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Predicting Security Returns in the Presence of Upper Price Limits

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Abstract

The objective of this paper is to develop a Bayesian forecasting model of security returns in a security market in which daily trading is subject to price limits. Traders and investors are assumed to be Bayesian decision makers under uncertainty, in the sense that they combine their a-priori information about the security returns with realized stock returns, to form the posterior and predictive distributions, on which trading decisions would be made. It allows traders and investors to adjust their long/short trading positions while considering the presence of upper price limits. In our model, security returns are assumed normally distributed, with an unknown mean and a known variance. Our traders and investors apply Bayes' rule to update their prior information to derive the security returns' predictive distribution. Our major contribution is the derivation of the stock returns' predictive distribution, which is analytically quite complex. To illustrate the usefulness of our forecasting model, we provide five numerical examples of the predictive distributions, in which we alter one of the parameters, ceteris paribus, and demonstrate its impact on the predictions. This would allow traders and investors to upgrade their trading positions in the presence of an upper price limits.

Keywords

Price Limits in the Stock Market, Bayesian Investors, Forecasting Stock Returns

1. Introduction

Upper or lower ***price limits*** are set by stock exchange such that when the security market hits the ***price limit***, it would trigger a trading halt. For a small daily price change the trading would be halted for a specific time period (one or two hours), but for a large price change, trading can be suspended until the end of the trading day. A security market that reaches its daily price limit is referred to as a **locked market**. Price limits are quite common in various Asian stock exchanges, in many foreign currency exchanges, and in all futures exchanges in the United States. The primary function of imposing price limits is to reduce security market volatility and prevent market manipulation and panic.

The objective of this paper is to extend the Bayesian methodology of Harel and Harpaz (2020). They used a Bayesian forecasting model (Lee, 2012) to predict the levels of the S & P 500 Index in the presence of circuit breakers. In this paper, it is assumed that traders and investors behave as Bayesian decision makers under uncertainty in a market that imposes an upper daily price limit. Our traders and investors combine their prior knowledge about the security which is based on historical and subjective information with the realized security returns, via Bayes' rule, to update the posterior and predictive distributions on which trading decisions would be made.

Prior studies of the behavior of investors in stock exchanges that imposed daily trading limits, were all empirical ones that focused primarily on the impact of the presence of price limits on market volatility, for example, Ayesha and Christo (2018), Berkman and Lee (2002), Chen, Rui and Wang (2005), and Dong and Li (2019). In particular, there were a sequence of empirical studies conducted by Kim (2001), Kim and Limpaphayom (2000), Kim, Liu and Yang (2013), Kim and Rhee (1997), to name but a few. It should be noted that the null hypothesis that imposing stock price limits reduces security market volatility is still empirically inconclusive. We must emphasize that none of above-mentioned empirical studies tried to construct a theoretical model to forecast stock returns in the presence of price limits. Consequently, our model is unique, novel and completely different from all historically conducted empirical studies.

Our paper is organized as follows. Section 2 introduces the Bayesian forecasting model, when the security returns are assumed normally distributed with an unknown mean and a known variance, but the stock exchange imposes an upper price limit. Our traders and investors are assumed to be Bayesian decision makers under uncertainty. Our original contribution is the formulation of the security returns' predictive distribution in a security market with an upper price limit. Our constructed predictive distribution is analytically quite complex. Section 3 introduces five distinct numerical examples of the security returns' predictive distributions for various parameter values. The last section provides the conclusion.

2. The Bayesian Forecasting Model

Various Asian stock exchanges and all the futures exchanges in the United States, set an upper limit return UL, whereby if the security return exceeds the upper limit UL, it would trigger a temporary trading halt for a specific period, and sometimes trading can be suspended until the end-of-the trading day. In our Bayesian model (Lee, 2012) the return on the security is assumed to be normally distributed, with an unknown return μ, and a known variance σ^2. Thus, the return likelihood function is given by,

$$l(r \mid \mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(r-\mu)^2}{2\sigma^2}\right], \tag{1}$$

Whereby, it is assumed that the mean return μ cannot exceed the upper limit *UL*, set by the stock exchange. In addition, it is assumed that the prior distribution is Normal but truncated at *UL*, and it has known a mean m_0, and a known variance σ_0^2.

Thus, the truncated prior return distribution $f_0(\mu \mid r, m_0, \sigma_0, UL)$ is given by,

$$f_0(\mu \mid r, m_0, \sigma_0, UL) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}(UL-m_0)/\sigma_0}\exp\left[-\dfrac{(r-m_0)^2}{2\sigma_0^2}\right], & \text{if } \mu \le UL \\ 0, & \text{if } \mu > UL \end{cases} \tag{2}$$

The posterior distribution $f_1(\mu \mid r, m_0, \sigma_0, UL)$ can be calculated by using Bays' rule (Lee, 2012), and it is proportional to the product of the prior distribution $f_0(\mu \mid r, m_0, \sigma_0, UL)$ and the likelihood function $l(r \mid \mu,\sigma)$, we obtain:

$$\begin{aligned} f_1(\mu \mid r, m_0, \sigma_0, UL) &\propto f_0(r \mid \mu)\, l(r \mid \mu,\sigma) \\ &\propto \exp\left[-\frac{(r-\mu)^2}{2\sigma^2}-\frac{(\mu-m_0)^2}{2\sigma_0^2}\right], \text{ if } \mu \le UL, \end{aligned} \tag{3}$$

with $f_1(\mu \mid r, m_0, \sigma_0, UL)=0$, if $\mu > UL$.

Combining the two terms in the right-hand side of Equation (3), deleting the constants which are intendent μ on and simplifying, the truncated posterior distribution f_1 of can be written as:

$$\begin{aligned} & f_1(\mu \mid r, m_0, \sigma_0, UL) \\ & \propto \exp\left[-\frac{1}{2\sigma^2\sigma_0^2/(\sigma^2+\sigma_0^2)}\left(\mu-\frac{\sigma^2 m_0+\sigma_0^2 r}{\sigma^2+\sigma_0^2}\right)^2\right], \text{ if } \mu \le UL, \end{aligned} \tag{4}$$

with $f_1(\mu \mid r, m_0, \sigma_0, UL)=0$, if $\mu > UL$.

The posterior distribution of $f_1(\cdot)$ in Equation (4), is truncated at *UL*, and it turn out to be a conjugate Normal distribution, with mean $m_1=\dfrac{\sigma^2 m_0+\sigma_0^2 r}{\sigma^2+\sigma_0^2}$, and variance $\sigma_1^2=\sigma^2\sigma_0^2/(\sigma^2+\sigma_0^2)$. The normalizing constant, $\Phi(x)$, of the posterior Normal posterior distribution $f_1(\mu \mid r, m_0, \sigma_0, UL)$ is given by:

$$\begin{aligned} \Phi(x) &\equiv \Phi\left(\frac{UL-(\sigma^2 m_0+\sigma_0^2 r)/(\sigma^2+\sigma_0^2)}{\sigma^2\sigma_0^2/(\sigma^2+\sigma_0^2)}\right) \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}y^2} e^{-\frac{1}{2}y^2}\,dy=\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{x}{\sqrt{2}}\right). \end{aligned} \tag{5}$$

where Φ is the cumulative Normal distribution.

Note that once the security return hits the price limit *UL*, trading would be halted. To predict the security return r in the presence of an upper limit UL, our Bayesian traders and investors are interested in the predictive distribution PR of the security return r, unconditional on μ. Thus, the predictive distribution of r is given by integrating the product of the posterior distribution $f_1(\mu \mid r, m_0, \sigma_0, UL)$ in Equation (4), and the likelihood function $l(r \mid \mu, \sigma)$ in Equation (1), where $\Phi(x)$ is the normalizing constant from Equation (6). We derive the security return's **predictive distribution** in the presence of an upper price limit UL, as a function of the stock return r (ceteris paribus),

$$
\begin{aligned}
&PR\left(r \mid m_0, r, \sigma_0^2, \sigma^2, UL\right) = \Phi(x)\int_{-\infty}^{UL} f_1(\cdot)\, l(r \mid \mu, \sigma)\,\mathrm{d}\mu \\
&= \Phi(x)\int_{-\infty}^{UL} \exp\left[-\frac{1}{2\sigma^2\sigma_0^2/\left(\sigma^2+\sigma_0^2\right)}\left(\mu - \frac{\sigma^2 m_0 + \sigma_0^2 r}{\sigma^2+\sigma_0^2}\right)^2\right] \\
&\quad * \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(r-\mu)^2}{2\sigma^2}\right]\mathrm{d}\mu \\
&= \frac{\Phi(x)\sigma\sigma_0\sqrt{2\pi}}{2\sqrt{\sigma^2+2\sigma_0^2}}\exp\left(-\frac{\sigma^2\left(m_0-r\right)^2}{2\left(\sigma^2+\sigma_0^2\right)\left(\sigma^2+2\sigma_0^2\right)}\right) \\
&\quad * \left[1+\operatorname{erf}\left(\frac{\left(\sigma^2+2\sigma_0^2\right)UL - 2\sigma_0^2 r - \sigma^2 m_0}{\sigma\sigma_0\sqrt{2}\sqrt{\sigma^2+2\sigma_0^2}}\right)\right] \\
&= \left[\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{\left(\sigma^2+2\sigma_0^2\right)UL - 2\sigma_0^2 r - \sigma^2 m_0}{\sigma\sigma_0\sqrt{2}}\right)\right] \\
&\quad * \frac{\sigma\sigma_0\sqrt{2\pi}}{2\sqrt{\sigma^2+2\sigma_0^2}}\exp\left(-\frac{\sigma^2\left(m_0-r\right)^2}{2\left(\sigma^2+\sigma_0^2\right)\left(\sigma^2+2\sigma_0^2\right)}\right) \qquad (6) \\
&\quad * \left[1+\operatorname{erf}\left(\frac{\left(\sigma^2+2\sigma_0^2\right)UL - 2\sigma_0^2 r - \sigma^2 m_0}{\sigma\sigma_0\sqrt{2}\sqrt{\sigma^2+2\sigma_0^2}}\right)\right]
\end{aligned}
$$

Note that the (error function) erf is defined as:

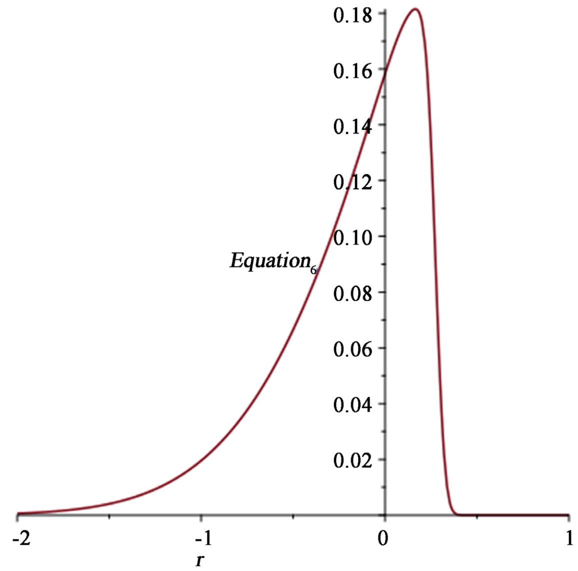
$$\operatorname{erf}(y) = \frac{2\left(\int_0^y \mathrm{e}^{-t^2}\,\mathrm{d}t\right)}{\sqrt{\pi}}.$$

Since the **predictive distribution** in Equation (6) is quite complex to analyze analytically, and in the next section we will provide several numerical examples.

3. Numerical Examples of the Predictive Distribution in Equation (6)

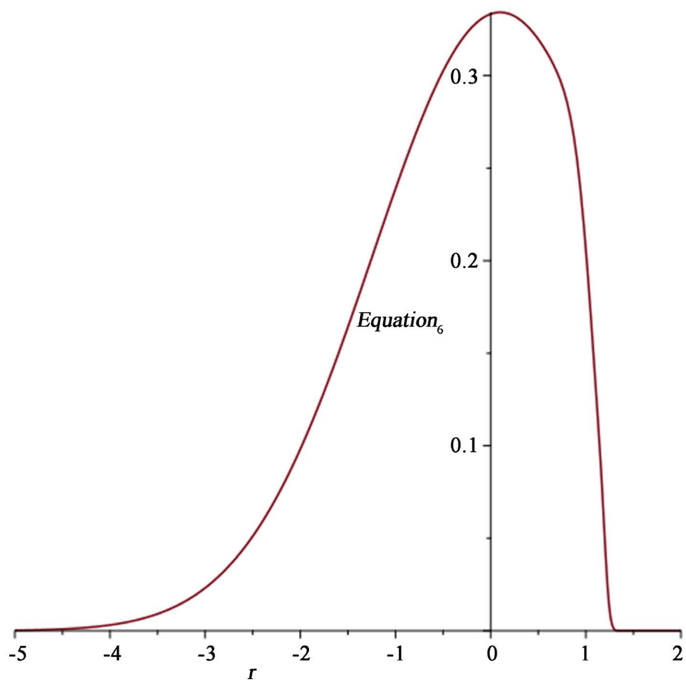
In this section we provide five numerical examples of the predictive distribution in Equation (6). In each of the following five examples (Graphs 1-5), we alter one parameter, ceteris paribus, and plot the corresponding predictive distributions.

Example 3.1: Consider the following model parameters: $\sigma = 0.2$, $\sigma_0 = 0.3$, $m_0 = 1$, and $UL = 0.5$, and substituting these values in Equation (6) and plotting it for the return interval: $-2 < r < 1$, we obtain the predictive distribution,



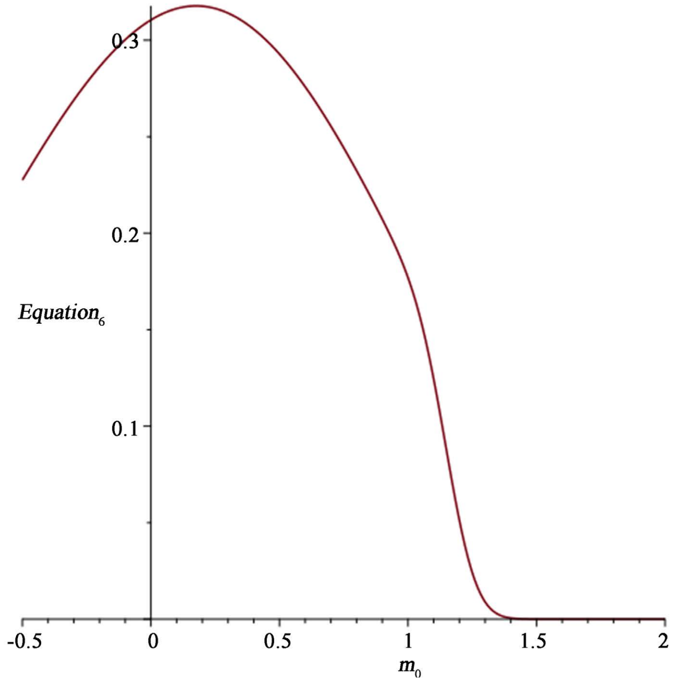
Graph 1. The Predictive Probability Distribution as a function of the security return, *r*. The model parameters are: $\sigma = 0.2$, $\sigma_0 = 0.3$, $m_0 = 1$, and $UL = 0.5$.

Example 3.2: Consider the following model parameters: $\sigma = 0.2$, $\sigma_0 = 0.4$, $m_0 = 0.1$, and $UL = 1$, and substituting these values in Equation (6) and plotting it for the return interval: $-5 < r < 2$, we obtain the predictive distribution,



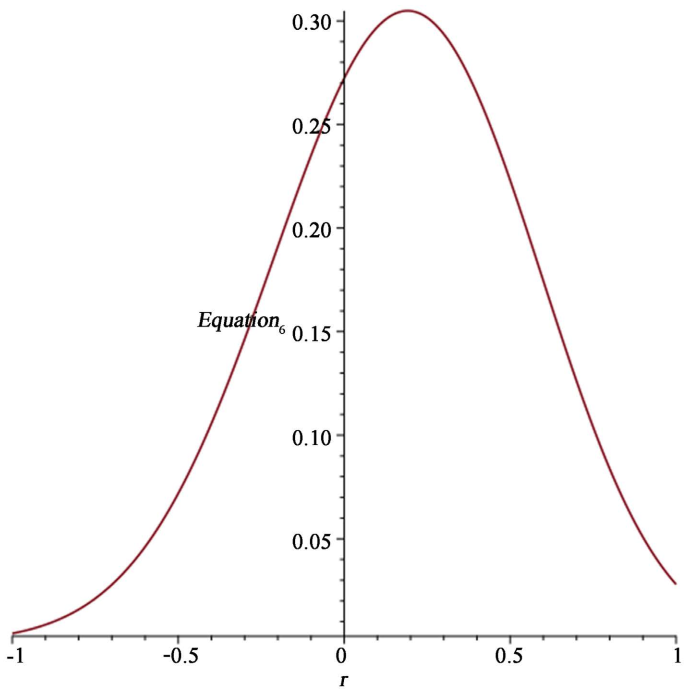
Graph 2. The Predictive Probability Distribution as a function of the security return, *r*. The model parameters are: $\sigma = 0.2$, $\sigma_0 = 0.4$, $m_0 = 0.1$, and $UL = 1$.

Example 3.3: Consider the following model parameters: $\sigma = 0.2$, $\sigma_0 = 0.3$, $r = 0.2$, and $UL = 0.5$, and substituting these values in Equation (6) and plotting it for m_0: $-0.5 < r < 2$, we obtain the predictive distribution,



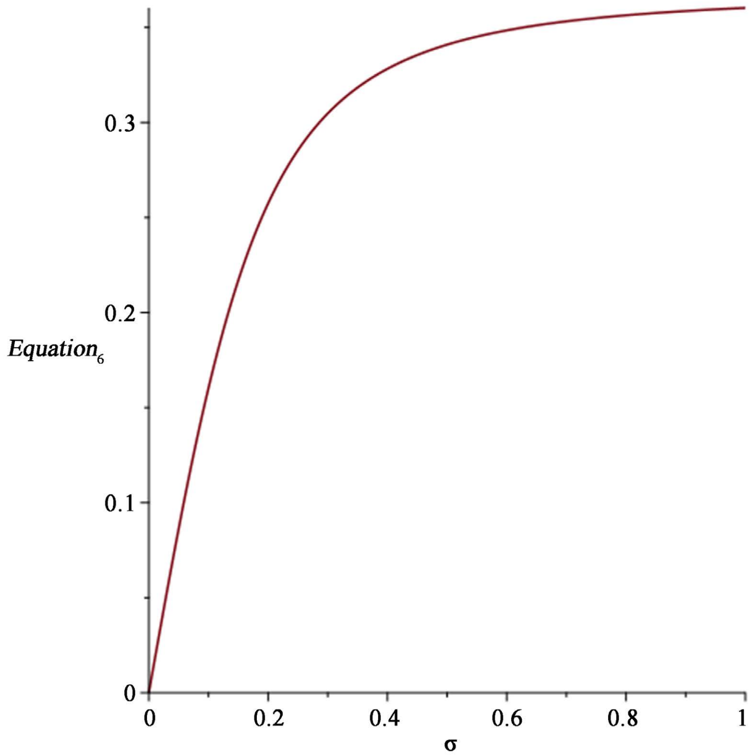
Graph 3. The Predictive Probability Distribution as a function of the security return, *r*. The model parameters are: $\sigma = 0.2$, $\sigma_0 = 0.3$, $r = 0.2$, and $UL = 0.5$.

Example 3.4: Consider the following model parameters: $\sigma = 0.3$, $\sigma_0 = 0.15$, $m_0 = 0.2$, and $UL = 0.5$, and substituting these values in Equation (6) and plotting it for the return interval: $-1 < r < 1$, we obtain the predictive distribution,



Graph 4. The Predictive Probability Distribution as a function of the security return, *r*. The model parameters are: $\sigma = 0.3$, $\sigma_0 = 0.15$, $m_0 = 0.2$, and $UL = 0.5$.

Example 3.5: Consider the following model parameters: $r = 0.2$, $\sigma_0 = 0.15$, $m_0 = 0.2$, and $UL = 0.5$, and substituting these values in Equation (6) and plotting it for $0 < \sigma < 1$, we obtain the predictive distribution,



Graph 5. The Predictive Probability Distribution as a function of the security return, *r*. The model parameters are: $r = 0.2$, $\sigma_0 = 0.15$, $m_0 = 0.2$, and $UL = 0.5$.

4. Conclusion

Our paper develops a Bayesian predictive model of security returns in markets that impose an upper price limit. When the security prices hit the upper price limit, trading is halted, and the market is referred to as **locked market**. Trading can be suspended for a few hours, and sometimes up to the end-of-the trading day. Various security exchanges in Asia, currency exchanges, and all futures exchanges in the United States impose price limits. The purpose of enforcing price limits is to mitigate market volatility and depress stock manipulation. In our paper, traders and investors are assumed to be Bayesian decision makers under uncertainty. They combine their historical and subjective knowledge regarding the security future returns with the realized returns, to update the posterior and predictive distributions. Security returns are assumed to be normally distributed with unknown mean returns and known variances. Traders and investors are fully aware of the presence of an upper price limit, when forming their expectations. Thus, traders and investors start their analysis by using a truncated normal prior distribution, which is revised using Bayes' rule to form the posterior and predictive distributions.

The major contribution of our paper is the formulation of the security returns' predictive distribution, contingent on which future trading decision would be made. The predictive distribution is quite complex to analyze analytically, and we provide 5 numerical examples, by altering one parameter, ceteris paribus, and investigate the corresponding distributions. Our model can be useful to traders and investors who make decisions under uncertainty in a security market that imposes an upper price limit. When revising their predictions, traders and investors consider the presence of an upper price limit, and this allows them to calibrate their forecasts.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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