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Investment Demand Beliefs and Involuntary Unemployment in a Stock Market Overlapping Generations Model

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Abstract

It is the aim of this paper to model involuntary unemployment in Magill and Quinzii's (2003) seminal stock market model with non-shiftable capital and affine equity-price expectations. In contrast to New-Keynesian macro-models, unemployment is not traced back to inflexible prices and wage rates, but to inflexible aggregate investment based on investors' "beliefs" (Farmer, 2020) about investment demand. After setting up the stock market model, sufficient conditions for the existence and dynamic stability of a Golden Rule steady state with involuntary unemployment are presented and the comparative dynamics of this steady state are investigated. While an increase in investor optimism decreases unemployment in the short and long run, a smaller savings rate does this only temporarily.

Keywords

Involuntary Unemployment, Investors' Beliefs, Stock Market OLG Model, Existence, Dynamic Stability and Comparative Dynamics of Steady States

1. Introduction

Mainstream macroeconomics adhering to the New-Keynesian approach of dynamic stochastic general equilibrium (DSGE) models trace involuntary unemployment back to inflexible prices and wage rates due to price adjustment costs, imperfect competition, and other forms of market failure (for a survey see Dixon, 2000). In contrast, a minority of macro-oriented general equilibrium modelers employ stylized, dynamic, intertemporal general equilibrium with perfect competition in factor and output markets in order to feature involuntary unemployment. In line with Morishima (1977) and more recently Magnani (2015), unemployment is not traced back to inflexible prices and wage rates, but to inflexible aggregate investment formed independently of aggregate savings. Magnani (2015) in Solow's (1956) neo-classical growth model and Farmer and Kuplen's (2018) in Diamond's (1965) overlapping generations (OLG) model claim that aggregate investment is macro-founded without any need for micro-foundations.

In contrast, there is Lucas Jr.'s (1972) magisterial claim that all macro-economic relationships should be micro-founded. This is obviously not the case with an independent aggregate investment function which is, however, decisive for the occurrence of involuntary unemployment in intertemporal equilibrium. Once Lucas Jr.'s (1972) claim is accepted, the research challenge then consists in providing micro-foundations for an independent aggregate investment function such that aggregate-output demand and not labor supply govern employment. It is the objective of the present paper to meet this research challenge within the confines of stylized intertemporal general equilibrium models, in particular the seminal stock market OLG model of Magill and Quinzii (2003) with non-shiftable capital and affine equity-price expectations.

While Magill and Quinzii (2003) provide micro-foundations for aggregate investment, the investment quantity of firms is optimally indeterminate and thus full employment can prevail. To be able to model unemployment within Magill and Quinzii's (2003) stock market model, Farmer (2023) cancels their labor market-clearing condition and endogenizes the unemployment rate in line with Magnani (2015)[1]. This procedure leaves the system of intertemporal-equilibrium equations under-determinate and provokes the need for a closing equation. As Farmer (2023) extensively discusses[2], a (degenerate) belief function by Farmer (2020) represents the closing equation. In particular, each investor forms a quantity belief about his/her investment demand. As Farmer (2020) forcefully argues, this belief function is to be seen as a primitive in addition to households' preferences, firms' production technologies and economy's resources.

Against this research background, our first contribution to literature is to modify Magill and Quinzii's (2003) stock market model such that the unemployment rate becomes endogenous and the beliefs of investors regarding investment demand govern firms' optimally indeterminate investment quantities.

Our second contribution to the literature is to show how the structure of the intertemporal equilibrium dynamics derived from households' and firms' optimization conditions, from government's budget constraint and the intertemporal market clearing conditions changes when firms' investment quantities are both optimally indeterminate and determined by investors' beliefs regarding investment demand. Moreover, the existence and the dynamic stability of a Golden Rule steady state of the intertemporal equilibrium will be shown.

Our third contribution to the literature consists in deriving analytically the steady-state effects on the endogenous variables of main parameter changes. This

[1]Tanaka (2020) models involuntary unemployment in a three-period OLG model, however without capital accumulation and investment.

[2]The present paper might be considered as an extremely condensed version of Farmer (2023).

is completed by a numerical calculation of the intertemporal equilibrium paths of the endogenous variables in response to small parameter changes.

The structure of the paper is as follows. The next section presents the model setup. This is followed by the derivation of the intertemporal-equilibrium dynamics and demonstration of sufficient conditions for the existence and dynamic stability of steady states. We then investigate the comparative dynamics of the steady-state responses of the capital-output ratio, the equity-price discount and the unemployment rate to the main parameter changes. A numerical specification of all model parameters is then used to calculate numerically the intertemporal-equilibrium paths of these dynamic variables in response to small parameter changes. The main conclusions are drawn in the final section of the paper.

2. The Stock Market Olg Model with Involuntary Unemployment

As in Magill and Quinzii (2003), we consider an economy of the infinite horizon which is composed of infinitely lived firms and finitely lived households. In addition to Magill and Quinzii (2003), we also assume an infinitely lived government with a balanced budget from period to period. In each period $t = 0,1,2,\cdots$ a new generation, called generation *t*, enters the economy. A continuum of $L_t > 0$ units of identical agents comprises generation *t*.

As in Diamond's (1965) seminal OLG model, and in line with Magill and Quinzii (2003), we assume exogenous growth of the population $g^L > -1$ which implies the following dynamics of population L_t: $L_{t+1} = G^L L_t$, $G^L \equiv 1 + g^L$, $L_0 = \underline{L} > 0$. In addition to Magill and Quinzii (2003), we also assume exogenous growth of labor productivity denoted by $g^a > -1$ which implies the following dynamics of labor productivity a_t: $a_{t+1} = G^a a_t$, $G^a \equiv 1 + g^a$, $a_0 = \underline{a} > 0$.

Each household consists of one agent and the agent is intergenerationally egoistic: The old agent has no concern for the young agent and the young agent has no concern for the old agent. They live two periods long, namely youth (adult) and old age. In contradistinction to the original Diamond's (1965) OLG model and Magill and Quinzii's (2003) full-employment, stock-market model, in our model economy there are also employed and (involuntarily) unemployed households. All households are endowed with one unit of labor but only the employed households are able to sell it inelastically to firms. In exchange for the labor supply each employed household of generation *t* obtains the real wage rate w_t, which denotes the units of the produced good per unit of labor. Thus, the labor supply in period *t* is not equal to L_t, but only to $(1-u_t)L_t$, where $0 \le u_t < 1$ denotes the unemployment rate. The number of unemployed households (= people) is thus $u_t L_t$. Since the unemployed are unable to obtain any labor income from the market, they are supported by the government through the unemployment benefit ς_t (per household) in each period.

In order to finance the unemployment benefit, the government collects taxes on wages, quoted as a fixed proportion of wage income, $\tau_t w_t h_t, 0 < \tau_t < 1$. The unemployed do not pay any taxes. Young, employed agents, denoted by superscript *E*, split the net wage income $(1-\tau_t)w_t$ each period between current consumption $c_t^{1,E}$ and savings s_t^E. Savings of the employed are invested in the shares of firms, where a share $\theta_t^{j,E}$ of firm $j=1,\cdots,J$ in period *t* is bought in the stock market at price Q_t^j by the younger households from the older households. Moreover, the younger households also invest their savings in bonds emitted by firms $j(=1,\cdots,J)$, denoted by $b_{t+1}^{j,E}$, with a rate of return i_{t+1}.

In old age, the employed household sells the shares at the price $Q_{t+1}^{j,E}$ to the then younger household in period $t+1$. The revenues from asset sales and the returns from holding assets one period long, $(1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J}\theta_t^{j,E}\left(D_{t+1}^j + Q_{t+1}^j\right)$, are used to finance retirement consumption $c_{t+1}^{2,E}$, where D_t^j denotes the dividend paid by firm *j* in period *t*. In old age, the previously young employed households consume their gross return on assets: $c_{t+1}^{2,E} = (1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J}\theta_t^{j,E}\left(D_{t+1}^j + Q_{t+1}^j\right)$. This is also true for the unemployed households who finance their retirement consumption through the returns on equity purchases and firm bonds in youth financed by unemployment benefits: $c_{t+1}^{2,U} = (1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,U} + \sum_{j=1}^{J}\theta_t^{j,U}\left(D_{t+1}^j + Q_{t+1}^j\right)$, where $c_{t+1}^{2,U}$, represents consumption of the unemployed in old age. To keep it all as simple as possible, we assume that the revenues from equity sales and dividends are not taxed.

The typical younger, employed household maximizes the following intertemporal utility function subject to the budget constraints of the active period (i) and of the retirement period (ii):

$$\text{Max} \rightarrow \varepsilon \ln c_t^{1,E} + \beta \ln c_{t+1}^{2,E}$$

subject to:

(i) $c_t^{1,E} + \sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} Q_t^J \theta_t^{j,E} = w_t(1-\tau_t)$,

(ii) $c_{t+1}^{2,E} = (1+i_{t+1})\sum_{j=1}^{J} b_{t+1}^{j,U} + \sum_{j=1}^{J}\theta_t^{j,U}\left(D_{t+1}^j + Q_{t+1}^j\right)$.

Here, $0<\varepsilon\leq 1$ depicts the utility elasticity of employed household's consumption in youth and $0<\beta<1$ denotes the subjective future utility discount factor. Needless to say, the intertemporally additive utility function involves the natural logarithm of employed household's consumption in youth weighted by ε, and the natural logarithm of employed household's consumption in old age weighted by $0<\beta<1$.

In order to obtain the first-order conditions for a maximum of the intertemporal utility function subject to the constraints (i) and (ii), we form the following Lagrangian:

$$L_t^E \equiv \varepsilon \ln c_t^{1,E} + \beta \ln c_{t+1}^{2,E} - \lambda_t^E\left(c_t^{1,E} + \sum_{j=1}^{J} b_{t+1}^{j,E} + \sum_{j=1}^{J} Q_t^j \theta_t^{j,E} - w_t(1-\tau_t)\right)$$

$$-\lambda_{t+1}^{E}\left(c_{t+1}^{2,E}-\left(1+i_{t+1}\right)\sum_{j=1}^{J}b_{t+1}^{j,E}-\sum_{j=1}^{J}\theta_{t}^{j,E}\left(D_{t+1}^{j}+Q_{t+1}^{j}\right)\right).$$

Differentiating the Lagrangian with respect to $c_t^{1,E}, c_{t+1}^{2,E}, b_{t+1}^{j,E}, \theta_t^{j,E}, j=1,\cdots,J$ yields the following first-order conditions for an intertemporal utility maximum:

$$c_t^{1,E}=\frac{\varepsilon}{\varepsilon+\beta}\left(1-\tau_t\right)w_t, \tag{1}$$

$$\frac{D_{t+1}^{j}+Q_{t+1}^{j}}{Q_t^{j}}=1+i_{t+1},\ j=1,\cdots,J, \tag{2}$$

$$c_{t+1}^{2,E}=\frac{\beta}{\varepsilon+\beta}\left(1+i_{t+1}\right)\left(1-\tau_t\right)w_t, \tag{3}$$

$$s_t^{E}=\frac{\beta}{\varepsilon+\beta}w_t\left(1-\tau_t\right),\ s_t^{E}\equiv\sum_{j=1}^{J}b_{t+1}^{j,E}+\sum_{j=1}^{J}\theta_t^{j,E}Q_t^{j}. \tag{4}$$

The typical younger, unemployed household maximizes the following intertemporal utility function subject to the budget constraints of the active period (i) and the retirement period (ii):

$$\text{Max}\rightarrow\varepsilon\ln c_t^{1,U}+\beta\ln_t c_{t+1}^{2,U}$$

subject to:

(i) $c_t^{1,U}+\sum_{j=1}^{J}b_{t+1}^{j,U}+\sum_{j=1}^{J}Q_t^{J}\theta_t^{j,U}=\varsigma_t,$

(ii) $c_{t+1}^{2,U}=\left(1+i_{t+1}\right)\sum_{j=1}^{J}b_{t+1}^{j,U}+\sum_{j=1}^{J}\theta_t^{j,U}\left(D_{t+1}^{j}+Q_{t+1}^{j}\right).$

Again, $0<\varepsilon\le 1$ denotes the utility elasticity of consumption in unemployed youth, while $0<\beta<1$ depicts the subjective future utility discount factor and ς_t denotes the unemployment benefit per capita unemployed.

Performing similar intermediate steps as above with respect to the younger, employed household yields the following first-order conditions for a constrained intertemporal utility maximum:

$$c_t^{1,U}=\frac{\varepsilon}{\varepsilon+\beta}\varsigma_t, \tag{5}$$

$$\frac{D_{t+1}^{j}+Q_{t+1}^{j}}{Q_t^{j}}=1+i_{t+1},\ j=1,\cdots,J, \tag{6}$$

$$c_{t+1}^{2,U}=\frac{\beta}{\varepsilon+\beta}\left(1+i_{t+1}\right)\varsigma_t, \tag{7}$$

$$s_t^{U}=\frac{\beta}{\varepsilon+\beta}\varsigma_t,\ s_t^{U}\equiv\sum_{j=1}^{J}b_{t+1}^{j,U}+\sum_{j=1}^{J}\theta_t^{j,U}Q_t^{j}. \tag{8}$$

All firms are endowed with an identical (linear-homogeneous) Cobb-Douglas production function which reads as follows:

$$Y_t^{j}=M\left(a_tN_t^{j}\right)^{1-\alpha}\left(K_t^{j}\right)^{\alpha},\ j=1,\cdots,J,\ 0<\alpha<1, M>0. \tag{9}$$

Here, Y_t^j denotes production output of firm $j=1,\cdots,J$, $M>0$ stands for total factor productivity (equal for all firms), N_t^j represents the number of employed laborers with firm *j*, with productivity of a_t each, while K_t^j denotes the input of capital services of firm *j*, all in period *t*, and $1-\alpha$ (α) depicts the production elasticity (= production share) of labor (capital) services, also equal for all firms. In line with the seminal paper of Magill and Quinzii (2003), we assume that (physical) capital is durable, depreciates at the rate $0<\delta<1$, and needs to be installed one period before it is used. Thus capital K_t^j used by firm *j* is the capital stock that has been carried over from the period before, i.e. period $t-1$. Moreover, we assume "that capital once installed in a firm cannot be 'unbolted' and transformed back into the homogeneous current output or transferred to another firm, without incurring significant adjustment costs—which for simplicity we take to be infinite" (Magill & Quinzii, 2003: p. 242). As a consequence, such firm-specific capital has limited value in a resale market. In the extreme, it is completely firm-specific, so that no part of it has a positive value in the second-hand market.

"In such an economy capital accumulation will only take place if the market structure permits firms to be infinitely lived. Invested capital has…value only if the firm retains its identity as income generating unit in the economy. The natural market structure which permits short-lived agents to transfer ownership of long-lived firms from one generation to the next is an equity market for ownership shares of firms" (Magill & Quinzii, 2003: p. 243). Consistent with the firm specificity of capital is that each firm is a corporation with an infinite life where ownership shares are transmitted from one generation to the next through the stock market. As already introduced above, Q_t^j denotes the equity price of firm *j* at date *t*.

Firms are owned by the equity holders and are managed so as to maximize the payoff of their current owners. These are the younger households who buy the shares of firm *j* endowed with a capital of $(1-\delta)K_t^j$, from the older households for the price Q_t^j, and decide on the investment $I_t^j \geq 0$ to be made. Magill and Quinzii (2003: pp. 244-245) show that an investment quantity larger than zero is chosen such that the net present value of the investment is maximized:

$$\underbrace{\max}_{\{I_t^j,N_{t+1}^j\}}\left\{-I_t^j+\frac{1}{1+i_{t+1}}\left[\left(K_{t+1}^j\right)^\alpha\left(a_{t+1}N_{t+1}^j\right)^{1-\alpha}-w_{t+1}N_{t+1}^j+Q_{t+1}^j\left(\left(1-\delta\right)K_{t+1}^j\right)\right]\right\}$$

$$\Leftrightarrow \underbrace{\max}_{\{I_t^j,N_{t+1}^j\}}\left\{-I_t^j+\frac{1}{1+i_{t+1}}\left[\left(\left(1-\delta\right)K_t^j+I_t^j\right)^\alpha\left(a_{t+1}N_{t+1}^j\right)^{1-\alpha}-w_{t+1}N_{t+1}^j\right.\right. \tag{10}$$

$$\left.\left.+\left(1-\delta\right)^2K_t^j+\left(1-\delta\right)I_t^j-V_{t+1}^j\right]\right\}.$$

Here, the equivalence between the first and the second line comes from Magill and Quinzii's (2003: p. 244) insight that in an intertemporal equilibrium shareholders expect an affine (linear) relationship between the expected equity price of non-depreciated capital in period $t+1$, $Q_{t+1}^j\left(\left(1-\delta\right)K_{t+1}^j\right)$, and non-depreciated capital stock at that time, i.e.:

$$Q_{t+1}^{j}\left(\left(1-\delta\right)K_{t+1}^{j}\right)=\left(1-\delta\right)^{2}K_{t}^{j}+\left(1-\delta\right)I_{t}^{j}-V_{t+1}^{j},\ j=1,\cdots,J,V_{t+1}^{j}\geq 0, \tag{11}$$

where $V_{t+1}^{j}, j=1,\cdots,J$ denotes the discount on the equity price of firm *j* at time $t+1$ due to the non-shiftability of firm *j*'s capital stock.

Maximization of the net present value in the second line of Equation (10) implies the following first-order conditions:

$$\alpha M\left[\left(1-\delta\right)K_{t}^{j}+I_{t}^{j}\right]^{\alpha-1}\left(a_{t+1}N_{t+1}^{j}\right)^{1-\alpha}=\delta+i_{t+1}, \tag{12}$$

$$M\left[\left(1-\delta\right)K_{t}^{j}+I_{t}^{j}\right]^{\alpha}\left(a_{t+1}N_{t+1}^{j}\right)^{-\alpha}a_{t+1}=w_{t+1}. \tag{13}$$

Since all firms have the same production function (see Equation (9)) and the capital depreciation rate is the same with all firms, the optimal capital labor ratio will be the same for all firms: $\frac{K_{t}^{j}}{a_{t}N_{t}^{j}}=\frac{K_{t}^{j'}}{a_{t}N_{t}^{j'}}=\frac{K_{t}}{a_{t}N_{t}},\ j\neq j'=1,\cdots,J$. Moreover, since the number of employed workers is $N_{t}\equiv\sum_{j=1}^{J}N_{t}^{j}=L\left(1-u_{t}\right)$, we can rewrite the profit maximization conditions (12) and (13) as follows:

$$\alpha M\left[K_{t+1}\Big/\left(a_{t+1}L\left(1-u_{t+1}\right)\right)\right]^{\alpha-1}=\delta+i_{t+1}, \tag{14}$$

$$\left(1-\alpha\right)M\left[K_{t+1}\Big/\left(a_{t+1}L\left(1-u_{t+1}\right)\right)\right]^{\alpha}a_{t+1}=w_{t+1}. \tag{15}$$

Finally, the GDP function can be rewritten as follows:

$$Y_{t}\equiv\sum_{j=1}^{J}Y_{t}^{j}=M\left(a_{t}L\left(1-u_{t}\right)\right)^{1-\alpha}\left(K_{t}\right)^{\alpha}. \tag{16}$$

As in Diamond (1965), the government does not optimize, but is subject to the following budget constraint period by period:

$$L_{t}u_{t}\varsigma_{t}=\tau_{t}\left(1-u_{t}\right)w_{t}L_{t}, \tag{17}$$

where, for the sake of simplicity, it is assumed that the government does not have any other expenditures than the unemployment benefits and that there is no government debt.

As Magnani (2015: pp. 13-14) rightly states, aggregate investment in Solow's (1956) neoclassical growth model is not micro-, but macro-founded since it is determined by aggregate savings. The same holds true in Diamond's (1965) OLG model of neoclassical growth where perfectly flexible aggregate investment is also determined by aggregate savings of households. As already mentioned in the Introduction above, and as the first-order conditions for optimal investment of younger shareholders (14) and (15) show, optimal investment is indeterminate and thus also perfectly flexible in the stock market model of Magill and Quinzii (2003). This is most easily seen if we rewrite Equations (14) and (15) as follows:

$$\alpha M\left(\frac{K_{t+1}}{a_{t+1}L_{t+1}}\right)^{\alpha-1}\left(1-u_{t+1}\right)^{1-\alpha}=\delta+i_{t+1}, \tag{18}$$

$$(1-\alpha)M\left(\frac{K_{t+1}}{a_{t+1}L_{t+1}}\right)^{\alpha}(1-u_{t+1})^{-\alpha}=w_{t+1}. \tag{19}$$

Equations (18) and (19) do not allow for determination of the optimal firm investment quantity.

Morishima (1977) and more recently Magnani (2015), both deviate from neoclassical growth models in maintaining that an independent investment function is needed to determine the level of investment in intertemporal equilibrium models of involuntary unemployment. The big question, however, is where does this function come from in a general equilibrium model with an active stock market and an explicit firm maximization calculus to determine investment quantities?

In order to provide an answer to this question we recall the no-arbitrage condition between shares and corporation bonds (2) $\left(D_{t+1}^{j}+Q_{t+1}^{j}\right)/Q_{t}^{j}=1+i_{t+1},\ j=1,\cdots,J$, with $D_{t+1}^{j}=M\left(K_{t+1}^{j}\right)^{\alpha}\left(a_{t+1}N_{t+1}^{j}\right)^{1-\alpha}-w_{t+1}N_{t+1}^{j}-(1+i_{t+1})I_{t}^{j},\ j=1,\cdots,J$. Respecting the first-order conditions for net present value maximization (18) and (19), and assuming that affine equity price expectations are rational, i.e. Equation (11) holds, then we can show, following Magill and Quinzii (2003: p. 247), that

$$\frac{D_{t+1}^{j}+Q_{t+1}^{j}}{Q_{t}^{j}}=\frac{K_{t+1}^{j}(\delta+i_{t+1})-(1+i_{t+1})I_{t}^{j}+(1-\delta)K_{t+1}^{j}-V_{t+1}^{j}}{(1-\delta)K_{t}^{j}-V_{t}^{j}},$$

if and only if

$$V_{t+1}^{j}=(1+i_{t+1})V_{t}^{j},\ j=1,\cdots,J,\forall t\geq 0. \tag{20}$$

In order to make the intertemporal equilibrium equations determinate, the following degenerate "belief" function (Farmer, 2020) which states that firm *j*'s investment quantity is equal to an exogenous, time-stationary constant $\Phi^{j},\ j=1,\cdots,J$, and reflects "Keynesian investors' animal spirits" with respect to investment demand (Magnani, 2015: p.14):

$$I_{t}^{J}=\Phi^{j},\ j=1,\cdots,J. \tag{21}$$

In addition to the restrictions imposed by household and firm optimizations and by the government budget constraint, markets for labor, firm bonds, and equity, ought to clear in all periods (the market for the output of production is cleared by means of Walras' law[3]).

$$L_{t}(1-u_{t})=\sum_{j=1}^{J}N_{t}^{j}=N_{t},\forall t. \tag{22}$$

$$L(1-u_{t})b_{t+1}^{E}+Lu_{t}b_{t+1}^{U}=\sum_{j=1}^{J}b_{t+1}^{j},\ \forall t. \tag{23}$$

The demand of the younger employed and the unemployed households for firm bonds (left-hand side of Equation (23)) balance with their supply (right-hand side of Equation (23)). Firms finance their investments by the sales of bonds:

[3]The proof of Walras' law can be obtained upon request from the author.

$$\sum_{j=1}^{J} I_t^j = \sum_{j=1}^{J} b_{t+1}^j, \ \forall t. \tag{24}$$

The shares of employed and unemployed younger households sum to unity:

$$L(1-u_t)\theta_t^{j,E} + Lu_t\theta_t^{j,U} = 1, \ j = 1,\cdots,J, \ \forall t. \tag{25}$$

The sales of equity shares by employed and unemployed older households are equal to the share purchases of employed and unemployed younger households:

$$L_{t-1}(1-u_{t-1})\theta_{t-1}^{j,E} = L_t(1-u_t)\theta_t^{j,E}, \ j = 1,\cdots,J, \ \forall t, \tag{26}$$

$$L_{t-1}(1-u_{t-1})\theta_{t-1}^{j,U} = L_t(1-u_t)\theta_t^{j,U}, \ j = 1,\cdots,J, \ \forall t. \tag{27}$$

Using the definition of savings for younger employed households in (4) and younger unemployed households in (8), together with the bond market clearing condition (23), the investment financing constraint (24) and condition (25), leads us to the following aggregate savings/investment equality:

$$L_t(1-u_t)s_t^E + L_t u_t s_t^U = \sum_{j=1}^{J} I_t^j + \sum_{j=1}^{J} Q_t^j. \tag{28}$$

On respecting Equation (21) and firm-specific accumulation equation

$$K_{t+1}^j = (1-\delta)K_t^j + I_t^j, \ j = 1,\cdots,J, \tag{29}$$

the following equilibrium equation results:

$$I_t^j = \Phi^j = K_{t+1}^j - (1-\delta)K_t^j, \ j = 1,\cdots,J, \ \forall t. \tag{30}$$

Equation (30) does not appear in Magill and Quinzii's (2003) stock market model, since they assume full employment of the labor force, which is equivalent to $u_t = 0, \forall t$ in our model. For $u_t > 0$ and u_t being endogenous, Equation (30) features as the equilibrium condition which makes the whole set of intertemporal equilibrium equations determinate. In contrast to Morishima (1977: pp. 117-119) and Magnani (2015: p. 14), inflexible firm-specific and aggregate investment is not simply assumed to be macro-founded but turns out to be consistent with an indeterminate, market-value maximizing investment quantity of firm *j*. In this restricted sense, we are entitled to claim that inflexible investment is micro-founded in our modified stock-market model of involuntary unemployment.

3. Intertemporal Equilibrium

To start with, assume in line with Magill and Quinzii (2003: p. 249), a balanced-growth intertemporal equilibrium in which firms exhibit at all times the same relative sizes and stock market values. Then, consider initial conditions $(K_0^j, V_0^j) = \nu_j(K_0, V_0)$ with $\nu_j > 0$ and $\sum_{j=1}^{J} \nu_j = 1$. If, for the sequence of (real) wage and interest rates $(w_t, i_{t+1})_{t\geq 0}$, aggregate discounts $(V_t) \geq 0$ and employment-investment decisions $(N_t, I_t)_{t\geq 0}$ satisfy Equations (11), (12), (13), (20), (29) and (30), then $(V_t^j, N_t^j, I_t^j) = \nu_j(V_t, N_t, I_t)$ also satisfy Equations (11), (12), (13), (29) and (30), such that for each firm $\left(N_t^j, I_t^j\right)$ is market-value maximizing, its market value is larger than zero, and the return on equity equals i_{t+1}. Hence, the optimal choices of individual firms can be depicted by the market-value maximizing choice of aggregate employment and capital.

Acknowledging the linear-homogeneity of firm production functions (16) and the underemployment equilibrium condition (22), we can switch to aggregate capital per efficient labor $k_t \equiv K_t/(a_t L_t)$ quantities, and rewrite the first-order conditions (18) and (19) as follows:

$$\alpha M\left(k_{t+1}\right)^{\alpha-1}\left(1-u_{t+1}\right)^{1-\alpha} = \delta + i_{t+1}, \tag{31}$$

$$\left(1-\alpha\right) M a_{t+1}\left(k_{t+1}\right)^{\alpha}\left(1-u_{t+1}\right)^{-\alpha} = w_{t+1}. \tag{32}$$

As a next step, the aggregate version of Equation (29) is solved for I_t and inserted into the savings/investment equality (28). Assuming that affine equity price expectations (11) also prevailed in period t, the savings/investment equality can be rewritten as follows:

$$L_t\left(1-u_t\right)s_t^E + L_t u_t s_t^U = K_{t+1} - \left(1-\delta\right)K_t + \left(1-\delta\right)K_t - V_t = K_{t+1} - V_t. \tag{33}$$

Next, insert into Equation (33) the optimal savings functions (4) and (8) and the government balanced budget condition (17):

$$\begin{aligned} &L_t\left(1-u_t\right)\sigma w_t\left(1-\tau_t\right) + L_t u_t \sigma \varsigma_t = L_t\left(1-u_t\right)\sigma w_t\left(1-\tau_t\right) + L_t\left(1-u_t\right)\sigma w_t \tau_t \\ &= L_t\left(1-u_t\right)\sigma w_t = K_{t+1} - V_t,\ \sigma \equiv \beta/\left(\varepsilon+\beta\right). \end{aligned} \tag{34}$$

Inserting into Equation (34) the first-order condition (32) for t, and dividing the resulting equation on both sides by $a_t L_t$, we obtain:

$$\begin{aligned} &\left(L_t\left(1-u_t\right)\sigma a_t\left(1-\alpha\right)M\left(k_t\right)^{\alpha}\left(1-u_t\right)^{-\alpha} = K_{t+1} - V_t\right)\frac{1}{a_t L_t} \\ &\Leftrightarrow \left(1-\alpha\right)\sigma M\left(k_t\right)^{\alpha}\left(1-u_t\right)^{1-\alpha} = \frac{K_{t+1}}{a_{t+1}L_{t+1}}\frac{a_{t+1}L_{t+1}}{a_t L_t} - \frac{V_t}{a_t L_t} = k_{t+1}G^n - v_t, \\ &G^n \equiv \frac{a_{t+1}L_{t+1}}{a_t L_t},\ v_t \equiv \frac{V_t}{a_t L_t}. \end{aligned} \tag{35}$$

By using the capital-output ratio $\kappa_t \equiv K_t/Y_t = K_t\Big/\left[M a_t L_t\left(1-u_t\right)^{1-\alpha}\left(k_t\right)^{\alpha}\right] = \left(k_t\right)^{1-\alpha}\Big/\left[M\left(1-u_t\right)^{1-\alpha}\right]$ or $k_t = M^{1/(1-\alpha)}\left(\kappa_t\right)^{1/(1-\alpha)}\left(1-u_t\right)$, Equation (35) can be transformed into Equation (36):

$$\left(1-\alpha\right)\sigma M^{1/(1-\alpha)}\left(\kappa_t\right)^{\alpha/(1-\alpha)}\omega_t = G^n M^{1/(1-\alpha)}\left(\kappa_{t+1}\right)^{1/(1-\alpha)}\omega_{t+1} - v_t,\ \omega_t \equiv 1-u_t,\ \forall t. \tag{36}$$

Equation (36) represents the first difference equation of the intertemporal equilibrium in our stock-market model of involuntary unemployment.

The second dynamic equation results from summing Equation (30) over all firms and dividing the resulting equation on both sides by $a_t L_t$:

$$\frac{\sum_{j=1}^{J}\Phi^j}{a_t L_t} \equiv \phi = G^n k_{t+1} - \left(1-\delta\right)k_t$$

$$= G^n M^{1/(1-\alpha)} \left(\kappa_{t+1}\right)^{1/(1-\alpha)} \omega_{t+1} - (1-\delta) M^{1/(1-\alpha)} \left(\kappa_t\right)^{1/(1-\alpha)} \omega_t. \tag{37}$$

The third equilibrium-dynamics equation pops up when Equation (20) is divided on both sides by $a_t L_t$, and when the definition of v_t and the first-order condition (31) are used:

$$\begin{aligned} G^n v_{t+1} &= \left[1-\delta+\alpha M\left(k_t\right)^{\alpha-1}\left(1-u_t\right)^{1-\alpha}\right] v_t \\ &= \left(1-\delta+\alpha/\kappa_t\right) v_t, \text{ with } 0 \le v_{t+1} \le (1-\delta)^2 k_t. \end{aligned} \tag{38}$$

The three-dimensional dynamic system (36)-(38) can be reduced to two dimensions by solving Equation (36) for $G^n M^{1/(1-\alpha)} \left(\kappa_{t+1}\right)^{1/(1-\alpha)} \omega_{t+1}$, inserting the result into Equation (37) and solving the resulting equation for ω_t:

$$\omega_t = 1-u_t = \frac{\phi - v_t}{M^{1/(1-\alpha)}\left[(1-\alpha)\sigma\left(\kappa_t\right)^{\alpha/(1-\alpha)} - (1-\delta)\left(\kappa_t\right)^{1/(1-\alpha)}\right]}. \tag{39}$$

Reinserting (39) for t and $t+1$ into Equation (37), generates the following two-dimensional dynamic system:

$$\begin{aligned} &\frac{G^n \left(\kappa_{t+1}\right)^{1/(1-\alpha)} \left(\phi - v_{t+1}\right)}{(1-\alpha)\sigma\left(\kappa_{t+1}\right)^{\alpha/(1-\alpha)} - (1-\delta)\left(\kappa_{t+1}\right)^{1/(1-\alpha)}} \\ &= \phi + \frac{(1-\delta)\left(\kappa_t\right)^{1/(1-\alpha)}\left(\phi - v_t\right)}{(1-\alpha)\sigma\left(\kappa_t\right)^{\alpha/(1-\alpha)} - (1-\delta)\left(\kappa_t\right)^{1/(1-\alpha)}}, \end{aligned} \tag{40}$$

$$G^n v_{t+1} = \left(1-\delta+\alpha/\kappa_t\right) v_t, \text{ with } 0 \le v_{t+1} \le (1-\delta)^2 M^{1/(1-\alpha)} \left(\kappa_t\right)^{1/(1-\alpha)} \left(1-u_t\right), \tag{41}$$

whereby $\kappa_0 = \underline{\kappa} > 0$ and $v_0 = \underline{v} > 0$, $\underline{\kappa}$ and $\underline{v}$ exogenously given.

4. Existence of Steady States

The steady states of the equilibrium dynamics depicted by the difference Equations (40) and (41) are defined as $\lim_{t\to\infty} v_t = v$ and $\lim_{t\to\infty} \kappa_t = \kappa$. Due to the relative simplicity of the dynamic system (40) and (41) explicit steady-state solutions are possible. As in Magill and Quinzii (2003), there are two different steady-state solutions of the equilibrium dynamics (40) and (41): (1) The zero-discount, or so-called Diamond-solution $\kappa_D > 0$ and $v_t = v = 0, \forall t$, and (2), the positive-discount steady state $\kappa > 0$ and $v_0 = \underline{v} > 0$, $v_t > 0, \forall t$, and $v > 0$. Here we focus on solution (2). This leads us to the following Proposition 1:

Proposition 1. Suppose that $G^n / \left[G^n - (1-\delta)\right] > (1-\alpha)\sigma/\alpha$ and $\phi < M^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \left[G^n - (1-\delta)\right]^{\frac{-\alpha}{1-\alpha}}$. Then, the following steady solution for $(\kappa, v) > 0$ and $0 < u < 1$ exists:

$$\kappa = \frac{\alpha}{G^n - (1-\delta)}, \tag{42}$$

$$v = \phi\left\{\frac{G^n}{G^n - (1-\delta)} - \frac{(1-\alpha)\sigma}{\alpha}\right\}, \tag{43}$$

$$u = 1 - \frac{\phi}{M^{1/(1-\alpha)}\kappa^{1/(1-\alpha)}\left[G^n - (1-\delta)\right]}. \tag{44}$$

Remark: Rearranging the steady-state solution (42) brings forth: $1-\delta+\alpha/\kappa = 1+i = G^n$. This means that the positive-discount steady state features the so-called Golden-Rule, intertemporal-consumption allocation which is long-run efficient.

5. Dynamic Stability of the Positive-Discount Steady State

The next step is to investigate the local dynamic stability of the unique, positive-discount, steady-state solution. To this end, the intertemporal equilibrium Equations (40) and (41) are totally differentiated with respect to $\kappa_{t+1}, v_{t+1}, \kappa_t, v_t$. Then, the Jacobian matrix $J(\kappa, v)$ of all partial differentials with respect to κ_t and v_t is formed as follows:

$$J(\kappa, v) \equiv \begin{bmatrix} \frac{\partial \kappa_{t+1}}{\partial \kappa_t}(\kappa, v) & \frac{\partial \kappa_{t+1}}{\partial v_t}(\kappa, v) \\ \frac{\partial v_{t+1}}{\partial \kappa_t}(\kappa, v) & \frac{\partial w_{t+1}}{\partial v_t}(\kappa, v) \end{bmatrix}, \tag{45}$$

with

$$\frac{\partial \kappa_{t+1}}{\partial \kappa_t} \equiv j_{11} = 1 - \frac{\alpha}{(1-\alpha)\sigma},$$

$$\frac{\partial \kappa_{t+1}}{\partial v_t} \equiv j_{12} = \frac{\alpha^2}{(1-\alpha)\sigma G^n \phi},$$

$$\frac{\partial v_{t+1}}{\partial \kappa_t} \equiv j_{21} = \frac{\left[G^n - (1-\delta)\right]\phi\left\{\sigma\left[G^n - (1-\delta)\right] - \alpha\left\{G^n + \sigma\left[G^n - (1-\delta)\right]\right\}\right\}}{\alpha^2 G^n},$$

$$\frac{\partial v_{t+1}}{\partial v_t} = 1.$$

Due to the simplicity of the elements of the Jacobian (45), its eigenvalues φ_1 and φ_2 can be directly calculated as follows:

$$\varphi_1 = \frac{1-\delta}{G^n}, \tag{46}$$

$$\varphi_2 = 1 + \frac{G^n - (1-\delta)}{G^n} - \frac{\alpha}{(1-\alpha)\sigma}. \tag{47}$$

Proposition 2. Suppose the assumptions of Proposition 1 and additionally $\frac{(1-\alpha)\sigma}{\alpha} > \frac{G^n}{2G^n - (1-\delta)}$ hold. Then, the eigenvalues φ_1 and φ_2 of the Jacobian (45) at the steady-state solution (42)-(45) are strictly larger than zero and smaller than unity ($0 < \varphi_1 < 1$, $0 < \varphi_2 < 1$) which implies that the equilibrium dynamics in the neighborhood of the steady state is (locally) asymptotically stable.

Proof. Since $0<\delta<1$ and $G^n>1$, $0<\varphi_1<1$ is obvious. $\phi_2>0 \Leftrightarrow \frac{2G^n-(1-\delta)}{G^n}>\frac{\alpha}{(1-\alpha)\sigma} \quad \Leftrightarrow \frac{(1-\alpha)\sigma}{\alpha}>\frac{G^n}{2G^n-(1-\delta)}$ on account of the additional assumption in Proposition 2. $\phi_2<1 \Leftrightarrow \frac{G^n-(1-\delta)}{G^n}-\frac{\alpha}{(1-\alpha)\sigma}<0$ $\Leftrightarrow \frac{(1-\alpha)\sigma}{\alpha}<\frac{G^n}{G^n-(1-\delta)}$ on account of the assumption in Proposition 1.

In other words: the dynamics with initial values $\kappa_0=\underline{\kappa}>0$ and $v_0=\underline{v}>0$ in the neighborhood of the positive-discount, steady-state solution in our stock market model with involuntary unemployment is non-oscillating and converges towards the steady state as time approaches infinity.

6. Comparative Dynamics of the Steady-State Solution and the Intertemporal Equilibrium Dynamics

Before concluding it is apt to investigate firstly the comparative dynamics of the positive-discount steady state. The effects of infinitesimal, isolated parameter changes on the positive-discount steady-state solution (42)-(44) are summarized in the following Proposition 3.

Proposition 3. Suppose that the assumptions of Propositions 1 and hold. Then, the effects of infinitesimal, isolated changes of main model parameters on the positive-discount steady-state solution (42)-(44) read as follows:

$$\begin{aligned}
&\frac{\partial\kappa}{\partial\alpha}=\frac{1}{G^n-(1-\delta)}>0,\ \frac{\partial\kappa}{\partial G^n}=-\frac{\alpha}{\left[G^n-(1-\delta)\right]^2}<0,\\
&\frac{\partial\kappa}{\partial\delta}=-\frac{\alpha}{\left[G^n-(1-\delta)\right]^2}<0,
\end{aligned} \tag{48}$$

$$\begin{aligned}
&\frac{\partial v}{\partial\phi}=\left\{\frac{G^n}{\left[G^n-(1-\delta)\right]}-\frac{(1-\alpha)\sigma}{\alpha}\right\}>0,\ \frac{\partial v}{\partial G^n}=\frac{-(1-\delta)\phi}{\left[G^n-(1-\delta)\right]^2}<0,\\
&\frac{\partial v}{\partial\delta}=\frac{-G^n\phi}{\left[G^n-(1-\delta)\right]^2}<0,\ \frac{\partial v}{\partial\alpha}=\frac{\phi\sigma}{\alpha^2}>0,\ \frac{\partial v}{\partial\sigma}=\frac{-(1-\alpha)\phi}{\alpha}<0.
\end{aligned} \tag{49}$$

$$\begin{aligned}
&\frac{\partial u}{\partial\phi}=\frac{-1}{M^{1/(1-\alpha)}\kappa^{1/(1-\alpha)}\left[G^n-(1-\delta)\right]}<0,\\
&\frac{\partial u}{\partial G^n}=\frac{-\alpha\kappa^{1/(\alpha-1)}M^{1/(\alpha-1)}\phi}{(1-\alpha)\left[G^n-(1-\delta)\right]^2}<0,\\
&\frac{\partial u}{\partial\delta}=\frac{-\alpha\kappa^{1/(\alpha-1)}M^{1/(\alpha-1)}\phi}{(1-\alpha)\left[G^n-(1-\delta)\right]^2}<0,\\
&\frac{\partial u}{\partial\alpha}=\frac{\kappa^{(2-\alpha)/(\alpha-1)}\phi M^{-1/(1-\alpha)}\left[1-\alpha+\alpha(\log\kappa+\log M)\right]}{(1-\alpha)^2\left[G^n-(1-\delta)\right]^2}>0.
\end{aligned} \tag{50}$$

Considering the results of the comparative-dynamics experiment in (48)-(50)

one encounters well-known and not so familiar findings. It is well-known from the theory of exogenous growth that a higher capital income share ($d\alpha > 0$), a lower natural growth rate $d\left(G^n - 1\right) < 0$ and a lower capital depreciation rate ($d\delta < 0$) increase the capital-output ratio ($d\kappa > 0$). Moreover, marginal changes of the saving rate (σ) do not impact the steady-state capital-output ratio. New are the findings with respect to the effects of all parameters on the steady-state discount (see the partial derivatives in (49)). More investors' optimism ($d\phi > 0$) and a higher capital income share ($d\alpha > 0$) increase the steady-state discount $(dv > 0)$, while a higher natural growth rate $d\left(G^n - 1\right) > 0$, a larger capital depreciation rate ($d\delta > 0$) and a higher saving rate ($d\sigma > 0$) decrease the discount ($dv < 0$). Also new, and most important for the topic of this paper, are the effects of marginal parameter changes on the unemployment rate. Here, the partial derivatives in (50) show that only a larger capital income share ($d\alpha > 0$) increases the unemployment rate ($du > 0$), while more investor's optimism ($d\phi > 0$), a larger natural growth rate ($d\left(G^n - 1\right) > 0$) and a larger depreciation rate ($d\delta > 0$) decrease the steady-state unemployment rate ($du < 0$). Notice this typical "Keynesian" result in our neo-classical growth model: more optimistic investors reduce the steady-state unemployment rate. Notice also that an altered saving rate does not change the steady-state unemployment rate.

Thus, it remains to see whether and if yes how the saving rate impacts the unemployment rate along the intertemporal-equilibrium path towards the new steady state. In order to be able to answer these questions we switch to a numerical specification of our stock market model of involuntary unemployment. The main model parameters are chosen such that the assumptions of Propositions 1 and 2 hold. Moreover, we choose the following "typical"[4] parameter set which accords rather well with medium-term stylized facts regarding the growth rate of gross domestic product, the interest rate, the savings ratio, the investment ratio and the unemployment rate of the global economy averaged over the time period between 1990 and 2020 (see IMF, 2008, 2014, 2020): $G^n = 2.1$, $\beta = 0.5$, $\delta = 0.7$, $\varepsilon = 0.9$, $M = 10$, $\phi = 2.622$. Inserting into the steady-state Equations (42)-(44) these parameter values, these equations generate the following steady-state solution: $\kappa = 0.1389$, $v = 0.2497$, $u = 0.06$.

Consider now a small positive and unexpected shock on ε from 0.9 towards 0.91 implying a small decrease of the saving rate. Then, **Table 1** exhibits the intertemporal equilibrium path of main endogenous variables towards the new steady state: $\kappa = 0.1389$, $v = 0.2688$, $u = 0.06$.

A glance in **Table 1** reveals that a small reduction of the saving rate temporarily reduces the capital-output ratio and the unemployment rate, while the equity-price discount increases. After theoretically infinite periods (practically after 80 periods) the capital-output ratio and the unemployment rate return towards the pre-shock values, while the equity price increases. That the unemployment rate temporarily (in the short-term) decreases with a lower saving rate sounds

[4]Why this parameter set is typical, is more extensively discussed in Farmer (2022).

again "Keynesian" in our neo-classical growth model with involuntary unemployment.

Starting again from the same steady-state solution as before the saving-rate shock, we increase now the "animal spirits" parameter from $\phi = 2.622$ towards $\phi = 2.65$: all other parameters remain on their pre-saving-rate-shock values. The effects of this small, positive investment shock on the capital-output ratio, the equity-price discount and on the unemployment rate along the intertemporal-equilibrium path are depicted in **Table 2**.

As **Table 2** reveals, the positive shock on investment temporarily decreases the capital-output ratio and (rather starkly) the unemployment rate, while the equity-price discount increases in the short- and long-term. While the unemployment rate increases again along the intertemporal equilibrium path, it turns out to be lower in the new steady state: A Keynes-like result even in the long run.

Our last shock experiment concerns the natural growth rate (the qualitative impacts of a higher depreciation rate are similar). Starting once more from the

Table 1. Intertemporal equilibrium path of $(\kappa_t, v_t, u_t)_{t>1}$ after a small negative saving-rate shock.

t	0	1	2	3	4	5	6	…	40
κ_t	0.1389	0.1380	0.1379	0.1380	0.1381	0.1382	0.1382	…	0.1389
v_t	0.2497	0.2497	0.2510	0.2524	0.2538	0.2545	0.2561	…	0.2688
u_t	0.06	0.0520	0.0516	0.0523	0.0529	0.0534	0.0539	…	0.06

Source: Author's own calculation.

Table 2. Intertemporal equilibrium path of $(\kappa_t, v_t, u_t)_{t>1}$ after a small positive "animal-spirits" shock.

t	0	1	2	3	4	5	6	…	40
κ_t	0.1389	0.1388	0.1387	0.1387	0.1387	0.1388	0.1388	…	0.1389
v_t	0.2497	0.2497	0.2498	0.2501	0.2502	0.2504	0.2506	…	0.2524
u_t	0.06	0.0488	0.0488	0.0489	0.0490	0.0491	0.0491	…	0.0499

Source: Author's own calculation.

Table 3. Intertemporal equilibrium path of $(\kappa_t, v_t, u_t)_{t>1}$ after a small positive natural-growth shock.

t	0	1	2	3	4	5	6	…	40
κ_t	0.1389	0.1358	0.1354	0.1353	0.1353	0.1353	0.1353	…	0.1351
v_t	0.2497	0.2439	0.2428	0.2423	0.2419	0.2416	0.2414	…	0.2379
u_t	0.06	0.0546	0.0536	0.0534	0.0532	0.0531	0.0529	…	0.0514

Source: Author's own calculation.

steady state before the saving-rate shock and the parameters implying it, we increase the natural growth factor from $G^n = 2.1$ to $G^n = 2.15$. The impacts on the capital-output ratio, the equity-price discount and the unemployment rate along the intertemporal equilibrium path are depicted in **Table 3**.

A marginally higher natural growth rate decreases temporarily and permanently the capital-output ratio, the equity-price discount and the unemployment rate. A similar effect results from a higher depreciation rate.

7. Conclusion

This paper introduces an endogenous unemployment rate and investors' beliefs ("animal spirits") about the magnitude of investment demand into Magill and Quinzii's (2003) stock market OLG model with non-shiftable capital and affine equity-price expectations. The model parameter representing investors' demand beliefs can be seen as a degenerate belief function by Farmer (2020). Moreover, this belief's determined investment quantity is consistent with optimally indeterminate firm-level investment. In that sense, inflexible aggregate investment is micro-founded in our stock market model with involuntary unemployment.

In contradistinction to Magill and Quinzii's (2003) full employment model, in our model, the unemployment rate appears as an additional dynamic variable with the consequence that the intertemporal-equilibrium dynamics are in principle three instead of two-dimensional as in Magill and Quinzii (2003). The step-by-step derivation of the intertemporal-equilibrium equations from the first-order conditions for intertemporal utility and market value maxima, the government budget constraint, the degenerate belief function of investors and the market-clearing conditions brings forth that the unemployment rate is not a slow-moving dynamic variable but a sort of a jump variable. Slowly moving or truly dynamic variables are the capital-output ratio and the equity-price discount as in Magill and Quinzii (2003), making our intertemporal equilibrium dynamics also two-dimensional. Knowing the intertemporal equilibrium path of these truly dynamic variables, the unemployment rate in each period can in principle be calculated from a combination of the savings/investment- and the capital-accumulation equation.

We then investigate the existence of steady-state solutions whereby the capital-output ratio and the equity price discount do not change over time. As in Magill and Quinzii (2003), there are two steady-state solutions: 1) the zero-discount or diamond steady state and 2) the positive-discount steady state whereby the capital-output ratio accords to the Golden rule of intertemporal consumption allocation: one plus the interest rate equals the natural growth rate. We focus on the second steady state and find in Proposition 1 that a positive-discount steady state exists if the natural growth factor divided by the sum of the natural growth rate plus the depreciation rate is larger than the aggregate saving rate (= wage share times saving rate of younger households) over the capital income share, and the animal-spirits parameter is not too large, made precise in Proposition 1.

In order to be able to perform comparative dynamics of the effects of parameter shocks on main variables, we then check the dynamic stability of the equilibrium dynamics in the neighborhood of the positive-discount steady state. We find that local asymptotic stability of the equilibrium dynamics is ensured when the existence condition holds, and the natural growth factor divided by the sum of 2 times the natural growth and the depreciation rate is smaller than the aggregate savings rate over the capital income share. Both eigenvalues are then larger than zero and smaller than unity.

Having proven in Propositions 1 and 2, the existence and dynamic stability of the positive-discount steady state, we are entitled to perform local, comparative-dynamic experiments whereby we investigate the impacts of infinitesimal changes of the main model parameters on the steady-state capital-output ratio, the equity-price discount and the unemployment rate. We find that a higher capital income share increases the capital-output ratio, while both a higher natural growth rate and a higher depreciation rate decrease the capital-output ratio. In comparison to these well-known responses of the capital-output ratio, the reactions of the equity-price discount are more interesting while new: more investor optimism and a higher capital-income share increase the equity-price discount, a higher natural growth rate, a larger depreciation rate and a higher saving rate decrease the equity-price discount. Most interesting are the responses of the steady-state unemployment rate which increases with a larger capital-income share and decreases with higher natural growth, a larger depreciation rate and more investor optimism. This last result accords well with short-term Keynesian insights, and it turns out to be valid even in the long run.

Completely, in accordance with the insights from neo-classical growth theory, variations of the saving rate do neither impact the steady-state capital-output ratio nor the steady-state unemployment rate. Thus, we finally investigate the effects of saving-rate changes on the intertemporal-equilibrium path of the capital-output ratio, the equity-price discount, and the unemployment rate. Due to the analytical complexity of the algebra of the partial derivatives of these dynamic variables with respect to marginal parameter variations, we resort to a numerical specification of main model parameters which are in line with the assumptions of Propositions 1 and 2 and are representative of "typical" numerical parameter values within this sort of stylized intertemporal equilibrium models. We find that a marginally smaller saving rate temporarily reduces the capital-output ratio and the unemployment rate, while the equity-price discount increases. After about 80 periods (theoretically after an infinite number of time periods), the capital-output ratio and the unemployment rate return to their pre-shock values, while the equity-price discount permanently increases.

Moreover, we also investigate the intertemporal-equilibrium effects of more investor optimism and a larger natural growth rate. We find that the former temporarily decreases the capital-output ratio and rather strongly the unemployment rate, while the equity-price discount increases in the short and long run. Moreover, the positive investment shock reduces the unemployment rate also in the long run. Finally, a marginally higher natural growth rate decreases temporarily and permanently the capital-output ratio, the equity-price discount and the unemployment rate. A similar effect results from a higher depreciation rate.

Obviously, there is ample space for future research. The highest on the agenda in this respect is the search for a non-degenerate belief function that is consistent with intertemporal equilibrium in our modified stock-market model of involuntary unemployment.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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