Theoretical Economics Letters, 2022, 12, 1539-1558
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Not Enough R&D? Or Maybe Too Much?

Intensity of Knowledge Spillovers and Optimal R&D Policy in Schumpeterian Growth Theory

Elie Gray*

Abstract

This paper presents an endogenous growth model *à* Aghion & Howitt (1992) in which we explicitly formalize knowledge spillovers in the innovation process. We revisit the issue of the Pareto non-optimality of the Schumpeterian equilibrium by revealing the part played by the *intensity of knowledge spillovers*. Basically, we highlight that the market incompleteness characterizing this type of decentralized economy (knowledge is not priced) is all the more likely to lead to an under-optimal (*resp.* over-optimal) R&D effort as the intensity of knowledge spillovers is high (*resp.* low). The reason behind this is that the effects of the distortion of R&D incentives resulting from market incompleteness are amplified all the more as this intensity is strong. Complementarily, we derive the optimal tool dedicated to correct the market failure caused by market incompleteness, and we demonstrate that it clearly depends on the intensity of knowledge spillovers: the higher (*resp.* lower) the intensity of knowledge spillovers is, the more likely this policy tool should consist in a subsidy (*resp.* tax). Moreover, if this optimal tool happens to be a subsidy, then this subsidy will be all the larger as the intensity is high.

Keywords: Schumpeterian growth theory; Pareto sub-optimality; Market incompleteness; Knowledge spillovers; R&D incentives; Optimal policy tools

How to cite this paper: Gray, E. (2022) Not Enough RD? Or Maybe Too Much? *Theoretical Economics Letters, 12*, **1539-1558. https://doi.org/10.4236/tel.2022.126084**

Received: July 27, 2022
Accepted: November 6, 2022
Published: November 9, 2022

*Corresponding author - TBS Business School, Universitéde Toulouse, Toulouse, France. 20 Bd. Lascrosses, BP 7010 31068 Toulouse Cedex 7, France. Tel.: +33 (0)5.61.29.49.25 . Email: e.gray@tbs-education.fr.

1 Introduction

The theory of endogenous growth based on innovation underlines the presence of a voluntary mechanism at the origin of the accumulation of knowledge from which stems technical progress, engine of long-run growth. Knowledge is a non-rival good - a stock of intellectual capital distinct from physical capital or human capital - the creation of which depends on a specific and endogenous investment: knowledge accumulates through the activity of research and development (R&D). The process of knowledge accumulation has been formalized by two seminal paradigms which basically differ in how new knowledge is interpreted. Whereas in Romer (1990), it consists of new goods or new production processes (one generally refers to "horizontal" knowledge accumulation), in Aghion & Howitt (1992), it consists of an improvement of the quality of an already existing good or process (one generally refers to "vertical" knowledge accumulation).

In order to deal with the non-rivalry property of knowledge, the fundamental papers by Romer (1990) and Aghion & Howitt (1992), focused on decentralized economies with incomplete markets and imperfect competition. A market and a price are specified for goods that incorporate knowledge, but not for knowledge itself. Incentives to invest in the creation of knowledge go through the fact that agents investing R&D expect to get some market power. Indeed, R&D activity is indirectly funded by monopoly profits resulting from intellectual property rights granted to innovators. Romer (1990) considers an equilibrium in which each innovator obtains an infinitely-lived patent on the good embodying his innovation. In Aghion & Howitt (1992) on the other hand, the equilibrium is inspired by Schumpeter's creative destruction mechanism insofar as the firm which manages to innovate in a sector replaces the previous innovator and monopolizes this sector until the next innovation occurs. Hence, such equilibria are characterized by two market failures: firstly, *market power* ensuing from the presence of a monopoly on each intermediate good incorporating an innovation, and secondly, a positive externality which consists of the *market incompleteness* resulting from the fact that there is no market for knowledge since knowledge is not priced. Because of these two market failures, Pareto non-optimality is likely to arise in the *laisser faire* equilibrium.

Endogenous growth theory has long emphasized this issue of Pareto non-optimality. In particular, many R&D-based endogenous growth models predict that in the absence of public policies, the decentralized economy can lead to an either *insufficient* or *excessive* level of resources allocated to R&D activity, and thus to a either *sub-optimal* or *over-optimal* growth rate of the economy. This well-known result has been extensively discussed in the growth literature, both in vertical differentiation class of models (*e.g.*, Grossman & Helpman 1991; Aghion & Howitt 1992; Segerstrom 1998; Li 2003) and in expanding variety models *à la* Romer (*e.g.*, Benassy 1998; Jones & Williams 2000; Alvarez-Pelaez & Groth 2005).

In the present paper, we develop a standard endogenous growth model *à* Aghion & Howitt (1992) in which we make knowledge accumulation and knowledge diffusion explicit in the innovation process; this first novelty enables us to clearly formalize what we coin *"intensity of knowledge spillovers"*. In the endogenous growth literature, the expression *"knowledge spillovers"* refers simultaneously to two intertwined issues which have been studied in a large body of literature (see, for instance, Romer, 1990; Aghion & Howitt 1992, 1998, 2009; Segerstrom, 1998; Li, 2002; Peretto & Smulders, 2002; Jones, 2005; Sener, 2008; or Acemoglu, 2009). The first issue relates to the technology used in endogenous growth models: knowledge production functions generally assume that the knowledge previously created in a particular sector spreads ("spills over") into the economy, thus enhancing the creation of new knowledge in other sectors. The second issue somehow ensues from the first one: it relates to the considered decentralized economy which is characterized by the presence of a positive externality entailed by market incompleteness since knowledge is not priced. This article obviously considers both issues, nevertheless, we use the expression "knowledge spillovers" specifically to refer to the fact that the knowledge inherent in any innovation diffuses across sectors. In particular, we introduce the concept of *"intensity of knowledge spillovers"* to formalize the fact that knowledge may diffuse more or less broadly.

Once the model developed, we compute the associated first-best social optimum; then, we study the outcome of a classic Schumpeterian equilibrium. The developed framework is no exception to the rule: in the *laisser faire* Schumpeterian equilibrium, the R&D effort (represented by the quantity of labor used in R&D in the present model) can be sub-optimal or over-optimal as may be the case in standard Schumpeterian growth frameworks descending from the seminal paper by Aghion & Howitt (1992). Several complementary approaches trying to understand why the R&D effort can either be sub-optimal or over-optimal can be found. Aghion & Howitt (1992, 1998) focus on the various market failures involves by the equilibrium considered in order to understand why Pareto non-optimality may arise; they explain that the problems of surplus appropriability and of knowledge spillovers both tend to lead towards under-investment in R&D, and that the effects of creative destruction and of duplication both tend to lead towards over-investment in R&D. A complementary approach relates the fact that there is too little or too much R&D to the "size of innovations", that is to the height of the jumps on the quality ladder. According to Grossman & Helpman (1991), only intermediate-size innovations should be subsidized; small-size and large-size innovations should be taxed; while according to Segerstrom (1998), it is optimal to subsidize small-size innovations and to tax large-size innovations. These approaches focus on intra-sectoral knowledge spillovers but do not consider inter-sectoral knowledge spillovers. Models by Li (2003) and by Sener (2008) generalize the analysis of Segerstrom (1998) by taking them into account. Sener (2008) confirms the results of Segerstrom (1998), whereas Li (2003) shows that when inter-sectoral knowledge spillovers effects are sufficiently large, R&D activities should be subsidized. As a matter of fact, "it is easy to find more papers in this literature with major differences in R&D policy recommendations" (Sener, 2008).

The analysis conducted in our paper sheds new light on these issues insofar as the model we develop enables us to identify a clear link between the intensity of knowledge spillovers, the Pareto non-optimality of the seminal Schumpeterian equilibrium initially introduced by Aghion & Howitt (1992), and the optimal R&D public policy. The key point made in this paper lies in that the market failure involved by market incompleteness characterizing the Schumpeterian equilibrium is all the more likely to lead to an under-optimal R&D effort as the intensity of knowledge spillovers is high. Conversely, it is all the more likely to lead to an over-optimal R&D effort as the intensity is low. In fact, we reveal that the effects of the distortion of R&D incentives resulting from market incompleteness are amplified all the more as the intensity of knowledge spillovers is high. This issue is also reflected in the fact that the optimal tool dedicated to correct the market failure caused by market incompleteness is clearly positively dependent on the intensity of knowledge spillovers. Indeed, we show that the higher (*resp.* lower) the intensity of knowledge spillovers is, the more likely this policy tool should consist in a subsidy (*resp.* tax); and that if it is a subsidy, then this subsidy will be all the larger as the intensity is high.

The remainder of the paper is organized as follows. In Section 2, we explicitly formalize knowledge spillovers in an endogenous growth model with vertical innovations in line with Aghion & Howitt (1992) and we compute the first-best social optimum. In Section 3, we study the seminal decentralized economy introduced by Aghion & Howitt (1992); specifically, we define, characterize, and compute a standard Schumpeterian equilibrium. In Section 4, we revisit the issue of Pareto non-optimality of the Schumpeterian equilibrium by considering the part played by the intensity of knowledge spillovers; in particular, we compare the *laisser faire* equilibrium with the first-best social optimum, we implement the first-best social optimum in the Schumpeterian decentralized economy by characterizing the optimal public tools, and we study the properties of the optimal tool dedicated to correct the market failure caused by market incompleteness. We conclude in Section 5.

2 Model and First-best Social Optimum

This section displays a canonical continuous-time endogenous Schumpeterian growth model, in which we explicitly formalize knowledge spillovers. In Section 2.1, we present the technologies and the preferences; in particular, we detail the mechanisms underlying the process of innovation, namely the part played by knowledge accumulation and diffusion. Then, in Section 2.2, we compute the first-best social optimum.

2.1 Technologies and Preferences

We consider a standard endogenous growth model with vertical innovations in line with Grossman & Helpman (1991) and Aghion & Howitt (1992) in which we introduce explicitly knowledge spillovers. For that purpose, we derive a general law of knowledge accumulation, in which knowledge spillovers manifest through two channels. Firstly, in any type of sector, the R&D activity produces innovations using the knowledge inherent in previously created innovations. These innovations may have been produced within this sector; or in other sectors, which can be more or less technologically distant from the aforesaid sector. Secondly and reciprocally, the knowledge produced in any given sector spills over into R&D activity of other sectors; this diffusion being all the more likely that sectors are technologically close. To formalize these knowledge spillovers, we use the circular product differentiation model of Salop (1979): there is a continuum $\mathcal{Q}$, of measure N, of intermediate sectors uniformly distributed on a clockwise oriented circle. At each date t, in each sector i, $i \in \mathcal{Q}$, an intermediate good i is produced in quantity x_{it}; besides, each of these intermediate goods is associated with a specific stock of knowledge κ_{it}. The whole stock of knowledge in the economy at date t is[1]

$$\mathcal{K}_t = \int_{\mathcal{Q}} \kappa_{it} \, di. \tag{1}$$

Each sector has its own R&D activity which uses two inputs: a rival one (labor) and a non rival one (a stock of knowledge).[2] Let us present the set of basic assumptions underlying the innovation process in our model. First, as in most standard Schumpeterian growth models, the innovation process is uncertain:

Assumption 1. *In any intermediate sector i, $i \in \mathcal{Q}$, innovations occur randomly with a Poisson arrival rate λl_{it}, $\lambda > 0$, where l_{it} is the amount of labor devoted to R&D at date t.*

Second, an innovation at date t in any given sector i consists in an enhancement of the quality of the intermediate good produced in this sector. In other words, an innovation corresponds to an increase in the stock of knowledge κ_{it} and to the incorporation of this new stock of knowledge in the intermediate good i. This appears in Assumption 2 below, which formalizes that the innovation process goes along with the fact that, in each sector i, the R&D activity produces innovations (and thus new knowledge) by making use of a pool comprising previously created knowledge:

Assumption 2. *In any intermediate sector i, $i \in \mathcal{Q}$, if an innovation occurs at date t, the increase in knowledge is $\Delta\kappa_{it} = \theta\mathcal{P}_{it}$, $\theta > 0$, where $\mathcal{P}_{it}$ is the pool of knowledge from which this sector's R&D activity can draw from in order to innovate.*

Third, for any given sector i, the composition of the pool of knowledge $\mathcal{P}_{it}$ at the disposal of R&D activity i basically depends on the usability of the knowledge created by the R&D activities of all the other sectors. The importance of the influence that the R&D activities of various sectors can have on each other has often been stressed by empirical studies (*e.g.*, Griliches, 1992 and 1995; Hall, Mairesse & Mohnen, 2010; Hall, 2004) and has been at the core of the seminal endogenous growth theory (*e.g.*, Romer, 1990; Aghion & Howitt, 1992 and 1998; Howitt, 1999; Jones, 1999). In particular, it has been emphasized that the R&D activity of one sector is likely to entail positive spillovers effects in other sectors; moreover, "such spillovers are all the more likely and significant as the sender and the receiver are closely related" (Hall, Mairesse & Mohnen, 2010).

Based on these ideas, we propose a simple formalization of how these pools of knowledge are formed. In any sector i, the R&D activity is both receiving and sending knowledge. Indeed, any given R&D activity i makes use of the knowledge generated by the innovation process occurring in other sectors. Let $\mathcal{Q}_i^{\mathcal{R}}$ denote the subset of sectors of $\mathcal{Q}$ producing knowledge which enters the pool $\mathcal{P}_{it}$. Besides, through its innovation process, any given R&D activity i produces knowledge that spills over into R&D activities of other sectors. Let $\mathcal{Q}_i^{\mathcal{S}}$ denote the subset of sectors of $\mathcal{Q}$ that can use the knowledge produced by R&D activity i; we name the measure of $\mathcal{Q}_i^{\mathcal{S}}$ the "scope of diffusion of knowledge κ_{it}". We make the following assumptions on the way knowledge diffuses across sectors:

Assumption 3. *In any intermediate sector i, $i \in \mathcal{Q}$, when an innovation occurs, knowledge spills symmetrically over the circle $\mathcal{Q}$.*

Assumption 4. *The scope of diffusion of knowledge is identical for all sectors; it is denoted by ϑ, $1 \leq \vartheta \leq N$.*

Not only these two assumptions enables us to mitigate intricacy, but they also ensue from the assumption of symmetry across sectors commonly made in endogenous growth models.[3] Consequently, the subset of $\mathcal{Q}$ comprising the sectors that use the knowledge κ_{it} produced by R&D activity i is $\mathcal{Q}_i^{\mathcal{S}} = [i - \vartheta/2\,;\, i + \vartheta/2]$. Besides, R&D activity i makes use of the knowledge produced by the sectors belonging to the subset $\mathcal{Q}_i^{\mathcal{R}} = [i - \vartheta/2\,;\, i + \vartheta/2]$; in other words, the knowledge produced by the R&D activities of any sector $j \in [i - \vartheta/2\,;\, i + \vartheta/2]$ contributes

[1] Knowledge is assumed to be an homogenous good. Besides, its initial stock, $\mathcal{K}_0$, is normalized to one.

[2] In this paper, the rival good used in R&D is labor; alternatively, one could consider the use of physical capital, or of the final good (see, for instance, in Barro & Sala-i-Martin, 2003). As detailed below, the composition of the stock of knowledge may include only the knowledge produced with the sector, all the knowledge available in the economy, or any case in between these two polar cases (see the comments after Lemma 1).

[3] We provide more details on the symmetry assumption below, at the end of this section. As usual, this standard assumption is used in the present paper to compute the first-best social optimum (see the proof of Proposition 1 in Section 2.2) and the Schumpeterian equilibrium (see Section 3.2).

to the pool of knowledge used by R&D activity of sector i.[4] Hence, at each date t, in any intermediate sector i, the pool of knowledge used by the R&D activity is

$$\mathcal{P}_{it} = \int_{\mathcal{Q}_i^{\mathcal{R}}} \kappa_{ht} dh, \forall i \in \mathcal{Q}, \text{ with } \mathcal{Q}_i^{\mathcal{R}} = [i - \vartheta/2\,;\, i + \vartheta/2]\,. \tag{2}$$

The law of knowledge accumulation in any sector i is derived from Assumptions 1, 2, 3, and 4; it is characterized in Lemma 1 below.

Lemma 1. *At each date t, in any intermediate sector i, knowledge is produced along with*

$$\dot{\kappa}_{it} = \lambda\theta l_{it}\mathcal{P}_{it}, \forall i \in \mathcal{Q}, \text{ where } \mathcal{P}_{it} \text{ is given in (2).} \tag{3}$$

Proof. Let k, $k \in \mathbb{N}$, be the number of innovations occurring in a given intermediate sector i, $i \in \mathcal{Q}$, during a time interval $(t, t + \Delta t)$. The stock of knowledge accumulated in sector i at the beginning of the period is κ_{it}. Under Assumptions 1 and 2, the stock of knowledge at the end of the period, $\kappa_{i\,t+\Delta t}$, is a random variable taking the values

$$\{\kappa_{it} + k\theta\mathcal{P}_{it}\}_{k\in\mathbb{N}} \text{ with associated probabilities } \left\{ \frac{\left(\int_t^{t+\Delta t} \lambda l_{iu} du\right)^k}{k!} e^{-\int_t^{t+\Delta t} \lambda l_{iu} du} \right\}_{k\in\mathbb{N}},$$

where $\mathcal{P}_{it}$ ensues from Assumptions 3 and 4, and is given in (2). The expected stock of knowledge at date $t + \Delta t$ is thus

$$\mathbb{E}\left[\kappa_{i\,t+\Delta t}\right] = \sum_{k=0}^{\infty} \frac{\left(\int_t^{t+\Delta t} \lambda l_{iu} du\right)^k}{k!} e^{-\int_t^{t+\Delta t} \lambda l_{iu} du} \left[\kappa_{it} + k\theta\mathcal{P}_{it}\right]$$

$$= \left[\kappa_{it} \sum_{k=0}^{\infty} \frac{\left(\int_t^{t+\Delta t} \lambda l_{iu} du\right)^k}{k!} + \theta\mathcal{P}_{it} \left(\int_t^{t+\Delta t} \lambda l_{iu} du \right) \sum_{k=1}^{\infty} \frac{\left(\int_t^{t+\Delta t} \lambda l_{iu} du\right)^{k-1}}{(k-1)!} \right] e^{-\int_t^{t+\Delta t} \lambda l_{iu} du}$$

$$= \left[\kappa_{it} e^{\int_t^{t+\Delta t} \lambda l_{iu} du} + \theta\mathcal{P}_{it} \left(\int_t^{t+\Delta t} \lambda l_{iu} du \right) e^{\int_t^{t+\Delta t} \lambda l_{iu} du} \right] e^{-\int_t^{t+\Delta t} \lambda l_{iu} du}$$

$$= \kappa_{it} + \lambda\theta \left(\int_t^{t+\Delta t} l_{iu} du \right) \mathcal{P}_{it}.$$

Hence, denoting by Λ_{iu} a primitive of l_{iu} with respect to the time variable u, one has

$$\frac{\mathbb{E}\left[\kappa_{i\,t+\Delta t}\right] - \kappa_{it}}{\Delta t} = \lambda\theta \frac{\Lambda_{it+\Delta t} - \Lambda_{it}}{\Delta t} \mathcal{P}_{it}.$$

Letting Δt tend to zero in the Newton's difference quotients of $\mathbb{E}\left[\kappa_{it}\right]$ and of Λ_{it}, one gets

$$\dot{\kappa}_{it} \equiv \frac{\partial \mathbb{E}\left[\kappa_{it}\right]}{\partial t} = \lambda\theta l_{it}\mathcal{P}_{it}.$$

This shows that the expected knowledge in any sector i, $i \in \mathcal{Q}$, is a differentiable function of time. Its derivative gives the law of knowledge accumulation in sector i as exhibited in Lemma 1 (the expectation operator is dropped to simplify notations). □

The formalization presented above generalizes the standard innovation-based endogenous growth theory insofar as the law of knowledge accumulation derived in Lemma 1 is quite general. Indeed, many standard laws of knowledge accumulation considered in the endogenous growth theory literature turn out to be particular cases

[4] Because of the assumptions of symmetry, one has $\mathcal{Q}_i^{\mathcal{S}} = \mathcal{Q}_i^{\mathcal{R}}$. Considering a more general framework in which each sector i has a specific ϑ_i, one would have $\mathcal{Q}_i^{\mathcal{S}} = \left\{ h \in \mathcal{Q} / \left| i - h \right| \leq \frac{\vartheta_i}{2} \right\}$ and $\mathcal{Q}_i^{\mathcal{R}} = \left\{ h \in \mathcal{Q} / \left| i - h \right| \leq \frac{\vartheta_h}{2} \right\}$.

of (3). As explained in Aghion & Howitt (1998), Howitt (1999), Jones (1999), Laincz & Peretto (2006), or Dinopoulos & Sener (2007), most growth models differ mainly in the specification of the knowledge production technology. In fact, Lemma 1 underlines that the main distinction is to be found in the constitution of the pools of knowledge used by R&D activities, and in the knowledge spillovers they stem from.

In particular, Lemma 1 illustrates the fact that the R&D activity of a given sector always uses the knowledge accumulated so far in this sector and potentially captures part of the mass of the ideas created in all others; this subset of $\mathcal{K}_t$ is more or less important, as the scope of knowledge diffusion ϑ is more or less wide. Eventually, depending on the choice of the parameter ϑ, one obtains a large collection of pools, $\mathcal{P}_{it}$, and thus models considering various types of knowledge spillovers. Let us display two polar cases.

Global knowledge spillovers. It can be assumed that all knowledge systematically spills into the whole economy. Formally, if one assumes that the scope of knowledge diffusion is maximal ($\vartheta = N$), one has $\mathcal{Q}_i^{\mathcal{S}} = \mathcal{Q}_i^{\mathcal{R}} = \mathcal{Q}, \forall i \in \mathcal{Q}$. Each sector i makes use of the whole stock of knowledge available in the economy: rewriting (2) and using (1), one has $\mathcal{P}_{it} = \int_{\mathcal{Q}} \kappa_{ht} dh = \mathcal{K}_t, \forall i \in \mathcal{Q}$. Then, the resulting knowledge production function in each sector i is $\dot{\kappa}_{it} = \lambda\theta l_{it}\mathcal{K}_t, \forall i \in \mathcal{Q}$. This first polar case exhibits global knowledge spillovers.

It is interesting to note that this law of knowledge accumulation relates to the one originally introduced in the seminal paper of Romer (1990). Indeed, the present expression of the law of knowledge accumulation - which is here endogenously derived from assumptions made in a stochastic quality ladders model - leads to a law of motion of the whole disposable knowledge formally identical to the knowledge production function initially introduced by Romer (1990).[5]

No inter-sectoral knowledge spillovers but only intra-sectoral knowledge spillovers. The models of Grossman & Helpman (1991), Segerstrom (1998), Peretto (1999), Acemoglu (2009 - Ch. 14), or Aghion & Howitt (2009 - Ch. 4) implicitly consider spillovers only within each sector. In each sector, the pool of knowledge used by the R&D activity includes exclusively the knowledge previously accumulated within this sector. In our formalization, this second polar case amounts to assuming that the scope of knowledge diffusion is minimal ($\vartheta = 1$); then one has $\mathcal{P}_{it} = \kappa_{it}, \forall i \in \mathcal{Q}$. The resulting law of knowledge accumulation is $\dot{\kappa}_{it} = \lambda\theta l_{it}\kappa_{it}, \forall i \in \mathcal{Q}$.

Furthermore, the law of knowledge accumulation derived in Lemma 1 also allows to consider other standard laws of knowledge accumulation used in the endogenous growth theory by specifying the pools of knowledge $\mathcal{P}_{it}$ on an ad hoc basis.

"Leading-edge technology". In the models of Aghion & Howitt (1992), Young (1998), Howitt (1999), Segerstrom (2000), or Garner (2010), it is assumed that knowledge spillovers depend on the knowledge level reached by the frontier firms (*i.e.* reached in the most advanced sector). In our formalization, this amounts to assuming that $\mathcal{P}_{it} = \kappa_t^{max} = \max\{\kappa_{it}, i \in \mathcal{Q}\}, \forall i \in \mathcal{Q}$; then, one has $\dot{\kappa}_{it} = \lambda\theta l_{it}\kappa_t^{max}, \forall i \in \mathcal{Q}$.

No knowledge spillovers. In Barro & Sala-i-Martin (2003 - Ch. 6) or in Peretto (2007), the knowledge production technology uses final good only; it is thus considered that there is neither inter-sectoral nor even intra-sectoral knowledge spillovers. Our formalization encompasses a similar framework in which knowledge is produced only with private inputs (labor in the present model); for this purpose, it can be assumed that $\mathcal{P}_{it} = 1, \forall i \in \mathcal{Q}$; thus one has $\dot{\kappa}_{it} = \lambda\theta l_{it}, \forall i \in \mathcal{Q}$.

Before presenting the remainder of the model, let us provide some comments on Assumptions 1, 2, 3, 4, and on Lemma 1. Each of the tree parameters λ, θ, and ϑ accounts for the productivity of R&D activities. As seen in Assumption 1, λ stands for the productivity of the labor devoted to the creation of innovations. As seen in Assumption 2, θ stands for the productivity of the pool of knowledge in the production of innovations (*i.e.* of new knowledge): θ accounts for the extent to which the pool of knowledge $\mathcal{P}_{it}$ contributes to the increases in knowledge $\Delta\kappa_{it}$ resulting from an innovation ($\Delta\kappa_{it} = \theta\mathcal{P}_{it}$). In other words, θ *partly measures the intensity of knowledge spillovers.* As seen in Assumptions 3 and 4, and in the expression given in (2) of the pool of knowledge used by each R&D activity, the scope of knowledge diffusion, ϑ, stands for the overall influence of the knowledge inherent in any innovation on the constitution of the pools of knowledge. Accordingly, ϑ *also partly measures the intensity of knowledge spillovers.* We will return to these points below (see in the comments of Proposition 1).

The rest of the model is fairly standard and in line with seminal endogenous growth models. We consider

[5] Indeed, differentiating (1) with respect to time, one gets $\dot{\mathcal{K}}_t = \int_{\mathcal{Q}} \dot{\kappa}_{it} di = \lambda\theta \left(\int_{\mathcal{Q}} l_{it} di\right) \mathcal{K}_t \Leftrightarrow \dot{\mathcal{K}}_t = \lambda\theta L_t^R \mathcal{K}_t$, where $L_t^R = \int_{\mathcal{Q}} l_{it} di$ is the total amount of labor used in R&D.

an infinitely lived representative household who has the following intertemporal preferences:

$$\mathcal{U} = \int_0^\infty u(c_t)e^{-\rho t}dt, \tag{4}$$

where c_t is the per capital consumption, $u(c_t)$ is the associated instantaneous utility at date t, and ρ is the subjective discount rate. At each date t, there are L identical households, each one being endowed with one unit of labor which is supplied inelastically. In order to limit the number of parameters, we make the following simplifying assumptions. We consider $u(c_t) = \ln(c_t)$; besides, there is no population growth, and the population size is normalized to one.[6] Labor is used in the R&D activity of each sector i ($l_{it}, i \in \mathcal{Q}$) and to produce the final good (L_t^Y); hence, the constraint on the labor market is

$$1 = L_t^R + L_t^Y, \text{ where } L_t^R = \int_{\mathcal{Q}} l_{it}di \text{ is the total quantity of labor used in R\&D activity.} \tag{5}$$

An homogeneous final good is produced with labor combined with all available intermediate goods and the knowledge incorporated in each of them:

$$Y_t = (L_t^Y)^{1-\alpha} \int_{\mathcal{Q}} \kappa_{it}(x_{it})^\alpha di, \; 0 < \alpha < 1. \tag{6}$$

At each date t, the final good is either consumed by the representative household (c_t) or used to produce the intermediate goods. It is often assumed, that the production function of any given intermediate good $i, i \in \mathcal{Q}$, is characterized by increasing complexity (*i.e.* the larger the stock of knowledge inherent in intermediate good i, the more final good is needed to produce this intermediate good). We assume likewise:

$$x_{it} = \frac{y_{it}}{\kappa_{it}}, \; \forall i \in \mathcal{Q}, \tag{7}$$

where y_{it} is the quantity of final good used to produce x_{it} units of intermediate good i. Thus, the constraint on the final good market writes

$$Y_t = c_t + \int_{\mathcal{Q}} y_{it}di. \tag{8}$$

As usual in endogenous growth theory, computing the first-best social optimum as well as the Schumpeterian equilibrium requires at some point to make the standard assumption of symmetry across sectors (see, for instance, in Aghion & Howitt 1992 or 1998 - Ch. 3, or in Peretto & Smulders 2002).[7] Formally, when the time comes, it will be assumed both for computing the optimum (Section 2.2) and for computing the Schumpeterian equilibrium (Section 3.2), that all sectors have the same initial level of knowledge ($\kappa_{i0} = \kappa_0, \forall i \in \mathcal{Q}$), that $l_{it} = l_t, \forall i \in \mathcal{Q}$, and that $\kappa_{it} = \kappa_t, \forall i \in \mathcal{Q}$.[8] Finally, in all that follows, the rate of growth, $\frac{\dot{z}_t}{z_t}$, of any variable z_t, is denoted g_{z_t}.

2.2 First-best Social Optimum

The first-best social optimum is derived by maximizing the representative household's discounted utility (4) subject to (1), (2), (3), (5), (6), (7), and (8). The control variables are $x_{it}, i \in \mathcal{Q}$, L_t^Y, $l_{it}, i \in \mathcal{Q}$, and c_t; besides,

[6]The key results of the paper are maintained if one uses a C.E.S. instantaneous utility function of parameter ε, $u(c_t) = c_t^{1-\varepsilon}/(1-\varepsilon)$ and/or if one assumes constant population growth. These more general assumptions introduce additional parameters but do not add any relevant insight to the issues addressed in the present paper. Furthermore, considering constant population growth can easily be done without the model exhibiting the non desirable scale effects property. Indeed, to consider a scale-invariant fully endogenous growth model, it is sufficient to allow for expansion in the number of sectors (see, for example, Dinopoulos & Thompson 1998, Peretto 1998, Young 1998, Howitt 1999, Peretto 1999, or Aghion & Howitt 2009 - Ch. 4); then, removing scale effects in the present framework could be done by assuming that, at each date t, the number of sectors N_t and the size of the population L_t are proportional (*e.g.*, $N_t = \gamma L_t, \gamma > 0$). This commonly used assumption has been justified both theoretically and empirically (Aghion & Howitt 1998 - Ch. 12, Jones 1999, Segerstrom 2000, Laincz & Peretto 2006, or Dinopoulos & Sener 2007). See in Gray & Grimaud (2016) for a more general version of the present model.

[7]For more details on this assumption of symmetry across sectors, see for instance in Peretto (1998, 1999), or in Cozzi, Giordani & Zamparelli (2007).

[8]By way of example, let us rewrite (1), (2), and (3) under the symmetry assumption. The whole stock of knowledge in the economy is $\mathcal{K}_t = \int_{\mathcal{Q}} \kappa_{it}di = N\kappa_t$. The pool of knowledge used in sector i is $\mathcal{P}_{it} = \mathcal{P}_t = \vartheta\kappa_t, \forall i \in \mathcal{Q}$. The resulting law of knowledge accumulation is $\dot{\kappa}_{it} = \dot{\kappa}_t = \lambda\theta\vartheta l_t\kappa_t, \forall i \in \mathcal{Q}$.

there is a continuum of state variables κ_{it}, $i \in \mathcal{Q}$ (in fact, there is a continuum of constraints relative to the law of motion of knowledge). The Hamiltonian of the dynamic optimization problem can be written as

$$\mathcal{H} = \ln(c_t)e^{-\rho t} + \mu_t \left[Y_t - c_t - \int_{\mathcal{Q}} \kappa_{it} x_{it} di \right] + \nu_t \left[1 - L_t^Y - \int_{\mathcal{Q}} l_{it} di \right] + \int_{\mathcal{Q}} \zeta_{it} \left[\dot{\kappa}_{it} \right] di,$$

where $Y_t = (L_t^Y)^{1-\alpha} \int_{\mathcal{Q}} \kappa_{it}(x_{it})^\alpha di$, $\dot{\kappa}_{it} = \lambda\theta l_{it}\mathcal{P}_{it}, \forall i \in \mathcal{Q}$, $\mathcal{P}_{it} = \int_{\mathcal{Q}_i^{\mathcal{R}}} \kappa_{ht} dh$; and where μ_t, ν_t, and $\zeta_{it}, i \in \mathcal{Q}$, are the co-state variables associated with the final good resource constraint, the labor constraint, and the continuum of state variables, respectively.

The first-best social optimum is characterized in the following Proposition. From now on, the superscript "o" is used for "'first-best social optimum". Furthermore, let us introduce the notation $\Theta \equiv \theta\vartheta$; as detailed below in the comments of Proposition 1, Θ is defined as the measure of the *"intensity of knowledge spillovers"*.

Proposition 1. *At the first-best social optimum, the partition of labor, the quantity of each intermediate good i, and the growth rates are*

$$L_t^{Yo} = L^{Yo} = \frac{\rho N}{\lambda\Theta};\ l_{it}^o = l^o = \frac{1}{N} - \frac{\rho}{\lambda\Theta}, \forall i \in \mathcal{Q};\ x_{it}^o = x^o = \alpha^{\frac{1}{1-\alpha}} \frac{\rho N}{\lambda\Theta}, \forall i \in \mathcal{Q};$$

$$\text{and } g_{c_t}^o = g_{Y_t}^o = g_{\mathcal{K}_t}^o = g_{\kappa_{it}}^o = g^o = \lambda\Theta\, l^o = \frac{\lambda}{N}\Theta - \rho, \forall i \in \mathcal{Q}, \forall t, \text{ with } \Theta \equiv \theta\vartheta.$$

Proof. The first-order conditions of the maximization program are[9]

$$\frac{\partial \mathcal{H}}{\partial x_{it}} = 0, \forall i \in \mathcal{Q} \Leftrightarrow \mu_t \left[\alpha (L_t^Y)^{1-\alpha} \kappa_{it} (x_{it})^{\alpha-1} - \kappa_{it} \right] = 0, \forall i \in \mathcal{Q}, \tag{9}$$

$$\frac{\partial \mathcal{H}}{\partial L_t^Y} = 0 \Leftrightarrow \mu_t (1-\alpha) \frac{Y_t}{L_t^Y} = \nu_t, \tag{10}$$

$$\frac{\partial \mathcal{H}}{\partial l_{it}} = 0, \forall i \in \mathcal{Q} \Leftrightarrow \zeta_{it} \lambda\theta \int_{\mathcal{Q}_i^{\mathcal{R}}} \kappa_{ht} dh = \nu_t \Leftrightarrow \zeta_{it}\lambda\theta\mathcal{P}_{it} = \nu_t, \forall i \in \mathcal{Q}, \tag{11}$$

$$\frac{\partial \mathcal{H}}{\partial c_t} = 0 \Leftrightarrow c_t^{-1} e^{-\rho t} = \mu_t, \tag{12}$$

$$\frac{\partial \mathcal{H}}{\partial \kappa_{it}} = -\dot{\zeta}_{it}, \forall i \in \mathcal{Q} \Leftrightarrow \mu_t \left[(L_t^Y)^{1-\alpha} (x_{it})^\alpha - x_{it} \right] + \int_{\mathcal{Q}_i^{\mathcal{R}}} \zeta_{ht} \lambda\theta l_{ht} dh = -\dot{\zeta}_{it}, \forall i \in \mathcal{Q}. \tag{13}$$

From (9), one gets the usual symmetric use of intermediate goods:

$$x_{it} = x_t = \alpha^{\frac{1}{1-\alpha}} L_t^Y,\ \forall i \in \mathcal{Q}. \tag{14}$$

Using (1) and (14), the final good production function (6) rewrites

$$Y_t = \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t. \tag{15}$$

Then, from (8), (14), and (15), one obtains

$$Y_t = c_t + \alpha^{\frac{1}{1-\alpha}} L_t^Y \mathcal{K}_t \Leftrightarrow Y_t = c_t + \alpha Y_t \Leftrightarrow \frac{c_t}{Y_t} = 1 - \alpha. \tag{16}$$

Log-differentiating (12), (15), and (16) gives

$$g_{c_t} + \rho = -g_{\mu_t},\ g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t}, \text{ and } g_{c_t} = g_{Y_t}, \text{ respectively.} \tag{17}$$

Besides, plugging (15) in (10) and in (13), one gets

$$\mu_t (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t = \nu_t. \tag{18}$$

$$\text{and } \mu_t (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} L_t^Y + \lambda\theta \int_{\mathcal{Q}_i^{\mathcal{R}}} \zeta_{ht} l_{ht} dh = -\dot{\zeta}_{it}, \forall i \in \mathcal{Q}. \tag{19}$$

[9]Plus the usual transversality condition which will be used to characterize the steady-state optimum.

Let us now use the usual assumption of symmetry across sectors ($l_{it} = l_t$ and $\kappa_{it} = \kappa_t$, $\forall i \in \mathcal{Q}$).[10] The whole stock of knowledge (1) and the pool of knowledge in sector i (2) then write

$$\mathcal{K}_t = \int_{\mathcal{Q}} \kappa_{it} di = N\kappa_t, \text{ and } \mathcal{P}_{it} = \int_{\mathcal{Q}_i^{\mathcal{R}}} \kappa_{ht}\, dh = \mathcal{P}_t = \vartheta\kappa_t, \forall i \in \mathcal{Q}, \text{ respectively.} \tag{20}$$

Hence, we obtain the growth rate of the stock of knowledge in any sector i and of the whole stock of knowledge in the economy:

$$g_{\kappa_{it}} = g_{\kappa_t} = g_{\mathcal{K}_t} = \lambda\theta\vartheta l_t, \forall i \in \mathcal{Q}. \tag{21}$$

Besides, the labor constraint (5) rewrites

$$Nl_t = 1 - L_t^Y. \tag{22}$$

Under the symmetry assumption, (11) can be expressed as

$$\zeta_{it} = \zeta_t = \frac{\nu_t}{\lambda\theta\vartheta\kappa_t}, \forall i \in \mathcal{Q}. \tag{23}$$

Then, using (18), (20), and (23), one obtains

$$\nu_t = \mu_t(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} N\kappa_t = \zeta_t\lambda\theta\vartheta\kappa_t \Leftrightarrow \frac{\mu_t}{\zeta_t} = \frac{\lambda\theta\vartheta}{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} N}. \tag{24}$$

Log-differentiating (24), one obtains

$$g_{\mu_t} = g_{\zeta_t}. \tag{25}$$

Furthermore, rewriting (19) under the symmetry assumption gives $\frac{\mu_t}{\zeta_t}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} L_t^Y + \lambda\theta\vartheta l_t = -\frac{\dot{\zeta}_t}{\zeta_t}$; then, using (24) one gets $g_{\zeta_t} = \frac{\dot{\zeta}_t}{\zeta_t} = -\frac{\lambda\theta\vartheta}{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} N}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} L_t^Y + \lambda\theta\vartheta l_t = -\frac{\lambda\theta\vartheta}{N}\left(L_t^Y + Nl_t\right)$. Finally, using (22) and (25) gives

$$g_{\mu_t} = g_{\zeta_t} = -\frac{\lambda\theta\vartheta}{N}. \tag{26}$$

Then, the optimal growth rate of per-capita consumption, g_c^o, can be derived using (17) and (26):

$$g_c^o = \frac{\lambda}{N}\theta\vartheta - \rho. \tag{27}$$

Finally, the first-best social optimum is the solution of the following system of equations, which summarize the conditions obtained in (14), (17), (21), (22), and (27):

$$\begin{cases} g_c^o = \frac{\lambda\theta\vartheta}{N} - \rho & (a) \\ g_Y^o = g_c^o & (b) \\ g_{L_t^Y}^o = g_Y^o - g_{\mathcal{K}_t}^o & (c) \\ g_{\kappa_{it}^o} = g_{\kappa_t^o} = g_{\mathcal{K}_t^o} = \lambda\theta\vartheta l_t^o, \forall i \in \mathcal{Q} & (d) \\ Nl_t^o = 1 - L_t^{Yo} & (e) \\ x_{it}^o = x_t^o = \alpha^{\frac{1}{1-\alpha}} L_t^{Yo}, \forall i \in \mathcal{Q} & (f) \end{cases}$$

From equations (a), (b), (c), (d) and (e), one obtains the following differential equation in L_t^{Yo}:

$$g_{L_t^Y}^o = \frac{\lambda\theta\vartheta}{N} - \rho - \lambda\theta\vartheta\frac{1 - L_t^{Yo}}{N} \Leftrightarrow g_{L_t^Y}^o = \frac{\lambda\theta\vartheta}{N} L_t^{Yo} - \rho.$$

Let $X_t = 1/L_t^{Yo}$, one gets a first-order linear differential equation in X_t, $\dot{X}_t - \rho X_t = -\frac{\lambda\theta\vartheta}{N}$, the solution of which is

$$X_t = \left(X_0 - \frac{\lambda\theta\vartheta}{\rho N}\right) e^{\rho t} + \frac{\lambda\theta\vartheta}{\rho N} \Leftrightarrow L_t^{Yo} = \frac{1}{\left(\frac{1}{L_0^{Yo}} - \frac{\lambda\theta\vartheta}{\rho N}\right) e^{\rho t} + \frac{\lambda\theta\vartheta}{\rho N}}.$$

[10] See the comment above (end of Section 2.1) on the symmetry assumption commonly made in endogenous growth models.

Using the transversality condition, we show that $L_t^{Y o}$ instantly reaches its steady-state level $L^{Y o^{ss}} = \rho N/\lambda\theta\vartheta$; and thus, one has $L_t^{Y o} = L^{Y o^{ss}}, \forall t$. Hence, one obtains the optimal partition of labor, and optimal quantity of intermediate goods:

$$L^{Y o} = \frac{\rho N}{\lambda\theta\vartheta},\ l_i^o = l^o = \frac{1}{N} - \frac{\rho}{\lambda\theta\vartheta}, \forall i \in \mathcal{Q}, \text{ and } x_i^o = x^o = \alpha^{\frac{1}{1-\alpha}} \frac{\rho N}{\lambda\theta\vartheta}, \forall i \in \mathcal{Q}. \tag{28}$$

The optimal growth rate of knowledge accumulation in any sector i and in the whole economy are given by

$$g_{\kappa_i^o} = g_{\kappa^o} = g_{\mathcal{K}^o} = \frac{\lambda\theta\vartheta}{N} - \rho, \forall i \in \mathcal{Q}. \tag{29}$$

Finally, (27), (28), and (29) prove Proposition 1. □

As explained above in the comments to Assumptions 1, 2, 3, 4, and to Lemma 1, the tree parameters λ, θ, and ϑ all determine the productivity of R&D activities. Hence, as shown by the results obtained in Proposition 1, these parameters obviously determine the optimal growth rate of the economy. At the first-best optimum, an increase in λ, θ, and/or ϑ implies a reallocation of labor from the final good production toward R&D activity; thus leading to a higher growth rate.

The part played by λ is rather evident as it determines the productivity of labor in R&D (Assumption 1): the more productive the R&D is, the more innovations occur, and thus the higher growth rate is.

The roles played by the parameters θ and ϑ are closely related: θ determines the productivity of the pool of knowledge in the production of new knowledge in a given sector (Assumption 2), and ϑ determines the overall influence of the knowledge produced in a particular sector on the constitution of the pools of knowledge used by the R&D activities of the other sectors (Assumptions 3 and 4, and equation (2)). When an innovation occurs in a given sector, θ measures the impact of the knowledge inherent in this innovation on the production of new knowledge in other sectors; whereas ϑ measures the subset of sectors which will be using the knowledge stemming from this innovation. Nevertheless, θ and ϑ both clearly measure the intensity of knowledge spillovers. In the present model, as illustrated in Proposition 1, these two parameters often appear in the form of the product $\theta\vartheta$; therefore, we have introduced the notation $\Theta \equiv \theta\vartheta$, to stand for the *"intensity of knowledge spillovers"*, which is a key determinant in the optimal growth rate. One has the following Corollary to Proposition 1.

Corollary. *The stronger the intensity of knowledge spillovers* Θ *in the economy, the higher the optimal R&D effort* $l^o = \frac{1}{N} - \frac{\rho}{\lambda\Theta}$*, and thus the higher the optimal growth rate* $g^o = \frac{\lambda}{N}\Theta - \rho$.

To conclude this section, let us note that in the endogenous growth literature, the denomination "knowledge spillovers" relates simultaneously to two issues. First, a "technology-related" issue as it refers to the mechanism by which knowledge previously created spills over into the economy, thus enhancing the creation of new knowledge. Second, an "equilibirum-related" issue as it refers to market incompleteness: in the decentralized economies generally considered (*e.g.*, the equilibria studied by Romer, 1990, or by Aghion & Howitt, 1992), knowledge is not priced. As detailed in the introduction of this paper, knowledge spillovers related issues have been extensively studied (*e.g.*, Romer, 1990; Aghion & Howitt 1992, 1998, 2009; Segerstrom, 1998; Li, 2002; Peretto & Smulders, 2002; Jones, 2005; Sener, 2008; Acemoglu, 2009). In this section, we have introduced explicitly a formalization of knowledge spillovers in the technology of production of knowledge (see Assumptions 2, 3, and 4, and Lemma 1). In Section 3 below, we study a Schumpeterian equilibrium *à* Aghion & Howitt (1992), and we shed a new light on the link between knowledge spillovers and the Pareto non-optimality of the Schumpeterian equilibrium which results from market incompleteness.

3 Decentralized Economy: Schumpeterian Equilibrium

In this section, we study a decentralized economy in which the funding of R&D activity (and thus of the production of knowledge) is based on assumptions inspired by Schumpeter's creative destruction mechanism. Formally, we define, characterize, and compute the standard Schumpeterian equilibrium *à* Aghion & Howitt (1992).

3.1 Definition of the Schumpeterian Equilibrium and Agents' Behaviors

Consider any sector i, $i \in \mathcal{Q}$. Once an innovation occurs in sector i, its producer is given an infinitely-lived patent. Then, in each sector i, the latest innovator has a monopoly on the intermediate good i until replaced by the next innovator upgrading the quality of this intermediate good. Hence, the R&D activity of each sector - which basically produce knowledge - is indirectly financed by a succession of monopoly on an intermediate good the quality of which sequentially increases as the knowledge it incorporates accumulates in the sector.

Such a decentralized economy is characterized by two market failures. First, market power implied by the presence of a monopoly on each intermediate good market. Second, market incompleteness: knowledge creation is indirectly funded by monopoly profits since there is no market for knowledge (knowledge is not priced). Hence, the Schumpeterian equilibrium considered in the present paper is likely to be Pareto non-optimal; in particular, at the *laisser faire* equilibrium, the quantity of labor used in R&D (*i.e.* the R&D effort) can either be sub-optimal or over-optimal. We thus consider two tools dedicated to mitigate these two market failures. Let Υ^P denote the tool to correct market power; it is well known that monopoly power can be corrected by an *ad valorem* subsidy on each intermediate good demand. Let Υ^I denote the public tool to alleviate market incompleteness; as it will be proven below, this tool can consist either in a subsidy or in a tax on the profits of each R&D activity.

Let us now define formally the set of Schumpeterian equilibria as functions of the policy tools vector $\left(\Upsilon^P, \Upsilon^I\right)$. We denote the wage by w_t, the price of intermediate good i by $q_{it}, i \in \mathcal{Q}$, the interest rate by r_t; and we normalize the price of the final good to one.

Definition. *At each vector of public policy tools $\left(\Upsilon^P, \Upsilon^I\right)$ is associated a particular Schumpeterian equilibrium which consists of time paths of set of prices*

$$\left\{\left(w_t\left(\Upsilon^P, \Upsilon^I\right), \left\{q_{it}\left(\Upsilon^P, \Upsilon^I\right)\right\}_{i\in\mathcal{Q}}, r_t\left(\Upsilon^P, \Upsilon^I\right)\right)\right\}_{t=0}^{\infty}$$

and of quantities

$$\left\{\left(L_t^Y\left(\Upsilon^P, \Upsilon^I\right), \left\{l_{it}\left(\Upsilon^P, \Upsilon^I\right)\right\}_{i\in\mathcal{Q}}, \left\{\kappa_{it}\left(\Upsilon^P, \Upsilon^I\right)\right\}_{i\in\mathcal{Q}}, \left\{x_{it}\left(\Upsilon^P, \Upsilon^I\right)\right\}_{i\in\mathcal{Q}}, c_t\left(\Upsilon^P, \Upsilon^I\right), Y_t\left(\Upsilon^P, \Upsilon^I\right)\right)\right\}_{t=0}^{\infty}$$

such that: the representative household maximizes his utility; firms maximize their profits; the labor market, the financial market, and the final good market are perfectly competitive and clear; on each intermediate good market, the innovator is granted an infinitely-lived patent and monopolizes the production and sale until replaced by the next innovator; and there is free entry on each R&D activity (i.e. the zero profit condition holds for each R&D activity).

In order to fully characterize the decentralized economy, we now present in detail the agents' behaviors. For the purpose of simplifying notations throughout the following computations, we momentarily drop the $\left(\Upsilon^P, \Upsilon^I\right)$ arguments for all variables.

Representative household.

The representative household maximizes his intertemporal utility given by (4) subject to his budget constraint, $\dot{b}_t = w_t + r_t b_t - c_t - T_t$, where b_t denotes the per capita financial asset and T_t is a lump-sum tax used by the government to finance public policies. One gets the usual Keynes-Ramsey condition:

$$r_t = g_{c_t} + \rho. \tag{30}$$

Final good producer.

The final good market is assumed to be competitive; the profit of the final good producer (recall, the price of the final good is normalized to one) writes

$$\pi_t^Y = (L_t^Y)^{1-\alpha} \int_{\mathcal{Q}} \kappa_{it}(x_{it})^{\alpha} di - w_t L_t^Y - \int_{\mathcal{Q}} (1 - \Upsilon^P) q_{it} x_{it} di.$$

The first-order conditions of the profit maximization program are

$$\frac{\partial \pi_t^Y}{\partial L_t^Y} = 0 \Leftrightarrow w_t = (1-\alpha)\frac{Y_t}{L_t^Y}, \text{ and} \tag{31}$$

$$\frac{\partial \pi_t^Y}{\partial x_{it}} = 0, \forall i \in \mathcal{Q} \Leftrightarrow \left(1 - \Upsilon^P\right) q_{it} = \alpha (L_t^Y)^{1-\alpha} \kappa_{it}(x_{it})^{\alpha-1}, \forall i \in \mathcal{Q}, \forall i \in \mathcal{Q}. \tag{32}$$

Intermediate goods producers.

In each sector i, $i \in \mathcal{Q}$, the latest innovator has a monopoly on the production and sale of intermediate good i. Given the public intervention on R&D activity (formalized by the public tool Υ^I introduced above), the incumbent monopoly maximizes the instantaneous net profit

$$\pi_t^{x_i} = (1+\Upsilon^I)\left(q_{it}x_{it} - y_{it}\right), \tag{33}$$

where the demand for intermediate good i, x_{it}, is given by (32). Using (7), the monopoly maximization program can be written:

$$\text{Max}_{x_{it}}\ \pi_t^{x_i} = (1+\Upsilon^I)\left(q_{it}x_{it} - x_{it}\kappa_{it}\right) \text{ subject to } \left(1-\Upsilon^P\right)q_{it} = \alpha(L_t^Y)^{1-\alpha}\kappa_{it}(x_{it})^{\alpha-1}.$$

The first-order condition with respect to x_{it} is

$$\frac{\partial \pi_t^{x_i}}{\partial x_{it}} = 0 \Leftrightarrow \frac{\partial}{\partial x_{it}}\left[(1+\Upsilon^I)\left(\frac{\alpha(L_t^Y)^{1-\alpha}\kappa_{it}(x_{it})^{\alpha}}{1-\Upsilon^P} - x_{it}\kappa_{it}\right)\right] = 0 \Leftrightarrow x_{it} = \left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{1}{1-\alpha}} L_t^Y;$$

replacing in (32), one gets

$$q_{it} = \frac{\alpha}{1-\Upsilon^P}\left(\frac{L_t^Y}{x_{it}}\right)^{1-\alpha}\kappa_{it} = \frac{\alpha}{1-\Upsilon^P}\left(\left(\frac{1-\Upsilon^P}{\alpha^2}\right)^{\frac{1}{1-\alpha}}\right)^{1-\alpha}\kappa_{it} = \frac{\kappa_{it}}{\alpha}.$$

Therefore, one obtains the usual symmetric use of intermediate goods in the final good production and the usual mark-up on the price of intermediate goods:

$$x_{it} = x_t = \left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{1}{1-\alpha}} L_t^Y \text{ and } q_{it} = \frac{\kappa_{it}}{\alpha},\ \forall i \in \mathcal{Q}. \tag{34}$$

Then, using (34) and (1), we can rewrite the monopoly profit in any sector i (33), the final good production function (6), and the expression of the wage given in (31) as:

$$\pi_t^{x_i} = (1+\Upsilon^I)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{1}{1-\alpha}} L_t^Y\kappa_{it}, \forall i \in \mathcal{Q}, \tag{35}$$

$$Y_t = \left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y\mathcal{K}_t, \text{ and} \tag{36}$$

$$w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t. \tag{37}$$

R&D activities.

In each sector i, $i \in \mathcal{Q}$, the incumbent innovator having innovated at date t has a monopoly on intermediate good i, and receives, at any date $\tau > t$, the instantaneous net profit $\pi_\tau^{x_i}$ with probability $e^{-\int_t^\tau \lambda l_{iu}du}$ (*i.e.* as long as there is no other innovation in sector i between date t and date τ).[11] We denote by $\Pi_t^{x_i}$ the value at date t of the latest innovation in sector i, it is the sum of the present values of the incumbent's expected net profits on the sale of intermediate good i:

$$\Pi_t^{x_i} = \int_t^\infty \pi_\tau^{x_i} e^{-\int_t^\tau (r_u+\lambda l_{iu})du}d\tau, \text{ where } \pi_\tau^{x_i} \text{ is given by (33).} \tag{38}$$

Then, the arbitrage condition in R&D activity i is obtained by differentiating (38) with respect to time; one gets

$$r_t + \lambda l_{it} = \frac{\dot{\Pi}_t^{x_i}}{\Pi_t^{x_i}} + \frac{\pi_t^{x_i}}{\Pi_t^{x_i}}, \forall i \in \mathcal{Q}. \tag{39}$$

[11] As detailed in Assumption 1, innovations in sector i occurs according to a Poisson arrival rate λl_{it}.

Given Assumption 1, innovations arrive according to a Poisson process of rate λl_{it}; thus the total expected revenue at date t when one unit of labor is invested in R&D is $\lambda \Pi_{it}^{x}$. Besides, the cost of one unit of labor is w_t. Consequently, the free entry condition in any R&D activity i is

$$w_t = \lambda \Pi_t^{x_i}. \tag{40}$$

Then, using (37), one gets the following value of the latest innovation in sector i at date t:

$$\Pi_t^{x_i} = \Pi_t^{x} = \frac{1-\alpha}{\lambda}\left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t, \forall i \in \mathcal{Q}. \tag{41}$$

3.2 Characterization of the Schumpeterian Equilibrium

Log-differentiating (36), one has

$$g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t}. \tag{42}$$

Using (1), (7), and (34), the constraint on the final good market (8) rewrites

$$Y_t = c_t + \left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{1}{1-\alpha}} L_t^Y \mathcal{K}_t. \tag{43}$$

Then, from (36) and (43), one obtains

$$c_t = \left(1 - \frac{\alpha^2}{1-\Upsilon^P}\right) Y_t. \tag{44}$$

Log-differentiating this expression gives

$$g_{c_t} = g_{Y_t}. \tag{45}$$

Furthermore, log-differentiating (41) gives $\frac{\dot{\Pi}_t^{x_i}}{\Pi_t^{x_i}} = g_{\mathcal{K}_t}$; then, using (35) and (41), the arbitrage condition (39) can then be rewritten

$$r_t + \lambda l_{it} = g_{\mathcal{K}_t} + \frac{1+\Upsilon^I}{1-\Upsilon^P} \lambda \alpha L_t^Y \frac{\kappa_{it}}{\mathcal{K}_t}, \forall i \in \mathcal{Q}. \tag{46}$$

As explained above, computing the Schumpeterian equilibrium requires to consider the standard assumption of symmetry across sector: $l_{it} = l_t, \forall i \in \mathcal{Q}$ and $\kappa_{it} = \kappa_t, \forall i \in \mathcal{Q}$.[12] Accordingly, the whole stock of knowledge (1), the pool of knowledge in any sector i (2), the growth rates of these stocks of knowledge, and the labor constraint (5) can be rewritten as follows:[13]

$$\mathcal{K}_t = \int_{\mathcal{Q}} \kappa_{it} di = N\kappa_t; \, \mathcal{P}_{it} = \int_{\mathcal{Q}_i^{\mathcal{R}}} \kappa_{ht}\, dh = \mathcal{P}_t = \vartheta \kappa_t, \forall i \in \mathcal{Q}; \tag{47}$$

$$g_{\kappa_{it}} = g_{\kappa_t} = g_{\mathcal{K}_t} = \lambda \theta \vartheta l_t, \forall i \in \mathcal{Q}; \tag{48}$$

$$N l_t = 1 - L_t^Y. \tag{49}$$

Hence, in any sector i, $i \in \mathcal{Q}$, the arbitrage condition in R&D (46) rewrites

$$r_t + \lambda l_t = \lambda \theta \vartheta l_t + \left(\frac{1+\Upsilon^I}{1-\Upsilon^P}\right) \frac{\lambda \alpha}{N} L_t^Y. \tag{50}$$

Eventually, the set of Schumpeterian equilibria is characterized by (30), (34), (36), (37), (42), (45), (48), (49), and (50); it is displayed in Proposition 2 below.

Proposition 2. *At each date t, the set of Schumpeterian equilibria* à la *Aghion & Howitt is characterized as follows.*

[12]See, at the end of Section 2.1, the comment on the symmetry assumption commonly made in endogenous growth models. Furthermore, the relevancy of the symmetric equilibrium is discussed in Cozzi, Giordani & Zamparelli (2007).

[13]The same reasoning has been made when computing the first-best social optimum; see, (22), (20), and (21).

- *The labor partition and the quantities of intermediate goods are*

$$L_t^Y\left(\Upsilon^P,\Upsilon^I\right)=L^Y\left(\Upsilon^P,\Upsilon^I\right)=\left(\rho+\frac{\lambda}{N}\right)\left[\frac{\lambda}{N}\left(1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right)\right]^{-1},\forall t;$$

$$l_{it}\left(\Upsilon^P,\Upsilon^I\right)=l\left(\Upsilon^P,\Upsilon^I\right)=\frac{1}{N}\left[1-L^Y\left(\Upsilon^P,\Upsilon^I\right)\right],\forall i\in\mathcal{Q},\forall t;$$

$$x_{it}\left(\Upsilon^P,\Upsilon^I\right)=x\left(\Upsilon^P,\Upsilon^I\right)=\left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{1}{1-\alpha}}L^Y\left(\Upsilon^P,\Upsilon^I\right),\forall i\in\mathcal{Q},\forall t.$$

- *The growth rates of per capita consumption, of the final good output, of the whole stock of knowledge in the economy, and of the stock of knowledge in each sector are*

$$g_{c_t}\left(\Upsilon^P,\Upsilon^I\right)=g_{Y_t}\left(\Upsilon^P,\Upsilon^I\right)=g_{\mathcal{K}_t}\left(\Upsilon^P,\Upsilon^I\right)=g\left(\Upsilon^P,\Upsilon^I\right),\forall t;$$

$$g_{\kappa_{it}}\left(\Upsilon^P,\Upsilon^I\right)=g\left(\Upsilon^P,\Upsilon^I\right),\forall i\in\mathcal{Q},\forall t;\ \textit{where } g\left(\Upsilon^P,\Upsilon^I\right)=\lambda\Theta l\left(\Upsilon^P,\Upsilon^I\right).$$

- *The stock of knowledge in the economy, the quantity of final good, and the level of per capita consumption are*

$$\mathcal{K}_t\left(\Upsilon^P,\Upsilon^I\right)=e^{g\left(\Upsilon^P,\Upsilon^I\right)t},\forall t,$$

$$Y_t\left(\Upsilon^P,\Upsilon^I\right)=\left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{\alpha}{1-\alpha}}L^Y\left(\Upsilon^P,\Upsilon^I\right)\mathcal{K}_t\left(\Upsilon^P,\Upsilon^I\right),\forall t,$$

$$\textit{and } c_t\left(\Upsilon^P,\Upsilon^I\right)=\left(1-\frac{\alpha^2}{1-\Upsilon^P}\right)Y_t\left(\Upsilon^P,\Upsilon^I\right),\forall t.$$

- *The prices (wage, price of intermediate goods, and interest rate) are*

$$w_t\left(\Upsilon^P,\Upsilon^I\right)=(1-\alpha)\left(\frac{\alpha^2}{1-\Upsilon^P}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t\left(\Upsilon^P,\Upsilon^I\right),\forall t,$$

$$q_{it}\left(\Upsilon^P,\Upsilon^I\right)=q_t\left(\Upsilon^P,\Upsilon^I\right)=\frac{\mathcal{K}_t\left(\Upsilon^P,\Upsilon^I\right)}{\alpha N},\forall i\in\mathcal{Q},\forall t,$$

$$\textit{and } r_t\left(\Upsilon^P,\Upsilon^I\right)=r\left(\Upsilon^P,\Upsilon^I\right)=g\left(\Upsilon^P,\Upsilon^I\right)+\rho,\forall t.$$

Proof. From (30), (42), (45), (48), and (50), one obtains

$$g_{L_t^Y}+\lambda\theta\vartheta l_t+\rho+\lambda l_t=\lambda\theta\vartheta l_t+\left(\frac{1+\Upsilon^I}{1-\Upsilon^P}\right)\frac{\lambda\alpha}{N}L_t^Y.$$

Using (49) and rearranging the terms results in the following differential equation in L_t^Y:

$$g_{L_t^Y}+\rho+\frac{\lambda}{N}(1-L_t^Y)=\left(\frac{1+\Upsilon^I}{1-\Upsilon^P}\right)\frac{\lambda\alpha}{N}L_t^Y\Leftrightarrow g_{L_t^Y}-\frac{\lambda}{N}\left(1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right)L_t^Y=-\left(\rho+\frac{\lambda}{N}\right).$$

Using the variable substitution $X_t=1/L_t^Y$, one obtains

$$\dot{X}_t-\left(\rho+\frac{\lambda}{N}\right)X_t=-\frac{\lambda}{N}\left(1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right).$$

The solution of this first-order linear differential equation is

$$X_t=e^{\left(\rho+\frac{\lambda}{N}\right)t}\left(X_0-\frac{1}{\rho+\frac{\lambda}{N}}\frac{\lambda}{N}\left[1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right]\right)+\frac{1}{\rho+\frac{\lambda}{N}}\frac{\lambda}{N}\left[1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right].$$

Consequently, one has

$$L_t^Y = \frac{1}{e^{\left(\rho+\frac{\lambda}{N}\right)t}\left(\frac{1}{L_0^Y} - \frac{1}{\rho+\frac{\lambda}{N}}\frac{\lambda}{N}\left[1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right]\right) + \frac{1}{\rho+\frac{\lambda}{N}}\frac{\lambda}{N}\left[1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right]}.$$

The transversality condition in the program of the representative household implies that L_t^Y immediately jumps to its steady-state level $L^{Y^{ss}}$; one has $L_t^Y = L^{Y^{ss}}, \forall t$. Reintroducing the $\left(\Upsilon^P, \Upsilon^I\right)$ arguments, the equilibrium quantity of labor in the final good production is

$$L_t^Y\left(\Upsilon^P, \Upsilon^I\right) = L^Y\left(\Upsilon^P, \Upsilon^I\right) = \left(\rho + \frac{\lambda}{N}\right)\left[\frac{\lambda}{N}\left(1+\frac{1+\Upsilon^I}{1-\Upsilon^P}\alpha\right)\right]^{-1}, \forall t. \quad (51)$$

Hence, one has $g_{L_t^Y} = 0$. Therefore, one can now derive all the *equilibrium quantities, growth rates, and prices.* Replacing (51) in (49) gives the quantity of labor used in each sector R&D activity,

$$l_{it}\left(\Upsilon^P, \Upsilon^I\right) = l\left(\Upsilon^P, \Upsilon^I\right) = \frac{1}{N}\left[1 - L^Y\left(\Upsilon^P, \Upsilon^I\right)\right], \forall i \in \mathcal{Q}, \forall t. \quad (52)$$

Replacing (51) in (34) gives the quantity of each intermediate good used in the final good production, $x_{it}\left(\Upsilon^P, \Upsilon^I\right) = x\left(\Upsilon^P, \Upsilon^I\right), \forall i \in \mathcal{Q}$. From (45), (48), and (52), one gets the growth rate of the stock of knowledge in each sector, $g_{\kappa_{it}}\left(\Upsilon^P, \Upsilon^I\right) = g\left(\Upsilon^P, \Upsilon^I\right), \forall i \in \mathcal{Q}$; the growth rate of the whole stock of knowledge in the economy, $g_{\mathcal{K}_t}\left(\Upsilon^P, \Upsilon^I\right) = g\left(\Upsilon^P, \Upsilon^I\right)$; the growth rate of final good, $g_{Y_t}\left(\Upsilon^P, \Upsilon^I\right) = g\left(\Upsilon^P, \Upsilon^I\right)$; and the growth rate of per capita consumption, $g_{c_t}\left(\Upsilon^P, \Upsilon^I\right) = g\left(\Upsilon^P, \Upsilon^I\right)$. Then, the whole stock of knowledge in the economy is

$$\mathcal{K}_t\left(\Upsilon^P, \Upsilon^I\right) = e^{g\left(\Upsilon^P, \Upsilon^I\right)t}, \forall t, \text{ where } g\left(\Upsilon^P, \Upsilon^I\right) = \lambda\theta\vartheta\, l\left(\Upsilon^P, \Upsilon^I\right). \quad (53)$$

From (36), (44), (51), and (53), one gets the equilibrium levels of final good and of per capita consumption, $Y_t\left(\Upsilon^P, \Upsilon^I\right)$ and $c_t\left(\Upsilon^P, \Upsilon^I\right)$. Replacing (53) in (37) and in (34), one gets the equilibrium wage, $w_t\left(\Upsilon^P, \Upsilon^I\right)$, and the equilibrium price of intermediate goods $q_{it}\left(\Upsilon^P, \Upsilon^I\right) = q_t\left(\Upsilon^P, \Upsilon^I\right), \forall i \in \mathcal{Q}$. Finally, using (30) and (53), one gets the equilibrium interest rate $r_t\left(\Upsilon^P, \Upsilon^I\right) = r\left(\Upsilon^P, \Upsilon^I\right)$. This proves Proposition 2. □

4 Schumpeterian Equilibrium and Pareto Non-Optimality

As mentioned in 3.1, in the absence of public policies, the decentralized economy considered in this paper is likely to be Pareto non-optimal. The present section addresses this issue. Notably, in 4.1, we revisit the fact that in the *laisser faire* equilibrium, the R&D effort (*i.e.* the quantity of labor used in R&D) can either be sub-optimal or over-optimal. Then, in 4.2, we implement the first-best social optimum in the Schumpeterian decentralized economy by characterizing the optimal policy tools; this enables us to show how the optimal tool dedicated to correct the market failure caused by market incompleteness depends on the intensity of knowledge spillovers. Finally, in 4.3, we highlight the link between the issue of Pareto non-optimality and the intensity of knowledge spillovers.

4.1 *Laisser faire* Schumpeterian equilibrium

From Proposition 2, we can derive straightforwardly the *laisser faire* Schumpeterian equilibrium by setting down $\left(\Upsilon^P, \Upsilon^I\right) = (0, 0)$.

Corollary. *At each date t, the* laisser faire *Schumpeterian equilibrium* à la *Aghion & Howitt is characterized as follows.*

- *The labor partition and the quantities of intermediate goods are*

$$L^{Ylf} = L^Y(0,0) = \frac{\rho + \frac{\lambda}{N}}{\frac{\lambda}{N}(1+\alpha)} = \frac{\frac{\rho N}{\lambda} + 1}{1+\alpha};$$

$$l_i^{lf} = l^{lf} = l(0,0) = \frac{1}{N} - \frac{\rho + \frac{\lambda}{N}}{\lambda(1+\alpha)}, \forall i \in \mathcal{Q};$$

$$x_i^{lf} = x^{lf} = x(0,0) = \alpha^{\frac{2}{1-\alpha}}\frac{\rho + \frac{\lambda}{N}}{\frac{\lambda}{N}(1+\alpha)}, \forall i \in \mathcal{Q}.$$

- *The growth rates of per capita consumption, of the final good output, and of the whole stock of knowledge in the economy are*

$$g_{c_t}^{lf} = g_{Y_t}^{lf} = g_{\mathcal{K}_t}^{lf} = g^{lf} = g(0,0) = \lambda\Theta l(0,0) = \lambda\Theta\left(\frac{1}{N} - \frac{\rho + \frac{\lambda}{N}}{\lambda(1+\alpha)}\right).$$

The comparison of the *laisser faire* Schumpeterian equilibrium obtained in this Corollary with the first-best social optimum (derived in Proposition 1) enables us to revisit the issue of Pareto non-optimality of the Schumpeterian equilibrium by highlighting the central role played by the intensity of knowledge spillovers Θ.

The Schumpeterian equilibrium *laisser faire* growth rate, $g^{lf} = \lambda\Theta\, l^{lf}$, and the optimal growth rate, $g^o = \lambda\Theta\, l^o$, both depend positively on Θ but in a different way because the partition of labor, and thus the quantities of labor allocated to R&D l^{lf} and l^o differ from each other. The quantity of labor in the *laisser faire* Schumpeterian equilibrium, l^{lf}, does not depend on the intensity of knowledge spillovers Θ. On the contrary, the optimal quantity of labor in R&D, l^o, clearly depends positively on Θ. Hence, the higher Θ is, the higher the R&D effort should be in order to maintain optimality. However, l^{lf} is independent of Θ.

Because the Schumpeterian equilibrium exhibits incomplete markets, each R&D activity does not internalize the positive impact of the knowledge it creates on other R&D activities; moreover, this impact is all the more significant as the intensity of knowledge spillovers Θ is strong. Therefore, if Θ is high (*resp.* low), it is likely that R&D effort will be insufficient (*resp.* excessive) with respect to what would be its optimal level. Eventually, this explains why the *laisser faire* growth rate g^{lf} can be lower or higher than the optimal growth rate g^o.

Furthermore, it is possible to determine a threshold $\tilde{\Theta}$ such that g^{lf} is lower (*resp.* greater) than g^o if and only if Θ is above (*resp.* below) this threshold:[14]

$$g^{lf} \lesseqqgtr g^o \Leftrightarrow \Theta \gtreqqless \tilde{\Theta}, \text{ where } \tilde{\Theta} = \frac{(1+\alpha)\rho}{\rho + \frac{\lambda}{N}}. \tag{54}$$

These results are illustrated in Figure 1.

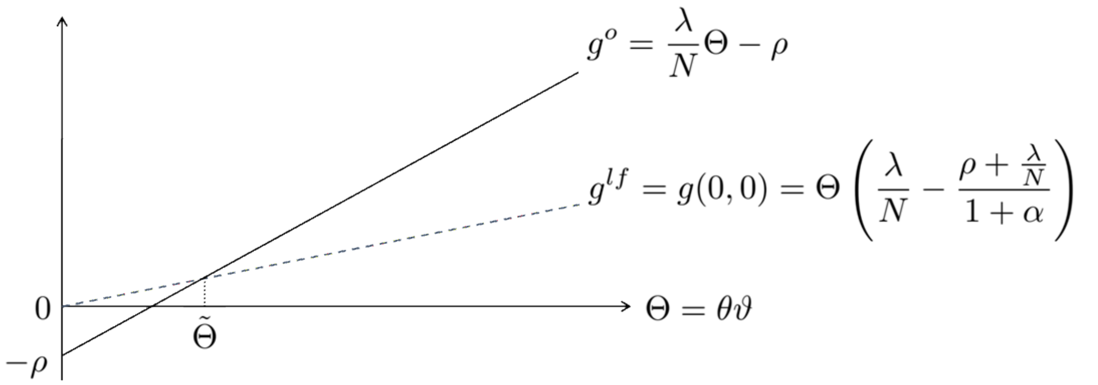

Figure 1. Pareto non-optimality of the *Laisser faire* Schumpeterian equilibrium and intensity of knowledge spillovers.

4.2 Implementation of the first-best social optimal

As detailed above, the Schumpeterian equilibrium is likely to be Pareto non-optimal because it involves two market failures; thenceforth, the first-best social optimal can be implemented by the use of two tools. The optimal set of tools $\left(\Upsilon^{Po}, \Upsilon^{Io}\right)$ can, for instance, be determined by identifying the equilibrium quantities of intermediate goods and of labor in R&D, $x\left(\Upsilon^P, \Upsilon^I\right)$ and $l\left(\Upsilon^P, \Upsilon^I\right)$, with the optimal ones, x^o and l^o. One gets Proposition 3 below.

Proposition 3. *The first-best social optimum can be implemented in the Schumpeterian equilibrium. The optimal set of public tools* $\left(\Upsilon^{Po}, \Upsilon^{Io}\right)$ *is given by*

$$\Upsilon^{Po} = 1 - \alpha \text{ and } \Upsilon^{Io} = \Theta\left(1 + \frac{\lambda}{\rho N}\right) - 2. \tag{55}$$

[14]The proof is straightforward: $g^{lf} \lesseqqgtr g^o \Leftrightarrow \lambda\Theta\left(\frac{1}{N} - \frac{\rho+\frac{\lambda}{N}}{\lambda(1+\alpha)}\right) \lesseqqgtr \frac{\lambda}{N}\Theta - \rho \Leftrightarrow \Theta \gtreqqless \frac{(1+\alpha)\rho}{\rho+\frac{\lambda}{N}} \equiv \tilde{\Theta}$.

Proof. In Proposition 2, we have characterized $x\left(\Upsilon^P,\Upsilon^I\right)$ and $l\left(\Upsilon^P,\Upsilon^I\right)$; in Proposition 1, we have computed x^o and l^o. The optimal tools Υ^{Po} and Υ^{Io} must satisfy

$$x\left(\Upsilon^{Po},\Upsilon^{Io}\right)=x^o \text{ and } l\left(\Upsilon^{Po},\Upsilon^{Io}\right)=l^o.$$

From $x\left(\Upsilon^{Po},\Upsilon^{Io}\right)=x^o$, one gets

$$\left(\frac{\alpha^2}{1-\Upsilon^{Po}}\right)^{\frac{1}{1-\alpha}}L^Y\left(\Upsilon^{Po},\Upsilon^{Io}\right)=\alpha^{\frac{1}{1-\alpha}}\frac{\rho N}{\lambda\theta\vartheta}, \text{where } L^Y\left(\Upsilon^{Po},\Upsilon^{Io}\right)=L^{Yo}=\frac{\rho N}{\lambda\theta\vartheta}$$
$$\Leftrightarrow\left(\frac{\alpha^2}{1-\Upsilon^{Po}}\right)^{\frac{1}{1-\alpha}}\frac{\rho N}{\lambda\theta\vartheta}=\alpha^{\frac{1}{1-\alpha}}\frac{\rho N}{\lambda\theta\vartheta}\Leftrightarrow\frac{\alpha^2}{1-\Upsilon^{Po}}=\alpha\Leftrightarrow\Upsilon^{Po}=1-\alpha.$$

From, $l\left(\Upsilon^{Po},\Upsilon^{Io}\right)=l^o$ one gets

$$\frac{1}{N}\left(1-\left(\rho+\frac{\lambda}{N}\right)\left[\frac{\lambda}{N}\left(1+\frac{1+\Upsilon^{Io}}{1-\Upsilon^{Po}}\alpha\right)\right]^{-1}\right)=\frac{1}{N}-\frac{\rho}{\lambda\theta\vartheta}$$
$$\Leftrightarrow\rho+\frac{\lambda}{N}=\frac{\rho}{\theta\vartheta}\left(1+\frac{1+\Upsilon^{Io}}{1-\Upsilon^{Po}}\alpha\right);$$

then, using $\Upsilon^{Po}=1-\alpha$, one obtains

$$\rho+\frac{\lambda}{N}=\frac{\rho}{\theta\vartheta}\left(1+\frac{1+\Upsilon^{Io}}{\alpha}\alpha\right)\Leftrightarrow\Upsilon^{Io}=\Theta\left(1+\frac{\lambda}{\rho N}\right)-2.$$

This proves Proposition 3. □

The results derived in Proposition 3 corroborate the analysis of the *laisser faire* conducted in 4.1. Regarding the optimal tool to correct the market failure entailed by the presence of a monopoly, Υ^{Po}, we recover a result which is standard in the endogenous growth literature. With regard to the optimal tool to mend the externality resulting from market incompleteness; we have shown in Proposition 3 that - as expected - it can consist in a subsidy ($\Upsilon^{Io}>0$) or in a tax ($\Upsilon^{Io}<0$), depending on the parameters of the model.

This property of the optimal tool Υ^{Io} echoes to the fact that, as explained above, in standard Schumpeterian growth models, the decentralized R&D effort can either be sub-optimal or over-optimal. The key point revealed here lies in that Υ^{Io} is an increasing function of Θ. This highlights the key role played by the intensity of knowledge spillovers. Indeed, we obtain the critical result according to which the stronger the intensity of knowledge spillovers Θ is, the more likely the optimal tool Υ^{Io} dedicated to R&D should consist in a subsidy; furthermore, this subsidy will be all the higher as Θ is high. The reason for this lies in the fact that R&D incentives are skewed by market incompleteness: if Θ is high (*resp.* low), the R&D effort will more likely be insufficient (*resp.* excessive), this is why R&D should probably be subsidized (*resp.* taxed).

4.3 Optimal tool to correct market incompleteness, Pareto optimality, and intensity of knowledge spillovers

Let us now study in more details the properties of the optimal tool Υ^{Io}. For that purpose, let us focus on the Schumpeterian equilibrium in which the monopoly distortion is optimally corrected by setting $\Upsilon^P=\Upsilon^{Po}=1-\alpha$, and in which there is *laisser faire* regarding R&D ($\Upsilon^I=0$). The associated growth rate writes

$$g\left(\Upsilon^{Po},0\right)=\lambda\Theta l\left(\Upsilon^{Po},0\right)=\lambda\Theta\frac{1}{N}\left[1-L^Y\left(\Upsilon^{Po},0\right)\right]=\Theta\left(\frac{\lambda}{N}-\frac{\rho+\frac{\lambda}{N}}{2}\right).$$

The comparison with the optimal growth rate yields the following result:[15]

$$g\left(\Upsilon^{Po},0\right)\lesseqqgtr g^o\Leftrightarrow\Upsilon^{Io}\gtreqqless 0. \tag{56}$$

[15]The proof is straightforward:

$$g\left(\Upsilon^{Po},0\right)\lesseqqgtr g^o\Leftrightarrow\Theta\left(\frac{\lambda}{N}-\frac{\rho+\frac{\lambda}{N}}{2}\right)\lesseqqgtr\frac{\lambda}{N}\Theta-\rho\Leftrightarrow\rho-\frac{\rho+\frac{\lambda}{N}}{2}\Theta\lesseqqgtr 0\Leftrightarrow\Theta\left(1+\frac{\lambda}{\rho N}\right)-2\gtreqqless 0\Leftrightarrow\Upsilon^{Io}\gtreqqless 0.$$

In (56), we prove the intuitive result that once the market failure entailed by market power is optimally corrected, the optimal tool to correct market incompleteness, Υ^{Io}, is a subsidy (*resp.* a tax) if and only if the allocation of labor in R&D activity - and thus the growth rate - is sub-optimal (*resp.* over-optimal). Again, the underlying reason lies in that market incompleteness distorts R&D incentives leading to a non optimal R&D effort (too little or too much labor used in R&D).

Besides, similarly as in 4.1, here also, one can determine a threshold $\tilde{\tilde{\Theta}}$ such that[16]

$$g\left(\Upsilon^{Po},0\right) \lesseqgtr g^o \Leftrightarrow \Theta \gtreqless \tilde{\tilde{\Theta}}, \text{ where } \tilde{\tilde{\Theta}} = \frac{2\rho}{\rho + \frac{\lambda}{N}}. \quad (57)$$

The results obtained in (56) and (57) are summarized in Proposition 4 and illustrated in Figure 2.

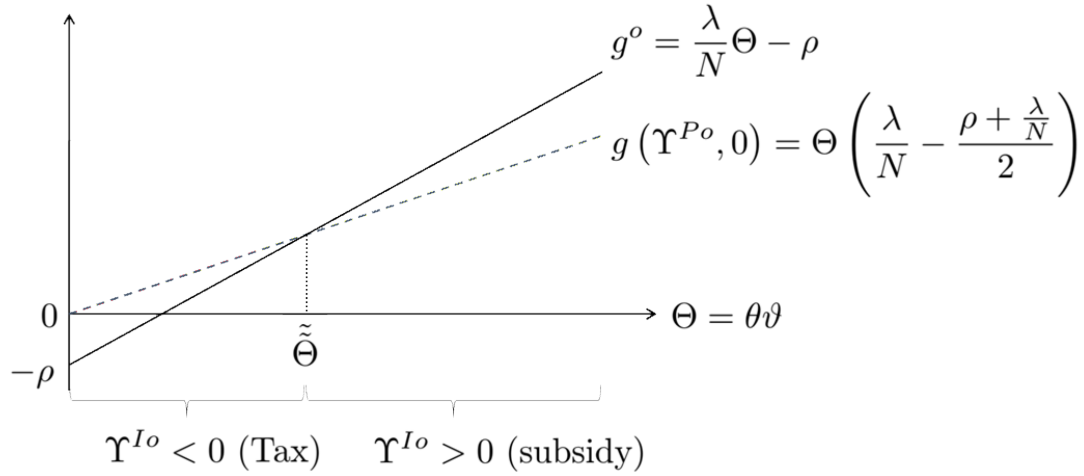

Figure 2. Optimal tool to correct market incompleteness and intensity of knowledge spillovers.

Proposition 4. *There exists a level of intensity of knowledge spillovers* $\tilde{\tilde{\Theta}}$ *such that*

$$\Theta \gtreqless \tilde{\tilde{\Theta}} = \frac{2\rho}{\rho + \frac{\lambda}{N}} \Leftrightarrow g\left(\Upsilon^{Po},0\right) \lesseqgtr g^o \Leftrightarrow \Upsilon^{Io} \gtreqless 0. \quad (58)$$

These final results highlight the fact already argued above that the intensity of knowledge spillovers is a key determinant in the issue of Pareto non-optimality of the Schumpeterian equilibrium and thus in the characterization of the optimal tool dedicated to compensate for market incompleteness.

5 Conclusion

In this paper, we developed a standard endogenous growth model *à* Aghion & Howitt (1992) in which we explicitly introduced (in a fairly simple way) the concept of intensity of knowledge spillovers. The subsequent analysis enabled us to shed new light on the issue of Pareto non-optimality of the seminal Schumpeterian equilibrium initially introduced by Aghion & Howitt (1992) by revealing how the intensity of knowledge spillovers determines the impact of the distortion of R&D incentives due to market incompleteness (no market for knowledge is considered in traditional Schumpeterian equilibria). The key result derived in this paper is that the higher (*resp.* lower) the intensity of knowledge spillovers is, the more likely market incompleteness will induce an under-optimal (*resp.* over-optimal) R&D effort, and thus the more likely the optimal policy aiming at correcting this market failure should be to subsidy (*resp.* to tax) the R&D activities. Furthermore, we showed that if the optimal tool does consist in a subsidy, the level of this subsidy should be all the higher as the intensity of knowledge spillovers is strong.

The formalization developed in the paper remains somehow simple insofar as we consider homogeneity in the intensity of knowledge spillovers across sectors (that is, homogeneity both in the increment in knowledge

[16]The proof is straightforward: $g\left(\Upsilon^{Po},0\right) \lesseqgtr g^o \Leftrightarrow \Theta\left(\frac{\lambda}{N} - \frac{\rho+\frac{\lambda}{N}}{2}\right) \lesseqgtr \frac{\lambda}{N}\Theta - \rho \Leftrightarrow \Theta \gtreqless \frac{2\rho}{\rho+\frac{\lambda}{N}} \equiv \tilde{\tilde{\Theta}}$.

resulting from an innovation and in the scope of diffusion of knowledge in the economy). Nevertheless, the results obtained can still be seen as a first step in developing basic arguments in support of the fact that various R&D activities should probably be targeted by different public policies, depending on the intensity of knowledge spillovers emanating from them. For example, considering an extension of this model in which each sector would be characterized by a specific level of intensity of knowledge spillovers could enables us to obtain analytically results in line with Akcigit, Hanley, & Serrano-Velarde (2016) who show quantitatively that a type-dependent R&D subsidy policy enables the social planner to achieve higher levels of welfare.

References

[1] Acemoglu D (2009) Modern economic growth. Princeton University Press, Princeton NJ

[2] Aghion P, Howitt P (1992) A model of growth through creative destruction. Econometrica 60(2):323-351

[3] Aghion P, Howitt P (1998) Endogenous growth theory. MIT Press, Cambridge MA

[4] Aghion P, Howitt P (2009) The economics of growth. MIT Press, Cambridge MA

[5] Akcigit U, Hanley D, Serrano-Velarde N (2016) Back to Basics: Basic Research Spillovers, Innovation Policy and Growth. CEPR Discussion Paper No 11707

[6] Alvarez-Pelaez MJ, Groth C (2005) Too little or too much R&D? European Economic Review 49(2):437-456

[7] Barro R, Sala-i-Martin X (2003) Economic Growth, second edition. MIT Press, Cambridge MA

[8] Benassy JP (1998) Is there always too little research in endogenous growth with expanding product variety? European Economic Review 42(1):61-69

[9] Cozzi G, Giordani PE, Zamparelli L (2007) The refoundation of the symmetric equilibrium in Schumpeterian growth models. Journal of Economic Theory 136(1):788-797

[10] Dinopoulos E, Sener F (2007) New directions in Schumpeterian growth theory. In: Hanusch H, Pyka A (eds) The elgar companion to neo-Schumpeterian economics, Edward Elgar, Cheltenham

[11] Dinopoulos E, Thompson P (1998) Schumpeterian growth without scale effects. Journal of Economic Growth 3(4): 313-335

[12] Garner P (2010) A note on endogenous growth and scale effects. Economics Letters 106(2):98-100

[13] Gray E, Grimaud A (2016) Using the Salop Circle to Study Scale Effects in Schumpeterian Growth Models: Why Inter-sectoral Knowledge Diffusion Matters. CESifo Working Paper Series No. 6021 (http://dx.doi.org/10.2139/ssrn.2829265)

[14] Griliches Z (1992) The search for R&D spillovers. Scandinavian Journal of Economics 94(supplement): 29-47

[15] Griliches Z (1995) R&D and productivity: econometric results and measurement issues. In: Stoneman P (ed) Handbook of the economics of innovation and technical change, Blackwell Handbooks in Economics

[16] Grossman G, Helpman E (1991) Quality ladders in the theory of growth. Review of Economic Studies 58(1):43-61

[17] Hall B (2004) Innovation and diffusion. In: Fagerberg J, Mowery DC, Nelson RR (eds) Handbook on innovation, Oxford University Press, Oxford

[18] Hall B, Mairesse J, Mohnen P (2010) Measuring the returns to R&D. In Hall B, Rosenberg N (eds) Handbook of the economics of innovation, Elsevier

[19] Howitt P (1999) Steady endogenous growth with population and R&D inputs growing. Journal of Political Economy 107(4):715-730

[20] Jones C (1999) Growth: with or without scale effects? American Economic Review Papers and Proceedings 89(2):139-144

[21] Jones C (2005) Growth and ideas. In: Aghion P, Durlauf S (eds) Handbook of Economic Growth, Elsevier Volume 1B, 1063-1111

[22] Jones C, Williams J (2000) Too much of a good thing? The economics of investment in R&D. Journal of Economic Growth 5(1):65-85

[23] Laincz C, Peretto P (2006) Scale effects in endogenous growth theory: an error of aggregation not specification. Journal of Economic Growth 11(3):263-288

[24] Li CW (2002) Growth and scale effects: the role of knowledge spillovers. Economics Letters 74:177-185

[25] Li CW (2003) Endogenous growth without scale effects: a comment. American Economic Review 93(3):1009-1017

[26] Peretto P (1998) Technological change and population growth. Journal of Economic Growth 3(4):283-311

[27] Peretto P (1999) Cost reduction, entry, and the interdependence of market structure and economic growth. Journal of Monetary Economics 43(1):173-195

[28] Peretto P (2007) Corporate taxes, growth and welfare in a Schumpeterian economy. Journal of Economic Theory 137(1):353-382

[29] Peretto P, Smulders S (2002) Technological distance, growth and scale Effects. Economic Journal 112(481):603-624

[30] Romer P (1990) Endogenous technological change. Journal of Political Economy 98(5):71-102

[31] Salop S (1979) Monopolistic competition with outside goods. Bell Journal of Economics 10(1):141-156

[32] Sener F (2008) R&D policies, endogenous growth and scale effects. Journal of Economic Dynamics and Control 32(12):3895-3916

[33] Segerstrom P (1998) Endogenous growth without scale effects. American Economic Review 88(5):1290-1310

[34] Segerstrom P (2000) The long-run growth effects of R&D subsidies. Journal of Economic Growth 5(3):277-305

[35] Young A (1998) Growth without scale effects. Journal of Political Economy 106(1):41-63