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Shapley Value and International Trade

Robert A. Agnew

Palm Coast, Florida, USA
Email: raagnew1@gmail.com

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Abstract

International trade is modeled as a multinational cooperative game where Shapley value leads to fair distribution of total trade gains. This fair-trade solution entails side payments among trading nations, essentially international trade adjustment assistance. These Shapley side payments provide balance to the often contentious free-trade solution, which is essentially open-door and hands-off. We develop requisite calculations and illustrate them with a hypothetical four-nation trade game. We conclude that Shapley fair-trade estimates and calculations should be an integral part of any international trade negotiation.

Keywords

International Trade, Cooperative Game Theory, Shapley Value

1. Introduction

Most international agreements encompass free trade among member nations, perhaps with side conditions related to worker safety and environmental pollution. Moreover, per economists Krugman and Obstfeld (2003) and Porter (1990), free trade fosters international competition, innovation, and prosperity. However, free trade can be quite contentious in developed nations with high wage structures and these nations have significant bargaining power by virtue of their vast domestic markets.

Shapley value of cooperative game theory provides a fair distribution benchmark in all sorts of economic and political negotiations, *e.g.* Agnew (2022). In the international trade arena, it provides a means of balancing overall trade benefits among nations while maintaining the transactional benefits of free trade. This is accomplished by side payments among participating nations. Shapley side payments among trading nations can be considered as *international* trade adjustment assistance (TAA). We've had intranational TAA for displaced workers in the U.S. for decades, but it has recently been terminated (https://www.dol.gov/agencies/eta/tradeact). Shapley fair trade suggests international TAA as a balancing mechanism for free trade.

Recent reviews of game theory application to international trade have been provided by Khurana (2022) and Mughwai (2020). However, these articles are focused on noncooperative game theory, particularly on the prisoner's dilemma. Krugman and Obstfeld (2003) also highlight the prisoner's dilemma in the context of trade policy. Kraphol, Ocelík, and Walendek (2021) present results from adaptive prisoner's dilemma iterations, but again the focus is on noncooperative strategies. Shanaev (2015) does focus on cooperative game theory and he uses Shapley value as the basis for "fair" pricing across international goods markets. Our approach is similar, but we focus specifically on the structure of a trade agreement that enables free trade and which adapts to Shapley fair-trade sharing of total trade gains via side payments among participating nations.

In the next section, we review requisite elements of cooperative game theory utilizing concepts and notation from Owen (2001), a widely referenced game theory textbook. The following section contains the structure of our international trade game, followed by a four-nation illustration.

2. Cooperative Games and Shapley Value

We have a finite set of players $N = \{1, \cdots, n\}$ and a superadditive characteristic function v defined on subsets of N with $v(\phi) = 0$. Nonempty subsets of N are called coalitions. Superadditivity requires that $v(S \cup T) \geq v(S) + v(T)$ whenever $S \cap T = \phi$. In addition, a game is monotone if $v(S) \leq v(T)$ whenever $S \subset T$ and a game is convex if $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$ for all $S, T \subset N$. We assume throughout that $n \geq 2$ and that v reflects transferable currency like the U.S. dollar.

An imputation is a vector $x = (x_1, \cdots, x_n)$ such that $x_i \geq v(\{i\})$ and $\sum_{i=1}^{n} x_i = v(N)$. The idea here is that a player must receive at least what he can achieve on his own and that the grand coalition N will ultimately form so the issue is fair distribution of the total pie. There are various solution concepts for cooperative games in terms of imputations but we will focus on two, Shapley value and the core.

We say that imputation y dominates imputation z if $y_i > z_i$ for all i in some nonempty $S \subset N$ and $\sum_{i \in S} y_i \leq v(S)$. The core $C(v)$ is the set of undominated imputations and is characterized as the set of imputations x satisfying $\sum_{i \in S} x_i \geq v(S)$ for all $S \subset N$ and $\sum_{i \in N} x_i = v(N)$. If the core is nonempty, imputations outside of it are inherently unstable.

Shapley value is the particular imputation $\phi[v]$ defined for characteristic function v by $\phi_i[v] = \sum_{\substack{S \subset N \\ i \in S}} \frac{(s-1)!(n-s)!}{n!} \left[v(S) - v(S - \{i\}) \right]$ where $s = |S|$ =

number of elements in set S and $\gamma(S)=\frac{(s-1)!(n-s)!}{n!}=\frac{1}{n\binom{n-1}{s-1}}$ depends only on the size of S. Shapley value is derived axiomatically but it has a heuristic expected value interpretation involving randomly permuted arrivals, all with the same probability $1/n!$. If player i arrives and finds coalition $S-\{i\}$ already there, he receives his marginal value $v(S)-v(S-\{i\})$. Shapley value $\phi_i[v]$ is the expected payoff to player i under this randomization scheme. Shapley value is widely viewed as distributionally fair. If the game is convex, then Shapley value is in the core $C(v)$.

3. International Trade Game

Consider a set of n nations who are negotiating a trade agreement on behalf of their citizens. For each nonempty $S\subset N$, let $g_i(S)$ be the gain from trade to nation i from inclusion in coalition S, where $g_i(\{i\})=0$ since there is no gain from going alone and $g_i(S)=0$ if $i\notin S$. Furthermore, we assume the monotonicity condition $g_i(S)\le g_i(T)$ for every i when $S\subset T$. And once again, we assume that these gains are denominated in transferable currency.

We define characteristic function $v(S)=\sum_{i\in S} g_i(S)$ and $y=(g_1(N),\cdots,g_n(N))$ represents the free-trade imputation associated with the grand coalition. We denote $z=\phi[v]$ as the fair-trade Shapley imputation. Then $z-y=(z_1-y_1,\cdots,z_n-y_n)$ defines side payments among nations, or international TAA, to achieve fairness.

Proposition. The free-trade imputation y is in the core $C(v)$. If the game is convex, the Shapley fair-trade imputation z is also in the core.

Proof. By monotonicity, $g_i(N)=\max_{S\subset N} g_i(S)$ for every i. Hence, $\sum_{i\in S} y_i \ge \sum_{i\in S} g_i(S)=v(S)$ for all $S\subset N$ and $\sum_{i\in N} y_i=v(N)$. If the game is convex, which it normally would be, it is known that Shapley value is in the core. ■

Before illustrating a four-nation game, we want to highlight the simple, but important two-nation case with $n=2$ and $N=\{1,2\}$. In this case, all imputations are in the core so there is no issue of dominance. The free-trade imputation $y=(y_1,y_2)=(g_1(N),g_2(N))$ can be unbalanced but the Shapley fair-trade imputation $z=(z_1,z_2)=\left(\frac{v(N)}{2},\frac{v(N)}{2}\right)=\left(\frac{y_1+y_2}{2},\frac{y_1+y_2}{2}\right)$ is balanced with 50-50 sharing of total gain from trade. This makes sense because there is no trading gain at all unless the two parties agree, *i.e.* it's all or nothing. 50-50 is a common structure for business joint ventures. It also corresponds to the Nash bargaining solution in Owen (2001), Chapter 9.

Having said all that, the free-trade solution y has the advantage that it just falls out. The fair-trade solution z requires ongoing side payments to achieve Shapley fairness, *i.e.* if $y_1<y_2$ then the Shapley 50-50 split entails an ongoing side payment

of $z_1 - y_1 = \frac{y_2 - y_1}{2}$ from Nation 2 to Nation 1 for the duration of the trade agreement. This of course will be a hard sell. Shapley fairness requires significant negotiation and administration beyond champagne toasts and dusty documents.

4. Four-Nation Trade Game

We illustrate Shapley calculations with a hypothetical trade game encompassing four nations of varying development and size. The entries in Table 1 were created

Table 1. Four-nation trade game.

Nation	GDP ($ Billion)	Description
1	$15,000	Large Developed
2	$2000	Medium Developed
3	$1000	Large Developing
4	$400	Medium Developing
Total	$18,400	

National GDP Gain ($ Billion) from Trade					
Coalition S	**1**	**2**	**3**	**4**	**Total $v(S)$**
Φ	$0	$0	$0	$0	$0
{1}	$0	$0	$0	$0	$0
{2}	$0	$0	$0	$0	$0
{3}	$0	$0	$0	$0	$0
{4}	$0	$0	$0	$0	$0
{1, 2}	$60	$60	$0	$0	$120
{1, 3}	$45	$0	$40	$0	$85
{1, 4}	$15	$0	$0	$40	$55
{2, 3}	$0	$20	$20	$0	$40
{2, 4}	$0	$12	$0	$16	$28
{3, 4}	$0	$0	$10	$8	$18
{1, 2, 3}	$90	$80	$70	$0	$240
{1, 2, 4}	$75	$70	$0	$60	$205
{1, 3, 4}	$60	$0	$50	$52	$162
{2, 3, 4}	$0	$30	$25	$20	$75
{1, 2, 3, 4} Free Trade	$105	$100	$80	$80	$365
% of Total	28.8%	27.4%	21.9%	21.9%	100%
% GDP Gain	0.70%	5.00%	8.00%	20.00%	1.98%
Shapley Fair Trade	$137.6	$96.6	$74.8	$56.1	$365.0
% of Total	37.7%	26.5%	20.5%	15.4%	100%
% GDP Gain	0.92%	4.83%	7.48%	14.02%	1.98%
Side Payment	$32.6	−$3.4	−$5.3	−$23.9	$0.0

by the author for illustration; they seem reasonable but they obviously don't correspond to any real-world trade agreement. It is easy to verify that this game is monotonic and it can be verified that it is also convex.

Table 1 stipulates GDP gains for the various possible coalitions along with free-trade and Shapley fair-trade distributions of total grand coalition GDP gain. To illustrate the Shapley calculation for Nation 1, we note that $\gamma(S)=\frac{1}{12}$ when $|S|=2$ or 3 and $\gamma(S)=\frac{1}{4}$ when $|S|=4$. Then, focusing on the Total column in Table 1 and ignoring irrelevant zeros, we have

$$\begin{aligned} z_1 &= \frac{v(\{1,2\})-v(\{2\})}{12}+\frac{v(\{1,3\})-v(\{3\})}{12}+\frac{v(\{1,4\})-v(\{4\})}{12} \\ &+\frac{v(\{1,2,3\})-v(\{2,3\})}{12}+\frac{v(\{1,2,4\})-v(\{2,4\})}{12} \\ &+\frac{v(\{1,3,4\})-v(\{3,4\})}{12}+\frac{v(\{1,2,3,4\})-v(\{2,3,4\})}{4} \\ &= \frac{120+85+55+240-40+205-28+162-18}{12}+\frac{365-75}{4} \\ &= 137.6 \end{aligned}$$

Even though this game is monotonic and convex, and hence both the free-trade and fair-trade imputations are in the core, Table 2 utilizes matrix multiplication to verify that both $\sum_{i\in S} y_i \ge v(S)$ and $\sum_{i\in S} z_i \ge v(S)$ for all relevant proper subsets $S \subset N$.

Once again, the free-trade solution just falls out and is essentially hands-off. The Shapley fair-trade solution, however, entails ongoing side payments among the four nations for the duration of the trade agreement. This may seem to stiff

Table 2. Matrix multiplication verifies that both free-trade and fair-trade imputations are in the core.

Coalition S					Free Trade Imputation	Free Trade Product	Fair Trade Imputation	Fair Trade Product		$v(S)$
{1, 2}	1	1	0	0	\$105	\$205	\$137.6	\$234.2	≥	\$120
{1, 3}	1	0	1	0	\$100	\$185	\$96.6	\$212.3	≥	\$85
{1, 4}	1	0	0	1	\$80	\$185	\$74.8	\$193.7	≥	\$55
{2, 3}	0	1	1	0	\$80	\$180	\$56.1	\$171.3	≥	\$40
{2, 4}	0	1	0	1		\$180		\$152.7	≥	\$28
{3, 4}	0	0	1	1		\$160		\$130.8	≥	\$18
{1, 2, 3}	1	1	1	0		\$285		\$308.9	≥	\$240
{1, 2, 4}	1	1	0	1		\$285		\$290.3	≥	\$205
{1, 3, 4}	1	0	1	1		\$265		\$268.4	≥	\$162
{2, 3, 4}	0	1	1	1		\$260		\$227.4	≥	\$75

small Nation 4, but it still enjoys a large percentage GDP gain from trade. Large Nation 1 benefits because it has more significant coalition options and hence considerably more bargaining power. It doesn't have to simply accept what falls out. Its side payment represents the price of entry to its developed market and a potential TAA fund for its displaced workers.

5. Conclusion

Shapley value is utilized for fair apportionment of value in a wide variety of economic and political settings. In the international trade setting, it yields a sensible split of total gains from trade, but it requires ongoing side payments among the parties to an agreement, unlike free trade which is essentially open-door and hands-off. However, untrammeled free trade can be destructive to wage structures in developed nations, leading to significant political backlash. Shapley fair trade can in principle balance the scales while still enabling global competition and development. Shapley estimates and calculations should be an integral part of every international trade negotiation. Negotiated Shapley side payments can ensure ongoing fairness and stability to resulting agreements.

Conflicts of Interest

No potential conflict of interest was reported by the author.

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