Money and Economic Growth Revisited: A Note

Eric Kam[1], John Smithin[2]

[1]Ryerson University, Toronto, Canada
[2]York University, Toronto, Canada
Email: erickam@ryerson.ca, jsmithin@yorku.ca

Abstract

In an important but neglected paper, Begg (1980) attempted to solve the puzzle of monetary super-neutrality in the steady-state. Super-neutrality was shown to depend on two sufficient conditions, only one of which is necessary. Begg argued that a more general specification restores monetary *non*-super-neutrality. This note suggests an additional sufficient condition for super-neutrality. The demand for real balances must be modeled as a decreasing function of the real interest rate. This has implications for models assuming a steady-state. Harkness (1978) had already shown that the extra sufficient condition is a *necessary* condition for existence.

Keywords

Monetary Super-Neutrality, Money and Growth, Rational Expectations, Real Interest Rate, Wealth Effect

1. Introduction

In an important but neglected paper, Begg (1980) attempted to solve the puzzle of monetary super-neutrality in the steady-state. The proposition of super-neutrality, that changes in the rate of growth of the money supply do not affect real economic variables—which was taken for granted in the rational expectations (RE) literature (Lucas, 1981)—was shown to depend on two sufficient conditions, only one of which is necessary. Begg argued that a more general specification restores the argument for monetary *non*-super-neutrality made earlier by such writers as Mundell (1963) and Tobin (1965).

This note argues there is an additional sufficient condition for monetary super-neutrality in the class of models described by Begg. Money is super-neutral if the demand for real balances is a decreasing function of the real interest rate. This possibility was never considered in the macroeconomic literature of the period. It should have been. Two years before Begg's paper Harkness (1978) had

shown that this additional sufficient condition is a *necessary* condition for the existence of the steady-state in neoclassical models.

The argument does not end debate over whether monetary policy is neutral or super-neutral. During these discussions in the twentieth century monetary policy was conceived of, naively, as entailing only deliberate changes in the rate of money supply growth. No attention was paid to such issues as endogenous money, endogenous time preference, or the rate of interest as a policy instrument. More careful consideration of these topics has the potential to affect the results dramatically (Kam, 2005; Kam, Smithin and Tabassum, 2019). However, Harkness's proof *is* decisive for the entire class of models under discussion at the time.

Section 2 describes Begg's two sufficient conditions for monetary super-neutrality. Section 3 constructs a macroeconomic model to formally derive them. Section 4 derives the third sufficient condition. The concluding section offers a summary.

2. Sufficient Conditions for Monetary Super-Neutrality

Monetary super-neutrality exists when changes in the rate of money supply growth do not affect the real economy (Sidrauski, 1967a, 1967b). Begg revealed an inconsistency in this respect between two competing literatures, the analysis of macroeconomic models featuring RE (Lucas, 1972, 1981; Sargent and Wallace, 1975; Barro, 1976) *versus* an earlier literature on money in neoclassical growth models (Mundell, 1963; Tobin, 1965; Johnson, 1967). The RE literature portrayed monetary policy as super-neutral in the steady-state. The neoclassical money and growth model (NMG) described monetary policy as *non*-super-neutral. Begg tried to solve the puzzle by deriving a general specification which violates super-neutrality.

Begg's argument was consistent with a central feature of RE namely that, "in a steady state, any expectations generating mechanism…yield[s] correct predictions" (Begg, 1980). Therefore, NMG should be interpreted as a special case of RE, thus making the inconsistency between the two the more surprising. Begg concluded that RE models lacked one important feature of NMG models, the presence of a wealth effect in the consumption function.

3. A Simple Formal Model

In Smithin's notation (Smithin, 1980), Begg's argument is;

$$y = c(y, w) + \mathrm{d}k/\mathrm{d}t + \delta k, \quad 0 < c_y < 1, c_w > 0 \tag{1}$$

$$m = l(y, i), \quad l_y > 0, l_i < 0 \tag{2}$$

$$y = f(k), \quad f_k > 0, f_{kk} < 0 \tag{3}$$

$$w = k + m \tag{4}$$

$$i = r + \pi \tag{5}$$

$$r = f_k - \delta \tag{6}$$

$$\mathrm{d}m/\mathrm{d}t = m(\theta - \pi) \tag{7}$$

Here y is real output, c the consumption function, w total real wealth, k the capital stock, m the real money stock, l the liquidity preference function, f the production function, i the nominal interest rate, r the real interest rate, and θ the rate of monetary growth. The symbols π, δ and θ stand for inflation, the depreciation rate and the rate of monetary growth. In the steady-state;

$$\mathrm{d}k/\mathrm{d}t = \mathrm{d}m/\mathrm{d}t = 0 \tag{8}$$

The steady-state capital stock, k^*, is then found by solving the following equation[1],

$$f\left(k^*\right) = c\left\{f\left[k^*\right], k^* + l\left[f\left(k^*\right), f_k\left(k^*\right) - \delta - \theta\right]\right\} + \delta k^* \tag{9}$$

The effect of a change in the monetary growth rate, θ, is thus given by;

$$\mathrm{d}k/\mathrm{d}\theta = c_w l_i \Big/ \left[f_k\left(1 - c_y - c_w l_y\right) - c_w\left(1 + l_i f_{kk}\right) - \delta \right] \tag{10}$$

If money is super-neutral, $\mathrm{d}k^*/\mathrm{d}\theta = 0$. Sufficient conditions for super-neutrality are therefore;

$$c_w = 0 \tag{11}$$

$$l_i = 0 \tag{12}$$

The first condition implies the absence of a wealth effect, the second eliminates the interest rate term in the demand for money function. Begg (1980) rejects the first, and dismisses the second as "…deny[ing] Keynesian liquidity preference".

The sign of $\mathrm{d}k^*/\mathrm{d}\theta$ is ambiguous, but Begg resolves this by examining the dynamic properties of the system, appealing to Samuelson's correspondence principle. If the system is to be "saddle-point stable" (Sargent, 1973) $\mathrm{d}k^*/\mathrm{d}\theta$ must be positive.

4. A Third Sufficient Condition

The upshot of Harkness's (1978) argument about existence was to bring into question the conventional specification of the demand for money function. Agents supposedly perceive the return on real balances to be the negative of the inflation rate. The opportunity cost of holding real balances is therefore the rate of return foregone by not holding other assets *plus* the inflation rate. If λ stands for the opportunity cost of holding real balances;

$$\lambda = r - (-\pi) = r + \pi \tag{13}$$

However, Harkness went into this question more deeply by examining the existence conditions for the steady-state in a model in which new money is injected *via* direct transfer payments to the economic agents. The transfers can ei-

---

[1]The main analytical result of the NMG model was a solution for capital market equilibrium. The growth rate itself was determined by exogenous factors, such as technical progress or the rate of growth of the labour force.

ther be completely random (as in Friedman's famous "helicopter") or tied to initial holdings at a rate which may, or may not, differ across individuals.

Harkness was able to demonstrate that a random distribution "is inconsistent with the existence of stable steady-state equilibrium". Also the only transfer rate that will avoid "distributional effects" (Harkness, 1978) is the same for all individuals and equal to the rate of monetary growth adjusted for population growth. What is the opportunity cost of holding real balances in such a world (Smithin, 1983)? Letting τ stand for the transfer rate, the opportunity cost of holding real money balances turns out to be;

$$\lambda = r + \pi - \tau \tag{14}$$

In Begg's model, the rate of population growth is zero and the transfer rate is $\tau = \theta$. In equilibrium the inflation rate is $\pi = \theta$. The opportunity cost of holding real balances is;

$$\lambda = r + \theta - \theta = r \tag{15}$$

This, therefore, is the appropriate argument in the demand for money function. Replacing $m = l(y, i)$ with $m = l(y, r)$, where $l_y > 0$ and $l_r < 0$, modify Equation (9) to read;

$$f\left(k^*\right) = c\left\{f\left[k^*\right], k^* + l\left[f\left(k^*\right), i - \theta\right]\right\} + \delta k^* \tag{16}$$

Totally differentiating;

$$f_k \, \mathrm{d}k^* = c_y f_k \, \mathrm{d}k^* + c_w l_r \, \mathrm{d}k^* + c_w l_r f_k \, \mathrm{d}k^* + c_w l_r \, \mathrm{d}i - c_w l_r \, \mathrm{d}\theta + \delta \, \mathrm{d}k^* \tag{17}$$

In the steady-state;

$$\mathrm{d}i = f_{kk} \, \mathrm{d}k^* - \delta \, \mathrm{d}k^* + \mathrm{d}\theta \tag{18}$$

Then substituting (18) into (17);

$$\mathrm{d}k^* / \mathrm{d}\theta = \left(c_w l_r - c_w l_r\right) / \left[f_k\left(1 - c_y - c_w l_y\right) - c_w\left(1 + l_r f_{kk}\right)\right] = 0 \tag{19}$$

This is a third sufficient condition guaranteeing super-neutrality independent of either the values or signs of c_w and l_r.

5. Conclusion

Given common roots in the neoclassical growth model neither the NMG literature nor RE models account for monetary *non*-super-neutrality. In spite of the claims of Mundell, Tobin (and later Begg) it is not possible to restore the idea of "forced saving" (Kam, 2005) in that framework. In order to establish these results, this note has been able to integrate the literature on neutrality and super-neutrality with that on the existence conditions for equilibrium in the neoclassical money and growth model.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Barro, R. (1976). Rational Expectations and the Role of Monetary Policy. *Journal of Monetary Economics, 2,* 1-32. https://doi.org/10.1016/0304-3932(76)90002-7

Begg, D. (1980). Rational Expectations and the Non-Neutrality of Systematic Monetary Policy. *Review of Economic Studies, 48,* 293-303. https://doi.org/10.2307/2296993

Harkness, J. (1978). The Neutrality of Money in Neoclassical Growth Models. *Canadian Journal of Economics, 11,* 701-713. https://doi.org/10.2307/134374

Johnson, H. (1967). Money in a Growth Model. In H. Johnson (Ed.), *Essays in Monetary Economics* (pp. 161-178). Harvard University Press.

Kam, E. (2005). A Note on Time Preference and the Tobin Effect. *Economics Letters, 89,* 127-132. https://doi.org/10.1016/j.econlet.2005.05.021

Kam, E., Smithin, J., & Tabassum, A. (2019). The Long-Run Non-Neutrality of Monetary Policy: A General Statement in a Dynamic General Equilibrium (DGE) Model. *Review of Political Economy, 31,* 202-217. https://doi.org/10.1080/09538259.2019.1642551

Lucas, R. (1972). Expectations and the Neutrality of Money. *Journal of Economic Theory, 4,* 103-124. https://doi.org/10.1016/0022-0531(72)90142-1

Lucas, R. (1981). Tobin and Monetarism: A Review Article. *Journal of Economic Literature, 19,* 558-567.

Mundell, R. (1963). Inflation and Real Interest. *Journal of Political Economy, 71,* 280-283. https://doi.org/10.1086/258771

Sargent, T. (1973). The Stability of Models of Money and Growth with Perfect Foresight. *Econometrica, 41,* 1043-1048. https://doi.org/10.2307/1914034

Sargent, T., & Wallace, N. (1975). Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule. *Journal of Political Economy, 83,* 241-254. https://doi.org/10.1086/260321

Sidrauski, M. (1967a). Inflation and Economic Growth. *Journal of Political Economy, 75,* 796-810. https://doi.org/10.1086/259360

Sidrauski, M. (1967b). Rational Choice and Patterns of Growth in a Monetary Economy. *American Economic Review Papers and Proceedings, 57,* 534-544.

Smithin, J. (1980). *On the Sources of the Super-Neutrality of Money in the Steady-State, Working Paper 80-14.* Department of Economics, McMaster University.

Smithin, J. (1983). A Note on the Welfare Cost of Perfectly Anticipated Inflation. *Bulletin of Economic Research, 35,* 65-69. https://doi.org/10.1111/j.1467-8586.1983.tb00447.x

Tobin, J. (1965). Money and Economic Growth. *Econometrica, 33,* 671-668. https://doi.org/10.2307/1910352