1365100599011025

Chatterjee, S., & Cooper, R. (1989). Multiplicity of Equilibria and Fluctuations in Dynamic Imperfectly Competitive Economies. *American Economic Review, 79,* 353-357.

Chatterjee, S., Cooper, R., & Ravikumar, B. (1993). Strategic Complementarity in Business Formation: Aggregate Fluctuations and Sunspot Equilibria. *Review of Economic Studies, 60,* 795-811. https://doi.org/10.2307/2298100

Coimbra, R., Lloyd-Braga, T., & Modesto, L. (2005). Endogenous Fluctuations in Unionized Economies with Productive Externalities. *Economic Theory, 26,* 629-649. https://doi.org/10.1007/s00199-004-0510-8

De La Croix, D., & Michel, P. (2002). *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*. Cambridge University Press. https://doi.org/10.1017/CBO9780511606434

Dos Santos Ferreira, R., & Lloyd-Braga, T. (2002). Can Market Power Sustain Endogenous Growth in Overlapping-Generations Economies? *Economic Theory, 20,* 199-205. https://doi.org/10.1007/s001990100190

Dos Santos Ferreira, R., & Lloyd-Braga, T. (2005). Non-Linear Endogenous Fluctuations with Free Entry and Variable Markups. *Journal of Economic Dynamics and Control, 29,* 847-871. https://doi.org/10.1016/j.jedc.2004.04.003

Dos Santos Ferreira, R., & Lloyd-Braga, T. (2008). Business Cycles with Free Entry Ruled by Animal Spirits. *Journal of Economic Dynamics and Control, 32,* 3502-3519. https://doi.org/10.1016/j.jedc.2007.06.018

Goenka, A., Kelly, D. L., & Spear, S. E. (1998). Endogenous Strategic Business Cycles. *Journal of Economic Theory, 81,* 97-125. https://doi.org/10.1006/jeth.1997.2380

Jacobsen, H. J. (2000). Endogenous, Imperfectly Competitive Business Cycles. *European Economic Review, 44,* 305-336. https://doi.org/10.1016/S0014-2921(98)00060-9

Kaas, L., & Madden, P. (2005). Imperfectly Competitive Cycles with Keynesian and Walrasian Features. *European Economic Review, 49,* 861-886. https://doi.org/10.1016/j.euroecorev.2003.08.008

Laitner, J. (1982). Monopoly and Long-Run Capital Accumulation. *Bell Journal of Economics, 13,* 143-157. https://doi.org/10.2307/3003436

Rivard, B. A. (1994). Monopolistic Competition, Increasing Returns, and Self-Fulfilling Prophecies. *Journal of Economic Theory, 62,* 346-362. https://doi.org/10.1006/jeth.1994.1019

Appendix. Proofs

Proof of Lemma 1. Implicit differentiation of (6) yields

$$G'(k)=\frac{\frac{\partial s}{\partial w}\left(w(k),1+f'(G(k))-\delta\right)w'(k)}{\frac{1}{n}-\frac{\partial s}{\partial R^e}\left(w(k),1+f'(G(k))-\delta\right)f''(G(k))}. \tag{12}$$

Assumption 1 implies that $\partial s(w,R^e)/\partial w>0$ and, if the Arrow-Pratt coefficient lies between zero and unity, also that $\partial s(w,R^e)/\partial R^e\geq 0$. Since, by Assumption 2, $w'>0$ and $f''<0$, we can conclude that (12) is strictly positive, proving that $G'>0$. □

Proof of Lemma 2. The first-order condition for Problem (8) is (9). The r.h.s. in (9) is strictly positive since, by Assumption 2, $f'>0$ and $\varepsilon_1>-1$. The Inada conditions imposed on u and v in Assumption 1 imply that for $s^i\to 0$, the l.h.s. in (9) converges to zero whereas it converges to infinity for $s^i\to w_t$. By the Intermediate Value Theorem, a solution $0<s_t^i<w_t$ to (9) thus exists. We next show that the objective function in Problem (8) is strictly concave such that the maximizer s_t^i is uniquely determined. The second-order condition for Problem (8) can be written as

$$\begin{aligned}&u''\left(w_t-s^i\right)+v''\left(\left[1+f'\left(\kappa_t\left(s^i\right)\right)-\delta\right]s^i\right)X_t\left(s^i,s_t^{-i}\right)^2\\&+v'\left(\left[1+f'\left(\kappa_t\left(s^i\right)\right)-\delta\right]s^i\right)\frac{f''\left(\kappa_t\left(s^i\right)\right)}{N_{t+1}}\left[2+\frac{s^i}{s^i+s_t^{-i}}\varepsilon_2\left(\kappa_t\left(s^i\right)\right)\right]\overset{!}{<}0,\end{aligned} \tag{13}$$

where, for notational convenience,

$$X_t\left(s^i,s_t^{-i}\right):=1+f'\left(\kappa_t\left(s^i\right)\right)-\delta+\frac{f''\left(\kappa_t\left(s^i\right)\right)}{N_{t+1}}s^i.$$

By Assumption 1, $u',v'>0$ as well as $u'',v''<0$. Since, by Assumption 2, $f''<0$ and $\varepsilon_2>-2$, we can infer that the l.h.s. in (13) is strictly negative. Hence, (13) is satisfied for all $s^i\geq 0$, implying that the objective function is indeed strictly concave. □

Proof of Proposition 3. *Part* (*i*). A rational expectation R_t^e is determined by

$$R_t^e=1+f'\left(ns\left(w_t,R_t^e\right)\right)-\delta. \tag{14}$$

From (14) and (2), it follows that the savings function $s\left(w,R_t^e\right)$, given a rational expectation R_t^e, solves

$$\frac{u'\left(w-s\left(w,R_t^e\right)\right)}{v'\left(\left[1+f'\left(ns\left(w,R_t^e\right)\right)-\delta\right]s\left(w,R_t^e\right)\right)}=1+f'\left(ns\left(w,R_t^e\right)\right)-\delta. \tag{15}$$

On the other hand, the function $\mathfrak{s}_t(w)$ is defined by

$$\frac{u'\left(w-\mathfrak{s}_t(w)\right)}{v'\left(\left[1+f'\left(n\mathfrak{s}_t(w)\right)-\delta\right]\mathfrak{s}_t(w)\right)}=1+f'\left(n\mathfrak{s}_t(w)\right)\left[1+\frac{\varepsilon_1\left(n\mathfrak{s}_t(w)\right)}{N_t}\right]-\delta. \tag{16}$$

Assumptions 1 and 2 imply that the marginal rate of intertemporal substitution on the l.h.s. of (15) and (16) is strictly increasing in savings. By contrast, the r.h.s. in (15) and (16) is strictly decreasing in savings owing to Assumption 2. Since

$$-1<\frac{\varepsilon_1\left(n\mathfrak{s}_t\left(w\right)\right)}{N_t}<0,$$

a comparison of (15) with (16) proves that $\mathfrak{s}_t\left(w\right)<s\left(w,R_t^e\right)$ for all $w>0,t\geq 0$. In particular, $\mathfrak{s}_t\left(w\right)\to s\left(w,R_t^e\right)$ if and only if $N_t\to\infty$ because (15) and (16) then coincide.

Part (ii). Implicit differentiation of (10) yields

$$\begin{aligned}\mathfrak{s}_t'\left(w\right)=u''\left(w-s_t\right)\Bigg(&u''\left(w-s_t\right)+v''\left(\left[1+f'\left(ns_t\right)-\delta\right]s_t\right)\\&\times\left[1+f'\left(ns_t\right)\left[1+\frac{\varepsilon_1\left(ns_t\right)}{N_t}\right]-\delta\right]\left[1+f'\left(ns_t\right)\left[1+\varepsilon_1\left(ns_t\right)\right]-\delta\right]\\&+v'\left(\left[1+f'\left(ns_t\right)-\delta\right]s_t\right)\frac{f''\left(ns_t\right)}{N_{t+1}}\left[1+N_t+\varepsilon_2\left(ns_t\right)\right]\Bigg)^{-1},\end{aligned}$$

where $s_t=\mathfrak{s}_t\left(w\right)$. By Assumption 1, $u',v'>0$ and $u'',v''<0$. Moreover, by Assumption 2, $f'<0$, $f''<0$, and $\varepsilon_2>-2$. Finally, since $N_t\geq 1$, we can conclude that $\mathfrak{s}_t'\left(w\right)>0$ for all $w,t\geq 0$.□

Proof of Corollary 1. The proof follows directly from the definition of G and $\mathcal{G}_t$ in (6) and (11), respectively, together with the properties of the savings functions s and $\mathfrak{s}_t$ established in Proposition 3 (i). □

Proof of Proposition 4. If $k^{\text{Nash}}>0$ exists, then it solves the fixed-point condition

$$k^{\text{Nash}}=n\mathfrak{s}_\infty\left(w\left(k^{\text{Nash}}\right)\right). \tag{17}$$

By contrast, $k^{\text{RE}}>0$ solves

$$k^{\text{RE}}=ns\left(w\left(k^{\text{RE}}\right),1+f'\left(G\left(k^{\text{RE}}\right)\right)-\delta\right). \tag{18}$$

Observe from (17) and (18) that $k^{\text{RE}}>k^{\text{Nash}}$ if and only if

$$s\left(w\left(k^{\text{RE}}\right),1+f'\left(G\left(k^{\text{RE}}\right)\right)-\delta\right)\overset{!}{>}\mathfrak{s}_\infty\left(w\left(k^{\text{Nash}}\right)\right). \tag{19}$$

Since $w'>0$ and $\mathfrak{s}_t'>0$ by Proposition 3 (ii), we have

$$\frac{\mathrm{d}\mathfrak{s}_t}{\mathrm{d}k}\left(w\left(k\right)\right)>0\quad\text{for all }k,t\geq 0$$

such that a sufficient condition for (19) is

$$s\left(w\left(k^{\text{RE}}\right),1+f'\left(G\left(k^{\text{RE}}\right)\right)-\delta\right)\overset{!}{>}\mathfrak{s}_\infty\left(w\left(k^{\text{RE}}\right)\right). \tag{20}$$

Since $R_\infty^e=1+f'\left(G_\infty\left(k^{\text{RE}}\right)\right)-\delta$ is a rational expectation, we can infer from Proposition 3 (i) that (20) is satisfied for any $k^{\text{RE}}>0$. Hence, $k^{\text{RE}}>k^{\text{Nash}}$. The competitive limit $k^{\text{RE}} = k^{\text{Nash}}$ obtains if and only if $\lim_{t\to\infty} N_t = \infty$ since (17) and (18) then coincide. □

Proof of Lemma 5. If $N_t = N_0$ for all $t \geq 0$, then the savings function $\mathfrak{s}_t$ defined by (10) becomes time-invariant and, therefore, also the capital accumulation law $\mathcal{G}_t$ defined in (11). Since $w' > 0$ and, by Proposition 3 (ii), $\mathfrak{s}' > 0$, we then obtain

$$\mathcal{G}'(k) = n\mathfrak{s}'(w(k))w'(k) > 0 \quad \text{for all } k \geq 0.$$

Hence, $\mathcal{G}' > 0$ such that the dynamics induced by (11) is monotonic. □