Nudging and Boosting: A Theoretical Framework for Policy Optimization

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Abstract
Problems of optimization are pervasive in the modern world. Policy Makers, indeed, have the aim of adopting the best policies mix, under their budget constraint, to maximize economic and social welfare. We exploit the classical Consumer Theory to introduce the Policy Maker optimization problem in steering or empowering good decisions. Specifically, we focus on two main behavioral policies: nudging and boosting. This framework allows us, indeed, to focus on the main building blocks of the so-defined evolutionary function. Since the policy mix depends on the cost of the different policies and their different elasticities, under the (debatable) assumption of rational citizens (i.e. constant returns to scale), this means that these latter are the most important variables that should be estimated. Specifically, by means of surveys, we suggest approximating the elasticities level.

Keywords
Nudging, Boosting, Consumer Theory, Evolutionary Function

1. Introduction and Literature Review
Problems of optimization are pervasive in the modern world, appearing in science, social science, engineering, and business. Recent developments in optimization theory have therefore had many important areas of application and promise to have even wider usage in the future (Intriligator, 2013). Policy Makers, indeed, have the aim of adopting the best policies mix, under their budget constraint, to maximize economic and social welfare. Neoclassical Theory focuses on the optimization problems of the aggregate production function and income distribution with a mathematical approach. A macroeconomic production function is a
mathematical expression that describes a systematic relationship between economic inputs and outputs, and the Cobb-Douglas and constant elasticity of substitution (CES) are two functions that have been used extensively (Miller, 2008). The debate over the Cobb-Douglas production function has been raging ever since the mathematician Charles Cobb teamed up in 1928 with the economist Paul Douglas and developed this famous model of aggregate production and distribution. Sowell emerges as the defender of the Cobb-Douglas, and Simon as the engaging critic. The correspondence demonstrates that the logical and empirical problems with the Cobb-Douglas were well-known by the most advanced minds of mainstream economics (Carter, 2014).

Existing uncertainties about the correct explanations for economic growth and business cycles cannot be settled by aggregative analysis within the neoclassical framework. Current disputes in theory rest largely on ad hoc, casually empirical, assumptions about departures from perfect rationality under uncertainty (Simon, 1984). However, our aim is not to discuss the (widely analyzed) weakness and strengths of the Neoclassical Theory. We simply exploit its mathematical model (i.e. the Cobb-Douglas production function) to introduce the Policy Maker optimization problem in steering or empowering good decisions. Specifically, we focus on two main behavioral policies: nudging and boosting.

Nudging is defined as any aspect of the choice architecture that alters people’s behavior in a predictable way without forbidding any options or significantly changing their economic incentives (Thaler & Sunstein, 2009). Among the most frequently applied nudging techniques, there are default options (Sunstein, 2013) and social norms (Cialdini et al., 2006; Bicchieri & Dimant, 2019). An alternative behavioral intervention, introduced more recently, is boosting. Boosting is based on the idea that individual awareness and skills can be improved, and people can learn how to overcome their biases through training (Hertwig & Ryall, 2019). Boosting interventions are distinguished into two macro-categories: short- and long-term boosting. Short-term boosts are aimed at enhancing a particular skill, but the positive effect on performance is confined to a specific context. Long-term boosts permanently change the cognitive and behavioral repertoire introducing new skills or enhancing the existing ones (Hertwig & Grüne-Yanoff, 2017). The latter creates a “capital stock” (Sunstein, 2016) that can be utilized intentionally and in various circumstances.

As explained in detail in the study by Hertwig & Grüne-Yanoff (2017), the two behavioral techniques differ in multiple aspects. Firstly, boosting does not target an individual’s behaviors, as nudges do, but skills. Moreover, while boosting recognizes bounds but identifies human skills and ways to foster them, nudging defines decision-makers as individuals subject to systematic cognitive biases (Kahneman, 2003). Furthermore, boosting and nudging interventions diverge in the causal path: while boosting improves skills through modifications in skills, knowledge, decision tools, or external environment, nudging harnesses cognitive and motivational deficiencies in tandem with changes in the external choice ar-
chitecture. In addition, according to boosting approach, the effects of the intervention should remain when the intervention is removed. On the other hand, nudging does not assume reversibility: when the intervention disappears, the behavior turns back to the preintervention status. Another interesting difference is that nudging techniques correct biases in specific contexts, while boosting equip individuals with domain-specific or generalizable skills. Finally, people that are nudged are not always aware and conscious of this, whereas boosting must be necessarily transparent and require cooperation and acceptance by boosted individuals.

In the following section, we exploit a classical model to introduce an unorthodox optimization problem.

2. Theoretical Framework: From the Consumer Theory to Behavioral Policy Optimization

We exploit the classical Consumer Theory to introduce a basic theoretical framework for Policy Maker optimization in steering or empowering good decisions. Consumer Theory is concerned with how a rational consumer would make consumption decisions (Levin & Milgrom, 2004). It seems quite reasonable to think of the Policy Maker as a rational consumer who wants to maximize his utility function given the available and manageable inputs (or levers). Therefore, we follow the neoclassical approach. It assumes rational economic agents whose objectives are expressed using quantitative functions, maximized subject to certain constraints. In this simple model, the rational agents are the Policy Maker (i.e. Government) and citizens. Let us define the Policy Maker objective function as “evolutionary function” (EF). The Policy Maker has two levers (or inputs) to optimize its objective function: nudging ($\eta$) and boosting ($\beta$) process. We define the evolutionary function, at time $t$ (i.e. single-period dimension), by using a Cobb-Douglas function with constant return to scale1 as follows:

$$E(\eta, \beta) = \eta^a \beta^b = \eta^a \beta^{1-a} \text{ with } (a + b) = 1 \rightarrow \text{constant returns to scale} \quad (1)$$

In the Consumer Theory, the Cobb-Douglas function is used to represent well-behaved preferences (i.e. monotonic and convex preferences). Monotonicity implies that the indifference curves (i.e. EFs) have a negative slope. Convexity supposes that the policy mix is a real available option, and the average mix is preferred to the extremes. Moreover, it satisfies the assumption of non-satiation (i.e. consumers never reach a point of saturation). These features are useful and reasonable for our theoretical purpose. The assumption of constant returns to scale is based on the idea that rational citizens should be at least able to incorporate one-to-one the implemented policies, steering or empowering their good decisions. Figure 1 shows the Policy Maker’s evolutionary function.

The slope of the evolutionary function is the marginal rate of substitution (MRS). It represents the rate at which the Policy Maker is willing to substitute

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1For constant returns to scale to occur, the relative change in the output should be equal to the proportionate change in the input.
one policy in favor of the other one to preserve the same level of evolution. Clearly, the negative slope associated with well-behaved preferences implies the renounce of one input in exchange of the other one. Since the evolutionary function is convex, the marginal rate of substitution is also decreasing. This means that the rate at which the Policy Maker is willing to exchange $\eta$ with $\beta$ decreases with increased $\eta$. Let us define the marginal evolution with respect to $\eta$ ($ME_\eta$) and $\beta$ ($ME_\beta$) as follow:

$$ME_\eta = \alpha \eta^{\alpha-1} \beta^{\alpha}$$  \hspace{1cm} (2)

$$ME_\beta = (1-\alpha) \eta^{\alpha} \beta^{-\alpha} - \frac{\alpha \eta^{\alpha}}{\beta^{\alpha}}$$  \hspace{1cm} (3)

Given (1), (2) and (3), let us calculate the MRS as follows:

$$MRS = -\frac{ME_\eta}{ME_\beta} = \frac{\Delta_\beta}{\Delta_\eta} = -\frac{\alpha}{1-\alpha} \frac{\beta}{\eta}$$  \hspace{1cm} (4)

Policy Maker choices the best mix of policies as a function of its budget constraint. Let us assume to know the cost of these policies, $(p_\eta, p_\beta)$, and the Policy Maker’s annual allocated budget, $I$. The Policy Maker’s budget constraint can be defined as follow:

$$p_\eta \eta + p_\beta \beta \leq I$$  \hspace{1cm} (5)

In this simplified framework, we suppose that the Policy Maker uses all the allocated funds to implement its policies. Therefore, the budget constraint becomes the following:

$$p_\eta \eta + p_\beta \beta = I$$  \hspace{1cm} (6)

The optimization process requires the choice of the feasible policy mix that maximizes the evolutionary function. In analytical terms, it is expressed by the following system:

$$\begin{cases}
MRS = \frac{p_\eta}{p_\beta} \\
p_\eta \eta + p_\beta \beta = I
\end{cases}$$  \hspace{1cm} (7)
Let us substitute in the system (7), Equation (4). We obtain that:

\[
\begin{align*}
\frac{\alpha}{\eta} \beta &= \frac{p_\eta}{p_\rho} \\
1 - \alpha \eta &= \frac{p_\eta}{p_\beta} \\
\eta \beta + p_\beta \beta &= I
\end{align*}
\]

(8)

Solving for \( \eta \) and \( \beta \) we find, respectively, the demand function of \( \eta \) and \( \beta \), that is:

\[
\eta = \frac{\alpha}{p_\eta}
\]

(9)

\[
\beta = (1 - \alpha) \frac{I}{p_\beta}
\]

(10)

Figure 2 shows the Policy Maker’s optimization process of the evolutionary function.

3. Brief Economic Discussion

Consumer behavior is complex and rarely follows traditional economic theories of decision-making. When choosing what products to buy or what services to use, people often think they are making smart decisions and behaving in ways that are highly rational and congruent with their values and intentions. Daily life illustrates that this is often not the case (Frederiks et al., 2015). The foundations of the Consumer Theory, however, provide rightful insights applied to this specific study of Behavioral Economics. It allows us, indeed, to focus on the main building blocks of the so-defined evolutionary function within a quantitative framework. Since the policy mix depends on the cost of the different policies and their different elasticities, under the (debatable) assumption of rational citizens (i.e. constant returns to scale), this means that these latter are the most important variables that should be estimated. Specifically, by means of surveys, we

\[
\begin{align*}
\eta &= \frac{a}{(a+b) p_\eta} \\
\beta &= \frac{b}{(a+b) p_\beta}
\end{align*}
\]

(11)
suggest approximating the elasticities level. Surveys could give us important information. The latter, indeed, are a key tool to evaluate which behavioral intervention is more important to be further investigated and applied to a specific target population with peculiar characteristics.

Estimating policies sensitivities, indeed, it is also possible to infer the path dependence and evaluate the degree of reactivity of citizens (i.e. returns to scale). Although we focus on the single-period dimension to illustrate the Policy Maker’s optimization process at time \( t \), we suppose the existence of a path dependence of these policies and its crucial role in affecting the relative sensitivities (under the assumption of CES at time \( t \)). The logical idea is that Policy Maker affects through investments (e.g. education, infrastructures, etc.) the degree of development of its factors, improving their future reaction functions. For instance, a Policy Maker, extremely attentive to the growth and development of its education system to boost the skills of citizens, will invest most of its financial resources to improve it. It is plausible to assume that this investing activity will generate positive spillover effects over time (i.e. a higher absolute and relative level of elasticity, although assuming a decreasing rate of growth over time and CES at time \( t \)). At the same time, it is likely that the investment costs are higher at the beginning of the process and are a function of past investments which have affected the current level of elasticity. That is why we highlight the key role of the policy path dependence in determining the current level of policies elasticities and relative costs. For the cost of policies, therefore, it is reasonable to assume the historical averages as a good proxy in our quantitative framework.

### 4. Conclusion

Policy Makers have the aim of adopting the best behavioral policies mix, under their budget constraint, to maximize economic and social welfare. We exploit the classical Consumer Theory to introduce a basic theoretical framework for Policy Maker optimization in steering or empowering good decisions. Consumer Theory is concerned with how a rational consumer would make consumption decisions (Levin & Milgrom, 2004). Therefore, we follow the neoclassical approach. It assumes rational economic agents whose objectives are optimized using quantitative functions, subject to certain constraints. In this simple model, the rational agents are the Policy Maker (i.e. Government) and citizens. We define the Policy Maker objective function as an “evolutionary function” (EF). The Policy Maker has two levers (or inputs) to optimize its objective function: nudging (\( \eta \)) and boosting (\( \beta \)) process. We represent the evolutionary function, at time \( t \) (i.e. single-period dimension), by using a Cobb-Douglas function with a constant return to scale. Since the policy mix depends on the cost of the different policies and their different elasticities, under the (debatable) assumption of rational citizens

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3For instance, in the energy sector, Bühren & Daskalakis (2020) investigate which green nudge is more appropriate to help citizens to save energy with survey involving 457 participants.

4Constant elasticity of substitution (CES).
(i.e. constant returns to scale), this means that these latter are the most important variables that should be estimated. Specifically, by means of surveys, we suggest approximating the elasticities level. Surveys could give us important information. The latter, indeed, is a key tool to evaluate which behavioral intervention is more important to be further investigated and applied to a specific target population with peculiar characteristics.

Estimating policy sensitivities, indeed, it is also possible to infer the path dependence and evaluate the degree of reactivity of citizens (i.e. returns to scale). Although we focus on the single-period dimension to illustrate the Policy Maker’s optimization process at time $t$, we suppose the existence of a path dependence of these policies and its crucial role in affecting the relative sensitivities (under the assumption of CES at time $t$). Although we acknowledge the pros and cons of the neoclassical foundations that characterize this study, our aim is to shed light on the Policy Maker behavioral policies optimization process using a simple quantitative framework. Empirical studies could stress test this theoretical structure. We suggest them as a natural development of future works.

**Conflicts of Interest**

The authors declare no conflicts of interest regarding the publication of this paper.

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