Theoretical Economics Letters, 2020, 10, 909-925
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

A Note on GDP- versus Consumption-Maximizing Growth

Yong He

CERDI/CNRS, Universite Clermont-Auvergne, Clermont-Ferrand, France
Email: yong.he@uca.fr

How to cite this paper: He, Y. (2020). A Note on GDP- versus Consumption-Maximizing Growth. *Theoretical Economics Letters, 10,* 909-925.
https://doi.org/10.4236/tel.2020.104054

Received: July 13, 2020
Accepted: August 21, 2020
Published: August 24, 2020

Abstract

Sparked by Baumol's revenue- versus profit-maximizing models of the firm, this note shows that if a nation seeks GDP-maximizing growth with capital expansion as driving force, the model could work only under the assumption that the consumers' aversion to under-consumption, an unavoidable consequence of over-investment, remains constant. Otherwise, it has to decelerate growth and ultimately converges to the neoclassical growth model with consumption optimality. The empirical evidence, especially the sustainability of the Chinese model, is examined to explore the extent to which the model captures the real world.

Keywords

GDP-Maximizing Growth, Aversion to Under-Consumption, Transition between Steady States, Sustainability

1. Introduction

Baumol (1959) has put forward the original idea of a firm that maximizes revenue instead of profit. The separation between ownership and control within the modern firm gives managers a discretional power to deviate from the goal of profit maximization fixed by the owners. The managers could seek to maximize revenue, because the sale performance is the prevailing indicator of the competitive position within the industry, and the increase in revenue is a sign of managerial success and on which managers' remunerations depend.

This idea is able to be extended to think about a nation's economic growth. In the neoclassical tradition, economic growth is analyzed in terms of consumption optimality. In a decentralized market economy, individuals, or households, have no other objective than consumption maximization, while capital is just treated as a means to generate consumption goods. The Ramsey (1928) model provided for an endogenous determination of the savings rate. It was further developed by Cass (1965) and Koopmans (1963), and became known as the Ramsey-Cass-Koopmans (RCK) model.

The government of a nation, or, in a conventional appellation, the social planner maximizes consumption if it adopts the households' choice, just like firm managers that obey the owners of the firm. It may also maximize GDP just as the firm maximizes revenue. In the absence of foreign exchange, GDP consists of capital and consumption. GDP maximization seemingly attributes equal importance to capital and consumption. Nonetheless, since capital is the single generator of GDP and consumption is, from the point of view of forming capital, a cost to minimize, capital formation through investment takes a leading role. Hence, GDP maximization is essentially capital maximization constrained by a certain level of consumption.

Dealing with this issue is not just an intellectual exercise, but has meaningful empirical implication. There has been a rich literature on the sources of growth of the Eastern Asian model, and more specifically, of the Chinese model. To cite a few more noteworthy, Krugman (1994) believed that Asian growth, like that of the Soviet Union in its high-growth era, was driven by extraordinary growth in inputs like labor and capital rather than by gains in efficiency. Young (1995) shared this judgment from the point of view of the weakness in the growth of total factor productivity in these countries. In the same vein, Kim and Lau (1994) found that, for the four major Eastern Asian countries, capital accumulation accounted for between 48 and 72 percent of their economic growth, in contrast to the case of 5 major industrialized countries, in which technical progress has accounted for between 46 and 71 percent of their economic growth.

Recognizing that these countries have resorted to high capital accumulation to realize high growth, we are facing a key theoretical question: the RCK growth model is also able to predict that during the early transitional stages, the economy has higher capital accumulation and GDP growth, and then they gradually reduce before arriving at the steady state. So what is the justification of building a GDP-maximizing model along with the RCK model?

We argue that consumption- and GDP-maximizing growth models are different given their difference in objective. GDP maximization is a deviation from the consumers' goal of consumption optimality. This deviation becomes possible due to the particular politico-economic processes in these countries. Assuming a government perfectly controlled by the electors in the sense that it will be dismissed at any moment it deviates from their goal, its centralized decision must be identical to the decentralized market solution. In the countries without or with less democratic control, the governments in general have higher motivation in GDP growth than just satisfying consumption given GDP growth is a generally recognized indicator of governance performance. This can reinforce the legitimacy of an autocratic government. Specially, such countries as ex-Soviet

Union and China had or have a goal of political and economic competition with the Western world. If following the Public choice view, even a democratic government under the majority rule could search for the goal of GDP maximization since sometimes its election depends on employment, which is linked with GDP growth. The representative government tends to become a revenue-maximizing Leviathan (Brennan & Buchanan, 1980), and the senior bureaucrats seek to maximize the budget and the output of the bureau (Niskanen, 1971).

Consequently, this paper addresses an issue that is different in nature with that in the RCK model. As will be shown subsequently, if the social planner seeks to maximize GDP, consumption level must be lower than that derived from a parallel-in-time RCK model. The objective of this study is to build a GDP-maximizing model (hereafter the GDP max model) to answer two key questions: 1) what would be its steady-state GDP, consumption, and capital relative to a neoclassical RCK model? 2) under what condition is the model sustainable?

The remainder of this study is organized as follows. Section 2 builds the GDP max model. Section 3 analyzes the model sustainability, adjustments and resulting adjusted transition trajectory as well as a numerical simulation. Section 4 introduces the empirical applications of the model. Section 5 presents conclusions.

2. The Model

Assume an economy with population = L and without population growth. We first introduce a RCK model, which is parallel in time with the ensuing GDP max model. The results of the RCK model will be used as the reference to compare with the GDP max model. The GDP max model is based on the social planner's preference, while the RCK model is based on the household consumption maximizing preference. The social planner, assumed knowing household's time discount choice, must run the RCK model also, to measure the household-preferred consumption level in order to determine the under-consumption rate, which will be the key for the working of the GDP max model.[1] To facilitate the comparison, the two models are made as close as possible in structure. The variables and parameters set are the same in both models, except a few specific to the GDP max model.

In what follows, the equations noted as A (e.g., 1A) are from the RCK model; those noted as B (e.g., 1B) are their equivalents in the GDP max model. To distinguish the variables in the two models, for instance, capital and investment are labeled as k and $\dot{k}$ in the GDP max model; their equivalents in the RCK model are labeled as $\bar{k}$ and $\dot{\bar{k}}$. Finally, all steady-state values are labeled * (e.g., k^* and $\bar{k}^*$).

[1]This is just a technical necessity to assume that the planner must also run the RCK model. It can be justified by the observations in the real world that the ruler, democratic or autocratic, needs to know the preferences, especially economic preferences, of the ruled, a necessary information to maintain the stability of the regime.

2.1. Outline of the RCK Model

A social planner, knowing perfectly the household preference, maximizes the household's time-discounted utility with the utility in t as $u(\bar{c}(t)) = \log(\bar{c}(t))$. His typical dynamical optimization problem takes the form:

$$U(0) = \int_0^\infty e^{-\rho t} \log\left[\bar{c}(t)\right] dt \tag{1}$$

$$\dot{\bar{k}}(t) = \bar{k}(t)^\alpha - \bar{c}(t) - \delta \bar{k}(t) \tag{1.A}$$

$$\bar{k}(0) = 1 \tag{2}$$

$$\lim_{t\to\infty}\left[\bar{k}(t) e^{-\bar{r}(t)t}\right] \geq 0 \tag{3}$$

where $\bar{c}$ is consumption, $\bar{k}$ is capital and $\bar{q} = \bar{k}^\alpha$ is production with a Cobb-Douglas production function. All of these variables are in terms of per capita. ρ is the rate of time preference, a larger ρ means that utilities are valued less the later they are received. δ is the capital discount rate. $\bar{r}$ is the average interest rate equal $(1/t)\int_0^t r(v) dv$.

The current-value Hamiltonian is then:

$$H\left[\bar{c}(t), \bar{k}(t), t, \bar{m}(t)\right] = \log\left[\bar{c}(t)\right] + \bar{m}(t)\left[\bar{k}(t)^\alpha - \bar{c}(t) - \delta \bar{k}(t)\right]$$

where $\bar{m} = \lambda e^{\rho t}$ is current-value of the shadow price λ.

The first-order conditions are: $\frac{\partial H}{\partial \bar{c}} = 0$, $\frac{\partial H}{\partial \bar{k}} = \rho \bar{m} - \dot{\bar{m}}$, and the transversality condition is $\lim_{t\to\infty} \bar{m}(t) e^{-\rho t} \bar{k}(t) = 0$. Using these conditions, another equation of motion can be derived:

$$\dot{\bar{c}} = \left[\alpha \bar{k}^{\alpha-1} - (\delta + \rho)\right]\bar{c} \tag{2.A}$$

The procedure to derive (2A) is omitted for briefness (cf. Barro & Sala-i-Martin 2003: A.3.8). The same procedure will be introduced in the subsequent derivation of this equation of motion for the GDP max model.

(2A) expresses the so-called Ramsey-Keynes's rule, which describes the consumption dynamics during the transition stages in terms of the trade-off between present and future consumptions. With higher time preference rate, or higher-valued present consumption, consumption growth rate will be lower. Coordinately, from (1A(a)), with larger present consumption, capital growth rate is also lower.

The steady-state values of $\bar{c}$ and $\bar{k}$ are found by setting $\dot{\bar{k}} = 0$ and $\dot{\bar{c}} = 0$ in (1A) and (2A), respectively. With $\dot{\bar{c}} = 0$, (2A) ensures that, at the steady state, the marginal product of $\bar{k}^*$ amounts to $\delta + \rho$. It follows that:

$$\bar{k}^* = \left(\frac{\alpha}{\delta + \rho}\right)^{\frac{1}{1-\alpha}} \tag{3.A}$$

$$\bar{c}^* = \bar{k}^{*\alpha} - \delta \bar{k}^* = \left(\frac{\alpha}{\delta + \rho}\right)^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{\alpha}{\delta + \rho}\right)^{\frac{1}{1-\alpha}} \tag{4A}$$

2.2. Structure of the Equations of GDP Max Model

First, the gdp function in the GDP max model is defined as:

$$q(t)=k(t)^{\alpha}-\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1}{\sigma}}\bar{c}(\tau)\left(0<\sigma\le 1;\left(1-\frac{c}{\bar{c}}\right)^{\frac{1}{\sigma}}=0\ \text{if}\ c>\bar{c}\right) \tag{5}$$

where the production per capita q is, like in the RCK model, a simplified version of GDP per capita, i.e., $q\equiv \mathtt{gdp}$.[2] The time notation for $\bar{c}$ is τ, in distinction with t, because it comes from the RCK model, and, hence, is not subject to derivation in the GDP max model. But, all the time, t and τ are in correspondence so that $\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)$ is the under-consumption rate of time t.

In this production function, in addition to k^{α}, a constraint, which can be called under-consumption constraint, is put. With c and $\bar{c}$ are consumptions in the GDP max and RCK models respectively, under-consumption is defined by $c/\bar{c}$. Concretely, the social planner uses both models to obtain the under-consumption level. The higher the $c/\bar{c}$, the lower is the under-consumption level. The term $1-\frac{c}{\bar{c}}=\frac{\bar{c}-c}{\bar{c}}$ is the under-consumption rate. Then $\left(1-\frac{c}{\bar{c}}\right)^{\frac{1}{\sigma}}$ scaled by $\bar{c}$ measures the loss in GDP due to the under-consumption effects. σ is a parameter measuring the degree of aversion to under-consumption.

Some justifications are required on the need to incorporate the under-consumption effect. While in an RCK model, the choice between maximizing current and future consumptions is a rational trade-off, in a GDP max model, the social planner has unlimited desire to expand capital and to minimize consumption since the unique driving force acting on GDP growth is investment. Some constraints must be set to make some trade-off effective on his choice. On the base of the real-world observations, our approach brings into the picture the negative under-consumption effects in terms of losses in GDP, to influence the social planner's calculations.

The set of an under-consumption constraint is motivated by two effects of under-consumption. The first is the disincentive effect. The aversion to under-consumption results in a direct production loss. This will be discussed more in detail in the subsequent section on empirical evidence. The second is the physiological effects due to under-consumption. The deficiencies in adequate health care reduce the effective labor force for production.

The values of $c/\bar{c}$ are in theory in two categories: $c/\bar{c}<1$ and $c/\bar{c}\ge 1$. $c/\bar{c}<1$ is the normal case in which the trade-off is effective for the social planner to choose an optimal value of c. $c/\bar{c}\ge 1$ is an extreme case in which under-consumption and the tradeoff disappear. It can be argued that whenever the social planner maximizes GDP and has a choice in c, c could not be higher than $\bar{c}$, because otherwise it would be a contradiction with the logic of GDP max-

[2]The income method is used here to calculate GDP.

imization, in which consumption must be constrained in favor of capital formation. For this reason, we must specify (5) as:

$$q(t)=k(t)^{\alpha}-\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1}{\sigma}}\bar{c}(\tau)\left(0<\sigma\leq 1;c\leq\bar{c}\right) \tag{6}$$

This production function is quite similar to a Stone-Geary function expressed as $q(t)=\left[k(t)-\Upsilon(t)\right]^{\alpha}$ where $\Upsilon(t)$ is a certain threshold level subject to various interpretations. In a sense, our model extends Stone-Geary function through giving $\Upsilon(t)$ a more precise interpretation, and, especially, a possibility to introduce some other factors that could affect its evolution.

With the gdp definition at hand, the utility function is set in the simplest form: $u\left[q(t)\right]=q(t)$ for the sake of tractability. The social planner's optimal problem is:

$$U(0)=\int_0^{\infty}\mathrm{e}^{-\rho t}q(t)\mathrm{d}t=\int_0^{\infty}\mathrm{e}^{-\rho t}\left[k(t)^{\alpha}-\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1}{\sigma}}\bar{c}(\tau)\right]\mathrm{d}t$$

(a) $\dot{k}(t)=k(t)^{\alpha}-\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1}{\sigma}}\bar{c}(\tau)-c(t)-\delta k(t)$ (1B)

(b) $k(0)=1$

(c) $\lim_{t\to\infty}\left[k(t)\mathrm{e}^{-\bar{r}(t)t}\right]\geq 0$

With the production function defined in (6), the equation of motion for investment is (IB) (a). As such, the social planner is facing a trade-off between under-consumption and the pro-investment effects of the control variable *c*.

The current-value Hamiltonian for GDP maximization is:

$$H\left[c(t),k(t),t,m(t)\right]$$
$$=k(t)^{\alpha}-\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1}{\sigma}}\bar{c}(\tau)+m(t)\left[k(t)^{\alpha}-\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1}{\sigma}}\bar{c}(\tau)-c(t)-\delta k(t)\right]$$

where $m=\lambda\mathrm{e}^{\rho t}$ is the shadow price λ in the current value.

The first-order conditions are: $\frac{\partial H}{\partial c}=0$, $\frac{\partial H}{\partial k}=\rho m-\dot{m}$, and the transversality condition is $\lim_{t\to\infty}m(t)\mathrm{e}^{-\rho t}k(t)=0$.

The first-order condition with respect to *c* is:

$$\frac{\partial H}{\partial c(t)}=\frac{1}{\sigma}\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1-\sigma}{\sigma}}+m(t)\left[\frac{1}{\sigma}\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1-\sigma}{\sigma}}-1\right]=0 \tag{7}$$

hence:

$$m(t)=B(t)/\left(1-B(t)\right) \tag{8}$$

where:

$$B(t)=\frac{1}{\sigma}\left(1-\frac{c(t)}{\bar{c}(\tau)}\right)^{\frac{1-\sigma}{\sigma}}>0 \quad \text{and} \quad \neq 1 \tag{9}$$

$B=\frac{\partial q}{\partial c}$ is the induced marginal loss of GDP by forgoing one unit of c due to under-consumption effect. B has central importance in our analysis. $B = 1$ is assumed not permissible; otherwise (8) will be undefined.

Using (8) and differentiating m with respect to t:

$$\dot{m}(t)=-\frac{\left(\frac{1-\sigma}{\sigma}\right)B(t)\dot{c}(t)}{\left(1-B(t)\right)^{2}\left(\bar{c}(\tau)-c(t)\right)} \tag{10}$$

Another first-order condition with respect to k is:

$$\frac{\partial H}{\partial k(t)}=\left(1+m(t)\right)\alpha k(t)^{\alpha-1}-m(t)(\delta+\rho)=-\dot{m}(t) \tag{11}$$

Putting (8) and (10) into (11) to replace m and $\dot{m}$, and rearranging, an equation of motion for c comparable to that in the RCK model (2A) is obtained as:

$$\dot{c}(t)\left[\frac{\left(\frac{1-\sigma}{\sigma}\right)B(t)}{\left(1-B(t)\right)^{2}\left(\bar{c}(\tau)-c(t)\right)}\right]=\frac{1}{1-B(t)}\alpha k(t)^{\alpha-1}-\frac{B(t)}{1-B(t)}(\delta+\rho) \tag{2B}$$

(2B) describes the consumption dynamics in the GDP max model. It is difficult to analyze it in comparison with (2A) without some specifications on σ and B. We are able, though, to note that, just as in (2A), time preference plus the capital discount rate also determine the consumption growth rate, and then, from (1B(a)), the capital growth rate is coordinately adjusted. In other words, Ramsey-Keyes's rule still works in the GDP max model. However, the specificity of the model comes from σ and B. To explore their role, we need to operate the steady state analysis for the sake of tractability.

2.3. The Steady State of GDP Max Model

Setting $\dot{c}$ and $\dot{k}=0$, the steady-state values of k^{*} and c^{*} are, respectively:

$$k^{*}=\left(\frac{1}{B^{*}}\right)^{\frac{1}{1-\alpha}}\bar{k}^{*} \tag{3B}$$

$$c^{*}=\left(\frac{1}{B^{*}}\right)^{\frac{\alpha}{1-\alpha}}\bar{k}^{*\alpha}-\left(\frac{1}{B^{*}}\right)^{\frac{1}{1-\alpha}}\delta\bar{k}^{*}-\left(1-\frac{c^{*}}{\bar{c}^{*}}\right)^{\frac{1}{\sigma}}\bar{c}^{*} \tag{4B}$$

where:

$$B^{*}=\frac{1}{\sigma}\left(1-\frac{c^{*}}{\bar{c}^{*}}\right)^{\frac{1-\sigma}{\sigma}} \tag{9'}$$

(3B) and (4B) are not in a reduced form. However, thanks to the similarity in the structure of the equations, the comparisons between the steady-state values of c and k in the two models become plausible.

3. Analysis

3.1. Sustainability

A least controversial concept of sustainability is adopted: the GDP max model is sustainable unless its GDP is higher than that in the RCK model. From (6) we know that once the consumption levers become the same in the two models, their GDPs will be the same, and the GDP max model converts to the RCK model. In other words, unsustainability means that the GDP max model must convert to consumption max model since the latter brings at least the same level of GDP.

Checking (3A), (3B), (4A) and (4B), the differences in the steady-states values of consumption and capital between the two models are mainly determined by B^*, defined by (9'). The steady-state GDP values of the two models are:

$$\overline{gdp}^* = \bar{k}^{*\alpha} = \left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}} \tag{12A}$$

$$gdp^* = k^{*\alpha} - \left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}} \bar{c}^* = \left(\frac{1}{B^*}\right)^{\frac{\alpha}{1-\alpha}} \overline{gdp}^* - \left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}} \bar{c}^* \tag{12B}$$

(12A) and (12B) give rise to a simpler expression of sustainability. In the case of consumption optimization, a share θ of $\overline{gdp}^*$ is used for $\bar{c}^*$. Then:

$$gdp^* = \left[\left(\frac{1}{B^*}\right)^{\frac{\alpha}{1-\alpha}} - \theta\left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}}\right]\overline{gdp}^* \tag{13}$$

Note that in the RCK model, from (4A) and (12A), $\theta = \bar{c}^* / \overline{gdp}^* = \left[(1-\alpha)\delta+\rho\right]/(\delta+\rho) < 1$.

Without taking into account the second terms in the bracket on the right hand of (13), at least, it must be $0 < B^* < 1$ for the GDP max model being sustainable. This result gives rise to:

Proposition 1

The sustainability of the GDP max model requires $0 < B^* < 1$.

Why must be $B^* < 1$? Given that $B = \partial gdp/\partial c$ is the induced marginal loss in GDP by one unit of foregone consumption due to under-consumption, by virtue of (8), this proposition fixes a boundary condition on the level of this loss to guarantee the positivity of the current shadow price at the steady state. The importance of Proposition 1 will become clearer when exploring the factors that affect it.

$B^* < 1$ is just a minimum condition. This condition is important because it results in a meaningful theoretical interpretation. From (13), the sufficient condition for the model being sustainable is: $\left(\frac{1}{B^*}\right)^{\frac{\alpha}{1-\alpha}} - \theta\left(1-\frac{c^*}{\bar{c}^*}\right)^{\frac{1}{\sigma}} > 1$. Under this condition, the boundary value of B^* for the sustainability is still lower than 1. To simplify the presentation in the following theoretical analysis, this boundary value is still referred as being one.

To compare the steady-state values of k and c between the two models, from (3B) $k^* = \left(\frac{1}{B^*}\right)^{\frac{1}{1-\alpha}} \bar{k}^*$, as $B^* < 1$, then $k^* > \bar{k}^*$ follows. Comparing c^* and $\bar{c}^*$ directly through (4A) and (4B) is not tractable due to the complicate non-reduced form of the latter. A feasible way for this comparison comes from the sustainability condition defined in Proposition 1. With $m^* = \frac{B^*}{1-B^*} > 0$, It follows that $B^* = \frac{1}{\sigma}\left(1 - \frac{c^*}{\bar{c}^*}\right)^{\frac{1-\sigma}{\sigma}} > 0$. Thus $c^* < \bar{c}^*$ follows. Therefore, in their steady states, capital is higher, and consumption is lower in the GDP max model than in the RCK model.

Given, from (13), the sustainability of the GDP max model is governed by B^*, the next is to know what determines the level of B^*? Checking (9'), on the one hand, B^* must be a positive function of σ. The lager the parameter scaling up the under-consumption effects, the larger is the marginal production loss due to under-consumption:

$$\frac{\partial B^*}{\partial \sigma} > 0 \tag{14}$$

On the other hand, B^* is a negative function of $c^*/\bar{c}^*$. The lower the under-consumption, the lower the loss in GDP due to under-consumption:

$$\frac{\partial B^*}{\partial \frac{c^*}{\bar{c}^*}} = -\frac{1-\sigma}{\sigma^2\left(1 - \frac{c^*}{\bar{c}^*}\right)} B^* < 0 \tag{15}$$

As $\frac{\partial B^*}{\partial \sigma} > 0$, it follows that, inasmuch as $\sigma < 1$ and is constant, to the extent that the negative effect of the increasing $c^*/\bar{c}^*$ sufficiently offsets the positive effect of σ on B^* so that $gdp^*/\overline{gdp}^* > 1$, the GDP max model could always remain sustainable. Its intuitive meaning is clear: the level of under-consumption fixed by the social planner can be higher whenever the aversion to under-consumption scaled by σ is lower. With a low σ, its corresponding $c^*/\bar{c}^*$ can be fixed at a low level; thus B^* stays at a value much lower than 1. With a higher σ, its corresponding $c^*/\bar{c}^*$ must be fixed at a higher level; thus, B^* approaches 1, but the model always maintains sustainability. Needless to say that once σ becomes very high, for instance, it is close to 1, then, $c^*/\bar{c}^*$ has to be set close to 1. In this case, the GDP max model becomes very close to the RCK model, even if in theory it is still sustainable.

3.2. Transition between Steady States with Increasing σ

Our discussion on the relationship between σ and $c^*/\bar{c}^*$ has so far been based on a constant σ. This, however, can be shown to be an unrealistic postulation. In what follows, we establish the channels by which this aversion is increasing, or σ is rising. This can be introduced in a formal way:

$$\sigma = f\left[CP(gdp), IDE(gdp)\right] \left(\frac{\partial\sigma}{\partial CP} > 0, \frac{\partial CP}{\partial gdp} > 0, \frac{\partial\sigma}{\partial IDE} > 0, \frac{\partial IDE}{\partial gdp} > 0\right) \quad (16)$$

where *CP*, *IDE*, *gdp* stand for the propensity for consumption, the international demonstration effect, and GDP per capita, respectively.

First, via the propensity for consumption, income growth over time raises the aversion to under-consumption. Along with income growth, the propensity for consumption rises because: 1) as the ultimate aim of production is consumption, the desire to consume increases; 2) the capability to afford a higher consumption level rises; and 3) the demand for necessary goods decreases, whereas the demand for high-quality, comfort-linked luxury and leisure goods increases. The latter being more expensive, the share of consumption in income must increase. Along with higher income, people are also more sensible to the "public bad" that affects their health and living environment. This is another category of consumption that becomes increasingly expensive. The share of consumption in income rises with income growth is supported by empirical tests.[3] Because of this increasing propensity, consumption must increase correspondingly. Otherwise, dissatisfaction increases. This is equivalent to increasing aversion to under-consumption: with the same share of consumption in income, consumers feel increasing unhappiness over time.

Second, via what is called the "international demonstration effect", income growth over time also positively affects the aversion to under-consumption. This effect was formulated by Nurkse (1953). He believed that, in developing nations, people "come into contact with superior goods or superior patterns of consumption, with new articles or new ways of meeting old wants." As a result, these people are "apt to feel after a while a certain restlessness and dissatisfaction. Their knowledge is extended, their imagination stimulated; new desires are aroused" (quoted in Kattel et al. 2009: p. 141). This international demonstration effect is intensified with internationalization, which correlates with income growth. Doubtlessly, this effect increases the aversion to under-consumption.

Whenever considering an evolving parameter σ, a long-term comparative dimension is introduced into the dynamic analysis. This junction is inductively appealing since a dynamic analysis involves long-term evolution, and the parameters that specify the model are more likely to evolve. This method of comparative dynamics in the growth models is the counterpart of comparative statics in general equilibrium analysis, and was originally applied by Aoki (1980). It is ex-

[3]Using the world development indicators (World Bank, 2014) for 198 countries from 1960 to 2012, it was found that, the consumption to GDP ratio rises along with the GDP per capita. In this regression, the countries with their GDP per capita (at a constant price) lower than 1000 USD were ruled out because inductively they must spend an unusually high share of income on necessary goods. The regression results are available upon request.

tended to analyze the transition between steady states after a change in a structural parameter (Novales et al., 2009).

Having shown the realism of the premise that σ rises over time, observing (13), and given $B^* = \frac{1}{\sigma}\left(1 - \frac{c^*}{\overline{c}^*}\right)^{\frac{1-\sigma}{\sigma}}$, the key process is how with rising σ, to keep maximizing gdp^*, the under-consumption rate $c^*/\overline{c}^*$ is moved.

As shown in Appendix, totally differentiating gdp^* with respect to σ and $c^*/\overline{c}^*$ and setting $\mathrm{d}gdp^* = 0$ yields $\mathrm{d}\frac{c^*}{\overline{c}^*}\Big/\mathrm{d}\sigma > 0$. This means that with maximized GDP, whenever σ rises, $c^*/\overline{c}^*$ always must rise. With rising σ, as the renewed steady-state values of c^* are successively higher, leading to higher B^*, the marginal loss in production. Consequently, the k^* (via (3B)) and gdp^* (via (13)) are successively lower. The GDP max model is no longer sustainable when the ratio $gdp^*/\overline{gdp}^*$ equals one. This ratio also implies the growth rates during the transitional stages before the steady state of the GDP max model of a certain σ level. The higher this ratio, the higher the growth rate of the GDP max model during the transitional stages. All of these define the shifting steady states and can be expressed by a new proposition:

Proposition 2

In the presence of an increasing σ, the shifting steady states of the GDP max model determine its growth rates to decelerate. Ultimately, when σ rises to a certain level, the GDP max model converges to the RCK model.

The sustainability requires B^* to be sufficiently low. Whenever σ rises, if $c^*/\overline{c}^*$ remains unchanged, B^* will increase towards the boundary value fixed by Proposition 1, and the sustainability of the model is at stake. Consequently, to keep maximizing GDP, consumption must be raised. In other words, with the rising aversion to under-consumption, the under-consumption level must be reduced. Otherwise, the loss in GDP due to this aversion offsets the gain from capital increase resulted from under-consumption.

When σ rises from a low base, or from a low aversion base, as under-consumption leads to low GDP loss, $c^*/\overline{c}^*$ is only required to be raised slightly. Accordingly, B^* is kept much lower, leading to a fairly modest decrease of k^*. The model maintains a GDP growth level much higher than that in the RCK model during the transitional stages. When σ rises from a medium base, since the ratio $c^*/\overline{c}^*$ is now required to attain a level higher relative to the case of a low σ, the model loses much of its force to maintain its GDP growth, and a slowdown in GDP growth is expected relating to the model with a low aversion base. But B^* is still lower than 1 and the sustainability is not a concern.

When σ rises from a high base, $c^*/\overline{c}^*$ is needed to come up to one. Thus, relating to the RCK model, there is no longer sufficient capital formation derived from under-consumption. As $B^* \to 1$, from 3(A) and 3(B), $k^* \to \overline{k}^*$, and from

(13), $gdp^* \to \overline{gdp}^*$. In other words, the model is no longer sustainable and converges to the RCK model.

In all above theoretical exploration, for the sake of tractability, there was not any unstable-state analysis. Nevertheless, the unstable-state results are implied there. With the rise of σ, for a steady state being able to go to another steady state, $c^*/\overline{c}^*$ must rise coordinately. Otherwise, the model gives rise to an unstable state. The appearance of the unstable-state results makes sense in the real world. As will be shown in subsequently, it helps to explain the economic breakdown of the Ex-Soviet Union.

3.3. A Numerical Simulation

The transition between steady states in the presence of an increasing σ is simulated with the results presented in Table 1. First, using (3A), (3B), (4A), (4B) and (9'), the optimal ratios $c^*/\overline{c}^*$ and $k^*/\overline{k}^*$ as well as B^* are obtained. Afterwards, using (13), the corresponding ratios $gdp^*/\overline{gdp}^*$ are obtained.

Table 1 tells us that, in line with Proposition 2, along with the rising σ, the ratio $c^*/\overline{c}^*$ progressively rises and converges to one, implying the convergence of the GDP max model to the RCK model. The decreases of $k^*/\overline{k}^*$ and

Table 1. Simulation results.

σ	$c^*/\overline{c}^*$	$k^*/\overline{k}^*$	$gdp^*/\overline{gdp}^*$	B^*
0.1	0.31066	5.71206	1.99109	0.35150
0.2	0.47407	4.96084	1.86996	0.38253
0.3	0.59178	4.38250	1.77112	0.41207
0.4	0.68501	3.89975	1.68515	0.44196
0.5	0.76329	3.47735	1.60766	0.47342
0.6	0.83191	3.09564	1.53618	0.50763
0.7	0.89409	2.74358	1.46949	0.54577
0.8	0.95098	2.42202	1.40864	0.58816
0.9	0.99433	2.18630	1.36516	0.62542
0.95	0.99986	2.00479	1.32071	0.65881
0.96	0.99994	1.83788	1.27561	0.69408
0.97	X	X	X	X

Notes: These results are calculated on the basis of (3A), (3B), (4A), (4B), (9') and (13) by using the representative values of $\delta = 0.07$, $\rho = 0.02$, and $\alpha = 0.4$. B^* is the steady-state marginal loss of GDP due to under-consumption. $c^*/\overline{c}^*$, $k^*/\overline{k}^*$, and $gdp^*/\overline{gdp}^*$ are the ratios of steady-state consumption per capita, capital per capita, and GDP per capita derived from the GDP max model to those from the RCK model, respectively. Both the computations of $gdp^*/\overline{gdp}^*$ from (13) and $c^*/\overline{c}^*$ from (4B) use $\theta = 0.69$, computed with $\theta = \left(\overline{gdp}^* - \delta\overline{k}^*\right)/\overline{gdp}^*$. To compute $c^*/\overline{c}^*$, (4B) is reformulated in: Min $\left(\frac{1}{B^*}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{1}{B^*}\right)^{\frac{1}{1-\alpha}}\delta\left(\frac{\alpha}{\delta+\rho}\right)^{\frac{1}{1-\alpha}} - \left(1-\frac{c^*}{\overline{c}^*}\right)^{\frac{1}{\sigma}}\overline{c}^* - c^*$, and the relationship $c^* = \frac{c^*}{\overline{c}^*}\theta\overline{gdp}^*$ is employed to replace the last termc^*. The X in the last line indicates that optimal solution is no longer possible.

$gdp^* / \overline{gdp}^*$ cease long before arriving at one, because when σ reaches a certain high level (to 0.97 in this case), the simulation is no longer able to produce an optimal solution. Nevertheless, the trend of convergence is perceived from the progressive decrease of $k^* / \overline{k}^*$ and $gdp^* / \overline{gdp}^*$, and the increase of B^* in the achieved simulation results.

4. Empirical Evidence: Slowdown or Breakdown?

To what extent does the GDP max model capture the real world? For answering this question, two criterions are applied to identify the countries having visible GDP-maximizing inclination: 1) they are free of, or meet weak democratic control (so that their governments are able to impose their choices over the market); 2) they have an unusual GDP growth period with high capital formation share in GDP.

Ex-Soviet Union is an extreme case of such model. It essentially consisted of minimizing population's consumption and mobilizing maximum economic resources for industrialization in order to win international competition, above all, military competition with the USA. In its early stages, industrialization was achieved through depriving peasants by collectivization and acquiring maximum foods and raw materials for industries (Nove, 1969). While the Soviet Union became one of the leading industrial nations in its last period, it suffered severe shortage of consumer goods. In this respect, the disincentive effects of under-consumption: the loafing on the job and absenteeism were determinant in the breakdown of the economy in the end 1980s (Filtzer, 1996). This breakdown is also explained by its ridged industrial structure to convert from heavy industry to production of consuming goods. In 1989, one-fourth of the entire Soviet population was engaged in military activities.[4] For Gaidar (2007), military industries were often "locked in" specific usage. With the economy unable to adjust its $c/\overline{c}$ ratios from the supply side, the economy led to the unstable state, and the breakdown.

The Eastern Asian model covering around ten countries in the region is close to the GDP max model. The two most eminent representatives are Japan and China. Japanese firms have been qualified as revenue-maximizing (Uekusa & Caves, 1976; Komiya, 1992; Blinder, 1992). The state had a pronounced pro-investment and GDP growth-seeking tendency during the early stage, because Japanese government was strong and the opposition was weak. Democracy in the true sense was thus not developed. The state influenced the economy through its privileged links with the dominant banking and industrial conglomerates (the zaibatsu), supported economic nationalism and helped Japanese firms with exportation and taking market shares in international competition (Morishima, 2000).

In current times, in terms of GDP growth China is the most representative rising star. Whereas Chinese economy has worked under a preliminary market

[4] http://en.wikipedia.org/wiki/Military_history_of_the_Soviet_Union.

system, with its double heritage from the central-planning system and Eastern Asian culture, and in the absence of democratic control, the state remains the major player in the economy. Chinese Government admits officially the existence of the "GDP worship" in China in the past (cf. Li, 2017).

In **Table 2**, the shares of capital formation in GDP of China and two countries historically having GDP maximization tendency: Japan and South Korea are presented in comparison with the USA and the World average level. It depicts that both Japan and South Korea had their capital to GDP ratios fall and approach the world mean level, implying the unsustainability and a stepping back to consumption optimization.

In most periods, not only were China's capital shares higher than the world mean level and that of the USA, but also its recent level was higher than that of Japan and South Korea in their growth peaks. This growing trend that began in the 1970s continues to maintain itself. As shown in **Table 2**, inversely correlated with the capital to GDP ratio, China's share of consumption in GDP has been in continuous decline and has reached below than 50% point in 2006-2010, significantly lower than the world mean level (over 75%).

Several factors could drive China to decelerate its capital expansion and, hence, GDP growth. First, as IMF (2012) estimated, its rate of utilization of production capacity has been as low as 60%. Second, one important measurement of under-consumption in a modern economy is the deficit in some basic public goods, especially those for health care, environment improvement, and poverty reduction. China has accumulated an enormous burden of health care deficiency and of environment degradation. According to World Health Organization data, in 2012, China's total health expenditures in GDP were 5.4%, while the mean value in the world was 10.1%. The government expenditures spent in the economies of energy and the environment in terms of GDP percentage were only 1.5% in 2012, lower than that of most countries (2-3%). Environmental deterioration has reached the dangerous level. The World Bank (2013: p. 249) estimated that in 2008, China's environmental degradation and resource depletion were valued at approximately 9% of GDP, over ten times higher than the corresponding levels in South Korea and Japan.

China has "economized" some public expenditures on consumption and put

Table 2. Shares of gross capital formation in GDP (%).

Period	71 - 75	76 - 80	81 - 85	86 - 90	91 - 95	96 - 00	01 - 05	06 - 10
China (Consumption/GDP)	29.10 (70.95)	32.66 (67.50)	34.91 (65.16)	37.50 (63.53)	39.72 (59.03)	38.21 (58.87)	39.30 (57.80)	44.55 (49.13)
Japan	36.25	31.71	29.40	29.91	30.10	26.79	23.21	21.75
South Korea	26.47	30.84	29.88	32.03	37.48	32.87	29.74	29.29
USA	21.98	23.14	23.51	23.01	20.72	22.48	22.43	20.93
World average level	22.40	24.29	24.16	21.92	22.56	22.52	21.77	23.93

Source: Calculated on the basis of the world development indicators (World Bank, 2014).

them all in financing their capital expansion. After more than 30 years of extensive growth through constraining people's consumption, it is now time for China to deal with its accumulated consequences. Expressed in an alternative manner, it has to return to joining the "normal" consumption-maximizing world. In 2019, China's GDP growth has decreased to 6.0 from above 10% in average during more than 30 years. According to our theoretical prediction, this decreasing trend will persist during the coming years.

5. Conclusion

The GDP-maximizing model constrained by consumers' rising aversion to under-consumption was shown to go from high growth to slow-down, and finally to converge to the RCK model that maximizes consumption. This model is able to explain what happens in a number of countries that seek exceptional GDP growth through the state intervention in capital expansion.

The limit of this study is having dealt with the rise of the aversion to under-consumption as the changes in parameter, instead of as a variable. Defining it as a variable, especially as an endogenous variable, will give rise to more theoretical insights, but will require some more sophistical modeling structure. It will be left for further research.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Aoki, M. (1980). A Comparative Dynamic Analysis of a Growth Model under Alternative Policy Regimes. *Journal of Macroeconomics, 2,* 1-39. https://doi.org/10.1016/0164-0704(80)90021-X

Barro, R., & Sala-i-Martin, X. (2003). *Economic Growth* (2nd ed.). Cambridge, MA: MIT Press.

Baumol, W. (1959). *Business Behavior, Value, and Growth.* New York: Macmillan.

Blinder, A. S. (1992). International Perspective: Trading with Japan: Why the US Loses Even on a Level Playing Field? *Business Economics, 27,* 25-29.

Brennan, H. G., & Buchanan, J. M. (1980). *The Power to Tax*. Cambridge: Cambridge University Press.

Cass, D. (1965). Optimum Growth in an Aggregative Model of Capital Accumulation. *Review of Economic Studies, 32,* 233-240. https://doi.org/10.2307/2295827

Filtzer, D. (1996). Labor Discipline and the Decline of the Soviet System. I*nternational Labor and Working-Class History, 50,* 9-28. https://doi.org/10.1017/S0147547900013181

Gaidar, Y. (2007). *Collapse of an Empire: Lessons for Modern Russia*. Washington DC: Brookings Institution Press.

IMF (2012). *People's Republic of China: Staff Report for the 2012 Article IV Consultation*. Washington DC: IMF.

Kattel, R., Kregel, J. A., & Reinert, E. S. (2009). *Radgnar Nurkse: Trade and Development*. London: Anthem Press. https://doi.org/10.7135/UPO9781843318187

Kim, J., & Lau, L. J. (1994). The Sources of Economic Growth of the East Asian Newly Industrialized Countries. *Journal of the Japanese and International Economies, 8,* 235-271. https://doi.org/10.1006/jjie.1994.1013

Komiya, R. (1992). *Business Enterprise in Japan*. Boston, MA: The MIT Press.

Koopmans, T. (1963). *On the Concept of Optimal Economic Growth*. Cowles Foundation Discussion Paper, 163.

Krugman, P. (1994). The Myth of Asia's Miracle. *Foreign Affairs, 73,* 62-78. https://doi.org/10.2307/20046929

Li, J. (2017). *Farewell to China's GDP Worship (China Studies)*. Singapore: World Scientific. https://doi.org/10.1142/10454

Morishima, M. (2000). *Japan at a Deadlock*. London and New York: Macmillan and St Martin's Press.

Niskanen, W. A. (1971). *Bureaucracy and Representative Government*. New York: Aldine-Atherton.

Novales, A., Fernandez, E., & Ruiz, J. (2009). *Economic Growth—Theory and Numerical Solution Methods*. Berlin Heidelberg: Springer-Verlag.

Nove, A. (1969). *An Economic History of the USSR*. Harmonds Worth: Penguin Books Ltd.

Nurkse, R. (1953). *Problems of Capital Formation in Underdeveloped Countries*. Oxford: Oxford University Press.

Ramsey, F. (1928). A Mathematical Theory of Saving. *Economic Journal, 38,* 543-559. https://doi.org/10.2307/2224098

Uekusa, M., & Caves, R. E. (1976). *Industrial Organization in Japan*. Washington DC: The Brooking Institution.

World Bank (2013). *China 2030, Building a Modern, Harmonious, and Creative Society*. Washington DC: World Bank.

World Bank (2014). *World Development Indicators 2014*. Washington DC: World Bank. https://doi.org/10.1596/978-1-4648-0163-1

Young, A. (1995). Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience. *The Quarterly Journal of Economics, 110,* 641-680. https://doi.org/10.2307/2946695

Appendix

To determine the relationship between σ and $c^*/\overline{c}^*$ at the steady state, from (13), totally differentiating gdp^* with respect to σ and $\frac{c^*}{\overline{c}^*}$, setting $gdp^* = 0$, and rearranging, the following equation is obtained:

$$\frac{\mathrm{d}\frac{c^*}{\overline{c}^*}}{\mathrm{d}\sigma} = \frac{-\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{\sigma}\left(\frac{1}{B^*}\right)^{\frac{1}{1-\alpha}}\left[1+\frac{1}{\sigma}\ln\left(1-\frac{c^*}{\overline{c}^*}\right)\right]-\frac{\theta}{\sigma}\left(1-\frac{c^*}{\overline{c}^*}\right)\ln\left(1-\frac{c^*}{\overline{c}^*}\right)}{\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1}{B^*}\right)^{\frac{2\alpha-1}{1-\alpha}}\left(1-\frac{c^*}{\overline{c}^*}\right)^{-1}+\theta} \quad \text{(A1)}$$

To prove its positivity, as by (14), $\partial B^*/\partial\sigma > 0$, and:

$$\frac{\partial B^*}{\partial\sigma} = -\frac{1}{\sigma}B^*\left[1+\frac{1}{\sigma}\ln\left(1-\frac{c^*}{\overline{c}^*}\right)\right] \quad \text{(A2)}$$

It must be that:

$$\frac{1}{\sigma}\ln\left(1-\frac{c^*}{\overline{c}^*}\right) < -1 \quad \text{(A3)}$$

Given $\ln\left(1-\frac{c^*}{\overline{c}^*}\right) < 0$, and by (A3), the numerator in (A1) is positive. It follows that, $\mathrm{d}\frac{c^*}{\overline{c}^*}\Big/\mathrm{d}\sigma > 0$.