

# European Options and Fixed Cost Spreads

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## Abstract

This paper revisits the put-call parity condition for European options, on both non-dividend paying and dividend paying stocks, in the presence of fixed costs spreads. It demonstrates that the put-call parity condition becomes a set of inequalities under these conditions. A model patterned on the Roll model for fixed cost bid-ask spreads is postulated, and its estimation using a generalized method of moments (GMM) approach, is suggested. Finally, the paper demonstrates that in the presence of bid-ask spreads, European options on non-dividend-paying stocks may have a negative time value, unlike the case of such options in the absence of spreads. Also, due to the presence of such spreads, there could be situations, where both calls and puts with the same exercise price are simultaneously exercised.

## Keywords

European Options, Put-Call Parity, Fixed Cost Spreads, Generalized Method of Moments Estimation, Negative Time Value for Deep-in-the-Money Calls

## 1. Introduction

The put-call parity relationship for European options has been derived by using a no-arbitrage argument, under the assumption that stocks and options trade in markets without any transaction costs like bid-ask spreads. The objective of this paper is to introduce fixed cost spreads, in both spot and derivative markets, and examine the implications for the put-call parity theorem. The paper also attempts to examine, whether the assumption of spreads, can cause us to reexamine, traditional notions of option premiums.

This paper derives the equivalent of the put-call parity relationship, when stock and derivative markets are characterized by the presence of bid-ask spreads. Subsequently, the paper postulates a model for the put-call parity relationship, and

its empirical test using a generalized method of moments (GMM) approach. The paper also demonstrates that in the presence of bid-ask spreads, deep-in-the-money European calls may have negative time values.

## 2. The Absence of Arbitrage

The key results in derivatives, hinge on the absence of arbitrage. All option prices must satisfy these properties, or else there will be arbitrage opportunities. The term arbitrage refers to the ability of a trader to make cost-less and risk-less profits. A costly but risky investment should yield a risk-adjusted expected rate of return. A strategy that entails an investment, but is devoid of risk, should yield a risk-less rate of return. The ability of an individual to earn a return without an investment of his own, in an environment devoid of risk, is referred to as arbitrage. In other words, a strategy that yields a cash inflow at certain points in time, and a zero cash flow at other points in time, can be termed as an arbitrage strategy, because it leads to non-negative returns for the investor without requiring an investment (which would have manifested itself as a cash outflow) at any point in time. Such an opportunity will be exploited by anyone who perceives it, to the maximum possible extent till it is eliminated.

## 3. Traditional Assumptions

In order to derive the put-call parity relationship, the following assumptions are made. Traders do not have to pay commissions or incur any form of transaction costs. There are no bid-ask spreads. There are no margin requirements. All traders are price takers. There are no taxes. There is a single risk-less rate of return in the economy and everyone can borrow or lend unlimited amounts at this rate. Securities are infinitely divisible, that is investors can trade in fractions of securities. Traders receive the full premium on options written by them. If a share is sold short, the trader is entitled to receive the full proceeds from the sale immediately. Investors can trade in the stock and options markets instantaneously. If an option written by an investor is exercised, he will get immediate notice of the assignment. Dividends are received on the ex-dividend date. The price decline on the ex-dividend date is exactly equal to the quantum of the dividend. Investors are non-satiated. That is, they are constantly seeking opportunities to increase their wealth. Finally, arbitrage opportunities are exploited as soon as they appear, and till they cease to exist.

## 4. Symbols

- $V_t$   $\equiv$  the stock price at the outset;
- $V_T$   $\equiv$  the stock price at the time of expiration of the options;
- $E$   $\equiv$  the exercise price of the options;
- $r$   $\equiv$  the risk-less rate of interest;
- $C_{E,t}$   $\equiv$  the price of a European call at time  $t$ ;
- $P_{E,t}$   $\equiv$  the price of a European put at time  $t$ .

## 5. Put-Call Parity for European Options

The Put-Call Parity theorem states that (see [Stoll, 1969](#))

$$C_{E,t} - P_{E,t} = V_t - \frac{E}{(1+r)^{T-t}},$$

where  $C_{E,t}$  represents the price of a European call with an exercise price of  $E$  and time to expiration equal to  $T-t$  periods, and  $P_{E,t}$  represents the price of a European put with the same exercise price and expiration date. A recent test of this relationship is [Steurer, Fahling, and Du \(2022\)](#).

## 6. European Options on a Dividend Paying Stock

The put-call parity relationship for European options on a stock which pays a known dividend  $D$  at time  $t^*$ , where  $t^*$  is a point in time before the expiration of the option such that  $t < t^* < T$ , may be stated as:

$$C_{E,t} - P_{E,t} = V_t - \frac{D}{(1+r)^{t^*-t}} - \frac{E}{(1+r)^{T-t}}.$$

## 7. Models of the Bid-Ask Spread

The quoted bid-ask spread is defined as the difference between the price at which a dealer is willing to sell the security, the ask, and the price at which he is willing to buy it, the bid. There are three major theories that purport to explain the existence of the spread. These are, order processing cost models, inventory cost models, and adverse selection models. The simplest of the three models are the order processing, or fixed cost models.

[Stoll \(1989\)](#) makes a distinction between the quoted spread, and the effective or realized spread. The realized spread is defined as the difference between the price at which the dealer sells a security, and the price at which he buys at an earlier instant. The order processing models, predict that the two spreads are equal.

One of the most popular models in this class is the Roll model ([Roll, 1984](#)), which was later extended by [Choi, Salandro, and Shastri \(1988\)](#). Its premise is that the spread compensates the dealer for the costs incurred in processing buy and sell transactions. In this framework, transactions at the bid are offset by transactions at the ask, and the difference between the two is used by the dealer to defray his expenses. In this paper, we study the order processing model, and its implications for European options. By focusing solely on the fixed cost model, we are ignoring the other two components of the spread, namely inventory and adverse selection costs. However, this does not imply that these costs are insignificant, although empirical research has shown that the fixed cost component constitutes the bulk of the spread<sup>1</sup>.

The literature on the estimation of the realized spread is fairly diverse. Roll estimates realized spreads in an order processing framework using daily and weekly data and attempts to relate them to firm size. Choi, Salandro and Shastri

<sup>1</sup>See [Parameswaran \(1991\)](#).

use transaction data for stock options to estimate the spread, and the conditional probability of a price continuation. [Stoll \(1989\)](#) estimates the relative components of the spread using transaction prices and bid-ask quotes. [Amihud and Mendelson \(1987\)](#), who study the impact of the trading mechanism on the return-generating process, incorporate a partial adjustment model for the evolution of asset prices.

## 8. Assumptions

The order processing model was developed under the following set of assumptions:

- Security prices at any given time, fully incorporate all relevant available information. What this means is that the price that would be observed in the absence of market imperfections accurately reflects the intrinsic value of the asset.
- Buy or sell orders are equally probable at any instant in time, on an unconditional basis, and order flows are serially independent.
- The underlying price of the asset is independent of the order flow in the market. This implicitly rules out any adverse information effects.
- The true value of the security is bracketed by the bid-ask spread and the spread is assumed to be constant and symmetric, at least for the time period for which the analysis is conducted.

## 9. Incorporation of Spreads

The quoted bid-ask spread is defined as the difference between the price at which a dealer is willing to sell the security, the ask, and the price at which he is willing to buy it, the bid. Now, we will introduce fixed cost spreads in all the three product markets:

- $s_s \equiv$  the bid-ask spread in the stock market;
- $s_c \equiv$  the bid-ask spread for call options;
- $s_p \equiv$  the bid-ask spread for put options.

The true asset price of, be it a stock or an option, is assumed to be the mid-point of the bid and ask prices. The arbitrageur can buy the stock or the options at the ask price and sell them at the corresponding bid. The bid prices are therefore  $V - \frac{s_s}{2}$ ,  $C - \frac{s_c}{2}$ , and  $P - \frac{s_p}{2}$ . The ask prices are  $V + \frac{s_s}{2}$ ,  $C + \frac{s_c}{2}$ , and  $P + \frac{s_p}{2}$ .

## 10. Revisiting Strategy-1 of the Put-Call Parity Proof in the Presence of Spreads

Consider the following strategy:

- Buy the stock;
- Sell the call;

- Buy the put;
- Borrow the present value of the exercise price.

The initial cash flow is:

$$-\left(V_t + \frac{S_s}{2}\right) + \left(C_{E,t} - \frac{S_c}{2}\right) - \left(P_{E,t} + \frac{S_p}{2}\right) + \frac{E}{(1+r)^{T-t}}.$$

There are three possibilities at expiration.

$$\text{Case-1: } E < V_T - \frac{S_s}{2} < V_T + \frac{S_s}{2}$$

The counterparty who has bought the call from the arbitrageur will exercise. If he wants the stock he would rather pay  $E$  than the ask price of the stock. If he does not want the stock he will exercise and pay  $E$ . The stock can then be sold at the bid which by assumption is higher. The put is out-of-the-money and will not be exercised. The net result is an inflow of  $E$ . When the amount borrowed, is paid off with interest, the net cash flow is zero.

$$\text{Case-2: } V_T - \frac{S_s}{2} < E < V_T + \frac{S_s}{2}$$

If the counterparty does not want the stock he will not exercise the option, because after acquiring the stock by paying  $E$ , he will have to dispose it off by selling it at the bid, which by assumption is lower. If the counterparty wants the stock then he will exercise, because paying  $E$  to buy it is cheaper than buying at the spot ask. Now consider the put. If the counterparty were to exercise the call, the stock will be handed over. The put will not be exercised because it is not profitable to buy the stock at the ask and then deliver it for  $E$ . If the counterparty does not exercise, the put will be exercised. This is because exercising and delivering under the put option at the exercise price, is more attractive than selling the stock at the spot bid. The net result, once again, is an inflow of  $E$ . The overall cash flow, after considering the initial borrowing is zero.

$$\text{Case-3: } V_T - \frac{S_s}{2} < V_T + \frac{S_s}{2} < E$$

The counterparty will not exercise the call. The arbitrageur will exercise the put because selling the stock for  $E$  is more attractive than selling it at the spot bid. As before, the net inflow is  $E$ . The net cash flow, after considering the initial borrowing is zero.

Hence irrespective of the relationship between the exercise price and the terminal stock price, the cumulative cash flow from the long stock, the short call, and the long put is  $E$ . The overall cash flow is therefore zero, in all three terminal scenarios, because the amount borrowed at the outset has to be repaid with interest. Thus, to rule out arbitrage:

$$\begin{aligned} &-\left(V_t + \frac{S_s}{2}\right) + \left(C_{E,t} - \frac{S_c}{2}\right) - \left(P_{E,t} + \frac{S_p}{2}\right) + \frac{E}{(1+r)^{T-t}} \leq 0, \\ \Rightarrow &C_{E,t} - P_{E,t} \leq V_t - \frac{E}{(1+r)^{T-t}} + \frac{S_s}{2} + \frac{S_c}{2} + \frac{S_p}{2}. \end{aligned}$$

## 11. Revisiting Strategy-2 of the Put-Call Parity Proof in the Presence of Spreads

Now, let us reverse the above strategy. The required actions are:

- Short sell the stock;
- Buy the call;
- Sell the put;
- Lend the present value of the exercise price.

The initial cash flow is

$$\left(V_t - \frac{s_s}{2}\right) - \left(C_{E,t} + \frac{s_c}{2}\right) + \left(P_{E,t} - \frac{s_p}{2}\right) - \frac{E}{(1+r)^{T-t}}.$$

There are three possibilities at expiration.

$$\text{Case-1: } E < V_T - \frac{s_s}{2} < V_T + \frac{s_s}{2}$$

The counter-party who has bought the put will not exercise. The arbitrageur will exercise the call. Buying the stock at  $E$  is more attractive than buying it at the spot ask to close out the short stock position. The net result is an outflow of  $E$ . The overall cash flow is zero.

$$\text{Case-2: } V_T - \frac{s_s}{2} < E < V_T + \frac{s_s}{2}$$

The counterparty who has bought the put will exercise if he has the stock, for the alternative is to sell it at the spot bid, which by assumption is lower. If he does not have the stock, he will not exercise because buying the stock at the spot ask and selling it at the exercise price will lead to a loss. If the counterparty exercises the put, the arbitrageur will use the stock to cover the short stock position. He will not exercise the call because paying  $E$  for the stock and then selling it at the spot bid will lead to a loss. If the counterparty does not exercise the put, the arbitrageur will exercise the call, for it is cheaper to cover the short position by paying  $E$ , than by buying the stock at the spot ask. The overall cash flow is an outflow of  $E$ , and the net cash flow is zero once again.

$$\text{Case-3: } V_T - \frac{s_s}{2} < V_T + \frac{s_s}{2} < E$$

The counterparty, who has bought the put, will definitely exercise. The stock received from him can be used to cover the short stock position. The call will not be exercised. The net result is an outflow of  $E$ .

Hence, irrespective of the relationship between the exercise price and the terminal stock price, the cumulative cash flow from the short stock, the long call, and the short put is  $-E$ . The overall cash flow is therefore zero, in all three terminal scenarios, because the amount lent at the outset, will be repaid with interest. Thus to rule out arbitrage:

$$\left(V_t - \frac{s_s}{2}\right) - \left(C_{E,t} + \frac{s_c}{2}\right) + \left(P_{E,t} - \frac{s_p}{2}\right) - \frac{E}{(1+r)^{T-t}} \leq 0,$$

$$\Rightarrow C_{E,t} - P_{E,t} \geq V_t - \frac{E}{(1+r)^{T-t}} - \frac{s_s}{2} - \frac{s_c}{2} - \frac{s_p}{2}.$$

Thus, in the presence of spreads, the put-call parity condition may be expressed as:

$$V_t - \frac{E}{(1+r)^{T-t}} - \frac{s_s}{2} - \frac{s_c}{2} - \frac{s_p}{2} \leq C_{E,t} - P_{E,t} \leq V_t - \frac{E}{(1+r)^{T-t}} + \frac{s_s}{2} + \frac{s_c}{2} + \frac{s_p}{2}.$$

The equivalent condition for a stock paying a known dividend  $D$  at time  $t^*$ , where  $t^*$  is a point in time before the expiration of the option such that  $t < t^* < T$ , may be stated as:

$$\begin{aligned} V_t - \frac{D}{(1+r)^{t^*-t}} - \frac{E}{(1+r)^{T-t}} - \frac{s_s}{2} - \frac{s_c}{2} - \frac{s_p}{2} &\leq C_{E,t} - P_{E,t} \\ &\leq V_t - \frac{D}{(1+r)^{t^*-t}} - \frac{E}{(1+r)^{T-t}} + \frac{s_s}{2} + \frac{s_c}{2} + \frac{s_p}{2} \end{aligned}$$

## 12. A Model for Put-Call Parity

Let  $S_t$  be the observed stock price at time  $t$ . Mathematically, the model may be stated as follows:

$$S_t = V_t + \frac{s_c}{2} Q_{s,t},$$

and,

$$V_t = V_{t-1} + \mu + \varepsilon_t,$$

where  $S_t$  is the observed price at time  $t$ ,  $V_t$  is the intrinsic value of the asset at time  $t$ , which would have been observed in the absence of the spread, and  $Q_{s,t}$  is the indicator of the transaction type.  $Q_{s,t} = 1$  if the observed stock price is at the ask, and  $-1$  if it is at the bid. The value innovations,  $\{\varepsilon\}$  are assumed to be serially uncorrelated, and have a zero mean and constant variance  $\sigma_\varepsilon^2$ . The drift in the true prices  $\mu$  is assumed to be constant over time, as is the spread  $s$ .

$$E(\Delta Q) = 0; \text{ Variance of } (\Delta Q) = 2; \text{ Covariance of } (\Delta Q_t, \Delta Q_{t-1}) = -1.$$

Thus, the variance of the stock price changes is  $\text{Var}(\Delta S_t) = \sigma_\varepsilon^2 + \frac{s^2}{2}$  and the covariance of the observed price changes equals minus the square of half the bid-ask spread, *i.e.*  $\text{Cov}(\Delta S_t, \Delta S_{t-1}) = -s^2/4$ , and the estimates of the covariance can be transformed to get the spreads<sup>3</sup>.

We can express the put-call parity condition for European options on a non-dividend paying stock as:

$$C_{E,t} - P_{E,t} = V_t - \frac{E}{(1+r)^{T-t}} + \frac{s_s}{2} Q_{s,t} + \frac{s_c}{2} Q_{c,t} + \frac{s_p}{2} Q_{p,t}.$$

<sup>2</sup>Researchers often assume that the  $\{\varepsilon\}$ 's are i.i.d. This, however, is not necessary for our study.

<sup>3</sup>The resulting estimate is subject to underestimation due to Jensen's inequality, because the transformation uses a concave function.

Like  $Q_{s,t}, Q_{c,t} = 1$  if the observed call option price is at the ask, and  $-1$  if it is at the bid. A similar condition holds for  $Q_{p,t}$  in the case of put options. This model satisfies the put-call parity relationship derived earlier.

Let us define  $C^*$  as the observed call price and  $P^*$  as the observed put price. The observed call premium is  $C_{E,t}^* = C_{E,t} + \frac{s_c}{2} Q_{c,t}$  and the observed put premium is  $P_{E,t}^* = P_{E,t} + \frac{s_p}{2} Q_{p,t}$ . We will define the difference between the observed call and put premiums as  $Z$ .

Thus

$$\begin{aligned} Z_t &= C_{E,t}^* - P_{E,t}^* = C_{E,t} - P_{E,t} + \frac{s_c}{2} Q_{c,t} - \frac{s_p}{2} Q_{p,t} \\ &= V_t - \frac{E}{(1+r)^{T-t}} + \frac{s_s}{2} Q_{s,t} + 2 \frac{s_c}{2} Q_{c,t} \end{aligned}$$

If we use intra-day data, the expression for the present value of the exercise price drops out when we compute  $\Delta Z$ .

$$\Delta Z_t = Z_t - Z_{t-1} = \mu + \varepsilon_t + \frac{s_s}{2} \Delta Q_{s,t} + s_c \Delta Q_{c,t},$$

$$\text{Cov}(\Delta Q_{s,t}, \Delta Q_{c,t}) = 0; \text{Cov}(\Delta Q_{s,t}, \Delta Q_{c,t-1}) = 0; \text{Cov}(\Delta Q_{s,t-1}, \Delta Q_{c,t}) = 0.$$

$$\text{Thus variance of } \Delta Z_t = \sigma_\varepsilon^2 + \frac{s_s^2}{2} + 2s_c^2,$$

$$\text{Cov}(\Delta Z_t, \Delta Z_{t-1}) = -\frac{s_s^2}{4} - s_c^2.$$

### 13. A Method of Moments Estimation

Given the assumptions of the model, a number of results may be derived. Among them are the following:

$$E(\Delta S_t) = \mu.$$

The expected change in the observed price of the security is a constant.

$$\text{Var}(\Delta S_t) = \sigma_\varepsilon^2 + \frac{s_s^2}{2}.$$

The variance of changes in the observed price of the asset is equal to the variance of true price changes plus one half the square of the bid-ask spread.

$$\text{Cov}(\Delta S_t, \Delta S_{t-1}) = -\frac{s_s^2}{4}.$$

The first order serial covariance of observed price changes is minus one fourth the square of the bid-ask spread.

$$\text{Var}(\Delta Z_t) = \sigma_\varepsilon^2 + \frac{s_s^2}{2} + 2s_c^2.$$

The variance of the difference between the observed call premium and the ob-



served put premium is equal to the variance of the true stock price changes, plus one half of the square of the spread in the stock market, plus twice the square of the spread for call options.

$$Cov(\Delta Z_t, \Delta Z_{t-1}) = -\frac{s_s^2}{4} - s_c^2.$$

The first order serial covariance of the difference between the observed call premium and the observed put premium, is minus the sum of one fourth the square of the spread in the stock market and the square of the spread for call options.

We have five moment conditions, and four unknowns,  $\mu, \sigma_\epsilon^2$  and  $s_s$  and  $s_c$ . The population moment conditions may be jointly expressed as,

$$E \begin{pmatrix} S_t - S_{t-1} - \mu \\ (S_t - S_{t-1} - \mu)^2 - \sigma_\epsilon^2 - \frac{s_s^2}{2} \\ (S_t - S_{t-1} - \mu)(S_{t-1} - S_{t-2} - \mu) + \frac{s_s^2}{4} \\ (Z_t - Z_{t-1} - \mu)^2 - \sigma_\epsilon^2 - \frac{s_s^2}{2} - 2s_c^2 \\ (Z_t - Z_{t-1} - \mu)(Z_{t-1} - Z_{t-2} - \mu) + \frac{s_s^2}{4} + s_c^2 \end{pmatrix}.$$

In a finite sample, we apply the test to (see Hansen, 1982),

$$g_T(\mu, \sigma_\epsilon^2, s_s^2, s_c^2) = \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} S_t - S_{t-1} - \mu \\ (S_t - S_{t-1} - \mu)^2 - \left(\sigma_\epsilon^2 + \frac{s_s^2}{2}\right) \\ (S_t - S_{t-1} - \mu)(S_{t-1} - S_{t-2} - \mu) + \left(\frac{s_s^2}{4}\right) \\ (Z_t - Z_{t-1} - \mu)^2 - \sigma_\epsilon^2 - \frac{s_s^2}{2} - 2s_c^2 \\ (Z_t - Z_{t-1} - \mu)(Z_{t-1} - Z_{t-2} - \mu) + \frac{s_s^2}{4} + s_c^2 \end{pmatrix}.$$

The over identifying statistic associated with this equation has an asymptotic chi-squared distribution, with one degree of freedom. That is,

$$Tg_T'W_0g_T \sim \chi^2(1),$$

where  $W_0$  is the inverse of the variance-covariance matrix of restrictions.

### 14. Simultaneous Exercise

Consider the case where  $V_T + \frac{s_s}{2} > E > V_T - \frac{s_s}{2}$ . Holders of calls who wish to acquire the stock will exercise. Holders of puts, who have the stock, and do not wish to hold it further, will also exercise. So in this range of the stock price, both calls and puts may be exercised.

## 15. Implications for the Time Value

Conventional option theory has demonstrated that deep-in-the-money European put options can have negative time values, whereas deep-in-the-money European calls cannot. The rationale, is that in the case of the former, the longer the wait, the greater the interest foregone by not receiving the exercise price, while in the case of the latter, a longer time to maturity, implies greater interest earned due to the delay in parting with the exercise price. Out-of-the-money calls and puts, cannot have negative time values, because the option premium cannot be negative.

Let us examine the lower bound for put-call parity.

$$\begin{aligned} C_{E,t} - P_{E,t} &= V_t - \frac{E}{(1+r)^{T-t}} - \frac{s_s}{2} - \frac{s_c}{2} - \frac{s_p}{2} \\ &= E - \frac{E}{(1+r)^{T-t}} + V_t - \left(E + \frac{s_s}{2}\right) - \frac{s_c}{2} - \frac{s_p}{2}, \\ \Rightarrow C_{E,t} &= P_{E,t} + E - \frac{E}{(1+r)^{T-t}} + V_t - \left(E + \frac{s_s}{2}\right) - \frac{s_c}{2} - \frac{s_p}{2}. \end{aligned}$$

If  $V_t - \left(E + \frac{s_s}{2}\right) > 0$  that is

$$\begin{aligned} V_t &> \left(E + \frac{s_s}{2}\right), \\ \Rightarrow V_t &> \left(E - \frac{s_s}{2}\right), \end{aligned}$$

the call is in the money, from the perspective of those who want to acquire the stock, as well as those who plan to acquire and sell the stock. The time value is

$$P_{E,t} + E - \frac{E}{(1+r)^{T-t}} - \frac{s_c}{2} - \frac{s_p}{2}.$$

If the call is substantially in the money then the put value will be close to zero. If the interest rate is low and the time to maturity is not high, the difference between the exercise price and the present value of the exercise price will be small. In such a situation, if the bid-ask spreads, for both call and put options are high, then the call may have a negative time value. This is because the higher the spread paid by the option buyer, the more the interest income foregone by the inability to invest it. In such cases, the more the time till expiration, the worse it is for the option holder. Deep-in-the-money puts can have negative time values, with or without spreads.

## 16. Conclusion

This paper derives the equivalent of the put-call parity relationship for European options, in the presence of fixed cost spreads in both stock and options markets. It demonstrates that the put-call parity equivalent in this case is a set of inequali-

ties. The paper then postulates a model for the difference between the call and put premiums, in the presence of bid-ask spreads in the stock as well as the options markets. Using this model, the paper develops a test for the model based on the generalized method of moments (GMM) approach. The paper demonstrates that, when the markets are characterized by spreads, European calls and European puts, with the same exercise price and expiration date, may both be exercised. The paper also shows that deep-in-the-money European call options may have a negative time value, when there are spreads in the stock and options markets.

### Data Availability Statement

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

### Conflicts of Interest

- 1) No funding was received to assist with the preparation of this manuscript.
- 2) All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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