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Gini Multi-Decomposition by Subpopulation in Fuzzy Poverty: Evidence of Chad

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Abstract

This study aims to analyze the configuration of inequalities in Chad through a multidimensional approach. The multidimensional approach adopted and based on the fuzzy set is also called the theory of fuzzy set. The multidimensional poverty indices are calculated by the formulations of Cerioli and Zani. These indices are broken down following the method of Camilo Dagum by groups. The data used come from the Household Budget and Consumption Surveys of 2003 of 6695 households and 2011 of 9259 households. The result shows that the Gini indices indicate less unequal situations which are around 0.263 in 2003 to 0.278 in 2011. Social policies for the reduction of multi-inequality must be directed towards the most disadvantaged households.

Keywords

Population Subgroup, Decomposition into Groups, Multidimensional Poverty, Fuzzy Inequality

1. Introduction

The continuous widening of inequalities in Africa is sometimes accompanied by an increase in poverty. The World Bank also recognizes that inequality can contribute decisively to poverty. Chad, like other African nations, has experienced high and increasing inequality over the years. However, in Chad, social policies for poverty reduction seem to result in a decline in income poverty: 54.8% in 2003 against 46.7% in 2011 (Gadom et al., 2019), but they are often not sufficient to significantly modify the structure of income distribution (Gini index: 39.82 in 2003 against 43.32 in 2011). They must now direct resources towards the underprivileged and target groups at risk. This involves analyzing the multi-inequality of different sub-groups of households on the basis of certain criteria such as place of residence, sex of the head of household. The construction of a profile of

poverty inequality facilitates the formulation of targeted measures in the context of the fight against poverty and inequality. Section 2 reviews previous works. In Section 3, we present the data sources and methodology. Section 4 presents the empirical results and Section 5 the conclusion.

2. Previous Works

Earlier empirical works on income inequality have tried to decompose using the additive decomposition method (Rao, 1969). The authors (Fei et al., 1978) applied in Taiwan (China) during the period 1964-1972 and Fields (1979) in urban Colombia (1967-68) the Rao's method.

Unlike previous approaches, Shorrocks (1982, 1984) proposed a method to decompose the inequality, as measured by the squared coefficient of variation. This method offers rules for decomposing measures of income inequality either by subgroup (subpopulation) or by source of income. The first method is limited to breaking down the structure of the index into an intra-group measure and an inter-group measure. When a population is divided into two groups (for example, men and women), the intragroup coefficient represents the intensity of income inequality that prevails within the groups. The between-group measure symbolizes the inequalities that exist between groups of the population. Shorrocks (1982) used this method in the USA (1968-1977). Jätti (1997) used Shorrocks (1982)'s decomposition of the squared coefficient of variation in Canada, the Netherlands, Sweden, the UK, and the USA in 1980. Garcıa-Penalosa & Orgiazzi (2013) extended Jätti (1997)'s in Canada, Germany, Norway, Sweden, the UK, and the USA for three decades.

The author introduced a method that can be simultaneously decomposed into subgroups and attributes (Chakravarty, 1988).

Otherwise, Mussard (2004) in Italy (1989-2000) and Mussard & Pi Alperin (2008) in Argentina (1998), Mussard & Savard (2012) in Philippines (1997-98) used the method of Camilo Dagum Camilo Dagum (Dagum, 1997) for decomposition of the Gini income inequality.

Based on a tool of cooperative game theory, Shorrocks (1999, 2013) and Chantreuil & Trannoy (2011) proposed the decomposition using the shapley value (Shapley, 1953). The authors applied this decomposition to the UK (1995) and to the US (1994) income distributions (Sastre & Trannoy, 2002).

We use for the decomposition the Gini index proposed by Dagum (1997), which allows the disaggregation of the total inequality into three components in fuzzy poverty. The first represents the contribution to the total inequality of inequalities within each group of a population; the second, contribution to the total inequality of the net inequalities between each pair of groups that prevails within the population. And last, the contribution of the intensity of the transvariation between groups, in other words, the contribution to the total inequality of the inequalities between groups derived from the overlap between the distributions.

3. Methodology

3.1. Data

The data sources come from harmonized surveys on household living conditions, an initiative of the WAEMU Commission. They are carried out by the National Statistical Institutes (INSEED). Samples of households surveyed: 9259 households in 2011 and 6695 in 2003. The samples are representative at the national level with stratification by place of residence (urban/rural) and by region. In addition, the household questionnaire is composed of nine main topics classified as follows: identification of household members, composition, education of different members, employment, housing characteristics, household assets, health of members, retrospective expenses as well as self-consumption.

3.2. Method

Fuzzy indices of poverty (multidimensional poverty) provide a framework for better understanding poverty through its multiple facets.

The fuzzy poverty ratio $\mu(a_i)$ of household a_i can be defined as a weighted average of x_{ik} on all attributes (Costa & De Angelis, 2008)

$$\mu(a_i) = \frac{\sum_{k=1}^{m} x_{ij} w_k}{\sum_{k=1}^{m} w_k} \tag{1}$$

where $w_k = \log\left(\frac{\sum_{i=1}^{n} f(a_i)}{\sum_{i=1}^{n} x_{ik} f(a_i)}\right)$ is the weight attached to the attribute (Cerioli, 1990) $X_k = (x_{1k}, x_{2k}, \cdots, x_{ik}, \cdots, x_{nk})$ and $f(a_i)$ is the weight of household a_i in the sample, by imposing:

$x_{ik} = 1$, if the household a_i does not have the attribute *j*, else $x_{ik} = 0$.

The fuzzy poverty index is defined as:

$$\mu = \frac{\sum_{i=1}^{n} \mu(a_i) f(a_i)}{\sum_{i=1}^{n} f(a_i)} \tag{2}$$

with $C_i = \frac{\mu(a_i) f(a_i)}{\sum_{i=1}^{n} f(a_i)}$ is the contribution of household a_j to the overall fuzzy poverty index

And the ration $\mu(X_k)$ reflects the degree of deprivation of attribute X_k for the population of n households.

Dagum & Costa (2004) introduced decomposition by attribute by demonstrating that it is possible to calculate the contribution of attribute X_k to the overall fuzzy poverty index. The fuzzy poverty index is defined as

$$\mu = \frac{\sum_{k=1}^{m} \mu(x_k) w_k}{\sum_{k=1}^{m} w_k} \tag{3}$$

The authors get the (absolute) contribution of attribute X_k to the multidi-

mensional poverty index:

$$\mu = \sum_{k=1}^{m} \mu_k \tag{4}$$

where $\mu_k = \frac{\mu(x_k) w_k}{\sum_{k=1}^{m} w_k}$ is the contribution of attribute X_k to the overall fuzzy poverty index

Consider a population P of n households whose levels of contribution of household i to fuzzy poverty are denoted $\mu(a_i)(i=1,\cdots,n)$, divided into k groups P_j of size n_j. Suppose f_j the frequency of individuals belonging to group P_j and s_j the fuzzy poverty index share of P_j in the arithmetic mean of the $\mu(a_i)$ in P. We denote by $\bar{\mu}$ the arithmetic mean of the (a_i) in P and $\bar{\mu}_j$ the arithmetic mean of the $\mu(a_i)$ in P_j.

Let, $f_j = \frac{n_j}{n}$ and $s_j = \frac{\bar{\mu}_j * n_j}{\bar{\mu} * n} = f_j * \frac{\bar{\mu}_j}{\bar{\mu}}$

The Gini coefficient is defined by:

$$G = \sum_{i=1}^{n} \sum_{r=1}^{n} \frac{\left|\mu(a_i) - \mu(a_r)\right|}{2n^2 \bar{\mu}} \tag{5}$$

This expression makes it possible to decompose the Gini measure into a intragroup component G_w and a "gross" intergroup component G_{gb} (Dagum, 1997):

$$G = \sum_{j=1}^{k} G_{jj} f_j s_j + \sum_{j=2}^{k} \sum_{h=1}^{j-1} G_{jh} \left(f_j s_h + f_h s_j\right) \tag{6}$$

with $G_w = \sum_{j=1}^{k} G_{jj} f_j s_j$ and $G_{gb} = \sum_{j=2}^{k} \sum_{h=1}^{j-1} G_{jh} \left(f_j s_h + f_h s_j\right)$

where, G_{jj} is the Gini measure measured on the group P_j and where G_{jh} is the Gini measure measuring the inequalities between the groups P_j and P_h.

The Gini measure between two groups (G_{jh}) is constructed by calculating the sum of the difference binaries of the fuzzy poverty index between people of groups different, then normalizing this sum:

$$G_{jh} = \sum_{i=1}^{n_j} \sum_{r=1}^{n_h} \frac{\left|\mu_j(a_i) - \mu_h(a_r)\right|}{n_j n_h \left(\bar{\mu}_j + \bar{\mu}_h\right)} \tag{7}$$

where $\mu_j(a_i)$ and $\mu_h(a_r)$ are respectively the fuzzy poverty index of individuals i and r of the P_j and P_h groups, with $G_{jh} \in [0,1]$.

The decomposed Gini measure indeed shows a third term that evaluates the inequalities coming from the overlap zone between their distributions of the different groups studied. Let D_{jh} be the directional economic distance (Dagum, 1997) allowing us to measure the proportion binary differences of the fuzzy poverty index calculated from each of the area non-overlapping distributions of fuzzy poverty index groups P_j and P_h. The decomposition of the Gini coefficient into three components:

$$G=\sum_{j=1}^{k}G_{jj}f_j s_j+\sum_{j=2}^{k}\sum_{h=1}^{j-1}G_{jh}D_{jh}\left(f_j s_h+f_h s_j\right)+\sum_{j=2}^{k}\sum_{h=1}^{j-1}G_{jh}\left(1-D_{jh}\right)\left(f_j s_h+f_h s_j\right) \tag{8}$$

With

$G_w=\sum_{j=1}^{k}G_{jj}f_j s_j$, $G_{nb}=\sum_{j=2}^{k}\sum_{h=1}^{j-1}G_{jh}D_{jh}\left(f_j s_h+f_h s_j\right)$ and $G_t=\sum_{j=2}^{k}\sum_{h=1}^{j-1}G_{jh}\left(1-D_{jh}\right)\left(f_j s_h+f_h s_j\right)$

where

$$\begin{aligned}G_{gb}&=\sum_{j=2}^{k}\sum_{h=1}^{j-1}G_{jh}D_{jh}\left(f_j s_h+f_h s_j\right)+\sum_{j=2}^{k}\sum_{h=1}^{j-1}G_{jh}\left(1-D_{jh}\right)\left(f_j s_h+f_h s_j\right)\\&=\sum_{j=2}^{k}\sum_{h=1}^{j-1}G_{jh}\left(f_j s_h+f_h s_j\right)\end{aligned}$$

where

$$D_{jh}=\frac{d_{ij}-p_{jh}}{d_{ij}+p_{jh}} \tag{9}$$

where d_{ij} is the raw directional distance which is a weighted average of the differences of fuzzy poverty index $\mu_j(a_i)-\mu_h(a_r)$ for each fuzzy poverty index $\mu_j(a_i)$ of a member of P_j greater than fuzzy poverty index $\mu_h(a_r)$ of a member of P_h ($\mu_j(a_i)\geq\mu_h(a_r)$ as $\overline{\mu}_j\geq\overline{\mu}_h$):

$$d_{jh}=\int_0^{\infty}\mathrm{d}F_j(x)\int_0^{x}(x-y)\mathrm{d}F_h(y)$$

where F_j and F_h and are respectively the distribution functions of the fuzzy poverty index in the groups P_j and P_h.

And p_{jh} is moment of order 1 of transvariation:

$p_{jh}=\int_0^{\infty}\mathrm{d}F_h(x)\int_0^{x}(x-y)\mathrm{d}F_j(y)$ which is the weighted average of the binary differences in fuzzy poverty index

$$\mu_h(a_r)-\mu_j(a_i)\quad(\mu_j(a_i)\leq\mu_h(a_r)\ \text{ as }\ \overline{\mu}_j\geq\overline{\mu}_h)$$

Therefore, it is possible to measure the contribution of inequalities at inside of the G_w/G groups, the net contribution of inequality between G_{nb}/G groups and the contribution of transvariation intensity between G_t/G groups.

The index measures the inequalities generated by the fuzzy poverty index of the wealthiest groups on average widening gaps with those less wealthy groups. Suppose that as soon as the groups have the same average the inequalities sharp between them, groups are null (this is possible since because if $\overline{\mu}_j=\overline{\mu}_h\Rightarrow D_{jh}=0$). The term G_t on the contrary measures inequalities resulting from the income of the less wealthy groups on average, which create gaps in income for individuals from the wealthiest groups on average.

3.3. Choice of Indicators and Cut-Offs

The question of the choice of deprivation indicators has been discussed at length by Cheli, Ghellini, Lemmi and Pannuzi in (Cheli et al., 1994) and by Cheli and

Lemmi in (Cheli & Lemmi, 1995). These authors note that the choice of deprivation indicators is of fundamental importance. Furthermore, they recommend, in the analysis, to clearly distinguish the effect variables and the cause variables of poverty. Finally, Miceli in (Miceli, 2006) points out that the choice of deprivation indicators is particularly delicate and cannot intervene without a dose of arbitrary more or less and that the fuzzy measurement obtained is ultimately conditioned by the data availability. The deprivation thresholds first identify the people experiencing deprivation in each selected indicator. Deprivations are dichotomous.

The selection of socio-economic attributes to study the state of poverty was made on the basis of multidimensional notions of poverty, information from the Ecosit3 in 2011 and Ecosit2 in 2003 surveys and the Sustainable Development Goals (SDGs). This selection is very important because each of the selected attributes explains the degree of deprivation and social exclusion of the households studied (Miceli, 2006; Mussard & Alperin, 2005; Ambapour, 2009). For questions related to the choice of dimensions and capacities, see (Sen, 1993; Atkinson, 2003; Alkire, 2011; Alkire & Foster, 2011).

The selected variables are in this **Table 1** below and the lack of the income dimension would be justified by the fact that it would already act on almost all the other dimensions selected (for example having a permanent home depends on its income, energy, etc.).

Table 1. Dimensions and attributes.

Dimensions	Attributes	Deprived if...
Education (X_1)	Literacy (x_{11})	None of its members can read, write or count
	Attendance (x_{21})	A member of school age does not attend
	Instruction (x_{31})	The household head is uneducated
Housing (X_2)	Roof (x_{12})	Roofs of his houses are not sheet metal/tile and concrete
	Wall (x_{22})	Wall is not made of cement
	Floor (x_{32})	House has a dirt floor
Health (x_3)	Access to health centers (x_{13})	No Access to health centers
	Morbidity (x_{23})	At least one child is sick once a week
Sanitation (X_4)	Household waste (x_{14})	No access to adequate sanitation
	Existence of WC (x_{24})	No WC
Drinking water (X_5)	Source of drinking water (x_{15})	No access to clean drinking water
	Access to water (x_{25})	No access to water within 30 minutes
Energy (X_6)	Existence of electricity (x_{116})	No electricity
	Combustible (x_{26})	household uses dirty cooking fuel

Note: Dimensions and attributes selected by the author.

4. Empirical Results

4.1. Analysis of Fuzzy Poverty in Chad

The weight w_k represents the intensity of deprivation linked to the dimension X_k. the weights of Cerioli & Zani (1990) defined according to an inverse relation of the average degree of deprivation relative to the indicator j. According to this weighting system, more weight is assigned to the most common indicators.

The fuzzy poverty indices in Chad are 0.4874 in 2003 and 0.5889 in 2011. In other words, Chadian households are 48.74% structurally poor in 2003 against 58.89% in 2011, structural poverty increased by 20.82%, while monetary poverty stood at 54.8% in 2003 and 46.7% in 2011, it fell by around 8% between these two periods.

We note from this Table 2 that we have a global view on the causes of poverty, the methods of decomposition give us more detailed and precise information on the true causes of the determination of the multidimensional phenomenon of poverty.

4.2. Fuzzy Poverty by Gender

Understanding the phenomenon of poverty through the sex of the head of the household can provide useful elements for targeting actions aimed at improving the living conditions of the poor. Indeed, to propose policies that can help reduce poverty, the authorities need to know whether the phenomenon of poverty is linked to the gender of the head of the household or not. In Table 3, for the breakdown by sex, we note the households with female heads of households are the poorest (50.92%) compared to households with male heads (47.31%) in 2003 and we have the same situation in 2011, the poverty of female households is at 64.18% while that of households headed by men is at 56.52%. As there are more Chadian households headed by men, they account for around 77.73% in 2003 and 76.10% in 2011.

Table 2. Fuzzy poverty index.

Dimensions	2003			2011		
	w_k	$\mu(k)$	μ_k	w_k	$\mu(k)$	μ_k
Education (X_1)	0.173	0.638	0.110	0.266	0.643	0.171
Housing (X_2)	0.108	0.808	0.088	0.101	0.833	0.084
Health (x_3)	0.317	0.389	0.123	0.319	0.445	0.142
Sanitation (X_4)	0.076	0.727	0.055	0.076	0.742	0.056
Drinking water (X_5)	0.316	0.322	0.102	0.233	0.558	0.130
Energy (X_6)	0.010	0.972	0.010	0.005	0.986	0.006
Chad	1.00	0.487	0.487	1.00	0.589	0.589

Note: Author's calculations, $k \in \{1,2,3,4,5,6\}$.

Table 3. Decomposition by gender.

subpopulations	2003			2011		
	μ_i	c_i	C_j/μ	μ_i	c_i	c_i/μ
Male	0.473	0.379	0.777	0.565	0.448	0.761
Female	0.509	0.108	0.223	0.642	0.141	0.239

Note: Author's calculations, i = (male, female).

4.3. Fuzzy Poverty of Residence

In Table 4, the decomposition by residence indicates that poverty is accentuated in rural areas with rates of 49.16% in 2003 and 60.46% in 2011 to 24.37% in 2003 and 12.06%. We observe that rural poverty increases unlike urban poverty between 2003 and 2011. Poverty during the two years are almost explained by rural poverty, their contributions are at 93.66% in 2003 and 87.94% in 2011.

4.4. Fuzzy Inequality Index at National Level

The results in Table 5 are obtained from the computer program for the decomposition of the Gini indicator and its derived measures of entropy (Mussard et al., 2002), indicating that in 2003 the Gini index is 0.263, and in 2011 is 0.278. Households headed by men (0.264 in 2003 and 0.292 in 2011) are more unequal than those of women (0.252 in 2003 and 0.226 in 2011). The rural environment is less unequal than the urban environment.

4.5. Fuzzy Gender Inequality

In Table 6, the inequalities of households headed by men have seen a slight increase: 0.264 in 2003 and 0.291 in 2011, households headed by women, on the other hand, have decreased: 0.252 in 2003 and 0.226 in 2011.

The breakdowns of the national Gini indicators in Table 5: 0.263 in 2003 and 0.278 in 2011 indicate that 65.33% and 61.89% are explained by intragroup inequalities against 34.67% and 38.11% for intergroup inequalities and those of transvariations between men and women are 27.15% in 2003 and 23.66% in 2011. Intragroup inequalities due to men are around 60.53% in 2003 against that due to women of 4.79% in 2003 and 56.10% due to men against 5.79% due to women in 2011.

4.6. Fuzzy Inequality by Place of Residence

The contributions to the national Gini index in Table 7 are: 53.45% in 2003 and 49.06% in 2011 and coming from the Gini index shows that it is because of the inequality due to the rural environment and the others indicate that it is the urban environment. In 2011, the Gini index indicates an almost equal share of intra-group and inter-group inequality.

Table 4. Decomposition by residence.

subpopulations	2003			2011		
	μ_i	c_i	C_j/μ	μ_i	c_i	c_i/μ
Rural	0.492	0.457	0.937	0.605	0.518	0.879
Urban	0.244	0.031	0.063	0.320	0.071	0.121

Note: Author's calculations, i = (rural, urban).

Table 5. Gini indices.

populations/subpopulations	2003	2011
National	0.263	0.278
Male	0.264	0.292
Female	0.252	0.226
Urban	0.300	0.297
Rural	0.213	0.170

Note: Author's calculations.

Table 6. Gini indices.

	G	G_w	G_b	G^*	Male	Female
2003	0.263	0.172	0.091	0.071	0.159	0.013
2011	0.278	0.172	0.106	0.066	0.156	0.016

Note: Author's calculations.

Table 7. Gini indices.

	G	G_w	G_b	G^*	Urban	Rural
2003	0.263	0.141	0.122	0.022	0.015	0.125
2011	0.278	0.137	0.142	0.029	0.112	0.024

Note: Author's calculations.

4.7. Conclusion

The results indicate that in 2003 the Gini index is 0.263, and in 2011 it is 0.278. Between 2003 and 2011, there is a slight multi-inequality increase of 5.84%. The Gini indices tell us that in 2003 and 2011, households were not egalitarian (gender and place of residence).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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