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Cost of Capital for Private Firms

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Abstract

This paper elaborates on a new default-based cost of capital estimation for private-held firms. We test the model's ability to incorporate systematic risk and size premium. Results highlight a positive and statistically significant effect of CAPM expected return and size premiums on this novel cost of capital measure. Beyond the utility in practice for private equity valuation, preliminary results are promising for application on a larger cross-country sample.

Keywords

Size Premium, Cost of Capital, Default Risk, Private Firms

1. Introduction

Most empirical studies on the cost of equity determinants focus on listed firms (see Wan, 2020 and Hmiden et al., 2022 among others) and, apart from comparable approaches (Abudy et al., 2016; Barg et al., 2021), private firms' cost of capital investigations are lacking.

This paper elaborates on a new default-based cost of capital estimation for private-held firms, implying the default probability of Italian Guarantee Fund rating. We test the ability of the model to incorporate both systematic risk and size premium, analyzing a sample of Italian equity valuation reports. We imply all the publicly available reports with sufficient data for determining the discount rate used in the estimation (43 documents). The sample size is in line with other studies on private firm valuation (see, for example Elnathan et al., 2010, based on 66 firms).

Results highlight a positive and statistically significant effect of CAPM expected return and size premiums on this novel cost of capital measure.

In the knowledge of authors, no attempts are given by literature relatively on the effect of size premium and systematic risk on the privately-held firm cost of capital. However, beyond the utility in practice for private equity valuation, preliminary results are promising for application on a larger cross-country sample.

The paper is structured as follows. Section 2 elaborates on the default-based cost of capital estimation and hypotheses. Section 3 presents the sample and the research design, while Section 4 reports the results. Section 5 concludes.

2. The Cost of Capital Estimation and Hypotheses

Private firms' cost of capital estimation is usually based on comparable stock data (Abudy et al., 2016 among others) or implies credit risk measures (Oricchio, 2012). An exception is the model of Cheung (1999) which is based on the same default probability, both for equity holders and debt-holders. However, these two categories have a different risk profiles. As a response, the following model recognizes the different risk positions for equity in respect of debt financing at the probability of default level.

In accordance with past literature (Solomon, 1963; Baxter, 1967; Turner, 2014), the basic idea is that for extremely high leverage ratio ($\frac{D}{V} \to 1, E \to 0$), the cost of debt approximates the cost of capital ($r_D \to r_0$). As a consequence, the cost of capital determination is just a special case of cost of debt estimation (Beltrame & Zorzi, 2022).

Given a certain stream of operating cash flows:

$$\left(F_0^{op}, F_1^{op}, \cdots, F_i^{op}, \cdots, F_N^{op}\right) \tag{1}$$

where a generic firm operating cash flow F_i^{op} can be viewed as the sum of equity cash flow and debt cash flow, since the part of operating cash flow not servicing the debt is distributed to equity-holders:

$$F_i^{op} = F_i^E + F_i^D \tag{2}$$

In the following, we report the assumptions for determining the cost of capital.

Assumption I: $F_i^E = 0$ *for every i*.

In order to set $F_i^E = 0$, $F_i^D \to F_i^{op}$, thus $F_i^{op} = F_i^D$.

The equality can be re-written, decomposing both F_i^{op} and F_i^D:

$$EBIT_i - \Delta IC_i = IE_i - \Delta BVD_i \tag{3}$$

where $EBIT$ (Earning before interests and taxes) is the firm operating income, IE are the interest expenses, ΔIC is the variation in invested capital and ΔBVD is the variation in the book value of financial debt.

Assumption II: $IE_i \to EBIT_i$ *and* $\Delta BVD_i \to \Delta IC$ *for every i*.

Imposing this restrictive assumption, we are able to extend $F_i^E = 0$ both for a stable stream of cash flows (steady-state framework) and for a time-varying cash flows. Note that in time = 0, the assumption II implies $BVD_0 = IC_0$, since $\Delta BVD_0 = BVD_0$ and $\Delta IC_0 = IC_0$.

A null F_i^E from $i = 0$ to $i = N$, both in steady state and non-steady state framework, implies an equity value equal to zero. And, as a consequence, an equity value equal to zero leads to $r_0 = r_D$. Using the $WACC$ formulae, for simplicity with no taxes, we have:

$$WACC = r_0 = r_E \frac{E}{V} + r_D \frac{D}{V} = r_E \frac{0}{D} + r_D \frac{D}{D} = r_D \quad (4)$$

Copeland et al. (2005) obtain the same result both with and with no taxes using a structural model.

Exploiting a risk-neutral framework and a recovery rate on equity equal to zero as in Cheung (1999), the cost of debt will be:

$$r_D = \frac{r_f + PD}{1 - PD} \quad (5)$$

where r_f is the risk-free rate and PD is the probability of default.

In the same way, we can estimate the unlevered cost of capital under Assumptions I and II:

$$r_{D,I,II} = r_0 = \frac{r_f + PD_{I,II}}{1 - PD_{I,II}} \quad (6)$$

where $PD_{I,II}$ is the probability of default calculated under I and II.

Our cost of capital estimation is directly dependent on $PD_{I,II}$ as a measure of default risk. Empirical literature shows that credit risk is affected by both idiosyncratic firm characteristics and systematic factors (Denis & Denis, 1995; Jorion & Zhang, 2009). This evidence leads to our first hypothesis:

H1. Systematic risk positively affects $r_{D,I,II}$.

Regarding the specific risk-cost of capital evidences, the size premium (Banz, 1981; Fama & French, 1992) can be shown as an additional idiosyncratic component rather than a systematic risk one (Lamoureux & Sanger, 1989). Extending the database of Fama and French (1992) to 2000 and implying a Fama-MacBeth regression, Malkiel and Xu (2004) show how an idiosyncratic risk measure absorbs the size effect. Since the cost of debt usually prices specific firm characteristics, we can formulate our second hypothesis:

H2. Size premium positively affects $r_{D,I,II}$.

3. Research Design, Sample and Cost of Capital Variable

3.1. Research Design

Our empirical analysis aims to test whether the measure of the cost of capital presented in the previous section can price both the systematic risk and size premium components. The deterministic part of the model can be defined as follows:

$$\text{TLA\_CoC} = g\left(\text{Systematic\_CoC}, \text{Size\_premium}, \text{Firm\_size}, \text{Other\_FixedEffects}\right) \quad (7)$$

where $g(\cdot)$ is a generic link function.

In Table 1, variables definitions and sources are reported.

The other fixed effects consider Sector (Industrial, Services, Commercial, Real estate, and Constructions) and year of observation.

Table 1. Variable definitions and sources.

Variable	Description	Source
TLA_CoC	The TLA_CoC indicates the alternative cost of capital measure based on a default probability of a fully levered firm and on a certain level of risk-free-rate used by analysist in the firm valuation. More in detail, the cost of capital is operationalized in four steps: 1) We calculate the ratios reported in the second column of the appropriate table in appendix (looking at the firm sector), linking coefficients, floor and cap to each ratio value; 2) We perform Equation (10) to have a final score; 3) Basing on the score range we associate the PD through Table 2 data; 4) Taking the risk-free selected by the expert in the valuation report and the PD we calculate the cost of capital through Equation (6).	Equity valuation reports and Amadeus Aida data base
Systematic_CoC	The Systematic_CoC is the CAPM-based calculation of the unlevered cost of capital. Unfortunately, in some reports Betas and market risk premium inputs are not indicated. The unlevered cost of capital is extrapolated from the firm equity value and the $WACC$ or the Cost of equity, accordingly to Modigliani and Miller (1963): $WACC = r_0\left(1-\frac{D}{V}t_c\right) \rightarrow r_0 = \frac{WACC}{\left(1-\frac{D}{V}t_c\right)}$, $r_E = r_0 + (r_0 - r_D)\frac{D}{E}(1-t_c) \rightarrow r_0 = \frac{r_E + r_D\frac{D}{E}(1-t_c)}{1+\frac{D}{E}(1-t_c)}$.	Equity valuation reports
Size_premium	The Size_premium is the spread applied by accounting experts in the equity report to price the firm size effect.	Equity valuation reports
Firm_size	Firm_size is the control variable and takes the value of the logarithm of assets in model 2 and the logarithm of revenues in Model 3.	Equity valuation reports

This table reports the variables used in the empirical analysis. Fixed effects are on Sector (Industrial, Services, Commercial, Real estate and Constructions) and year.

The model in Equation (7) considers a percentage measure as the response variable. For this reason, the classical linear model specification cannot be directly applied. We finally decided to consider a Beta regression as proposed by Ferrari and Cribari-Neto (2004). The parameterization proposed by these authors accounts for the specific behavior of the dependent variable. Y is supposed to be Beta distributed:

$$y \sim \mathcal{B}(\mu,\phi) \quad \text{with} \quad 0 < y < 1, \tag{8}$$

where $\mu \in (0,1)$ is the expected value for the distribution. The variance $VAR(y) = \mu(1-\mu)/(1+\phi)$ depends on both μ and $\phi > 0$ which represents the dispersion parameter (the larger it is the smaller the variance observed in the data). For Beta distribution, the variance of the response variable is a function of μ. This characteristic renders the regression model based on this parameterization is heteroskedastic.

The model, as in the generalized linear model class, considers the estimation of the population mean based on a link function that we considered to be the logit transformation and, in its basic formulation, presents a fixed dispersion parameter. The logit link function is as follows:

$$g(\mu) = \log\left(\frac{\mu}{1-\mu}\right). \tag{9}$$

The estimated parameters can be interpreted as log-odds ratios connected to the explicative variables given the model specification. In short, positive values correspond to a positive effect on the odds values and, consequently, on the estimated proportion (percentage). Negative parameters can be interpreted specularly.

The model estimation is obtained considering the maximum likelihood approach using R statistical software (R Core Team, 2022) and, in particular, "betareg" library described in Cribari-Neto and Zeileis (2010).

3.2. Sample

The financial data is hand collected, using Italian data of equity valuation reports of accounting experts. We explore and collect data on Google, digiting the appropriate keywords: we write "valutazione" (valuation), "perizia" (appraisal) and "capitale economico" (equity value), going until the last page of the Google results. We use all the utilizable reports available from 2003 to 2022, collecting 43 observations. It was often necessary to complete the reports using the financial data from the AIDA database.

3.3. The $PD_{I,II}$ Determination through the Italian Guarantee Fund Rating

The Guarantee Fund for SMEs is an instrument set up by the Italian Ministry of Economic Development through Law no. 662/96 to facilitate access to credit for small businesses. This support is favored through the concession of a public guarantee that replaces collateral and personal guarantees normally provided by companies and entrepreneurs. Guarantees are granted after a rating assessment, substantially in line with the rating systems commonly used by credit intermediaries (financial ratios, corrected for bank relationships' elements and other warning events). It is possible to elaborate a rating and a *PD* just through the financial ratios, calibrated for considering firm's legal form (sole owner firm, non-limited company and limited company), accounting type (simplified or ordinary) and sector (industrial firm, constructions, commercial firm, services and real estate). The financial ratios-based rating is the result of four steps:

1) Financial and economic ratios calculation;

2) Ratios normalization (i.e. to normalize a ratio denominator equal to zero);

3) Dummy calculations;

4) Final score calculation (the system multiplies normalized ratios/dummies and coefficients to obtain the total score). In formulas:

$$\text{Firm score} = \text{Constant} + \sum\nolimits_{i=1}^{N} \text{Variable}_i \times \text{Coefficient}_i \tag{10}$$

The firm score can be obtained through the use of a platform made available by the fund (https://fdg.mcc.it/rating/) or using the formulas reported in this Italian guarantee fund technical document:

https://www.fondidigaranzia.it/wp-content/uploads/2019/12/Specifiche-tecniche-per-il-calcolo-della-probabilità-di-inadempimento-dal-20200215.pdf.

In **Appendix**, we report **Table A1**, where we provide all the details useful for the score computation for each sector: constant, variables and coefficients for firm forms and sectors composing the sample of the study. The tables also report the scores under Assumptions I and II. Finally, the $PD_{I,II}$ can be associated by looking at the correspondences in **Table 2**.

Table 2. Scoring, probability of default of the Italian Guarantee Fund and descriptive statistics.

Rating	Score Range (low)	Score Range (High)	PD
1	−999,999	−4.7066745760	0.12%
2	−4.7066745760	−4.4338240620	0.33%
3	−4.4338240620	−4.2547779080	0.67%
4	−4.2547779080	−3.8889098170	1.02%
5	−3.8889098170	−3.4677848820	1.61%
6	−3.4677848820	−3.2130939960	2.87%
7	−3.2130939960	−2.8844139580	3.62%
8	−2.8844139580	−2.6198046210	5.18%
9	−2.6198046210	−2.1981980800	8.45%
10	−2.1981980800	−1.5324805970	9.43%*
11	−1.5324805970	999,999	16.30%*

Variable	Mean	Median	St. Dev.	Min	Max
TLA_Coc	0.077	0.067	0.040	0.021	0.200
Systematic_CoC	0.066	0.068	0.020	0.025	0.100
Size_Premium	0.008	0.000	0.014	0.000	0.045
LnRevenues	14.716	15.013	2.200	9.210	17.871
LnAssets	14.883	15.175	1.969	10.644	17.867
Year				2003	2021
	%				
Sector (*composition*)					
…*Commercial*	5%				
…*Construction*	2%				
…*Industrial*	21%				
…*Real estate*	2%				
…*Services*	70%				

The above part is an author elaboration from https://fdg.mcc.it/rating/. The table allows us to link the firm score and the firm score under I and II to *PD* and $PD_{I,II}$ respectively. *In the absence of non-accounting information, the original model attributes the PD of class 11 (16.30%) to class 10 and introduces a PD of class 12 (22.98%) attributed to class 11. In our empirical analysis, we preferred to keep the PDs of class 10 and 11 without making these adjustments, to avoid anomalous jumps in probability for riskier classes.

4. Results

On the right, Table 2 shows a summary description of the involved variables. The main result of this analysis is that data highlights a great presence of services firms in respect of other sectors. For this reason, a dummy variable is implied (1 = Service firm; 0 Otherwise) to better capture the effect and the magnitude of the Services sector. The response variable presents a range of observations that is shrunk toward zero and an approximately symmetric distribution (as suggested by the comparison of mean and median values). Similar behavior is observed for Systematic CoC. Size_Premium shows many null observations. The variables describing the firms' sizes (LnRevenues and LnAssets) have been transformed by logarithms to solve the asymmetry issues in their distributions, and they present a similar characterization. The Year of observation varies between 2003 and 2021. The number of observations by year ranges from one to eight. For the sake of simplicity, the year is finally considered as a linear trend in the model (but more flexible solutions, such as time polynomials and splines, have been tried too).

Table 3 shows the models' estimation results. To enhance the model interpretation, we multiplied the observed values of Systematic_CoC and Size_Premium by 100. This way, a unit variation in these variables corresponds to a change by a factor e^{β} in the odds. This also can be approximately interpreted as a variation in the probability measure.

Table 3. Size premium, systematic risk premium and cost of capital.

	(1)	(2)	(3)
(Intercept)	74.057*	65.373	73.519*
	(41.050)	(43.878)	(41.088)
Year	−0.038*	−0.034	−0.038*
	(0.020)	(0.022)	(0.020)
Dummy Service = 1	−0.124	−0.094	−0.112
	(0.157)	(0.168)	(0.161)
LnAssets		0.020	
		(0.042)	
LnRevenues			0.010
			(0.035)
Size_premium	0.109**	0.109**	0.111**
	(0.052)	(0.052)	(0.052)
Systematic_CoC	0.081**	0.077**	0.077**
	(0.037)	(0.038)	(0.039)
Pseudo R^2	0.234	0.240	0.237

The table reports the three models useful to test the effect of systematic and size premium on TLA Cost of capital. Model (1) is with no size effects, (2) with Size = LnAssets and (3) with Size = LnRevenues. ** and * denote statistical significance at the 5% and 10% levels, respectively. Standard errors are reported in parentheses.

All the models highlight a positive and statistically significant effect of CAPM systematic risk and size premium on the alternative measure of cost of capital (TLA_CoC), confirming hypotheses 1 and 2. Looking at the slope coefficients in the models, a 1% increase in the CAPM cost of capital due to a different business and operating risk profile get an 8.0% - 8.4% increase in the TLA Cost of capital. A similar argumentation can be considered for the Size_premium variable obtaining an estimated positive effect between 11.5% - 11.7%. On average the overall cost of capital is higher with respect to CAPM cost of capital (7.7% versus 6.6%), including both a size premium effect (0.83%) and the rest (7.7% - 6.6% - 0.83%) as a generic idiosyncratic premium.

The common measures of size cannot catch the true activity dimension and complexity of the firm operating process. Empirical results of our study support this view since (1) the Firm_size never affects the cost of capital, and rather (2) analysts are able to incorporate the true firm size (in terms of operating process, costs, etc.) in the Size_premium, affecting the overall cost of capital.

Models (1) and (3) highlight a negative relation between Year and TLA_CoC. The TLA_CoC is decreasing during the time range of this study.

5. Conclusion

The model presented in this paper recognizes the different risk position for equity in respect of debt financing at the probability of default level, exploiting a framework in line with past studies (Solomon, 1963; Baxter, 1967; Copeland et al., 2005; Turner, 2014; Beltrame et al., 2014; Beltrame & Zorzi, 2022). In addition, we empirically highlight the ability of the model to incorporate both systematic and size premiums.

Results highlight the usefulness of the model for private equity and investment project valuations. Moreover, these preliminary results could pose the basis for a future large cross-country empirical investigation. A limitation of the study is that the *PD* calculation model is tailored to Italian SMEs, it should be revised for application in other countries.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Abudy, M., Benninga, S., & Shust, E. (2016). The Cost of Equity for Private Firms. *Journal of Corporate Finance, 37,* 431-443. https://doi.org/10.1016/j.jcorpfin.2016.01.014

Banz, R. (1981). The Relationship between Return and Market Values of Common Stock. *Journal of Financial Economics, 9,* 3-18. https://doi.org/10.1016/0304-405X(81)90018-0

Barg, J. A., Drobetz, W., & Momtaz, P. P. (2021). Valuing Start-Up Firms: A Reverse-Engineering Approach for Fair-Value Multiples from Venture Capital Transactions. *Finance Research Letters, 43,* Article ID: 102008.

https://doi.org/10.1016/j.frl.2021.102008

Baxter, N. D. (1967). Leverage, Risk of Ruin and the Cost of Capital. *The Journal of Finance, 22,* 395-403. https://doi.org/10.1111/j.1540-6261.1967.tb02975.x

Beltrame, F., & Zorzi, G. (2022). Pricing the Idiosyncratic Risk in the Cost of Capital: A Comprehensive Model. *Theoretical Economics Letters, 12,* 1221-1226. https://doi.org/10.4236/tel.2022.125065

Beltrame, F., Cappelletto, R., & Toniolo, G. (2014). *Estimating SMEs Cost of Equity Using a Value at Risk Approach: The Capital at Risk Model*. Springer. https://doi.org/10.1057/9781137389305

Cheung, J. (1999). A Probability-Based Approach to Estimating Cost of Capital for Small Business. *Small Business Economics, 12,* 331-336. https://doi.org/10.1023/A:1008067301393

Copeland, T. E., Weston, J. F., & Shastri, K. (2005). *Financial Theory and Corporate Policy*. Pearson Addison Wesley.

Cribari-Neto, F., & Zeileis, A. (2010). Beta Regression in R. *Journal of Statistical Software, 34,* 1-24. https://doi.org/10.18637/jss.v034.i02

Denis, D. J., & Denis, D. K. (1995). Causes of Financial Distress Following Leveraged Recapitalizations. *Journal of Financial Economics, 37,* 129-157. https://doi.org/10.1016/0304-405X(94)00792-Y

Elnathan, D., Gavious, I., & Hauser, S. (2010). An Analysis of Private versus Public Firm Valuations and the Contribution of Financial Experts. *International Journal of Accounting, 45,* 387-412. https://doi.org/10.1016/j.intacc.2010.09.001

Fama, E. F., & French, K. R. (1992). The Cross-Section of Expected Stock Returns. *The Journal of Finance, 47,* 427-465. https://doi.org/10.1111/j.1540-6261.1992.tb04398.x

Ferrari, S., & Cribari-Neto, F. (2004). Beta Regression for Modelling Rates and Proportions. *Journal of Applied Statistics, 31,* 799-815. https://doi.org/10.1080/0266476042000214501

Hmiden, O. B., Rjiba, H., & Saadi, S. (2022). Competition through Environmental CSR Engagement and Cost of Equity Capital. *Finance Research Letters, 47,* Article ID: 102773. https://doi.org/10.1016/j.frl.2022.102773

Jorion, P., & Zhang, G. (2009). Credit Contagion from Counterparty Risk. *The Journal of Finance, 64,* 2053-2087. https://doi.org/10.1111/j.1540-6261.2009.01494.x

Lamoureux, C. G., & Sanger, G. C. (1989). Firm Size and Turn-of-the-Year Effects in the OTC/NASDAQ Market. *The Journal of Finance, 44,* 1219-1245. https://doi.org/10.1111/j.1540-6261.1989.tb02651.x

Malkiel, B. G., & Xu, Y. (2004). *Idiosyncratic Risk and Security Returns*.

Modigliani, F., Miller, M. H. (1963). Corporate Income Taxes and the Cost of Capital: A Correction. *American Economic Review, 53,* 433-443.

Oricchio, G. (2012). *Private Company Valuation: How Credit Risk Reshaped Equity Markets and Corporate Finance Valuation Tools*. Springer. https://doi.org/10.1057/9781137271785

R Core Team (2022). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing. https://www.R-project.org

Solomon, E. (1963). Leverage and the Cost of Capital. *The Journal of Finance, 18,* 273-279. https://doi.org/10.1111/j.1540-6261.1963.tb00723.x

Turner, J. A. (2014). Teaching the Effects of Risky Debt and Financial Distress Costs Using Consistent Examples. *Journal of Financial Education, 49,* 114-139.

Wan, Z. (2020) Financial Constraint, Property Right and Cost of Capital. *Modern Economy, 11,* 443-452. https://doi.org/10.4236/me.2020.112033

Appendix

Table A1 reports the ratios, dummies, and coefficients necessary to obtain the score for a single firm and for each sector. The value of a single ratio/dummy is normalized in term of denominator and range from a floor to a cap reference. The column "Variable under I and II assumptions" report the revised ratio and dummies for a fully levered firm; a net income equal to zero implies interests expenses equal to EBIT, an equity capital equal to zero implies an amount of debts equal to the effective debts plus book value of equity.

Table A1. Variables and coefficients by sectors.

Industrial sector					
Variable	**Variable under I, II**	**If Denom. = 0 Ratio Equal to:**	**Floor**	**Cap.**	**Coeff.**
Constant	Constant				−4.584023
Current Debts/Revenues	Current debts/Revenues	1	0.4	1.4	1.709764
Interests Expenses/EBITDA	EBIT/EBITDA	0.1	−1	1	1.006155
Interests Expenses/Debts	EBIT/(Debts + Book Value of Equity)	0.06	0.01	0.06	21.7339
Cash/Revenues	Cash/Revenues	0.2	0.01	0.3	−3.257.383
Revenues/Inventory	Revenues/Inventory	11	1.4	11	−0.035931
% Variation of Revenues − 0.1	% Variation of Revenues − 0.1	0.2	−0.4	0.6	0.874921
Book Value of Equity/Assets	Final value = 0	0.1	0	0.64	−1.842869
Dummy = Interest Expenses/ EBITDA If EBITDA < 0; 0 Otherwise	Dummy = EBIT/EBITDA If EBITDA < 0; 0 Otherwise				−1.380648
Dummy = 1 If EBITDA < 0; 0 Otherwise	Dummy = 1 If EBITDA < 0; 0 Otherwise				0.502537
Dummy = % Variation of Revenues If % Variation of Revenues < 0; 0 Otherwise	Dummy = % Variation of Revenues If % Variation of Revenues < 0; 0 Otherwise				−1.318575
Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise	Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise				0.925375
Current Debts/Revenues × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	Current Debts/Revenues × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)				−0.672704
Interests Expenses/Debts × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	EBIT/(Debts + Book Value of Equity) × (Dummy = 1 if Revenues ≤ 500,000; 0 Otherwise)				−11.51058
Cash/Revenues × (Dummy = 1 if Revenues ≤ 500,000; 0 Otherwise)	Cash/Revenues × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)				1.934049

Construction Sector					
Variable	**Variable under I, II**	**If Denom. = 0 Ratio Equal to:**	**Floor**	**Cap.**	**Coeff.**
Constant					−4.258458
Interests Expenses/EBITDA	EBIT/EBITDA	1	−1	1	0.37765

Continued

Interests Expenses/Debts	EBIT/(Debts + Book Value of Equity)	0.01	0	0.03	34.64145
Book Value of Equity/Assets	Final Value = 0	0.03	0	2	−1.882866
Debts/Value of Production	(Debts + Book Value of Equity)/ Value of Production	1	0	1	1.314629
Current Liabilities/Assets	Current Liabilities/Assets	0.8	0	1	0.448655
Net Income/Value of Production	Final Value = 0	0.05	0	0.07	−5.638927
Book Value of Equity/Fixed Assets	Final Value = 0	3	0	8	−0.05176
% Variation of Value of Production − 0.1	% Variation of Value of Production − 0.1	0.2	−0.6	1.6	0.329288
Dummy = Interest Expenses/ EBITDA If EBITDA < 0; 0 Otherwise	Dummy = EBIT/EBITDA If EBITDA < 0; 0 Otherwise				−0.779867
Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise	Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise				0.48568
Dummy = % Variation of Value of Production If % Variation of Value of Production < 0; 0 Otherwise	Dummy = % Variation of Value of Production if % Variation of Value of Production < 0; 0 Otherwise				−0.998434
Debts/Value of Production × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	(Debts + Book Value of Equity)/ Value of Production × (Dummy = 1 if Revenues ≤ 500,000; 0 Otherwise)				−0.655727

Real Estate Sector					
Variable	**Variable under I, II**	**If Denom. = 0 Ratio Equal to:**	**Floor**	**Cap。**	**Coeff.**
Constant					−2.569235
Interests Expenses/EBITDA	EBIT/EBITDA	0.8	−0.8	1	0.8130648
Interests Expenses/Debts	EBIT/(Debts + Book Value of Equity)	0.01	0	0.06	14.0119
Book Value of Equity/Assets	Final Value = 0	0.1	0	1	−2.721187
Value of Production/Current Assets	Value of Production/Current Assets	1.5	0.3	10	−0.1391083
Dummy = Interest Expenses/ EBITDA If EBITDA < 0; 0 Otherwise	Dummy = EBIT/EBITDA if EBITDA < 0; 0 Otherwise				−1.401464
Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise	Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise				−0.5688427
Book Value of Equity/Assets × (Dummy = 1 if Revenues ≤ 500,000; 0 Otherwise)	Final Value = 0				1.765224

Commercial Sector					
Variable	**Variable under I, II**	**If Denom. = 0 Ratio Equal to:**	**Floor**	**Cap.**	**Coeff.**
Constant					−1.88977

Continued

Interests Expenses/EBITDA	EBIT/EBITDA	1	−1	1	0.73753
Interests Expenses/Debts	EBIT/(Debts + Book Value of Equity)	0.06	0	0.08	16.97147
Cash/Revenues	Cash/Revenues	0.02	0.01	0.1	−3.97341
% Variation of Revenues − 0.06	% Variation of Revenues − 0.06	0.24	−0.36	0.54	1.446892
Book Value of Equity/Assets	Final Value = 0	0.04	0	1.6	−2.86327
EBITDA/(Interest Expenses + Debts)	EBITDA/(EBIT + Debts + Book Value of Equity)	0.2	0	0.3	−1.68061
(Current Assets-Inventory)/ Current Liabilities	(Current Assets-Inventory)/ Current Liabilities	2	0	2	−0.33307
Revenues/Assets	Revenues/Assets	0.9	0.5	1.7	−0.85672
Dummy = Interest Expenses/ EBITDA If EBITDA < 0; 0 Otherwise	Dummy = EBIT/EBITDA If EBITDA < 0; 0 Otherwise				−1.3164
Dummy = % Variation of Revenues If % Variation of Revenues < 0; 0 Otherwise	Dummy = % Variation of Revenues If % Variation of Revenues < 0; 0 Otherwise				−2.98436
Interests Expenses/Debts × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	EBIT/(Debts + Book Value of Equity) × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)				−8.28285
Book Value of Equity/Assets × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	Final Value = 0				1.368938
Revenues/Assets × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	Revenues/Assets × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)				0.207691

Services Sector					
Variable	**Variable under I, II**	**If Denominator = 0 Ratio Equal to:**	**Floor**	**Cap.**	**Coefficient**
Constant					−4.689249
Current Debts/Revenues	Current debts/Revenues	2	0.2	2.5	0.427293
Cash/Revenues	Cash/Revenues	0.04	0.01	0.16	−7.428313
% Variation of Revenues − 0.06	% Variation of Revenues − 0.06	0.14	−0.36	0.84	0.668981
Current Liabilities/Assets	Current Liabilities/Assets	0.8	0	1	0.82794
Interest expenses/Value of Production	EBIT/Value of Production	0.04	0	0.04	29.88155
Debts/Book Value of Equity	10 (Since Denominator Is Equal to Zero)	10	−2	20	0.031407
Variable = 1 If Interests Expenses/ EBITDA < 0 and EBITDA < 0; Otherwise Interests Expenses/EBITDA	Variable = 1 If EBIT/EBITDA < 0 and EBITDA < 0; Otherwise EBIT/EBITDA	Not Nec.	Not Nec.	Not Nec.	0.400514
Dummy = % Variation of Revenues If % Variation of Revenues < 0; 0 Otherwise	Dummy = % Variation of Revenues If %Variation of Revenues < 0; 0 Otherwise				−1.558519

Continued

Current debts/Revenues × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	Current Debts/Revenues × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	−0.245754
Cash/Revenues × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	Cash/Revenues × (Dummy = 1 If Revenues ≤ 500,000; 0 Otherwise)	5.362561
Dummy = 1 If Book Value of Equity < 0; 0 Otherwise	0 (Since the Book Value of Equity Is Zero in a Fully Levered Situation)	0.542214