

Nash Bargaining and Outsourcing in a Duopoly Market

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Abstract

In this paper, we clarify the impact of Nash bargaining between manufacturers and an input supplier on manufacturer outsourcing decisions. At a subgame perfect equilibrium, the manufacturers select in-house production, and no outsourcing equilibrium exists. In the intra-industry outsourcing literature, [Sinha \(2016\)](#) shows that manufacturers select in-house production in response to the existence of fixed costs. In contrast, we clarified that Nash bargaining leads to in-house production equilibrium even when the cost function is not associated with fixed costs.

Keywords

Duopoly Market, Cournot Competition, Outsourcing, In-House Production, Nash Bargaining, Subgame Perfect Equilibrium

1. Introduction

Canon develops CMOS image sensors, essential components in digital single-lens reflex cameras, in-house, while Nikon outsources their production. If each manufacturer can either outsource input production or produce inputs in-house, what production system will form within the industry?

Theoretical research on outsourcing selection in the industry includes studies by [Shy and Stenbacka \(2003\)](#) and [Buehler and Haucap \(2006\)](#). [Sinha \(2016\)](#) represents the leading edge of this literature.

[Sinha \(2016\)](#) has shown that no outsourcing equilibrium exists when outsourced production is associated with economies of scale. In this paper, we demonstrate that, even without economies of scale, Nash bargaining between manufacturers and the input supplier yields the same result as in [Sinha \(2016\)](#).

The structure of this paper is as follows. In Section 2, the basic model is established, and in Sections 3 and 4, the subgame perfect equilibrium in a three-stage

game is calculated. The game comprises outsourcing selection by the manufacturer(s), Nash bargaining between the manufacturer(s) and supplier, and Cournot competition between manufacturers. Finally, our conclusions are summarized in Section 5.

2. Model

Consider an industry consisting of two manufacturers (Firm 1 and Firm 2) producing a given product and one supplier (Firm 3) producing an input essential to the production of that product.

In a product market, Firm 1 and Firm 2 engage in Cournot competition. Both firms can choose whether to develop inputs in-house, or to procure them from Firm 3.

We assume that there is no cooperation between Firm 1 and Firm 2 before outsourcing input production to Firm 3.

We also assume that the inverse demand function for the product in the market is

$$p = a - Q, \quad (1)$$

where p denotes the price of the product, and $a > 0$ is a sufficiently large constant to guarantee positive profits. Furthermore, Q expresses the total quantity of production by Firm 1 and Firm 2.

For simplicity, the marginal cost for both Firm 1 and Firm 2 is assumed to be the constant $c > 0$. Additionally, we assume that one unit of the input is necessary to produce one unit of the product.

When Firm i ($i = 1, 2$) produces the input in-house, the production cost C^I is

$$C^I = cq_i, \quad (2)$$

where the superscript I represents the selection of in-house production. Moreover, q_i expresses the quantity of production by Firm i ($i = 1, 2$).

On the other hand, when Firm i ($i = 1, 2$) purchases input produced by Firm 3, the production cost C^O is

$$C^O = rq_i, \quad (3)$$

where the superscript O represents the selection of outsourcing. Meanwhile, r is the input price. When considering the input price, [Shy and Stenbacka \(2003\)](#) and [Sinha \(2016\)](#) assume the supplier is a monopolist, so that only Firm 3 can decide r . In our model, every firm can produce the input and so r is defined as the Nash bargaining solution.

Denoting outsourcing selection as k ($k = I, O$), the profit of Firm i ($i = 1, 2$) π_i^k is expressed as

$$\pi_i^k = pq_i - C^k, k = I, O. \quad (4)$$

In this paper, we analyze a subgame perfect equilibrium in the following three-stage game:

Stage 1: Selection of outsourcing;

Stage 2: Nash bargaining;

Stage 3: Cournot competition.

In Section 3, we will solve the game using backward induction.

3. Equilibrium Analysis

3.1. When Both Manufacturers Select Outsourcing

When both Firm 1 and Firm 2 outsource, the best-response function for Firm i ($i = 1, 2$) is as follows:

$$BR_i^O(q_j) = \frac{a - q_j - r}{2}, i, j = 1, 2, j \neq i. \quad (5)$$

Using Equation (5) to derive the equilibrium quantity of production by Firm i gives

$$q_i^* = \frac{a - r}{3}, i = 1, 2. \quad (6)$$

Since a is assumed to be sufficiently large, $q_i^* > 0$. Totaling the equilibrium quantity of production by Firm 1 and Firm 2 and substituting it into Equation (1), we can obtain the equilibrium price p^* as follows:

$$p^* = \frac{a + 2r}{3}. \quad (7)$$

The equilibrium price of the input is calculated below. For simplicity, it is assumed that the marginal cost of Firm 3 is also a constant c . Therefore, both the manufacturers and Firm 3 share a marginal cost of c .

Firm 3 receives orders for production from Firm 1 and Firm 2, so that the quantity of production totals Q^* , and produces the input at marginal cost c , selling it to Firm 1 and Firm 2 at price r . Thus, Firm 3 obtains profit

$$2(r - c)(a - r)/3.$$

Without orders for production of the input, the associated profit is 0, and so the additional profit obtained by Firm 3 through bargaining is also

$$2(r - c)(a - r)/3.$$

In contrast, Firm 1 and Firm 2 purchase the input by paying rq_i^* to Firm 3, respectively, and earn revenue $p^*q_i^*$ by selling the product. Hence, Firm 1 and Firm 2 can each obtain a profit of $(a - r)^2/9$.

If Firm 1 and Firm 2 do not place orders for the input, their profit is $(a + 2r - 3c)(a - r)/9$. Hence, for both manufacturers, the additional profit obtainable through bargaining is $(a - r)(c - r)/3$.

Firm 3 is assumed to have the same bargaining power as both manufacturers. In this case, the equilibrium price for the input r^* can be defined by the following equation:

$$r^* = \arg \max_r \left(\frac{2(r - c)(a - r)}{3} \right)^{1/2} \left(\frac{(a - r)(c - r)}{3} \right)^{1/2}. \quad (8)$$

Regarding the price for the input, there exist three equilibria, namely $r^* = a$, $r^* = c$ and $r^* = \frac{a+c}{2}$. In the first two solutions, neither firm (whether that placing an order for the input or that receiving an order for the input) can obtain additional profit through bargaining. Hence, we disregard these two solutions and use only

$$r^* = \frac{a+c}{2}. \quad (9)$$

When calculating the profit of Firm i in a Cournot equilibrium by inserting Equation (9) into Equation (6) and Equation (7), it is possible to obtain

$$\pi_1^o = \pi_2^o = \frac{(a-c)^2}{36}. \quad (10)$$

3.2. When One Manufacturer Outsources and the Other Opts for In-House Production

When Firm 1 outsources and Firm 2 opts for in-house production, the best-response functions of Firm 1 and Firm 2 are

$$BR_1^o(q_2) = \frac{a - q_2 - r}{2} \quad (11)$$

and

$$BR_2^I(q_1) = \frac{a - q_1 - c}{2}, \quad (12)$$

respectively.

Calculating the equilibrium quantity of production simultaneously with these two formulae provides

$$q_1^* = \frac{a+c-2r}{3} \quad (13)$$

and

$$q_2^* = \frac{a-2c+r}{3}. \quad (14)$$

From Equation (1), the equilibrium price of the product is

$$p^* = \frac{a+c+r}{3}. \quad (15)$$

Firm 3 receives an order to produce the input in quantity q_i^* and the marginal cost of production is c . By selling the input to the manufacturers at price r , Firm 3 can obtain profit $(r-c)(a+c-2r)/3$, which equals the additional profit Firm 3 can obtain through bargaining.

On the other hand, Firm 1 purchases the input by paying Firm 3 $r q_i^*$ and can obtain revenue $p^* q_i^*$ by selling the product. Therefore, Firm 1 can obtain a profit of $(a+c-2r)^2/9$. When Firm 1 places no order to produce the input, the profit is $(a+r-2c)(a-2r+c)/9$ and so the additional profit obtained by Firm 1 through bargaining is $(a+c-2r)(c-r)/3$.

Assuming Firm 3 and Firm 1 have equal bargaining power, the equilibrium price of the input r^* is defined by the following equation:

$$r^* = \arg \max_r \left(\frac{(r-c)(a+c-2r)}{3} \right)^{1/2} \left(\frac{(a+c-2r)(c-r)}{3} \right)^{1/2}. \quad (16)$$

In this case too, there are three equilibria prices for the input, namely $r^* = c$, $r^* = (a+c)/2$ and $r^* = (a+c)/4$. In the first two solutions, neither firm can increase its profit through bargaining, so we disregard these two and use only

$$r^* = \frac{a+c}{4}. \quad (17)$$

In this case, the equilibrium profits of Firm 1 and Firm 2 are

$$\pi_1^O = \frac{(a+c)^2}{36} \quad (18)$$

and

$$\pi_2^I = \frac{(5a-7c)^2}{144}, \quad (19)$$

respectively.

The result of the above analysis remains unchanged if Firm 2 only outsources to Firm 3.

3.3. When Both Manufacturers Opt for In-House Production

When Firm 1 and Firm 2 opt for in-house production, and the best response function for each manufacturer is as follows:

$$BR_i^I(q_j) = (a - q_j - c)/2, \quad i, j = 1, 2, j \neq i. \quad (20)$$

Therefore, the equilibrium production quantity is

$$q_1^* = q_2^* = \frac{a-c}{3}. \quad (21)$$

Accordingly, the equilibrium profit for each manufacturer is:

$$\pi_i^I = (a-c)^2/9, \quad i = 1, 2. \quad (22)$$

4. Outsourcing Selection

The results in Section 3 are summarized in **Figure 1**.

		Firm 2	
		Outsourcing	In-house production
Firm 1	Outsourcing	$(a-c)^2/36$	$(a+c)^2/36$ $(5a-7c)^2/144$
	In-house production	$(5a-7c)^2/144$	$(a+c)^2/36$ $(a-c)^2/9$ $(a-c)^2/9$

Figure 1. Outsourcing selection.

In **Figure 1**, it can be found that, for both manufacturers, in-house production is the dominant strategy. Therefore, the following proposition is established:

Proposition 1. In the three-stage game comprising outsourcing selection, Nash bargaining, and Cournot competition, both manufacturers select in-house production at a subgame-perfect equilibrium.

5. Conclusion

Sinha (2016) demonstrated that manufacturers select in-house production in response to the existence of plant-building costs (in other words, fixed costs). In contrast, in this study, we clarified that Nash bargaining leads to in-house production equilibrium even when the cost function is not associated with fixed costs.

We could obtain this result because, whereas previous research assumed the input supplier to be a monopolist, we considered the supplier to be negotiating on equal terms with the manufacturers.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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