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# One Plus One Equals Two: More or Less*

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## Abstract

Mathematics has many advanced applications in economics. By contrast, this note applies very simple instruments. We explore a range of cases where the proposition makes sense that one plus one is more or less than two. A first case drawn from commerce documents an example where units have more than one dimension. Examples from production economics (with reproduction and predator-prey type interactions) further illustrate how adding units affect the results of summation. The implied logical ambiguities described here can hamper human communication. Put differently, cognitive limitations may be the very reason why results of simple scalar additions are so often presented.

## Keywords

Vector Addition, Interacting Units, Economic Units with Multiple Dimensions

## JEL-Classification

C60, D91, Y8

## 1. Introduction

Mathematics has become a central instrument of analysis in many sciences. This certainly holds true for the field of economics. Key contributors to mathematical economics have reflected on this development over the years (Leontief, 1954; Kantorovich, 1989; Debreu, 1991; Romer, 2015). This note takes up a very basic mathematical concept, namely addition, to discuss variants of truths and insights in different applications. Discussing a fundamental concept like addition may seem arduous but the reader will quickly appreciate the interesting deviations from and extensions of the concept of addition[1]. Let us start with the obvious.

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[1]We do not deal here with the sort of systematic mistakes people tend to make when practicing arithmetic. See Cox (1975) for a good introduction into this field.

Adding natural numbers simply means taking one of the numbers and continuing to count as many times as the count of the other number[2]. The twist I want to discuss is the adding of units *other* than numbers. Naturally, the units to be added can be numbered. However, in contrast to mere numbers these units either possess more than one numerical dimension or they have the capacity to interact. We start with a case where adding units, despite obviously counting different things, works just like adding numbers. Consider units that belong to different groups, yet each unit qualifies as a unit within a more general category. Here is an example. Clearly, Mercedes and Volkswagen cars are both cars. Adding the number $c_m$ of Mercedes vehicles to the number $c_v$ of Volkswagens gives the number $c$ of cars

$$c = c_m + c_v . \tag{1}$$

Hence, nothing is at variance with simple addition. This case is displayed in the three dimensional **Figure 1**. The two axes spanning the horizontal plane indicate the two types of car while the vertical axis indicates the total number of cars. Clearly, one plus one equals two.

The same logic of simple addition obviously holds true when one adds vehicles of different types for a single manufacturer. So, car makers typically publish numbers of total vehicle production or total sales. The Ford Motor Company (2021: p. 5), e.g., reports a total (sum of) sales of all vehicles of 2.044 million units in 2020 and 1.905 million in 2021. Likewise there are national figures for the sum of sales of new cars, so, e.g., 14.972 million total units in 2021 in the U.S[3]. Obviously, this simple addition cannot reflect all aspects of the totals involved. Notably, the profit margins of various vehicle types differ, as do the $CO_2$ emissions of the various cars and trucks. This point directly brings us to the first class of cases where one plus one not necessarily equals two. Instead of continuing with the example of automobiles we choose a simple example from international finance.

## 2. Adding Units with a Second Dimension

In this example from the world of commerce we discuss adding units that have a second dimension. On a formal level the cases considered here are *vector additions*. Concretely, we look at a situation with two national currencies, say US Dollars ($) and British Pounds (£)[4]. Consider first just looking at the total number of coins of one currency unit each. If we add the numbers of coins of different currencies we have exactly the same situation as in our basic case with Mercedes and Volkswagen cars: simple addition holds and the corresponding display is the same as **Figure 1**. Formally, this is just

$$c = c_\$ + c_£ . \tag{2}$$

[2]Formally, this is linked to the Peano axioms for natural numbers (Hamilton, 1982). The classic reference to a rigorous treatment to the 1 + 1 = 2 issue is Russell and Whitehead (1913).

[3]See https://fred.stlouisfed.org/series/ALTSALES

[4]Nurkse (1944) is a classic reference for exchange rate economics.

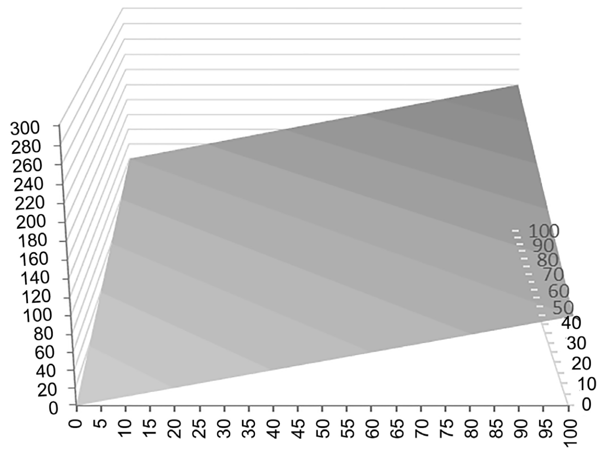



**Figure 1.** Plain addition.

Now, instead of the number of coins, consider expressing the value of the sum of these coins in *a single* currency. Let us define the exchange rate *e* as the number of £ units that have to be paid per unit of $. Except in the case where the exchange rate is one, we have a sum that is either more or less than the sum of units of currency[5]. Specifically, the outcome depends on which currency is chosen to express the common value. If it is valued in Pound Sterling then the sum expressed in £-value ( $cv_£$ ) is

$$cv_£ = ec_\$ + c_£ . \tag{3}$$

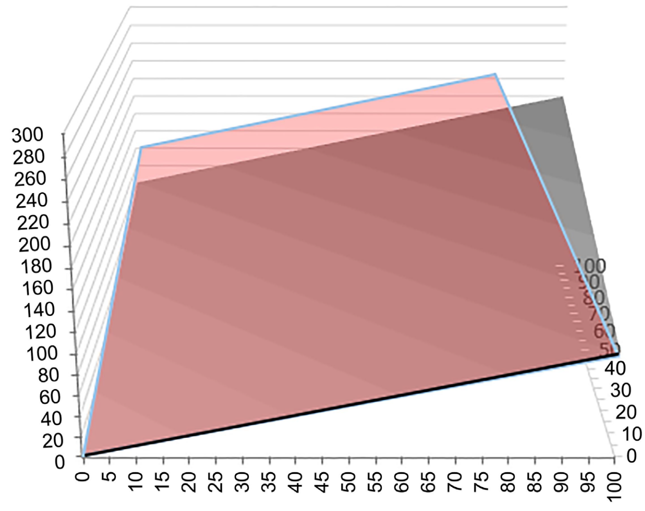
By contrast, if the value is expressed in US $ we have

$$cv_\$ = c_\$ + \frac{1}{e} c_£ . \tag{4}$$

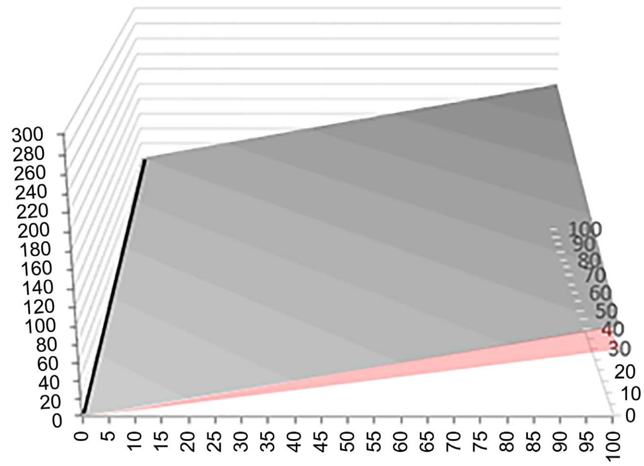
The two panels of **Figure 2** show these two cases. In each display the plane representing the simple sum of currency units is shown as the same plane also depicted in **Figure 1**. Since one unit of £ costs more than one unit of $, the sum of the $-value of coins is larger than the sum of currency units except in the case where there are zero Pounds. This is displayed in **Figure 2(a)** with the translucent plane. When the currency value is expressed in £ the reverse holds. The sum expressed as £-value is below the sum of currency units as shown in **Figure 2(b)**[6]. The two transparent planes displayed in the panels of **Figure 2(a)** and **Figure 2(b)** sandwich the plane shown in **Figure 1** and each of them has one (bold) line in common with it. Clearly, with exchange rates varying over time the planes displayed can change.

[5]Exchange rates values are rational numbers. In the example described here we take the Dollar-Pound rate as of the time of the writing which is approximately 1.35 $/£ (https://www.imf.org/external/np/fin/data/param_rms_mth.aspx). So this example is the first case considered where the sum will be a rational number.

[6]The value of a monetary unit in a common currency is clearly not the only relevant additional dimension. Consider weight: A dollar coin weighs 8.1 gram (https://www.usmint.gov/learn/coin-and-medal-programs/coin-specifications) whereas one pound sterling coin weighs 8.75 gram (https://www.royalmint.com/discover/uk-coins/coin-design-and-specifications/one-pound-coin/).




(a)




(b)

**Figure 2.** (a) The sum of two quantities of $ and £ expressed in US Dollars; (b) The sum of two quantities of $ and £ expressed in British Pounds.

The next section takes up a different sort of situation where one plus one does not necessarily add up to two. In particular, we explore situations with interactions between units. Two types of situation are studied where bringing into contact (adding) groups of units either increases or decreases the total of units.

## 3. Interaction of Units

Let us start with a description of examples of economic phenomena that are in this class of cases. Consider two important forms of interaction prevalent in the natural world: reproduction and predator-prey interaction. Both cases, besides their more general importance, appear in economics. In particular production economics concerned with animal husbandry can serve as an illustration. Clearly, the "adding" we have in mind here means physically bringing together units. In both cases we keep the modeling to the simplest possible. As part of the abstraction all issues of timing are ignored.

We start with the case of reproduction where one plus one will sum to more than two. In order to keep the analysis simple and the results comparable to the presentation in the earlier sections we assume that if one female and one male come together there will be one offspring[7]. Formally, we have for the number of animals ( $a$ )

$$a = a_f + a_m + \min\left(a_f, a_m\right), \tag{5}$$

where $a_f$ and $a_m$ are the number of females and males, respectively. **Figure 3** visualizes the outcome of this sort of addition. Only along the individual axes (either adding females to females or males to males) is it true here that one plus one equals two. By contrast, moving along the 45 degree line in the horizontal plane, one plus one is now three (e.g., one female and one male with one offspring sum to three)[8].

The next case is the adding of predators and prey animals, relevant, e.g., in fisheries management. Again, all dynamics are left out[9]. Here, we make the simple assumption that each predator devours one unit of prey. In this case the number of animals is

$$a = a_{pred} + a_{prey} - a_{prey}\left(\text{if } a_{pred} \geq a_{prey}\right) - a_{pred}\left(\text{if } a_{pred} < a_{prey}\right). \tag{6}$$

where $a$ is the number of animals with $a_{pred}$ and $a_{prey}$ the number of predators and prey, respectively. With the formulation of Equation (6) we assume that predators, at least for the moment, can survive without eating. **Figure 4** shows the outcome of this sort of addition. Moving along the 45 degree line in the horizontal plane, one plus one here adds up to one. As with the case before it

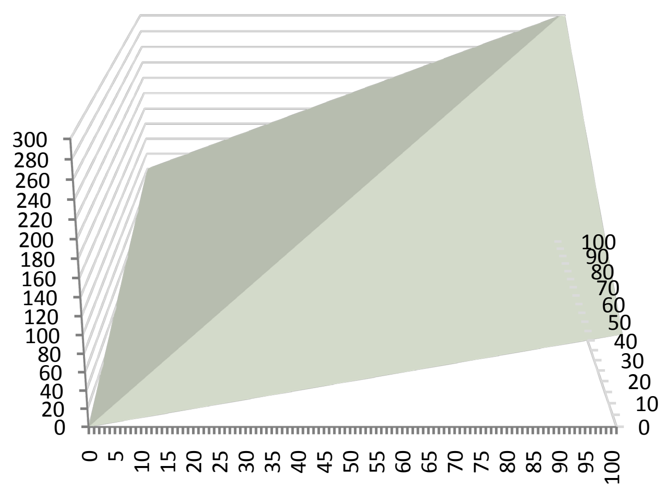



**Figure 3.** Reproduction.

[7]Fusco and Minelli (2019) offer a comprehensive treatment of reproduction biology.

[8]Note that in the case of hermaphroditic species, like some mollusks, the display would simplify to a plane with each extra individual adding more than one to the population.

[9]The classic contribution describing predator-prey dynamics is Lotka (1910). A good introduction to fisheries management is Flaaten (2016).

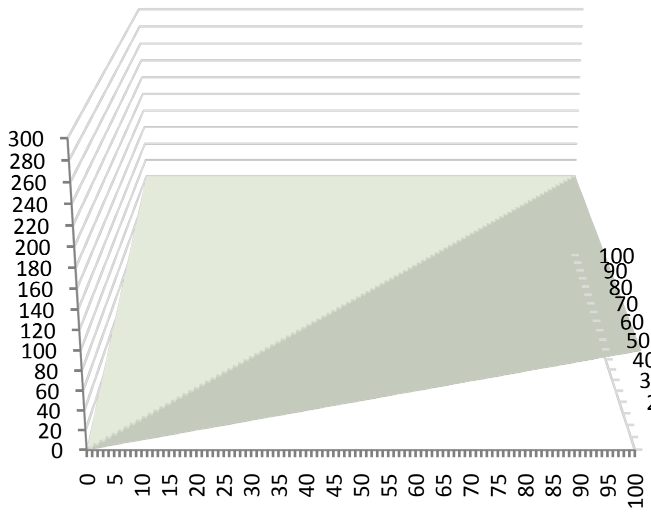



**Figure 4.** Predator and prey.

is only along the individual axes (either adding predators to predators or prey to prey) that one plus one actually equals two.

The planes displayed in Figure 3 and Figure 4 envelop the plane shown in Figure 1. Further, the space between the planes of Figure 3 (4) and Figure 1 mark the range of results where one plus one equals more (less) than two.

## 4. Conclusion and Discussion

This note discusses a range of cases where the proposition makes sense that one plus one may add up to more or less than two. A first case drawn from the world of international commerce documents an example where units have more than one dimension. The examples from production economics indicate that interaction between addable units can change also their sum. Clearly, the cases studied here are not limited to the field of economics. Vector addition appears in many other fields of applied mathematics and, e.g., predator-prey dynamics can be found throughout the natural world.

Once we get started on the sort of thinking outlined here we can easily think of a range of further cases for this sort of situation. Consider the example of "dice and dots". This is a case where the sum measured along the second dimension will be equal or larger than the sum of units. By contrast, the example "accused and convicts" marks a situation where the sum along the second dimension is smaller or equal to the sum along the first dimension. Further, "trees and leaves" is a case where the sum along the first dimension can be smaller, larger, or equal to the sum along the second dimension. Finally, it is obvious that adding two heaps of snow results in just one, albeit bigger, heap of snow. Whether and how these cases can illuminate economic phenomena is left to the imagination of the reader and to future research.

Further, and beyond the logical dimension of the topic discussed here, there are also behavioral aspects involved. First, humans may not succeed in agreeing on the results of numerical operations if they fail to first clarify the specifics of these operations[10]. Second, the very reason why results of simple scalar additions are often presented may be the result of cognitive heuristics. Why else should anybody care for a total of, e.g., vehicle sales in a country if not for the purpose of reducing complexity?

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

## References

Cox, L. S. (1975). Systematic Errors in the Four Vertical Algorithms in Normal and Handicapped Populations. *Journal for Research in Mathematics Education, 6,* 202-220. https://doi.org/10.2307/748696

Debreu, G. (1991). The Mathematization of Economic Theory. *American Economic Review, 81,* 1-7.

Ferreira, M. K. L. (1997). When 1 + 1 ≠ 2: Making Mathematics in Central Brazil. *American Ethnologist, 24,* 132-147. https://doi.org/10.1525/ae.1997.24.1.132

Flaaten, O. (2016). *Fisheries Economics and Management.*

Ford Motor Company (2021). *Annual Report.* https://www.annualreports.com/HostedData/AnnualReports/PDF/NYSE_F_2021.pdf

Fusco, G., & Minelli, A. (2019). *The Biology of Reproduction.* Cambridge University Press. https://doi.org/10.1017/9781108758970

Hamilton, A. G. (1982). *Numbers, Sets and Axioms: The Apparatus of Mathematics.* Cambridge University Press.

Kantorovich, L. V. (1989). Mathematics in Economics: Achievements, Difficulties, Perspectives. *American Economic Review, 79,* 18-22.

Lave, J. (1988). *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life.* Cambridge University Press. https://doi.org/10.1017/CBO9780511609268

Leontief, W. (1954). Mathematics in Economics. *Bulletin of the American Mathematical Society, 60,* 215-233. https://doi.org/10.1090/S0002-9904-1954-09791-4

Lotka, A. J. (1910). Contribution to the Theory of Periodic Reaction. *Journal of Physical Chemistry, 14,* 271-274. https://doi.org/10.1021/j150111a004

Nurkse, R. (1944). *International Currency Experience.* League of Nations.

Reed, H. J., & Lave, J. (1979). Interdisciplinary Anthropology. *American Ethnologist, 6,*

[10]The social element of cognition (i.e., the shared practices) has been documented to shape the use of arithmetic. In her ethnographic study Ferreira (1997) describes the various ways in which indigenous people in Brazil add and come to different results depending on the social situation. In anthropology, Reed and Lave (1979) and Lave (1988) have documented that various forms of doing arithmetic coexist as tools for professional problem solving and that the results of calculations, according to the experience of users, may differ.

568-582. https://doi.org/10.1525/ae.1979.6.3.02a00100

Romer, P. M. (2015). Mathiness in the Theory of Economic Growth. *American Economic Review, 105,* 89-93. https://doi.org/10.1257/aer.p20151066

Russell, B., & Whitehead, A. N. (1913). *Principia Mathematica.* Cambridge University Press.