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Two Simple Formulas Relating the Growth and Profit Rates

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Abstract

This article compares the magnitudes of the growth and profit rates within an economic model that represents two successive periods of production. Its main result is a pair of simple formulas determining respectively the (growth rate)/(profit rate) ratio and the (profit rate) – (growth rate) difference. The first formula permits to show that the ratio is a decreasing function of the capital/income ratio. The second formula allows us to point out that the difference, as a general rule, is an increasing function of the capital/income ratio although, under certain conditions, it may be otherwise. Both formulas consider the simplest case, when savings equals capital increase and, in addition, the capital/income ratio is constant. The general case in which these conditions are not necessarily satisfied is also considered. In the simplest case, the ratio between the two rates is equal to that between the savings rate and the capital share while the difference is equal to that between these same variables divided by the capital/income ratio. In the general case, the results just indicated are modified by a quotient having in both formulas the same numerator: the capital increase not due to savings (as a fraction of income) minus the increase in the capital/income ratio. In the first formula, the quotient is preceded by a positive sign and its denominator is the capital share while in the second one it is preceded by a negative sign and its denominator is the capital/income ratio. As illustrated by means of an empirical application, these results help to explain the inequality between the growth and profit rates whose importance for income distribution is underlined by Piketty.

Keywords

Growth Rate, Income Inequality, Piketty, Profit Rate

1. Introduction

One of the salient features of a modern economy, as demonstrated by Piketty

(2014, pp. 350-358), is that, as a general rule, the profit rate is higher than the growth rate. This may not be the case for a given country during long periods of time but it is still possible that some capital investments get a profit rate greater than the growth rate of the country in one such period. As the gap between the two rates becomes greater for a particular investment, the opportunity increases for the investor to enlarge his personal fraction of the country's capital stock. Furthermore, Piketty (2014: pp. 271-303) showed that, in higher income strata, income consists mostly of profits and also that greater capital investments tend to obtain higher profit rates (Piketty, 2014: pp. 430-467). These combined facts produce a tendency for income to concentrate at the higher strata. To emphasize its importance, Piketty (2014: pp. 25-27) presents the inequality as the main cause of greater income inequality. The relevance of Piketty's work has been widely recognized by the specialized literature (e.g., Boushey et al., 2017; Galbraith, 2014; Grantham, 2015; Lindert, 2014; Solow, 2014) and there have been also critical comments (e.g., Delsol et al., 2017; Giraud, 2014; Mankiew, 2015; Rognlie, 2015).

Concerning the origin of the inequality between the two rates Piketty (2014: p. 358) says, on the one hand, that it is a historical fact that derives from multiple causes and, in a later work Piketty (2015: p. 49) notes that this inequality holds true in the steady-state equilibrium of the most common economic models. On the other hand, Piketty (2011: p. 39) pointed out that in real-world economies, the profit share is usually much larger than the savings rate and this leads, in the Cobb-Douglas production function specification, to a steady-state situation where the profit rate is larger than the growth rate. On this regard, it is worth mentioning the "Second Fundamental Law of Capitalism" postulated in Piketty (2014: p. 358), that is, the long-term equality between the capital/income ratio and the (profit share)/(growth rate) ratio. It is probably due to the importance that he concedes to this principle that his discussions of the inequality between the profit and growth rates are mostly referred to the long-term setting without presenting a formula relating the profit and growth rates in the general case.

This article compares the magnitudes of the growth and profit rates within an economic model that represents two successive periods of production. It incorporates such factors as changes in relative prices and capital gains that normally play a role in the determination of growth rate. The main result is a couple of formulas determining respectively the growth rate)/(profit rate) ratio and the (profit rate) − (growth rate) difference. The first formula permits to show that the ratio is a decreasing function of the capital/income ratio. The second formula allows us to point out that the difference, as a general rule, is an increasing function of the capital/income ratio although, under certain conditions, it may be otherwise.

According to the first formula, the (growth rate)/(profit rate) ratio is equal to the sum of two fractions having both as denominator the capital share. The numerator of the first fraction is the savings rate and that of the second one, called here for short the capital term, is the capital increase not due to savings (measured with the national income) minus the increase in the capital/income ratio. In turn, according to the second formula, the (profit rate) − (growth rate) difference is determined by the difference between two fractions having both as denominator the capital/income ratio. The numerator of the first fraction is equal to the capital share minus the savings rate and that of the second one is the capital term. Both formulas allow for estimation, on the one hand, of the part due to each fraction in the determination of the corresponding function and, on the other hand, of the effect that a change taking place in each fraction has on the evolution of the corresponding function.

These results help to investigate the origin of the inequality between the growth and profit rates underlined by Piketty. As an example of this, we study the French economy during periods 1949-1979 and 1979-2009. We found, on the one hand, that the net effects of the capital/income ratio considered in the first formula were relatively small and, for this reason, the (growth rate)/(profit rate) ratio was predominantly determined in both periods by the (savings rate)/(capital share) ratio. On the other hand, the (profit rate) − (growth rate) difference was determined mainly by the capital/income ratio and by the difference between the capital share and the savings rate. Similar results were found concerning the evolution from one period to the other of both the (growth rate)/(profit rate) ratio and the (profit rate) − (growth rate) difference. On the basis of these results, two interesting tasks for future researches can be pointed out: on the one hand, to establish the extent to which conclusions similar to those presented here can be found in other cases. On the other hand, it is to study the determination of the saving rate and of the capital share in relation with the theme of this paper.

Including this introduction, the paper has 7 sections. Section 2 presents the model studied here while Sections 3 and 4 define certain relations between some of its main variables. Section 5 establishes the two formulas mentioned above and Section 6 applies them to empirical data in order to illustrate their utility. Some final comments are presented in the last section.

2. The Model

We consider an economy undergoing a succession of annual production cycles ending at dates $t = 1, 2, \cdots$ and, to identify each one of them, we refer to the date corresponding to the end of the production year. As usual, a variable indexed with a t represents its value in cycle t which may correspond to any date unless otherwise indicated. The notations Y_t and K_t represent respectively the national income obtained and the national capital used while β_t is the corresponding capital/income ratio. Thus,

$$\beta_t = \frac{K_t}{Y_t} \tag{1}$$

Furthermore, ΔK_t (for $t > 1$), S_t, Π_t, s_t, α_t and r_t denote respectively the capital increase, the amount of savings, the amount of profit, the savings rate, the capital share and the profit rate. Then:

$$\Delta K_t = K_t - K_{t-1} \tag{2}$$

$$s_t = \frac{S_t}{Y_t} \tag{3}$$

$$\alpha_t = \frac{\Pi_t}{Y_t} \tag{4}$$

$$r_t = \frac{\Pi_t}{K_t} \tag{5}$$

Finally, for $t > 1$, D_t is the difference between the capital increase in period *t* and the amount of savings in period $t-1$, d_t represents the ratio between this difference and the national income in period $t-1$ and g_t is the growth rate of national income from cycle $t-1$ to cycle *t*. Therefore,

$$D_t = \Delta K_t - S_{t-1} \tag{6}$$

$$d_t = \frac{D_t}{Y_{t-1}} \tag{7}$$

$$1 + g_t = \frac{Y_t}{Y_{t-1}} \tag{8}$$

3. Further Relations between the Main Variables

Equations (1), (2), (4), (6) and (8) imply respectively the following relations:

$$\frac{1}{\beta_t} = \frac{Y_t}{K_t} \tag{9}$$

$$K_t = K_{t-1} + \Delta K_t \tag{10}$$

$$\Pi_t = \alpha_t Y_t \tag{11}$$

$$\Delta K_t = S_{t-1} + D_t \tag{12}$$

$$Y_t = (1 + g_t) Y_{t-1} \tag{13}$$

In turn, Equations (5) and (11), taken together, imply that:

$$r_t = \alpha_t \left(\frac{Y_t}{K_t} \right) \tag{14}$$

while Equations (9) and (14), taken together, imply that:

$$r_t = \frac{\alpha_t}{\beta_t} \tag{15}$$

Now, dividing both sides of Equation (12) by Y_{t-1} we get:

$$\frac{\Delta K_t}{Y_{t-1}} = \frac{S_{t-1}}{Y_{t-1}} + \frac{D_t}{Y_{t-1}} \tag{16}$$

Substituting the first and second term on the right-hand side of this equation respectively by the left-hand sides of Equations (3) (corresponding to period

$t-1$) and (7) results in:

$$\frac{\Delta K_t}{Y_{t-1}} = s_{t-1} + d_t \tag{17}$$

$\Rightarrow$

$$\Delta K_t = \left(s_{t-1} + d_t\right) Y_{t-1} \tag{18}$$

4. The Growth Rate

In this Section, a formula is developed relating the growth rate to some of the main variables of the model.

4.1. The Growth Rate and Harrod's Fundamental Equation

Substituting the second term on the right-hand side of Equation (10) by the right-hand side of Equation (18) we get:

$$K_t = K_{t-1} + \left(s_{t-1} + d_t\right) Y_{t-1} \tag{19}$$

and substituting the numerator on the right-hand side of Equation (1) by the right-hand side of Equation (19) yields:

$$\beta_t = \frac{K_{t-1} + \left(s_{t-1} + d_t\right) Y_{t-1}}{Y_t} \tag{20}$$

Moreover, substituting the denominator on the right-hand side of this equation by the right-hand side of Equation (13) we obtain:

$$\beta_t = \frac{K_{t-1} + \left(s_{t-1} + d_t\right) Y_{t-1}}{\left(1 + g_t\right) Y_{t-1}} \tag{21}$$

Dividing both numerator and denominator on the right-hand side of this equation by Y_{t-1} results in:

$$\beta_t = \frac{\frac{K_{t-1}}{Y_{t-1}} + \left(s_{t-1} + d_t\right)}{1 + g_t} \tag{22}$$

Finally, substituting the first term of the numerator on the right-hand side of this equation by the left-hand side of Equation (1) corresponding to period $t-1$ yields:

$$\beta_t = \frac{\beta_{t-1} + \left(s_{t-1} + d_t\right)}{1 + g_t} \tag{23}$$

$\Rightarrow$

$$1 + g_t = \frac{\beta_{t-1} + \left(s_{t-1} + d_t\right)}{\beta_t} \tag{24}$$

$\Rightarrow$

$$g_t = \frac{\beta_{t-1} + \left(s_{t-1} + d_t\right)}{\beta_t} - 1 \tag{25}$$

$$= \frac{\beta_{t-1} + (s_{t-1} + d_t) - \beta_t}{\beta_t} \quad (26)$$

$$= \frac{s_{t-1} + d_t - (\beta_t - \beta_{t-1})}{\beta_t} \quad (27)$$

Let:

$$\vartheta_t = d_t - (\beta_t - \beta_{t-1}) \quad (28)$$

This term represents the net effect of three ratios relating capital and income in the numerator of Equation (27): on the one hand the ratio between the capital increase (excepting savings) and income, on the other hand, the capital/income ratios of the two periods of production. To simplify, I will refer to it as the capital term, it permits to write that equation as follows:

$$g_t = \frac{s_{t-1} + \vartheta_t}{\beta_t} \quad (29)$$

$\Rightarrow$

$$\vartheta_t = g_t \beta_t - s_{t-1} \quad (30)$$

As will be seen in Section 6, the last formula facilitates to calculate the capital term on the basis of data that is easily accessible. In an economy where capital increase is equal to savings and also the capital/income ratio is constant we have:

$$\vartheta_t = 0 \quad (31)$$

In this case, Equation (29) becomes:

$$g_t = \frac{s_{t-1}}{\beta_t} \quad (32)$$

Which is the Fundamental Equation obtained by Harrod (1939: p. 17), also called the Second Fundamental Law of Capitalism by Piketty (2014: p. 166) For this reason, Equation (29) can be useful to study the difference between the capital/income ratio and the (savings rate)/(growth rate) ratio within this model.

4.2. Proof of Equation (27)

To prove Equation (27) each term on the right-hand side will be substituted by its equivalence according with the corresponding definition:

$$\frac{s_{t-1} + d_t - (\beta_t - \beta_{t-1})}{\beta_t} = \frac{\frac{S_{t-1}}{Y_{t-1}} + \frac{D_t}{Y_{t-1}} - \left(\frac{K_t}{Y_t} - \frac{K_{t-1}}{Y_{t-1}}\right)}{\frac{K_t}{Y_t}} \quad (33)$$

$$= \frac{\frac{S_{t-1} + D_t}{Y_{t-1}} - \frac{K_t}{Y_t} + \frac{K_{t-1}}{Y_{t-1}}}{\frac{K_t}{Y_t}} \quad (34)$$

$$= \frac{\frac{\Delta K_t}{Y_{t-1}} - \frac{K_t}{Y_t} + \frac{K_{t-1}}{Y_{t-1}}}{\frac{K_t}{Y_t}} \quad (35)$$

$$= \frac{\frac{K_t - K_{t-1}}{Y_{t-1}} - \frac{K_t}{Y_t} + \frac{K_{t-1}}{Y_{t-1}}}{\frac{K_t}{Y_t}} \tag{36}$$

$$= \frac{\frac{K_t}{Y_{t-1}} - \frac{K_t}{Y_t}}{\frac{K_t}{Y_t}} \tag{37}$$

$$= \left[\frac{\frac{K_t}{Y_{t-1}} - \frac{K_t}{Y_t}}{\frac{K_t}{Y_t}}\right]\left[\frac{\left(\frac{Y_t}{K_t}\right)}{\left(\frac{Y_t}{K_t}\right)}\right] \tag{38}$$

$$= \frac{Y_t}{Y_{t-1}} - 1 \tag{39}$$

$$= g_t \tag{40}$$

5. The (Growth Rate)/(Profit Rate) Ratio and the (Profit Rate) – (Growth Rate) Difference

Dividing Equation (29) by Equation (15) results in:

$$\frac{g_t}{r_t} = \frac{\left[\frac{s_{t-1} + \vartheta_t}{\beta_t}\right]}{\left[\frac{\alpha_t}{\beta_t}\right]} \tag{41}$$

$$= \frac{s_{t-1}}{\alpha_t} + \frac{\vartheta_t}{\alpha_t} \tag{42}$$

Equation (42) permits to observe that the ratio between the growth and profit rates is smaller, equal or greater than the ratio between the savings rate and the capital share if ϑ_t is respectively smaller, equal or greater than zero. Furthermore, substituting in the last equation ϑ_t by the right-hand side of Equation (28) we get:

$$\frac{g_t}{r_t} = \frac{s_{t-1}}{\alpha_t} + \frac{d_t - (\beta_t - \beta_{t-1})}{\alpha_t} \tag{43}$$

$$= \left(\frac{1}{\alpha_t}\right)\left[s_{t-1} + d_t - (\beta_t - \beta_{t-1})\right] \tag{44}$$

Concerning the effect of an increase in the capital/income ratio from one period to the next one on the (growth rate)/(profit rate) ratio, this equation shows that the ratio is a monotonous decreasing function of the capital/income ratio.

Now, subtracting Equation (29) from Equation (15) results in:

$$r_t - g_t = \frac{\alpha_t}{\beta_t} - \frac{s_{t-1} + \vartheta_t}{\beta_t} \tag{45}$$

$$= \frac{\alpha_t - s_{t-1}}{\beta_t} - \frac{\vartheta_t}{\beta_t} \tag{46}$$

Substituting in this equation ϑ_t by the right-hand side of Equation (28) we obtain:

$$r_t - g_t = \frac{\alpha_t - s_{t-1}}{\beta_t} - \frac{d_t - (\beta_t - \beta_{t-1})}{\beta_t} \tag{47}$$

$$= \frac{\alpha_t - (s_{t-1} + \beta_{t-1} + d_t)}{\beta_t} + 1 \tag{48}$$

Concerning the effect of an increase in the capital/income ratio from one period to the next one on the difference between the profit and growth rates, this equation allows us to distinguish three possible cases because the numerator of the first term in the right-hand side of the equation can be greater, equal or less than zero. In first case, the difference between the two rates is a monotonous decreasing function of the capital/income ratio, in the second case it is constant and in the third one it is a monotonous increasing function of the capital/income ratio. Due to the fact that in modern economies normally $(s_{t-1}, \beta_{t-1}, d_t) > (0,1,0)$ while $\alpha_t < 1$ (see the data presented in the next section and in its sources), the third case must be prevalent. However, it may be otherwise if, for instance, β_{t-1} becomes small enough.

In an economy where capital increase is equal to savings and also the capital/income ratio is constant Equation (32) is true. For this reason, Equations (42) and (46) can be written, respectively, as follows:

$$\frac{g_t}{r_t} = \frac{s_{t-1}}{\alpha_t} \tag{49}$$

$$r_t - g_t = \frac{\alpha_t - s_{t-1}}{\beta_t} \tag{50}$$

6. A Numerical Example

Some of the formulas developed in the previous sections are used in this one to study the ratio and the difference between the growth and profit rates in the French economy during periods 1949-1979 and 1979-2009. Our purpose is to calculate the value of these functions and to identify the main causes of its changes. The country and the period have been chosen for no particular reason but to illustrate the previous developments. However, the results presented here constitute only a first approach on a subject that deserves more detailed analysis.

6.1. The Data and the Capital Term

Columns [1], [2], [5], [6] and [13] of Table FR.3c by Piketty and Zucman (2014) present the average values for the French economy corresponding to the periods 1949-1979 and 1979-2009 shown in **Table 1**.

Table 1. Average values for the French economy corresponding to the periods 1949-1979 and 1979-2009.

	g	β	α^*	r	s
1949-1979	4.8%	268%	21%	8.3%	14%
1979-2009	1.8%	379%	25%	6.6%	10%

However,

$$\frac{\alpha^*_{1949\text{-}1979}}{\beta_{1949\text{-}1979}} = \frac{0.21}{2.68} \tag{51}$$

$$= 0.07835 \tag{52}$$

a result contradicting the value reported in Table FR.3c for $r_{1949\text{-}1979}$. For this reason, in order to preserve the consistency of the data, we adopt $r_{1949\text{-}1979} = 7.83\%$ instead of 8.3%. Substituting successively each variable on the right-hand side of Equation (39) by its average value of periods 1949-1979 and 1979-2009 we get:

$$\vartheta_{1949\text{-}1979} = 0.048 \times 2.68 - 0.14 \tag{53}$$

$$= -0.01136 \tag{54}$$

$$\vartheta_{1979\text{-}2009} = 0.018 \times 3.79 - 0.1 \tag{55}$$

$$= -0.03178 \tag{56}$$

6.2. The Capital/Income and the (Savings Rate)/(Growth Rate) Ratios

Substituting successively in the right-hand side of Equation (31) each variable by its average value corresponding to periods 1949-1979 and 1979-2009 yields:

$$\frac{s_{1940\text{-}1979} + \vartheta_{1940\text{-}1979}}{g_{1940\text{-}1979}} = \frac{0.14}{0.48} + \frac{-0.01136}{0.48} \tag{57}$$

$$= 2.9116 - 0.2366 \tag{58}$$

$$= 2.675 \tag{59}$$

$$\frac{s_{1979\text{-}2009} + \vartheta_{1979\text{-}2009}}{g_{1979\text{-}2009}} = \frac{0.10}{0.018} + \frac{-0.03178}{0.018} \tag{60}$$

$$= 5.5555 - 1.7655 \tag{61}$$

$$= 3.7899 \tag{62}$$

According to these results, the (savings rate)/(growth rate) ratio was greater than the capital/income ratio in both periods. Dividing the difference between the two ratios by the first one yields $\frac{0.2366}{2.9116} = 0.0812$ in period 1949-1979 and $\frac{1.7655}{5.5555} = 0.3177$ in period 1949-2009. Therefore, the difference between the two ratios increased from being equivalent to 8.12% of the (savings rate)/(growth rate) ratio in the first period to 31.77% in the second period.

6.3. The (Growth Rate)/(Profit Rate) Ratio

The (growth rate)/(profit rate) ratios determined by the average values of these rates during periods 1949-1979 and 1979-2009 were, respectively:

$$\frac{g_{1949\text{-}1979}}{r_{1949\text{-}1979}} = \frac{0.048}{0.07835} \tag{63}$$

$$= 0.6126 \tag{64}$$

$$\frac{g_{1979\text{-}2009}}{r_{1979\text{-}2009}} = \frac{0.018}{0.066} \tag{65}$$

$$= 0.2727 \tag{66}$$

According to Equation (41), the first ratio results of the sum of the following fractions:

$$\frac{s_{1949\text{-}1979}}{\alpha^*_{1949\text{-}1979}} = \frac{0.14}{0.21} \tag{67}$$

$$= 0.6666 \tag{68}$$

$$\frac{\vartheta_{1949\text{-}1979}}{\alpha^*_{1949\text{-}1979}} = \frac{-0.01136}{0.21} \tag{69}$$

$$= -0.054 \tag{70}$$

while the second ratio results of the sum of the following fractions:

$$\frac{s_{1979\text{-}2009}}{\alpha^*_{1979\text{-}2009}} = \frac{0.1}{0.25} \tag{71}$$

$$= 0.4 \tag{72}$$

$$\frac{\vartheta_{1979\text{-}2009}}{\alpha^*_{1979\text{-}2009}} = \frac{-0.0318}{0.25} \tag{73}$$

$$= -0.1272 \tag{74}$$

These calculations allow us to make two remarks on the (growth rate)/(profit rate) ratio concerning respectively the determination of the value and the evolution of this function.

First, in both periods the ratio was determined mainly by the (savings rate)/(capital share) ratio. In period 1949-1979 the absolute value of each one of the two fractions in the right-hand side of Equation (41), compared to their sum, were respectively $\frac{0.6666}{0.6666+0.054} = 0.925$ and $\frac{0.054}{0.6666+0.054} = 0.0749$. In period 1979-2009 the corresponding proportions were respectively $\frac{0.4}{0.4+0.1272} = 0.7587$ and $\frac{0.1272}{0.4+0.1272} = 0.2412$. Therefore, in the first period, 92.5% of the value of the (growth rate)/(profit rate) ratio can be attributed to the (savings rate)/(capital share) ratio and 7.58% can be attributed to the (capital term)/(capital share) ratio. In turn, in the second period the corresponding percentages are 75.87% and 24.12%, respectively.

Second, the decrease of the ratio between the two periods was mainly due to the decrease of the (savings rate)/(capital share) ratio. Between the two periods considered the first fraction in the right-hand side of Equation (41) decreased from 0.6666 to 0.4, that is 0.2666, and the second fraction decreased from −0.054 to −0.1272, that is 0.0732. Therefore, in the decrease of $0.6126-0.2727=0.3399$ in the value of the (growth rate)/(profit rate) ratio that took place between the two periods considered, the part due to the decrease in the (savings rate)/(capital share) ratio is $\frac{0.2666}{0.3399}=0.7843$, that is 78.43%, and the part due to the decrease of the (capital term)/(capital share) ratio is $\frac{0.0732}{0.3399}=0.2153$, that is 21.53%.

It is worth adding that in both periods, the capital term was negative and the absolute value of the (capital term)/(capital share) ratio compared to the (savings rate)/(capital share) ratio, increased from $\frac{0.054}{0.6666}=0.081$ to $\frac{0.1272}{0.4}=0.318$, that is from 8.1% to 31.83% from one period to the other. Despite this fact, both the value of the (growth rate)/(profit rate) ratio as well as its evolution between the two periods considered was mainly due to the (savings rate)/(capital share) ratio.

6.4. The (Profit Rate) – (Growth Rate) Difference

The (profit rate) – (growth rate) difference determined by the average values of these rates during periods 1949-1979 and 1979-2009 were, respectively:

$$r_{1949\text{-}1979}-g_{1949\text{-}1979}=0.07835-0.048 \tag{75}$$

$$=0.03035 \tag{76}$$

$$r_{1979\text{-}2009}-g_{1979\text{-}2009}=0.066-0.018 \tag{77}$$

$$=0.048 \tag{78}$$

According to Equation (46), the first difference results of the sum of the following fractions:

$$\frac{\alpha^*_{1949\text{-}1979}-s_{1949\text{-}1979}}{\beta_{1949\text{-}1979}}=\frac{0.21-0.14}{2.68} \tag{79}$$

$$=0.02611 \tag{80}$$

$$-\frac{\vartheta_{1949\text{-}1979}}{\beta_{1949\text{-}1979}}=-\left(\frac{-0.01136}{2.68}\right) \tag{81}$$

$$=0.004238 \tag{82}$$

while the second difference results of the sum of the following fractions:

$$\frac{\alpha^*_{1979\text{-}2009}-s_{1979\text{-}2009}}{\beta_{1979\text{-}2009}}=\frac{0.25-0.1}{3.79} \tag{83}$$

$$=0.03956 \tag{84}$$

$$-\frac{\vartheta_{1979\text{-}2009}}{\beta_{1979\text{-}2009}}=-\left(\frac{-0.03178}{3.79}\right) \tag{85}$$

$$=0.00839 \tag{86}$$

These calculations allow us to make two remarks on the difference between the profit and growth rates, concerning respectively the determination of the value and the evolution of this function.

First, in both periods the difference was determined mainly by the [(capital share) − (savings rate)]/[capital/income] ratio. In period 1949-1979 the absolute values of each one of the two fractions in the right-hand side of Equation (46), compared to their sum, were respectively $\frac{0.02611}{0.02611+0.004238}=0.8603$ and $\frac{0.004238}{0.02611+0.004238}=0.1396$. In period 1979-2009 the corresponding proportions were respectively $\frac{0.03956}{0.03956+0.00839}=0.8250$ and $\frac{0.00835}{0.03956+0.00839}=0.1741$. Therefore, in the first period, 86.03% of the value of the difference between the two rates can be attributed to the [(capital share) − (savings rate)]/[capital/income] ratio and 13.96% can be attributed to the (capital term)/(capital share) ratio. In turn, in the second period the corresponding percentages are 82.5% and 17.41%, respectively.

Second, the increase of the difference between the two rates taking place from one period to the other was mainly due to the increase of the [(capital share) − (savings rate)]/[capital/income] ratio. Between the two periods considered the first fraction in the right-hand side of Equation (46) increased from 0.02611 to 0.03956, that is 0.01345, and the second fraction increased from 0.004238 to 0.00839, that is 0.004152. Therefore, in the increase of $0.048-0.03035=0.01765$ in the difference between the profit and growth rates that took place between the two periods considered, the part due to the increase in the [(capital share) − (savings rate)]/[capital/income] ratio is $\frac{0.01345}{0.01765}=0.7620$, that is 76.2%, and the part due to the increase of the (capital term)/(capital share) ratio is $\frac{0.004152}{0.01765}=0.2352$, that is 23.52%.

It is worth adding that in both periods, the capital term was negative and the absolute value of second fraction in the right-hand side of Equation (46), compared to the first one (which was positive in both periods), increased from $\frac{0.004238}{0.02611}=0.1623$ to $\frac{0.00839}{0.03956}=0.2120$, that is from 16.23% to 21.2% from one period to the other. Despite this fact, both the value of the difference between the profit and growth rates as well as the evolution of this value from the first to the second period considered was mainly due to the [(capital share) − (savings rate)]/[capital/income].

6.5. Additional Remarks

We found that the net effects of the capital/income ratio considered in Equation (41) were relatively small and, for this reason, the (growth rate)/(profit rate) ratio was predominantly determined by the (savings rate)/(capital share) ratio. In turn, the second term in the right-hand side of Equation (46) was relatively small too and, due to this fact, the difference between the profit and growth rates was determined mainly by this ratio and by the difference between the capital share and the savings rate.

Given the importance of both savings rate and capital share for the ratio as well as for the difference between the growth and profit rates, a better understanding of these relations may be reached in the future by studying the determination of the two indicated variables. Here, we will only add two brief comments on this matter.

First, to understand the behavior of the capital share, it is useful to consider it as the complement to unity of the labor share. Indeed, the labor share is equal to the (real labor income)/(productivity of labor) ratio (see Giovannoni 2014: p. 8). For this reason, the change of the capital share in the French Economy between the periods 1949-1979 and 1979-2009 can be viewed as resulting from the decrease of the real labor income relative to the productivity of labor.

Second, it is worth remembering that the amount of savings is equal to that of profits minus the part of profits consumed plus the part of wages saved. The capital share is greater, equal to or less than the savings rate if the sum of profits consumed is respectively greater, equal to or less than the sum of wages saved. Therefore, letting aside changes in income and capital, increasing the sum of profits consumed over the sum of wages saved also increases the profit rate relative to the growth rate. In turn, this increase favors, as noted in the introduction, the growth of the fraction of total income corresponding to the highest income levels.

7. Conclusion

This article presents two formulas expressing the (profit rate)/(growth rate) ratio and the (profit rate) − (growth rate) difference, respectively, as the sum and the difference of two particular pairs of fractions. In the first formula, the numerator of the first fraction is the savings rate and, in the second formula, the corresponding numerator is the (capital share) − (savings rate) difference. The denominator in the two fractions of the first formula is the capital share and in those of the second formula is the capital/income ratio while the numerator of the second fraction of the two formulas is the capital increase not due to savings (measured with the national income) minus the increase in the capital/income ratio. Because each one of the formula allows for an estimation of the relative importance of the corresponding two fractions, they help to explain the inequality between the two rates whose importance is underlined by Piketty. As an illustration of this fact, we studied the case of the French economy during periods 1949-1979 and 1979-2009. In this respect, we found on the one hand, that the net effects of the capital/income ratio considered in the first formula were relatively small and, for this reason, the (growth rate)/(profit rate) ratio was predominantly determined in both periods by the (savings rate)/(capital share) ratio. However, the relative importance of this ratio in the determination of the (growth rate)/(profit rate) ratio diminished from one period to the other. On the other hand, the (profit rate) – (growth rate) difference was determined mainly by the (capital share) – (savings rate) difference divided by the capital/income ratio. Nevertheless, the relative importance of this quotient in the determination of the (profit rate) – (growth rate) difference diminished from one period to the other. Similar results were found concerning the evolution from one period to the other of both the (growth rate)/(profit rate) ratio and the (profit rate) – (growth rate) difference.

Finally, it is worth mentioning what we consider to be the main limitations of the paper, one is the lack of a discussion of the literature on the determination of the profit and growth rates. Notwithstanding the modest scope of our work, that discussion may have been useful to evaluate the relative importance of its contributions. On the other hand, the limited number of cases taken into account in the empirical part of the paper reduced to an illustrative example. These limitations point out two interesting tasks for future researches.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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