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The Equity-Efficiency Conflict: Improving Pareto's Optimality Doctrine

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Abstract

A central issue in Public Economics is the appropriate design of a tax system that will succeed in reconciling the concepts of equity and efficiency. In the present study, the standard assumption of the household's utility being dependent on consumption (income) and labour (leisure) is adopted to arrive at a decision as to the nature of distortions and the fiscal measures required to eliminate them. The comparison of a utility function (with consumption and labour being treated as exogenous), that causes no distortions, with another utility function (with consumption depending on indirect taxes and labour supply on income taxes), that generates distortions, allows us to carry out a number of econometric, mathematical, and empirical tests, designed to redress the balance between the MRS and the MRT and to eliminate the distortions originating in the labour market and/or the commodities market.

Keywords

Equity-Efficiency Trade-Off, Pareto Optimal, Optimal Taxation, Income Distribution, Growth, Incentives

1. Introduction

One of the most important objectives of fiscal policy is to design a tax system to balance the various desirable, but often counteracting, attributes of taxation: To raise the required government revenue in a way that treats individuals fairly (equity) without inflicting serious damage to private agents' incentives to growth (efficiency).

These attributes are usually analyzed in an optimal taxation area of research, which is a normative approach to tax analysis based on the standard tools of welfare economics. However, the basic theorems of welfare economics must be applied in a world, where the first-best allocation of resources and income distribution can rarely be achieved. Hence, the practical applicability of welfare theorem is argued to be limited to date. This in turn leads to a divergence between optimal tax theory and practical tax design but, at the same time, it provides a scope for researchers to develop models and establish relationships which could help policy makers to reconcile opposing theoretical and practical considerations for appropriate tax schemes. One of the main objectives of the present article is to pave the way for reaching a compromise between tax designers and policy makers, when an optimal tax policy fails to be achieved, due to the existence of distortions arising from the response of consumers and wage earners to changes in (direct-indirect) tax rates.

What in particular we argue is that the standard welfare maximization approach may lead to a Pareto efficient outcome, which is not necessarily Pareto optimal[1], unless a range of conditions describing the reactions of private agents to tax-rate changes are met. This may happen when welfare maximization ignores the equity and efficiency effects that arise because taxes are often collected at some cost to various macro (micro) economic goals. Given that most analyses in the welfare maximization tradition ignore these features, their basic policy prescriptions are unlikely to result in improvements in welfare, when compared to those policy rules derived from a less formal but more realistic perspective (Slemrod, 1990; Bird, 1992).

In the third section, we replicate the standard welfare maximization model and present several of its more widely known results. This model is then substantially modified to incorporate some of the most important equity and efficiency considerations that have been largely ignored by recent literature. See, for example, Saez and Stantcheva (2018), Chiappori and Mazzocco (2017), Gayle and Shephard (2019) and Farhi and Gabaix (2020). However, before embarking on a detailed research to deal with the equity-efficiency trade-off, it would be constructive to set the limits to our study.

2. A General Outline of the Present Discussion

A few comments are made below to broaden the scope of the present study and look at more general matters.

1) The argument of this paper is that we can exploit the trade-off, provided that the political system can establish comprehensive means of income distribution and/or efficient resource allocation reflecting the choices of voters. The voting process is working in a responsive manner to minimize the influence of special interest groups, when they try to introduce market distortions through political actions outside the voting process. Such a voting system seems to negate

[1]According to Tillmann (2005), Pareto optimal allocations never exist in large economies because there is a lot of variation between individual preferences and abilities. Thus, in dealing with distributional equity, only second-best optimal allocations may be considered in egalitarian-type economies, where agents with the same preferences but with different abilities obtain the same consumption-labour bundle. Equity and efficiency are not in conflict if there is a fixed relation between preferences and abilities.

the value of the argument (e.g. Tillmann, 2005) that optimal income redistribution policies and Pareto optimal allocations can never co-exist, if there is variation in individual preferences. Therefore, we claim that government intervention in the form of employing equity and/or efficiency enhancing tax-rate adjustments can lead to Pareto optimality and competitive equilibrium.

2) Before modeling the optimization process, it is necessary to present several definitions which will provide a context for our discussion in Section 3.

The term "equity" is used in the literature in two conflicting senses:

- The egalitarian view (relative equity), according to which the policy makers' goal is to achieve the maximum equality of incomes (see, for example, Jencks, 1972).
- The alternative view shifts the emphasis to the absolute income of the poor, without regard to others' incomes (Rawls, 1971: p. 151).

Thus, absolute equity appears to be consistent with the Pareto criterion, whereas relative equity does not have this property (see Atkinson and Stiglitz, 1980: p. 341). In the discussion to follow, we will concern ourselves with a modified version of absolute equity. The underlying reason is that the absolute equity concept may lead to a Pareto optimal allocation through improving the utility level of at least one household but, in doing so, incentives and growth aspects—which are constituent parts of an efficient resource allocation—are completely ignored. In our modified version, income redistribution and growth prospects[2] are treated as distinct policy objectives which, however, can be achieved simultaneously by introducing suitable tax rate adjustments. We will not also constrain individual opinions to be identical; instead, we will assume that all preferences move in the same direction. Finally, we will maintain Becker's conclusion that it is in everyone's interest to maximize social welfare, even if certain individuals are selfish.

3) Turning to the conflict between equity and efficiency, we agree with Yew-Kwang's (1985) view that a Pareto optimal outcome may result in an unacceptably unequal distribution of income, with ambiguous effects on incentives and growth. The society may wish to achieve a more equal distribution of income while minimizing ineffiencies. No matter how one evaluates Pareto optimality, in this framework, distributional equality will be interpreted to mean vertical equity, in the sense that individuals with a diverse range of endowments will be taxed differently. This implies that the analysis of the horizontal distribution of income or fairness (horizontal equity)—i.e., individuals with identical endowments should be treated equally—is out of the scope of the present study.

[2]It should be noted that, according to the endogenous growth theory, the trade-off between equity and growth does not exist, when intergenerational transmission of human capital and wealth (learning by doing, stock of social knowledge, public services enhancing labour productivity, returns to scale, feedback effects on the cost of learning or innovation) are taken into account. The neoclassical approach to growth is claimed to be false because it emphasizes (as determinants of savings behaviour) the diminishing marginal productivity of additional increments to the capital stock and the exogeneity of technical progress. Thus, without change in technology, the growth rate of GDP tends to zero in the limit, due to the diminishing marginal return to additional savings.

4) In general, the concern for equity has long been an important aspect of economic analysis. However, recent research has focused on efficiency, rather than equity, dealing with issues such as the deadweight loss of taxation, the labour supply effects of direct taxes or of welfare transfers to the poor, the cost of government regulations and so on. The coexistence of two discrete fiscal objectives in the same analytical model structure gave rise to the question of whether equity and efficiency could be rivals or complementary factors in the design and implementation of economic policy. There are two opposing schools of thought that try to answer this question[3].

- A number of researchers support the view that equity and efficiency cannot be achieved simultaneously: a greater equity comes at the cost of a loss of efficiency. Policy situations that promote greater equity will have adverse effects on efficiency and *vice versa* (see Okun, 1975: p. 120). In particular, income redistribution causes changes in work effort, in savings and investment behaviour and in attitudes (motivation to acquire human capital), thus leading to less efficient use of resources.
- The above equity versus efficiency dictum predicts a positive relationship between inequality and real GDP growth, emphasizing the importance of economic incentives. It was however challenged by the incomplete markets and political outcomes theories of an inverse relationship between inequality and growth.

These theories find no empirical evidence for the argument that economic incentives are necessary for capital accumulation and growth or that inequality has any significant impact on investment (see, for example, Stevans, 2012). The resulting policy situation is one in which equity and efficiency complements each other.

The aim of the present study is not to give credit to any of these approaches. The indicators (proxy variables) which are employed in empirical or theoretical analyses to evaluate variations in income distribution and/or growth performance are not considered to be reliable measures of equity and efficiency (see Thurow, 1981). For example, comparing the top and the bottom quartiles of the income scale, before and after taxes, or putting the Gini coefficient into practice, tends to ignore the importance of the skill level as a determinant of income in-

[3]For example, Thurow (1981) compares efficiency (real per capita GDP growth) with equity (ratio of income top 20% to income bottom 20%) for the post-tax and pre-tax periods. The sample includes twelve advanced economies over the period 1960-1977. He finds that there is little or no rank order correlation between the degree of inequality and economic performance. Regarding incentives (growth), Thurow argues that countries with high savings rates have also equal distributions of income (e.g. Japan). Finally, the effect of inequity on the incentive to work effort varies, depending on the statistical strength of the determining factors: income effects (with a lower take-home income, one must work harder to achieve a similar standard of living), substitution effects (with a lower take-home wage rate, leisure becomes more attractive), backward bending labour supply curve, non-pecuniary determinants (prestige, power, fame, promotions) and so on. Okun (1975) has staked out an opposite position on this issue. He is generally credited with popularizing the idea of a "great trade-off" between equity and efficiency. He claims that such a trade-off is one of the many tough choices that economic agents have to make.

equalities. Similarly, the use of the rate of growth of GDP to measure variations in efficiency levels ignores the quality aspects of growth (environmental effects, incentives to the factors of production, potential growth, the use of human and physical resources, and so on), which enter the welfare function.

Instead of disputing over the validity of the equity and/or efficiency hypotheses, an attempt will be made to extend the analysis by producing argumentation in support of an alternative approach that overcomes the problem of the equity-efficiency dichotomy. Following the standard optimization process, a simple social welfare function is maximized with respect to consumption and labour, subject to the government budget constraint. If there are no distorting factors arising from the level or structure of taxation and affecting the behaviour of consumers and/or workers, the manipulation of the first-order conditions leads to Pareto optimality. In this case, the marginal rate of substitution (*MRS*) in consumption is equal to the marginal rate of transformation (*MRT*) in production and no government intervention is required. However, if such distortions occur and the *MRS* takes on a different value from the *MRT*, a scope is provided for policy makers to eliminate them via changes in the direct/indirect tax scheme.

The implicit assumption that is made throughout our analysis is that consumers' behaviour is significantly affected, if fiscal authorities make adjustments to the level of indirect tax rates[4]. On the other hand, the response of workers to taxation is mainly captured by the elasticity of the wage rate with respect to changes in direct tax rates. To simplify the analysis without impinging on the validity of our methodological approach, the response of entrepreneurs to increases (decreases) in the corporate profit tax rate will not be considered.

5) In such a conceptual framework as described above, the effects of tax policy on economic growth and on income distribution, and the trade-off between equity and efficiency, are evaluated in terms of whether tax-rate changes succeed in equating the *MRS* with the *MRT* (between consumption and leisure). As a consequence, we will not discuss other interesting topics on equity-efficiency, such as those briefly presented below:

- "Raising living standards and equally dividing the pie are mutually exclusive". Following this argument, policies that contribute to economic growth and

[4]Atkinson and Stiglitz (1976) argue that, with separable individual utility functions, governments will not wish to use commodity taxes, relying instead on an optimal income tax. However, their argument may not always be practicable. In many cases, policy makers may have extensive control over commodity prices via state-owned agencies or because a lot of goods are exported or imported through state controlled frontier points. Thus, indirect taxes can be important practical instruments for influencing income distribution and consumption. To approach the problem from a different perspective, indirect taxes are considered to cause distortions by "forcing wedges" between post-tax and pre-tax prices. To minimize distortions, it is necessary to tax goods which are in inelastic demand. Thus, efficiency requires that necessities should be taxed more heavily than luxuries. However, such an indirect tax policy contradicts the equity principle, unless we assume a single-consumer case. When labour is supplied exogenously, the single-consumer case would lead to a uniform commodity tax (Sandmo, 1976). Given that the interest in our study focuses on an endogenous labour-supply function, significant departures from the uniform taxation rule are shown to be produced.

those that redivide income separate the political parties of the right and the left.

- "The government contributes to growth not only directly, via introducing incentives to work and invest to private agents, but also indirectly by providing infrastructure, expanding educational opportunities, protecting property and contracts, providing for national defense and public health, and so on". Dealing with such indirect growth promoting government activities would diverge the discussion from the personal decision making framework and the rules of market exchange (positive externalities, government failures, crowding out effects).
- An increasing body of literature is related to the trade-off between income distribution and economic growth, as well as to the theory and empirical evidence on the macroeconomic determinants of growth and income inequality. See, for example, Kormendi and Meguire (1985), Grier and Tullock (1989), Dawson (1998), Barro (1990), Forbes (2000). Dealing with these issues would be incompatible with the basic argument of the present study that adjustments in the structure of the (in)direct tax system by the government can eliminate distortions in consumer demand and/or in the labour market.

6) The degree of competiveness is likely to play a prominent role in evaluating the trade-off hypothesis. In a perfectly competitive market, there would be no impact of inequality on productivity, unless fiscal authorities would adopt tax measures that distort incentives. For example, a more progressive tax system would reduce inequality but could also create a "deadweight" loss and diminish work effort. This view is challenged in the presence of incomplete markets (Alesina and Rodrik, 1994; Deininger and Squire, 1996). For instance, market failures, such as borrowing constraints, externalities, decreasing returns to capital and credit rationing lead to an inefficient allocation of resources, underinvestment and reduced productivity.

In addition to incomplete markets, the political process may also result in a negative relationship between inequality and growth. If, for example, the electorate vote against redistributive tax policies, greater income inequality would lead to higher tax rates in order to raise the poverty fine. As a consequence, the after-tax rate of return on capital, investment and growth is reduced.

Lastly, the equity-efficiency approach emphasizes the importance of incentives. Lowering the tax rates for the rich creates incentives to save and invest more, thus increasing capital formation and growth. However, this may not be the case when saver and investor are not the same person or when the investment schedule responds more to changes in income than to fluctuations in interest rates.

In sum, the evidence on the relationship between inequality and growth is far from conclusive. This relationship is predicted to be positive because of incentives, but it turns into negative in the context of incomplete markets and political theories (Forbes, 2000; Perotti, 1993; Barro, 1999). The question as to which prediction dominates is an empirical one, but it continues to be outside the scope of the present study.

Our central point of interest lies with comparing the results of maximizing a social welfare function (with fixed labour supply and consumption; i.e. without distortions) to the results of a model, in which market distortions are introduced. Then, tax-rate changes are employed to minimize these distortions.

As become evident from the preceding discussion, all the proposals that have been put forward are highly contentious. Thus, before proceeding, it is worth reminding the reader of some factors that our model omits and which may be distorting our conclusions.

Firstly, it is assumed that our model is static. All of the post-tax income is consumed immediately and consumption in excess of post-tax income is impossible. Agents with a low earning potential have no means to provide themselves with insurance. This could be true in practice, if our model's optimality conditions are derived under the assumption that a tax system links gross income directly to consumption.

Secondly, our model allows only one type of behavioural response to changes in tax rates: Labour supply response to direct tax rate changes and consumers' response to indirect tax rate changes. There is no scope for tax avoidance, for migration or for leaving the labour market. These omissions tend to understate the cost of altering the level or the structure of taxation (revenue losses to the government).

Thirdly, any optimal tax problem with heterogeneous agents must assume an objective criterion that allows changes in different agents' welfare to be compared against one another. In dealing with this issue, the results of the present study are likely to be partially dependent on the implicit assumption of taking into account the utilitarian criterion, aggregating concave utility functions.

In Section 3, we discuss the conventional optimal taxation methodology and present several of its more widely known standard results, which are set against the findings, derived from the modified version. Section 4 outlines some of the important considerations that have been largely ignored by the conventional analysis, by using simulations, numerical examples and mathematical tools to underline the practical implications of incorporating our theoretical work to the sphere of applied fiscal policy management. Finally, Section 5 concludes the discussion, laying out directions for further work.

3. Modeling Structure

3.1. The Standard Consumption-Leisure Optimization Model

Consider an economy with two commodities, a consumption good and a single labour service. A household's supply of the labour service is denoted by l, where $0 \le l \le 1$. Consumption of the good is denoted by c, where $c \ge 0$. Each household is characterized by their skill (ability) level, s, which gives the relative effectiveness or the marginal product of labour supplied per unit of time. In perfect competition, the marginal product of labour is identified as the real (hourly) wage rate, w, so a highly skilled household is more effective in production, earning higher wages than a low-skilled household.

Assuming that a household of ability s supplies l hours of labour, at a constant wage rate, the effective labour, $sl = wl$, is equal to the total productivity of worker, which in turn is equal to the pre-tax income. The underlying assumption is that the total of the post-tax income is spent on the purchase of the consumption good, i.e.

$$(1-t)wl = (1-t)y = pc \tag{1a}$$

where t stands for the proportional income tax rate, p represents the price level and y is the gross income. Normalizing the price of the consumption good at 1, Equation (1a) is written as

$$(1-t)wl = c \tag{1b}$$

It is further assumed that all households have the same strictly concave utility function, thus allowing for interpersonal comparability and a common utility function

$$U = U(c,l) \tag{2}$$

The next step is to maximize utility by choice of labour supply and consumer demand, subject to the budget constraint (1b), which is assumed to be the same for all households:

$$\mathscr{L}(c,l,\lambda) = U(c,l) + \lambda\left[(1-t)wl - c\right] \tag{3}$$

As already noted, the households have identical preferences over consumption and leisure. The utility function is continuously differentiable, strictly increasing in consumption and strictly decreasing in leisure:

$$U_c > 0, U_{cc} < 0, U_l < 0, U_l \to -\infty \text{ as } l \to 1$$

Assuming that the Lagrangian is formed as in (3), the resulting necessary conditions for the maximization can be combined to give

$$\left.\begin{aligned} \frac{\partial \mathscr{L}}{\partial c} &= \frac{\partial U}{\partial c} - \lambda = 0 \\ \frac{\partial \mathscr{L}}{\partial l} &= \frac{\partial U}{\partial l} + \lambda(1-t)w = 0 \end{aligned}\right\} \frac{\frac{\partial U}{\partial l}}{\frac{\partial U}{\partial c}} = -(1-t)w \tag{4a}$$

In the absence of taxation, Equation (4a) can be written as

$$\frac{\frac{\partial U}{\partial l}}{\frac{\partial U}{\partial c}}\left(= \frac{\partial c}{\partial l}\right) = -w \tag{4b}$$

Equation (4b) is the standard efficiency condition and ensures that, in equilibrium, the marginal rate of substitution (MRS) between labour and consumption is equal to (minus) the marginal rate of transformation (MRT) between labour and consumption. Remember that:

- The $MRT_{l,c}$ is the hourly cost of employing an extra unit of labour in the production of the consumption good (w = marginal product of labour), i.e. the cost of transforming an extra unit of labour into commodities in the production process.

- The $MRS_{l,c}$ measures the extent to which the utility of the household from the consumption of the good will increase if they are willing to sacrifice an extra unit of leisure, or to work an extra unit of time in the production of this good. In other words, a competitive equilibrium is reached when the marginal utility or the marginal benefit from the consumption of the good (demand function) is equated to the marginal cost of producing it (supply function).
- The *MRS* between consumption and labour is the equivalent of the *MRS* between consumption and income, or equivalent of the concept of the marginal propensity to consume (*MPC*), i.e. $MRS_{c,l} \cong MPC$ or $\frac{U_l}{U_c} = \frac{\partial C}{\partial Y}$.
- Given that, in the relation (1a), the wage rate is treated as constant and $l = \frac{y}{w}$, a utility function in terms of consumption and labor supply, could take the following form

$$U = U(c,l) = U\left(c, \frac{y}{w}\right), \text{ and}$$

$$\frac{\partial U}{\partial l} = \frac{\partial U}{\partial \frac{y}{w}} = \frac{\partial U}{\frac{1}{w}\partial y} = w\frac{\partial U}{\partial y}, \text{ or}$$

$$U_l = wU_y, \text{ or } \frac{1}{w}U_l = U_y,$$

The utility function is said to satisfy agent monotonicity if $\varphi = -\frac{U_y}{U_c}$ is a decreasing function of the wage rate; that is, if $\frac{\partial \varphi}{\partial w} < 0$.

Given that $\varphi = -\frac{U_y}{U_c} = -\frac{1}{w}\frac{U_l}{U_c}$ so that $\frac{\partial \varphi}{\partial w} < \frac{1}{w^2}\frac{\partial\left(\frac{U_l}{U_c}\right)}{\partial w}$, agent monotonicity is satisfied. This occurs because the second term on the right-hand side of the last relationship, $\frac{\partial\left(\frac{U_l}{U_c}\right)}{\partial w}$ is negative, in accordance with the three basic macroeconomic presumptions:

1) Labour supply increases with the wage rate (no backward bending labour supply curve), but at a decreasing rate (positive first derivative, negative second derivative).

2) Consumption rises with wage rate or income at a decreasing rate (positive first derivative, negative second derivative). This implies that consumption is not an inferior good, though it is subject to the rule of the diminishing marginal utility of consumption (income).

3) Since the *MRS* between consumption and pre-tax income is a decreasing function of income, agent monotonicity is equivalent to the condition that the

marginal propensity to consume declines as the household moves to higher income levels. This is an alternative definition of the diminishing marginal utility of consumption (income).

The foregoing discussion completes the analysis of the rules relating to the efficient allocation of resources between consumption and leisure, in the context of the standard simplified Pareto optimality setting. The maximization process, as described above, lead to a Pareto-efficient outcome in the absence of taxation and other distorting factors, coming from the potential reactions of both consumers to changes in indirect tax rates and workers to changes in direct tax rates. This motivates the study of feasible allocation mechanisms and the comparison of their outcomes to those of the standard modeling structure of Section 3.1. Thus, the natural question is whether government and/or market failures—that distort the static Pareto-equilibrium condition of equality between $MRS_{c,l}$ and $MRT_{c,l}$—can be eliminated by adjustments in the tax structure. Section 3.2 will go into detail over this issue.

3.2. The Revised Consumption-Leisure Optimality Model

To provide a reasonably simple derivation of the efficiency rule, it will be assumed that the economy consists of H households indexed $h = 1, \cdots, H$. Each household has a utility function

$$U^h = U^h\left[c^h\left(t_i, y^h\right), l^h\left(t_y, w\right)\right] \quad (5)$$

where c^h is the consumption of households h of the vector of private goods and l^h is the household's h supply of working time. Private consumption is taken to be a function of disposable income, y, and the indirect tax rate, t_i, whereas labour supply depends on an exogenously determined wage rate, w, and the direct tax rate, t_y.

To characterize the set of first-best or Pareto efficient allocations, each household chooses c^h and l^h, $h = 1, \cdots, H$, to maximize their utility level, constrained by the requirement that all the other households, 2 to H, obtain given utility levels and by the condition that the government will raise sufficient revenue to finance the provision of public goods. Varying the given utility levels for households 2 to H traces out the set of Pareto efficient allocations. The Lagrangian for this maximization problem is written

$$\begin{aligned}\mathscr{L} = {} & \mu^1 U^1\left[c^1\left(t_i, y^1\right), l^1\left(t_y, w\right)\right] + \sum\nolimits_{h=2}^{H} \mu^h \left\{U^h\left[c^h\left(t_y, y^h\right), l^h\left(t_y, w\right)\right] - \bar{U}^h\right\} \\ & - \lambda\left(t_i C + t_y wL - G\right)\end{aligned} \quad (6)$$

where $C = \sum\nolimits_{h=1}^{H} c^h$, $L = \sum\nolimits_{h=1}^{H} l^h$, and $\bar{U}^h$ is the utility level that must be achieved by $h = 2, \cdots, H$. The term μ^h may be interpreted as the social welfare weight that is given to each household ($\mu^h = 1$ for $h = 1$). Assuming that the specified utility levels can be reached simultaneously, the necessary conditions describing the optimal choice of consumption with respect to the indirect tax rate and the optimal choice of working time with respect to the direct tax rate are

$$\frac{\partial \mathscr{L}}{\partial t_i} = \mu^h \frac{\partial U^h}{\partial c^h} \frac{\partial c^h}{\partial t_i} - \lambda \left(C + t_i \frac{\partial C}{\partial t_i} \right) = 0 \tag{7a}$$

$$\frac{\partial \mathscr{L}}{\partial t_y} = \mu^h \frac{\partial U^h}{\partial l^h} \frac{\partial l^h}{\partial t_y} - \lambda \left(wL + t_y w \frac{\partial L}{\partial t_y} \right) = 0 \tag{7b}$$

dividing (7a) by (7b) and rearranging terms gives

$$\frac{\frac{\partial U^h}{\partial c^h}}{\frac{\partial U^h}{\partial l^h}} = \frac{\frac{c + t_i \left(\frac{\partial C}{\partial t_i} \right)}{\frac{\partial c^h}{\partial t_i}}}{\frac{wL + t_y w \left(\frac{\partial L}{\partial t_y} \right)}{\frac{\partial l^h}{\partial t_y}}} \tag{8}$$

It is presumed throughout that

1) responses of consumption spending to changes in indirect tax rates are equalized across households and to the population as a whole, i.e., $\frac{\partial C}{\partial t_i} = \frac{\partial c^h}{\partial t_i}$, and;

2) responses of labour supply to changes in direct tax rates are equalized across households and to the population as a whole, i.e. $\frac{\partial L}{\partial t_y} = \frac{\partial l^h}{\partial t_y}$.

Rearranging the terms in (8) gives

$$\frac{\frac{\partial U^h}{\partial c^h} \frac{\partial c^h}{\partial t_i}}{\frac{\partial U^h}{\partial l^h} \frac{\partial l^h}{\partial t_y}} = \frac{C + t_i \frac{\partial c^h}{\partial t_i}}{wL + t_y w \left(\frac{\partial l^h}{\partial t_y} \right)} \tag{9}$$

Given that the entire disposable income is assumed to be consumed, i.e. $wL = C$, and following the simple mathematical formula $\frac{1+A}{1+B} = 1 + A - B$, where $A = t_i \left(\frac{\partial c^h}{\partial t_i} \right)$ and $B = t_y w \left(\frac{\partial l^h}{\partial t_y} \right)$, Equation (9) takes the form

$$\begin{aligned} \frac{\frac{\partial U^h}{\partial c^h} \frac{\partial c^h}{\partial t_i}}{\frac{\partial U^h}{\partial l^h} \frac{\partial l^h}{\partial t_y}} &= \frac{C + A}{C + B} = \frac{1 + \frac{A}{C}}{1 + \frac{B}{C}} \\ &= 1 + \frac{A}{C} - \frac{B}{C} = \frac{C + A - B}{C} \\ &= \frac{C + t_i \left(\frac{\partial c^h}{\partial t_i} \right) - t_y w \left(\frac{\partial l^h}{\partial t_y} \right)}{C} \end{aligned} \tag{10}$$

Since the second term of the left-hand side of (10) is $\frac{\frac{\partial c^h}{\partial t_i}}{\frac{\partial l^h}{\partial t_y}} = \frac{\partial c^h}{\partial l^h}\frac{\partial t_y}{\partial t_i}$ and the term $\frac{\partial l^h}{\partial c^h}$ represents the marginal rate of transformation between consumption and labour, Equation (10) can be written

$$\frac{\frac{\partial U^h}{\partial c^h}}{\frac{\partial U^h}{\partial l^h}} = MRS_{c,l} = MRT_{c,l}\frac{\partial t_i}{\partial t_y}\frac{C + t_i\left(\frac{\partial c^h}{\partial t_i}\right) - t_y w\left(\frac{\partial l^h}{\partial t_y}\right)}{C} \tag{11}$$

Comparing Equation (4b)—Pareto efficiency in the absence of both taxation and other distortions—with Equation (11), it becomes evident that, if taxation and other distorting factors are introduced into the analysis, Pareto efficiency can be achieved ($MRS_{c,l} = MRT_{c,l}$) only when the term

$\frac{\partial t_i}{\partial t_y}\left[\frac{c + t_i\left(\frac{\partial c^h}{\partial t_i}\right) - t_y w\left(\frac{\partial l^h}{\partial t_y}\right)}{c}\right]$ is equal to 1, or when

$$\frac{\partial t_y}{\partial t_i} = 1 + \frac{t_i\left(\frac{\partial c^h}{\partial t_i}\right)}{c} - \frac{t_y w\left(\frac{\partial l^h}{\partial t_y}\right)}{c} \tag{12}$$

In Equation (12), we note that:

1) The term $\frac{\partial c^h}{\partial t_i}$ demonstrates the effect of a unit change in indirect tax rate on consumption. Since this relationship is expected to be negative, the term $t_i\left(\frac{\partial c^h}{\partial t_i}\right)$ may be interpreted as the marginal loss of indirect tax revenue resulting from a unit increase in indirect tax rate.

2) The term $w\left(\frac{\partial l^h}{\partial t_y}\right)$ represents the effect of a unit change in direct tax rate on labour supply (with the wage rate being treated as numeraire). Assuming that this relationship may bear a negative (positive) sign, the term $t_y w\left(\frac{\partial l^h}{\partial t_y}\right)$ is taken to measure the loss (gain) in terms of marginal direct-tax revenue that arises from a unit increase in the income tax rate (or equivalently from a relative decline in the wage rate).

3) In practice, the equilibrium condition $MRS_{c,l} = MRT_{c,l}$ can rarely be satisfied. This occurs only if the effects of indirect tax rates and indirect tax-rate changes on consumption, as well as the effects of direct tax rates and direct tax rate changes on labour supply, are exactly balanced out. In this case, which is an

exception to the rule, $\frac{\partial l^h}{\partial t_y} = -\frac{\partial c^h}{\partial t_i}$ and $t_i = t_y$, so that $\frac{\partial t_i}{\partial t_y} = 1$.

4) When these two kinds of effects differ in magnitude and given that reactions of both consumers and workers to taxation are not directly controlled by the government, it is only the fiscal instruments t_y and t_i that can be used by policy makers to ensure equilibrium between $MRS_{c,l}$ and $MRT_{c,l}$.

The extent to which direct and indirect tax rates can be re-combined to equalize $MRS_{c,l}$ and $MRT_{c,l}$ can be derived from the solution of (12) for t_y, in order to generate an equilibrium reaction function. Solving (12) for t_y results in a complicated formula, as shown in **Appendix 1**, which however cannot be easily manipulated by policy makers to achieve fiscal objectives. A readily manageable form of the reaction function is given below by Equation (13a),

$$t_y = \rho e^{-\frac{t_i}{C}\left(\frac{\partial l^h}{\partial t_y}\right)} + \frac{\frac{\partial c^h}{\partial t_i}}{\frac{\partial l^h}{\partial t_y}} t_i + C \frac{\frac{\partial l^h}{\partial t_y} - \frac{\partial c^h}{\partial t_i}}{\left(\frac{\partial l^h}{\partial t_y}\right)^2} \tag{13a}$$

where ρ stands for a constant. To simplify the analysis, we assume that labour supply is a linear function of the income tax rate, the wage rate and a set of other explanatory variables (X), i.e.,

$$l^h = b_0 + b_1 t_y + b_2 w + b_3 X$$

while consumption spending is a linear function of the indirect tax rate, the wage income, wl^h, and a set of its own explanatory variables (Z), i.e.,

$$c^h = a_0 + a_1 t_i + a_2 w l^h + a_3 Z$$

The partial derivatives of the above functions with respect to tax rates are the constant coefficients $b_1 = \frac{\partial l^h}{\partial t_y}$, and $a_1 = \frac{\partial c^h}{\partial t_i}$. Therefore, Equation (13a) can be re-written

$$t_y = \rho e^{-\frac{b_1}{C} t_i} + \frac{a_1}{b_1} t_i + \frac{C(b_1 - a_1)}{(b_1)^2} \tag{13b}$$

where ρ is a constant describing the initial conditions.

Following a similar procedure in solving (13a), a reaction function in which t_i is expressed in term of t_y may also be formulated.

Equation (13b) measures the required change in the direct tax rate following a one-percentage point increase (decrease) in the indirect tax rate, that is introduced by fiscal authorities in order to eliminate tax-induced distortions in demand and/or in labour supply. However, Equation (13b), even in its simplified form, cannot be easily subjected to closer study and our discussion will fail to emphasize those aspects of fiscal policy that are most relevant to elaborating the practical implications of employing tax-policy instruments, in order to handle distortions in the economy. This occurs because some technical aspects of the economy that are strictly necessary for later analysis are obscured by the quite complicated relationship between direct and indirect tax rates. In Section 3.3, we provide an analytical foundation for tax-policy intervention in the economy, in order to cope with distortions in consumer's and/or worker's behaviour, by using a non-linear utility function, which allows us to express the variables of interest in terms of elasticities rather than in terms of changes in absolute values.

3.3. Modifying the General Framework

Keeping all the assumptions of Section 3.2 in the modified framework, the utility function takes the form

$$U^h = U^h\left[\ln c^h(t_i), \ln l^h(t_y)\right]$$

while the Lagrangian for the maximization problem can be written

$$\mathscr{L} = U^h\left[\ln c^h(t_i), \ln l^h(t_y)\right] - \ln\left[t_i C(t_i)\right] - \ln\left[t_y wL(t_y)\right] \tag{14}$$

where

$$\ln c^h = a_0 + a_1 \ln t_i + a_2 \ln(wL) + a_3 \ln(CPI), \text{ and}$$

$$\ln l^h = b_0 + b_1 \ln(t_y) + b_2 \ln w$$

The necessary condition describing the choice of the indirect tax rate is

$$\frac{\partial \mathscr{L}}{\partial t_i} = \frac{\partial U^h}{\partial \ln c^h}\frac{\partial \ln c^h}{\partial \ln t_i} - \left(1 + \frac{\partial \ln C}{\partial \ln t_i}\right) = 0, \text{ or}$$

$$a_1 \frac{\partial U^h}{\partial \ln c^h} = 1 + a_1 \tag{15a}$$

For the choice of the level of the direct tax rate, optimizing with respect to $\ln t_y$ gives

$$\frac{\partial \mathscr{L}}{\partial t_y} = \frac{\partial U^h}{\partial \ln l^h}\frac{\partial \ln l^h}{\partial \ln t_y} - \left(1 + \frac{\partial \ln(wL)}{\partial \ln t_y}\right) = 0, \text{ or}$$

$$b_1 \frac{\partial U^h}{\partial \ln l^h} = 1 + wb_1 \tag{15b}$$

dividing (15b) by (15a) gives

$$\frac{b_1\left(\dfrac{\partial U^h}{\partial \ln l^h}\right)}{a_1\left(\dfrac{\partial U^h}{\partial \ln c^h}\right)} = \frac{1 + wb_1}{1 + a_1}$$

Since the wage rate is taken to be the numeraire having a unit price, the last equation takes the form

$$\frac{b_1}{a_1}\frac{\dfrac{\partial U^h}{\partial \ln l^h}}{\dfrac{\partial U^h}{\partial \ln c^h}} = \frac{1 + b_1}{1 + a_1} \tag{16}$$

It is well known from the preceding discussion that

1) $$\frac{\frac{\partial U^h}{\partial \ln l^h}}{\frac{\partial U^h}{\partial \ln c^h}}\left(=\frac{\partial \ln c^h}{\partial \ln l^h}\right)=MRS_{c^h,l^h}$$

2) $$\frac{a_1}{b_1}=\frac{\frac{\partial \ln C}{\partial \ln t_i}}{\frac{\partial \ln L}{\partial \ln t_y}}=\frac{\partial \ln C}{\partial \ln L}\frac{\partial \ln t_y}{\partial \ln t_i}=MRT_{C,L}\left(\frac{\partial \ln t_y}{\partial \ln t_i}\right)$$

Therefore, we can recast Equation (16) to make it consistent with Equation (11) of the linear model:

$$MRS_{c^h,l^h}=MRT_{C,L}\left[\frac{\partial \ln t_y}{\partial \ln t_i}\left(\frac{1+b_1}{1+a_1}\right)\right]=MRT_{C,L}\left[\frac{\partial \ln t_y}{\partial \ln t_i}\left(1+b_1-a_1\right)\right] \quad (17)$$

Equation (17) implies that market equilibrium exists (*MRS* = *MRT*) only if $\frac{\partial \ln t_y}{\partial \ln t_i}\left(1+b_1-a_1\right)=1$, that is, if

$$\frac{\partial \ln t_i}{\partial \ln t_y}=1+b_1-a_1 \quad (18)$$

The interpretation of (18) is quite similar to that of the linear model (see Equation (11)), that is,

1) Pareto efficiency results only if

- the direct-tax elasticity of labour supply and the indirect-tax elasticity of consumption goods exactly offset each other, and
- the percentage changes in both direct and indirect tax rates are exactly the same in size.

2) When the direct-tax elasticity of labour supply differs from the indirect-tax elasticity of consumption, then the only way to achieve Pareto efficiency is to change the structure of the tax system by placing greater emphasis on direct or indirect taxation.

In the usual case of asymmetric responsiveness of labour supply and consumption to changes in (in)direct tax rates, restructuring of the tax system is required to redress the balance. Policy makers have to reschedule the ratio of the proportional changes in the two sorts of tax rates in a way that eliminates the tax-induced distortions in demand and labour market. To establish a reasonable relationship between indirect and direct tax rates, Equation (18) is solved for t_i to generate the following reaction function:

$$\int \mathrm{d}\ln t_i=\left(1+b_1-a_1\right)\int \mathrm{d}\ln t_y \text{, or}$$

$$\ln t_i=\left(1+b_1-a_1\right)\ln t_y+k$$

where k is a constant that captures the initial conditions in the economy. From the last equation, we receive

$$t_i=kt_y^{1+b_1-a_1} \quad (19)$$

In order to measure the response of the indirect tax rate to changes in the direct tax rate in a practicable and manageable way, that would help fiscal authorities to properly re-design the appropriate tax structure, we must turn to the empirical investigation of our theoretical proposition by using market data and assigning numerical values (from the real economy) to the crucial variables.

4. Empirical Evidence and Simulations

Econometric estimates, simulations and numerical analysis of optimal (in)direct tax rates have become popular for two reasons:

- The tax rules derived in Section 2 suggest general observations about the structure of optimal tax rates, but they do not have precise implications. Empirical and numerical analyses may be viewed as providing a check on the interpretations and a means of examining them further.
- The motive of our analysis is to provide practical policy recommendations. This implies that the tax rules must be capable of being applied to data and to the values of the resulting optimal tax rates calculated. All the data series used in estimating the parameters of the above relationships have been taken from Ameco Database (Eurostat) and OECD Statistics.

To present refined estimates of optimal (in)direct tax rates, the first step is to maximize a social welfare function and manipulate the resulting first-order conditions with a view towards establishing a Pareto-efficient tax structure. The procedure used for this was discussed in Section 3. From now on, the main focus of interest will be Equation (19).

Equation (19) describes an infinite number of combinations of optimal direct and indirect tax rates. It measures the extent to which the indirect tax rate should change after a pre-determined percentage-point increase (decrease) in the direct tax rate to maintain equilibrium. The value of the constant, *k*, and the values of elasticities of demand and labour supply to indirect and direct taxes (i.e. a_1 and b_1), respectively, are treated as parameters.

A reverse relationship between the above two tax rates can also be found by dividing (15a) by (15b) and then by replicating the forgoing procedure (for deriving Equation (19)):

$$t_y = kt_i^{1+a_1-b_1} \tag{20}$$

Equation (20) may be interpreted as providing a map of indifference curves that present the preferences of policy makers for direct (indirect) tax rates over indirect (direct) tax rates, as a means of eliminating distortions coming from the labour market and/or from the market for consumer goods. It can be shown that, in Equation (20), the direct tax rate is a convex function of the indirect tax rate, if $1+a_1-b_1>1$, or $a_1>b_1$. In contrast, the indifference curves are concave to the origin if $1+a_1-b_1<1$, or $a_1<b_1$.

It is well understood that the design of a map *per se*, that includes an infinite number of indifference curves on the basis of (20), does not seem to comprehend the scale of the problem. It is clear that Equation (20) by itself cannot devise a mathematical method to prove the existence of equilibrium among indifference curves, unless a constraint on the direct/indirect tax-rate structure is placed. The capacity to use marked forms of limitations in the analysis is a developmental step, but the equality between the total tax revenue, T, and the sum of direct and indirect taxes ($T_y + T_i$) is considered to be a contextual and pragmatic constraint, i.e., $T = T_y + T_i$

or, dividing by national income,

$$\frac{T}{Y}\left(=\overline{\tau}\right)=\frac{T_y}{Y}+\frac{T_i}{Y}\left(=\overline{\tau}_y+\overline{\tau}_i\right)$$

or, dividing by the average tax rate,

$$1=\frac{\overline{\tau}_y}{\overline{\tau}}+\frac{\overline{\tau}_i}{\overline{\tau}} \tag{21}$$

Suppose now that there is a simple linear relationship between the average direct tax rate and the average indirect tax rate (a constant ratio equal to a), $\overline{\tau}_y = \alpha\overline{\tau}_i$ so that (21) takes the form

$$\frac{\left(1+a\right)\overline{\tau}_i}{\overline{\tau}}=1\text{, or}$$

$$\overline{\tau}_i=\frac{\overline{\tau}}{1+a} \tag{21a}$$

and, consequently,

$$\overline{\tau}_y=\frac{a}{1-a}\overline{\tau} \tag{21b}$$

Drawing at random two values for a, with the average tax rate being treated as parametric, allows us to determine two pairs of values for average direct and indirect tax rates. These pairs correspond to two discrete points on the (t_y, t_i) dimensional space and makes it possible to design the linear budget constraint. The tangency between the budget constraint and one of the indifference curves gives the equilibrium values for the direct and indirect tax rates.

The last step is to assign an appropriate value to the constant, k, and run two regressions for each of the six countries considered (France, UK, Italy, Germany, USA, Japan) in order to estimate the coefficient values of $a_1=\frac{\mathrm{d}\ln C}{\mathrm{d}\ln t_i}$ and $b_1=\frac{\mathrm{d}\ln L}{\mathrm{d}\ln t_y}$. The six sample countries have been chosen because they are industrialized with qualified and experienced fiscal policy makers who are capable of designing and implementing the advanced techniques proposed in our study. The logarithmic form of both the consumption and labour-supply functions is chosen because it provides currency-free elasticity estimates for both the indirect-tax induced changes in demand and the direct-tax induced changes in labour supply.

It should be stressed from the outset that it is beyond the scope of the present study to construct a fully-fledged system of equations (or an econometric model), which will be capable of capturing the effects of all of the explanatory variables, including (in)direct tax rates, on consumer's behaviour and labour supply incentives. Since we actually intend to describe how our proposed model could successfully function in practical, real world terms, as a rough guide to policy makers, we opt to employ *ad hoc* econometric estimates of the crucial parameters (k, a_1, b_1) for each country rather than numerical values or numerical examples. The logarithmic functions which were chosen for our estimates are

$$\ln C = a_0 + a_1 \ln t_i + a_2 \ln Y + a_3 \ln Z1$$

$$\ln L = b_0 + b_1 \ln t_y + b_2 \ln w + b_3 \ln Z2$$

where Y is GDP and $Z1$, $Z2$ are sets of additional determinants of consumption and labour supply, respectively. The results for the parameters of interest are presented in **Table 1** (values of remaining coefficients, significance levels and other diagnostic tests are provided on request).

The entries in column 4 of **Table 1** are the numerical values of the exponent in (20) for each of the sample countries. The use of a different methodological solution to the problem of estimating the value of the constant term can be justified as follows: The constant term stands for the initial conditions prevailing in an economy, in which tax-induced distortions that arise from the labour market and/or from consumer demand should be minimized via introducing carefully designed changes in the mix of direct-indirect tax rates.

A widely accepted indicator of alternative tax-rate combinations is argued to be the ratio of average direct to average indirect tax rates, $\frac{\bar{t}_y}{\bar{t}_i} = a$ (see Equation (21)). To determine the value of a, we run the following regression:

$$\ln \bar{\tau}_y = \ln a + \ln \bar{\tau}_i \tag{22}$$

Table 1. Estimates of the main coefficient values.

Country	$a_1 = \frac{\mathrm{d}\ln C}{\mathrm{d}\ln t_i}$	$b_1 = \frac{\mathrm{d}\ln L}{\mathrm{d}\ln t_y}$	$1 + a_1 - b_1$	Constant
(1)	(2)	(3)	(4)	(5)
France	−0.329	0.081	0.589	0.257
Germany	0.025	0.061	0.964	0.552
UK	0.554	−0.161	1.715	0.260
USA	−0.020	0.005	0.975	0.553
Japan	−0.256	−0.067	0.811	0.445
Italy	0.412	0.235	1.177	0.448

Source: Ameco database (Eurostat), OECD Statistics. Note: The parameters a_1 and b_1 in column (4) have been estimated on the basis of the functions $\ln C = a_0 + a_1 \ln t_i + a_2 \ln Y + a_3 \ln Z1$ and $\ln L = b_0 + b_1 \ln t_y + b_2 \ln w + b_3 \ln Z2$, respectively.

The inverse logarithm of a in (22) may be interpreted as representing the initial conditions in each country, as shown in Table 1 (column 5).

The final steps are:

1) To introduce the parameter values—as shown in columns 4 and 5 on Table 1—for each country into Equation (20), in order to obtain a numerically defined map of the indifference curves, describing the preferences of the policy makers over feasible combinations of direct and indirect tax rates.

2) To transform Equation (21) in terms of the direct tax rate

$$\overline{\tau}_y = \overline{\tau} - \overline{\tau}_i \tag{23}$$

where the average (total) tax rate, $\overline{\tau}$, is taken from the official government statistics of each country. The budget constraint (23) describes affordable direct-indirect tax-rate combinations.

3) To equate (20) with (23)

$$\overline{\tau} = kt_i^{1+a_1-b_1} + t_i \tag{24}$$

assuming that Equation (20) holds for any combination of feasible direct and indirect tax rates, including their average values.

Equation (24) can be easily solved in terms of the (average) indirect tax rate. Substituting the latter into (23) gives the (average) direct tax rate.

The pair of (in)direct tax rates derived from (24) and (23) determine the point, at which the government budget constraint is tangent to the potentially higher indifference curve and corresponds to the optimal combination of direct and indirect tax rates. Accordingly, such a mixture of (in)direct tax rates is argued to eliminate any distortion, originating in the labour market and/or in the market for consumer goods and to achieve equilibrium through permitting the equality between the marginal rate of substitution and the marginal rate of transformation.

Table 2 describes the details of calculating the optimal indirect tax rate, while Table 3 presents the actual *vis-a-vis* the optimal (in)direct tax rates.

A graphical representation of the budget constraint, the indifference curve, the equilibrium point and the optimal tax rates for each country is given in **Appendix 2** (Figures A1-A6).

The familiar conflict between equity and efficiency is illustrated in the above figures and in Table 3. If all the conditions for equilibrium are satisfied, the economy is at any point on the utility possibility curve defined by Equation (20), $t_y = kt_i^{1+a_1-b_1}$, in Figures A1-A6. According to the specific budget constraint depicted, the social welfare maximum point is at A, where the slope of one of the welfare contours is equal to the slope of the budget constraint, defined by Equation (21), $\overline{\tau} = \overline{\tau}_y + \overline{\tau}_i$. This is equivalent to saying that equilibrium in the economy is obtained when the marginal rate of substitution between direct and indirect taxes, given by (20), is equated to the marginal rate of transformation between these tax categories, as given by (21). Note that Table 3 analyses the graphical objects extracted from Figures A1-A6, for which the same criteria are

Table 2. Estimation of the optimal indirect tax rate.

France	$0.257t_i^{0.589} + t_i = 0.71$	$t_i = 0.533$
UK	$0.260t_i^{1.715} + t_i = 0.64$	$t_i = 0.547$
Italy	$0.448t_i^{1.177} + t_i = 1$	$t_i = 0.704$
Germany	$0.552t_i^{0.964} + t_i = 0.51$	$t_i = 0.324$
USA	$0.553t_i^{0.975} + t_i = 0.44$	$t_i = 0.280$
Japan	$0.445t_i^{0.811} + t_i = 0.38$	$t_i = 0.240$

Source: The above estimations of the table have been derived on the basis of the data provided by estimating the relation $\overline{\tau} = kt_i^{1+a_1-b_1} + t_i$.

Table 3. Actual and optimal direct and indirect tax rates.

	Average t_y	Average t_i	Average total tax rate	Optimal t_y	Optimal t_i
	(1)	(2)	(3)	(4)	(5)
France	0.300	0.405	0.705	0.177	0.533
UK	0.344	0.294	0.638	0.092	0.547
Italy	0.503	0.498	1.001	0.296	0.704
Germany	0.267	0.241	0.508	0.186	0.324
USA	0.276	0.160	0.430	0.160	0.280
Japan	0.214	0.170	0.384	0.140	0.240

Source: Ameco database (Eurostat). The above estimations of the table have been derived on the basis of the data provided by estimating the relation $t_y = kt_i^{1+a_1-b_1}$.

indicated. However, an algebraic representation, on the basis of Equation (24), could have greatly enhanced the exposition of the theory, while ensuring the same results, as shown in Table 2.

The resulting optimal equilibrium pairs of direct and indirect tax rates for each of the six countries considered are then used to construct Table 3, columns 4 and 5. Finally, the above optimal values for (in)direct tax rates are compared to the corresponding actual (average) tax rates (columns 1 and 2) to provide valuable information as to the required restructuring of the tax system in the direction of removing distortions originating in the labour market and/or in consumer demand.

Consider, for example, the case of the UK. In this country, the optimal direct tax rate (0.547) is higher than the actual (average) indirect tax rate (0.294), whereas the optimal direct tax rate (0.092) is lower than the actual (average) direct tax rate (0.344). This finding may be interpreted as follows (provided that the preliminary estimates of our model are not false):

1) Direct-tax incentives are not effective in encouraging people both to work harder and to save and invest more of their income.

2) Indirect taxation tends to provide an incentive to consume more and save less of their income.

Accordingly, at a given level of total tax revenue, optimal taxation rules recommend that more resources be devoted to efficiency criteria, via causing the tax burden to move from direct to indirect taxation.

The general conclusion that arises from the inspection of **Table 3** is that all the sample countries appear to assign a greater social welfare weight to equity considerations. Even though our findings seem to be in line with those of many other studies (see, for example, Sandmo, 1976; Forbes, 2000; Okun, 2015), it remains to be seen whether employing data from other countries or using alternative methodological procedures would differentiate the observed tendency of the tax systems in the sample countries to regard the fair distribution of income more highly than efficient allocation of resources. Last but not least, it should not escape our attention that the conclusions of the present study are based on the assumption that high direct tax rates distort optimal households' choices between work effort and leisure, whereas high indirect tax rates discourage consumption and cause a damage to the welfare state of the poor.

5. Concluding Remarks

Since the seminar work of Saez (2001), the focus of the tax literature has been on the direct role of labour supply elasticities, social preference parameters and the structure of the equity principle in shaping optimal income tax rates. It has been rare to frame these objects in terms of the ongoing trade-off discussion emphasized by the distortions and competing costs arising from efficiency considerations. This paper has presented a new characterization of the Mirrlees problem that is directly interpretable in terms of reaching an efficient trade-off of equity against efficiency and explores the practical insights that it provides.

Specifically, we defined two "distortion" variables that capture the cost of providing utility in a manner that allows the principle of equality between the marginal rate of substitution, $MRS_{c,l}$, and the marginal rate of transformation, $MRT_{c,l}$, to be adhered to more closely. We demonstrated that this object can be satisfied by introducing direct and indirect tax rate adjustments, which are capable of minimizing distortions in the labour market and/or in commodities' market. Moreover, our methodology can be used to analyze existing tax schedules for six developed countries, providing meaningful answers to questions such as: Is the cost of additional inefficiency too great to warrant improving the distribution of welfare or is the opposite true?

By employing a simple mathematical device, we showed how to operationalize such a question via determining appropriate combinations of (in)direct tax rates and using them to simulate alternative MRS-MRT equality conditions for policy makers. The results strongly implied that the tax systems of the sample countries are giving little weight to efficiency concerns relative to equity, in the sense that the efficiency gains would be greater than the opportunity cost associated with sacrificing part of the equity objectives.

These results are not definitive, though they do require an answer if the existing tax systems in the countries considered are to be defended. Two obvious criticisms are that they rely on the static version of the Mirrlees model and that they are sensitive to the precise specification of individual and social preferences. However, within our domain of ignoring possible dynamic extensions of the optimal policy problem and opting for ordinal preference maps and objective criteria—which rely on the curvature properties of the direct utility function—we believe that our conclusions do seem robust to most plausible parameterizations of the welfare maximization processes.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix 1

Estimating the Reaction Function

Consider the reaction function

$$\frac{\partial t_y}{\partial t_i} = 1 + \frac{t_i}{C}\frac{\partial c^h}{\partial t_i} + \frac{t_y}{C}\frac{\partial l^h}{\partial t_y} \tag{A1}$$

We have to agree to the following two crucial conditions as a prerequisite of solving Equation (A1):

1) The term $\frac{1}{C}\frac{\partial c^h}{\partial t_i}$ is a function solely of t_i, that is, $\frac{1}{C}\frac{\partial c^h}{\partial t_i} = a_1(t_i)$

2) The term $\frac{1}{C}\frac{\partial l^h}{\partial t_y}$ is a function solely of t_i, that is, $\frac{1}{C}\frac{\partial l^h}{\partial t_y} = a_2(t_i)$

If these conditions are met, Equation (A1) can take the following form $\frac{\partial t_y}{\partial t_i} = a_2(t_i)t_y + a(t_i)$ where $a(t_i) = 1 + t_i a_1(t_i)$, or

$$\frac{\partial t_y}{\partial t_i} = a_2(t_i)t_y + a(t_i) \tag{A2}$$

Equation (A2) is a first order linear differential equation, the solution of which is taken to express t_y as a function of t_i, as shown in the following relationship,

$$t_y(t_i) = \frac{1}{I(t_i)}\int_0^{t_i} I(t_i)a(t_i)\mathrm{d}t_i + t_y(0)\frac{1}{I(t_i)} \tag{A3}$$

where $I(t_i) = \mathrm{e}^{-\int_0^{t_i} a_2(t_i)\mathrm{d}t_i}$.

Appendix 2

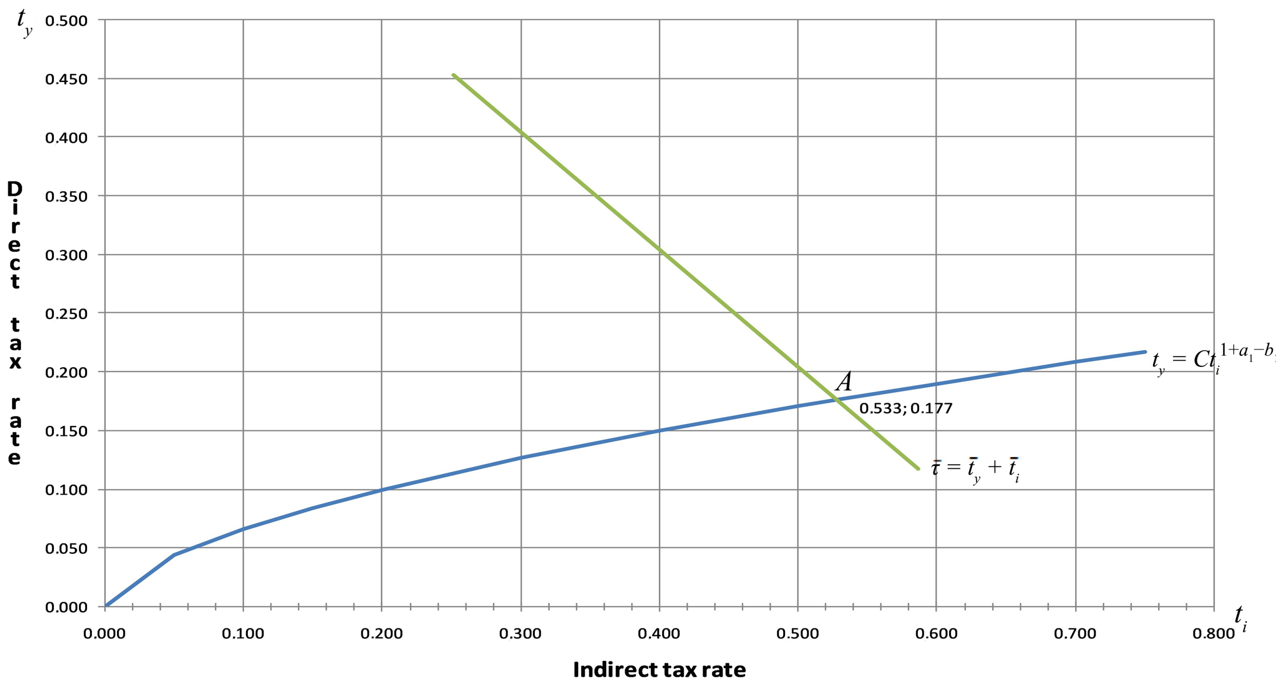

Figure A1. France: Optimal direct and indirect tax rates. Source: The above figure has been drawn on the basis of the data provided by estimating the relation $t_y = kt_i^{1+a_1-b_1}$ (Table 3).

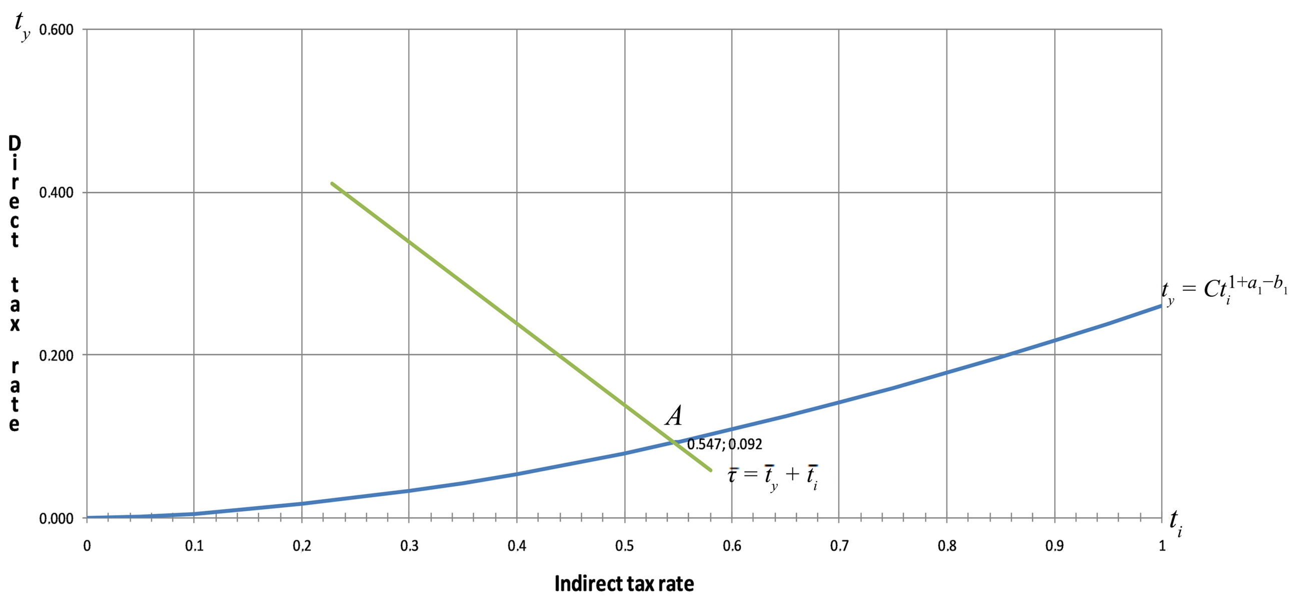

Figure A2. UK: Optimal direct and indirect tax rates. Source: The above figure has been drawn on the basis of the data provided by estimating the relation $t_y = kt_i^{1+a_1-b_1}$ (Table 3).

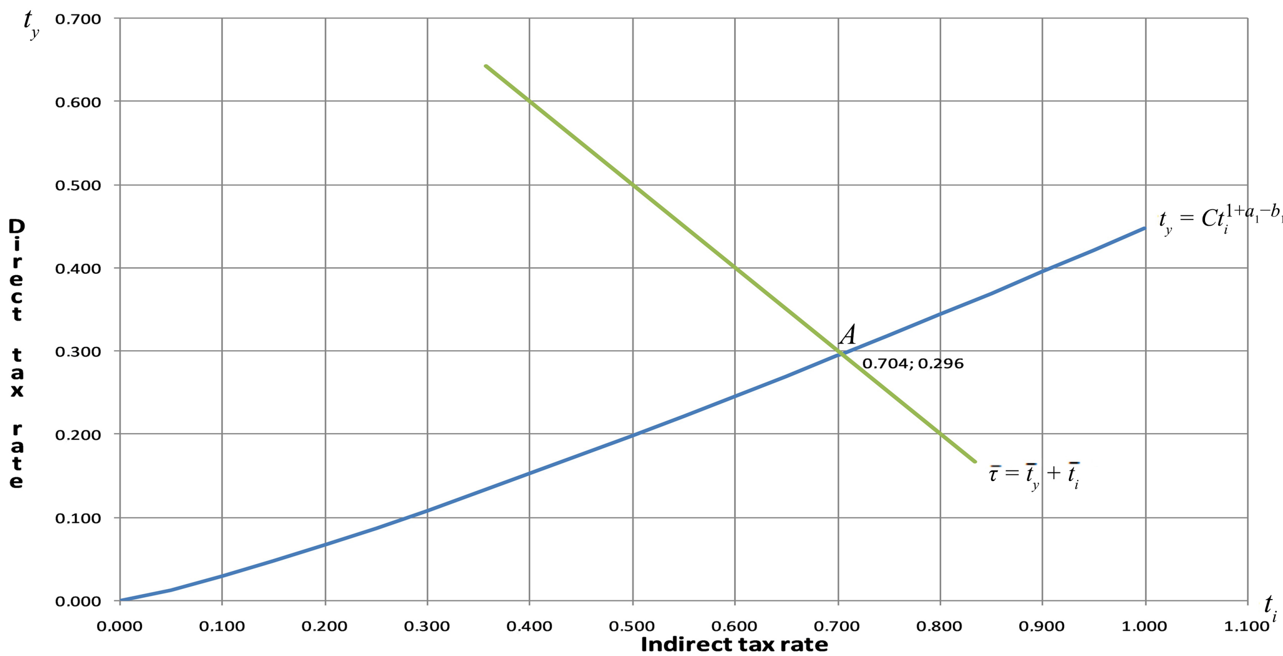

Figure A3. Italy: Optimal direct and indirect tax rates. Source: The above figure has been drawn on the basis of the data provided by estimating the relation $t_y = kt_i^{1+a_1-b_1}$ (Table 3).

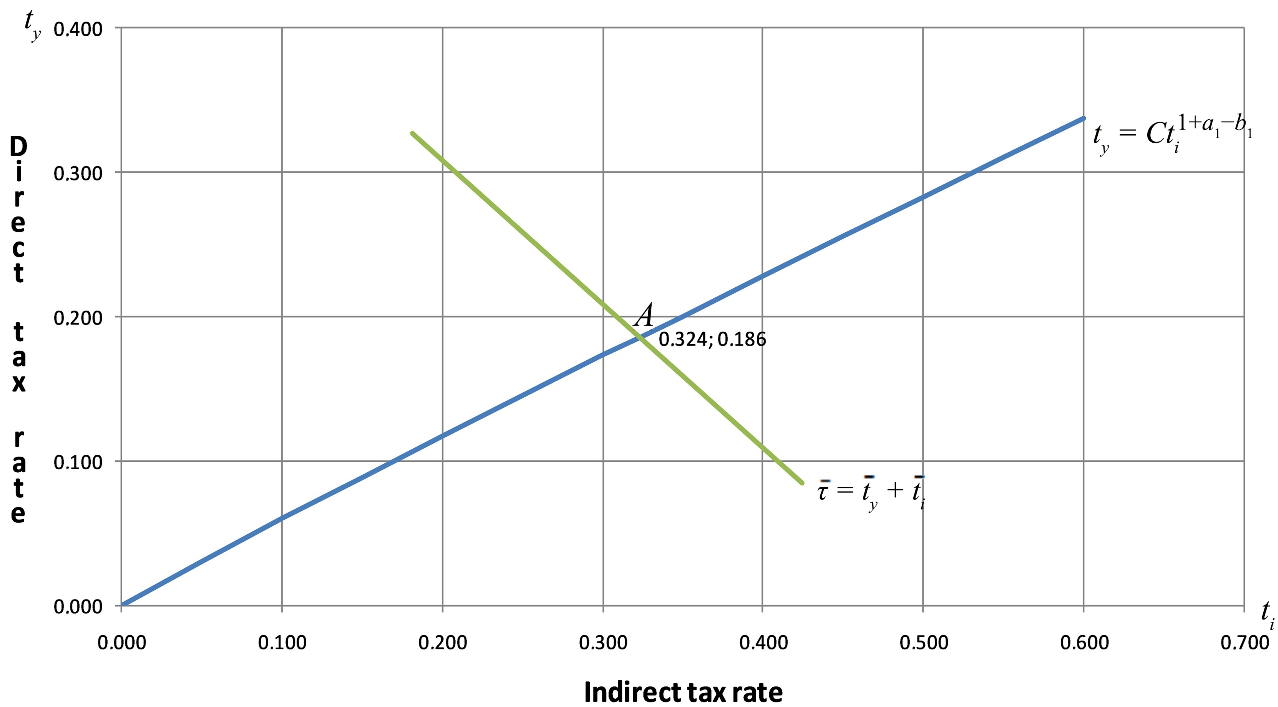

Figure A4. Germany: Optimal direct and indirect tax rates. Source: The above figure has been drawn on the basis of the data provided by estimating the relation $t_y = kt_i^{1+a_1-b_1}$ (Table 3).

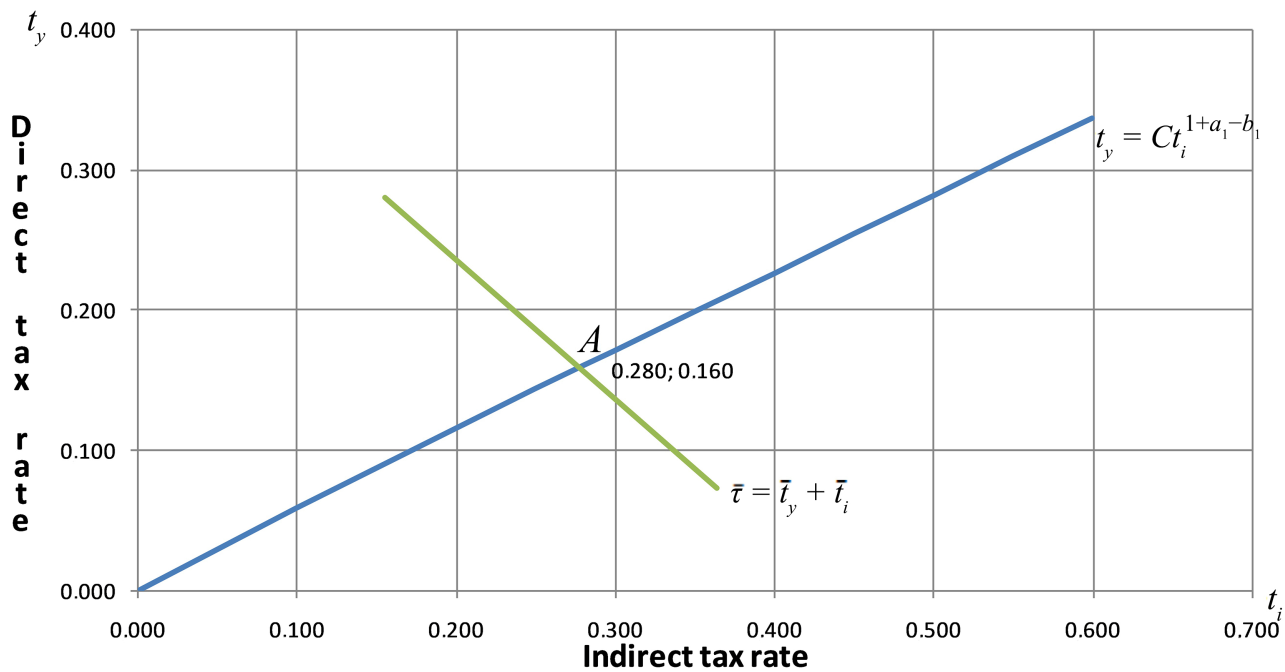

Figure A5. USA: Optimal direct and indirect tax rates. Source: The above figure has been drawn on the basis of the data provided by estimating the relation $t_y = kt_i^{1+a_1-b_1}$ (Table 3).

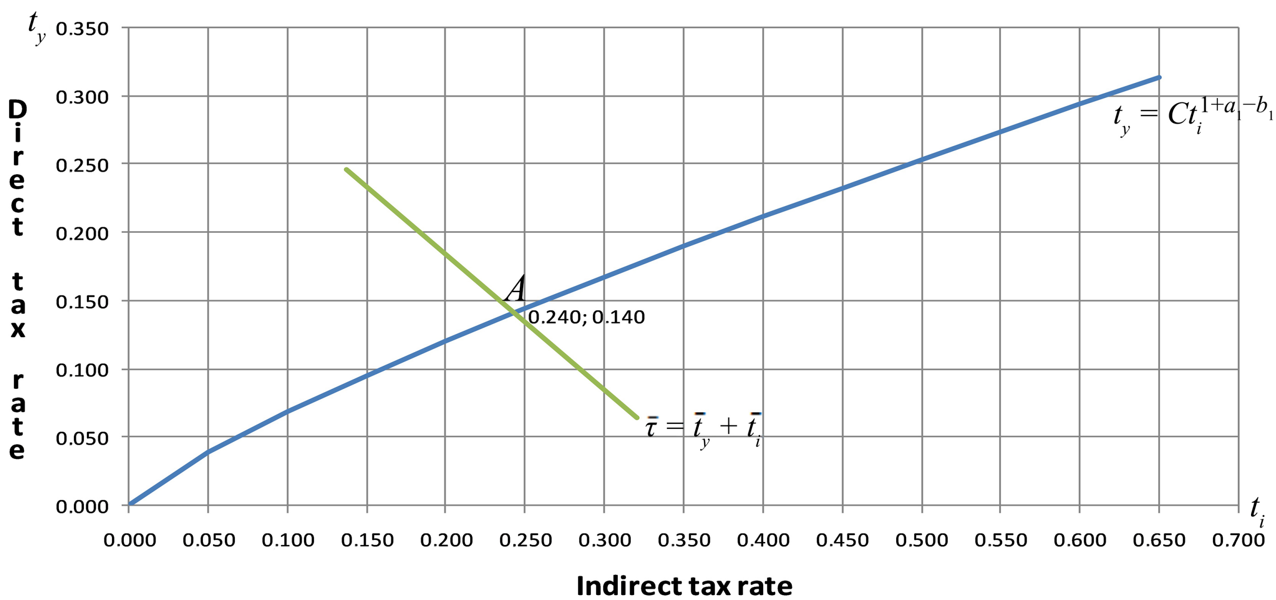

Figure A6. Japan: Optimal direct and indirect tax rates. Source: The above figure has been drawn on the basis of the data provided by estimating the relation $t_y = kt_i^{1+a_1-b_1}$ (Table 3).