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Effect of Expenditure Structure on Profitability Ratios

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Abstract

The aim of the study is to show how the company's expenditure structure affects the comparability of profitability ratios. The hypothesis is that the impact of the expenditure structure is significant, in which case it weakens the comparability of companies in terms of profitability. In addition to comparability across companies, the potential importance can also be seen in the time series data, where a change in the structure, for example due to investment or earnings management (EM), affects the ratios, although true profitability does not change. In this study, a simple steady mathematical model is developed, in which the profitability ratios are described by means of the internal rate of return (true profitability), growth rate and the lag between expenditure and revenue, as well as the expenditure structure. The importance of the expenditure structure is also investigated with the help of a numerical experiment and empirical data from 832 Finnish companies. The results show that, in accordance with the hypothesis, the expenditure structure has a significant effect on profitability ratios, especially those that are explicitly affected by the expenditure structure. The significant impact of the expenditure structure weakens comparability, and thus more attention should be paid to it when analyzing cross-sectional and time series data.

Keywords

Comparability, Profitability Ratios, Expenditure Structure, Steady Model, Numerical Analysis

1. Introduction

Expenditure structure can be defined as a mechanism for classifying in a systematic manner different types of expenditures and for collecting expenditure information. This information is used for accounting control, financial management, and budgeting purposes[1]. In this study, expenditure structure comprises the binary classification of the company's expenditures into short-term (current) and long-term (fixed) expenditures and is quantitatively measured by the share of current expenditures in all expenditures. The purpose of the study is to show how the expenditure structure, measured in this way, potentially affects the key profitability ratios used in financial statement analysis. This topic is of importance since financial statement analysis represents one of the financial world's most powerful and versatile accounting tools. Furthermore, profitability ratios are amongst the best-known financial statistics in this analysis (Gardiner, 1995). These ratios are a set of measurements used to determine the ability of a business to create earnings[2]. The use of these ratios has become so common that practically all financiers and companies use them in their financial decision-making. Therefore, it is of importance to understand how different factors, such as expenditure structure, affect the usability of profitability ratios.

The expenditure structure of a company has potentially a significant effect on profitability ratios, but this effect has not been analyzed systematically. This structure is an important accounting concept, as company management can influence its value in earnings management (EM). If the ratio is sensitive to the expenditure structure, EM can strongly influence its value. In addition, the validity of the profitability ratio can be questioned if it is particularly sensitive to the expenditure structure. Financial ratios have been intensively used by companies since the end of the 19th century (Horrigan, 1968; Foulke, 1968: Chapter 1; Lev, 1974). Therefore, also scientific research on financial statement analysis has a long history. However, this scientific research on financial ratios has been mostly empirical, so that all the theoretical weaknesses related to their use have not been resolved despite their long history (Horrigan, 1968; Lev & Sunder, 1979; Barnes, 1987). One of the main weakness of ratio analysis is that it should be limited to comparable firms, but the limits of the comparability have not investigated systematically (see Lev, 1974: 69-74)[3].

In practice, analysts typically classify companies in homogenic groups with respect to size, industry, age, and even growth to ensure (ratio) comparability (Lev, 1974: s. 38-39). In this study, it is argued that the expenditure structure as measured by the share of current expenditures provides us with an important additional factor of comparability. It is expected that the differences in the expenditure structure significantly affect the values of profitability ratios and, thus, weaken comparability between companies. This potential effect is important, since comparisons between companies can only be made effectively if the companies are mutually comparable. In comparable firms, profitability ratios strongly reflect the differences in the ability of a business to create earnings. The companies

[1]https://dpb.virginia.gov/forms/20070627-273/expenditurestructureFY2010.pdf.

[2]https://analystprep.com/cfa-level-1-exam/financial-reporting-and-analysis/capitalizing-expensing-effects-ratios/.

[3]Comparability is "the qualitative characteristic that enables users to identify and understand similarities in, and differences among, items" (Financial Accounting Standers Board (FASB) 2018, QC21).

in a sample are not comparable if the differences in profitability ratios are mainly due to differences in the expenditure structure[4].

Empirically, the most famous effect that weakens the comparability between companies is the industry effect. This effect means that financial ratios are influenced by the company's industry, and, therefore, financial ratios across different industries are not comparable (McDonald & Morris, 1984, 1985). In this case, the financial strength of a firm with a given set of financial ratios may thus depend on the industry of which it is a part. This suggests that profitability ratios of a firm reflect revenue/expenditure patterns (that is, expenditure structure) specific to its industry. For example, all profitability ratios expressed as a percentage of sales are typically much lower for retailing than manufacturing companies (Gombola & Ketz, 1983). Therefore, firms in different industries, even with similar financial ratio profile, have a different probability of failure (Ooghe & de Prijcker, 2008). It is clear that the differences in expenditure structure are not only limited to differences in industry but are specific to the company. Thus, in the same industry, differences in expenditure structure between companies may be significant but companies from different industries may have an identical structure. Thus, although being related to the industry effect, the expenditure structure effect is different and a more general concept. In business analysis, the expenditure structure is thus a very important concept, but at the same time it is a largely neglected in business research. In national economics research it however plays an important role (Liu, 2020).

Thus, in summary, the purpose of the study is to show how expenditure structure potentially affects the key profitability ratios used in financial statement analysis. The research hypothesis proposes that the effects of the expenditure structure are significant, since obviously they are visible, due to the industry effect, in the differences in profitability ratios across industries. However, this concept is not limited to industry differences but deals with any companies with different expenditure structure. If this hypothesis holds, it has clear implications to science and practice. In science, expenditure structure can provide us with a novel and interesting factor of comparability for further research of profitability analysis. In practice, it can provide us with an easy way to classify companies in comparable groups with respect to profitability ratios. This kind of classification may be more efficient than industry grouping including companies with potentially different expenditure structure in the same industry and vice versa.

The contents of this study are composed of five sections. In the introductory section, the background, motivation, and purpose of the research are presented. The second section introduces the theoretical model that has connections with the previous accounting frameworks presented by Laitinen (2017), Laitinen (2018), Laitinen & Laitinen (2022a), and Laitinen & Laitinen (2022b). In this

[4]"Financial statement comparability" is a broader concept referring to the level of standardization of accounting information that allows the financial statements of companies to be compared to each other. See de Franco, Kothari & Verdi (2011).

model, profitability ratios are derived with the aid of true profitability (internal rate of return), growth rate, average lag between expenditure and revenue, and the expenditure structure. The effects of the expenditure structure and the following effect on the average lag are associated with together using the chain rule. The effects on profitability ratios are analyzed using the partial derivatives of the ratios with respect to the factors in the model. In the third section, the effects of the expenditure structure are numerically analyzed using the numerical values and elasticities of the ratios. In the fourth section, empirical evidence from 832 Finnish firms is analyzed. Supporting the research hypothesis, the numerical and empirical analyzes show that the effects of the expenditure structure are strong, especially when the structure is high. However, the Return on Assets ratio is theoretically independent of the structure in this framework. Thus, this measure is useful in practice also when comparing profitability between companies, for example, from different industries. Finally, the last section presents a summary and conclusions.

2. Mathematical Framework for the Analysis

2.1. Basic Concepts

The theoretical results of the study are derived using a similar framework as used by researchers in analyzing the relationship between the return on investment (assets) ratio (ROI or ROA) and the internal rate of return (IRR). IRR is the central concept of profitability used by economists in discounting cash flows and it is also called the true yield (Solomon, 1966) or the economic rate of return (ERR) (Feenstra & Wang, 2000; Brief, 2013; Luckett, 1984). The models used in this research area are useful in depicting alternative accounting methods and concepts in the framework of profitability. The present study makes use of a similar model structure to derive the profitability ratios. This framework analyzes the revenue flows at the level of the firm describing revenue pattern by an infinite geometric lag structure between expenditure and the generated flow of total revenue (net sales). This revenue lag structure is important in differentiating between firms with different expenditure structure having different revenue lag. The parameter of the distribution describes the form of the structure and the length of the lag. For simplicity, the present framework makes a steady state assumption leading to that the growth rate is identical for all accounting concepts (expenditure, revenue, expenses, and assets). This is a general assumption of the models applied on this area of research (Feenstra & Wang, 2000; Luckett, 1984).

Following the general framework let us assume that total expenditure E generates an infinite geometric series of revenue following the lag parameter q. Let us assume that height of the series K is proportional to expenditure E which allows us to incorporate $IRR\ r$ in the model as follows:

$$E = E \cdot K \sum_{t=0}^{\infty} q^t (1+r)^{-t} \rightarrow K = \frac{1+r-q}{1+r} \qquad (1)$$

This result (1) shows that the height K is an increasing function of r and decreasing in q. The average time lag H can be calculated as $H = q/(1 - q)$.

Let us further assume that periodic expenditure E_t grows at a steady rate g and each periodic expenditure generates an identical flow of revenues as a relation to this expenditure. When the age of the company is assumed infinite, the following results for total revenues R_t is obtained:

$$R_t = E_0 (1+g)^t K \sum_{i=0}^{\infty} q^i (1+g)^{-i} = E_t K \frac{1+g}{1+g-q} \tag{2}$$

Thus, the revenue-expenditure ratio in period t is as follows:

$$\frac{R_t}{E_t} = \frac{(1+r-q)(1+g)}{(1+r)(1+g-q)} \tag{3}$$

This ratio (3) is symmetric with respect to r and g. If $r > g$, the ratio exceeds unity and if $g > q$, it is less that unity. For the special case $r = g$, the ratio equals unity. This ratio is also related to q and, consequently, implicitly to the expenditure structure L that determines q.

Let us assume that the periodic expenditure is expired as expense according to the same distribution as revenues are generated making expenses and revenues match. This simplifying assumption leads to the following result for the assets of the company A_t:

$$A_t = \sum_{i=0}^{\infty} E_0 (1+g)^{t-i} \sum_{j=i+1}^{\infty} q^j (1-q) = E_t \frac{q(1+g)}{1+g-q} \tag{4}$$

The accounting identity between expenses, assets and expenditure shows that expenses D_t can be solved using (4) in the following way:

$$D_t = A_{t-1} - A_t + E_t \rightarrow A_t = \frac{(E_t - D_t)(1+g)}{g} \quad g \neq 0 \tag{5}$$

Substituting now (4) in (5), the following result is obtained for expenses D_t:

$$D_t = E_t \frac{(1+g)(1-q)}{1+g-q} \tag{6}$$

In this framework, assets A_t and expenses D_t are independent of r (IRR).

If we want to analyze the effect of the expenditure structure on profitability ratios, the relation between the lag parameter and the structure must be described. Let us use the equation (1) to fix this relationship as an identity. Let us calculate the height of the lag distribution using a weighted average of the heights separately for the current and fixed expenditures as follows:

$$K = K^c \cdot L + K^f \cdot (1-L) \tag{7}$$

In this equation (7) K^c is the height for the lag distribution for current expenditure and K^f for fixed expenditure. The lagged revenue distributions for current and fixed expenditures are assumed separate with parameters q^c and q^f, respectively. L or the share of current expenditure is used as a weight in this weighted average.

Let us move K in (7) to the right-hand side of the equation and solve K^f to form the following identity G:

$$G = \frac{1+r-q}{1+r} - \frac{1+r-q^c}{1+r} \cdot L - \frac{(1+r)(1-L^o) - q + L^o q^c}{(1+r)(1-L^o)} \cdot (1-L) = 0 \tag{8}$$

In this equation (8) the solved K^f includes L as a parameter. However, when assessing the ceteris paribus relation between L and q, K^f (and thus L in this function) should be kept constant, as is indicated by the superscript o in L^o. Thus, only q and L are allowed to change in the identity $G = 0$. In this way, it is possible to assess the pure effect of L on q using the chain rule.

When the chain rule is adopted to (8), the following result is obtained:

$$\frac{\partial G}{\partial L} + \frac{\partial G}{\partial q} \cdot \frac{\partial q}{\partial L} = 0 \rightarrow \frac{\partial q}{\partial L} = -\frac{\partial G}{\partial L} \cdot \frac{\partial q}{\partial G} = \frac{q^c - q}{1-L} < 0 \tag{9}$$

Thus, the effect of L on q is always negative and, in absolute terms, an increasing function of L.

2.2. Profitability Ratios

The set of profitability ratios analyzed in this study consists of two cash-flow ratios and three accrual-based ratios. Cash-flow ratios often form a group of their own, because they are independent of the expensing method and are, therefore, not related to profit concepts (see Gombola & Ketz, 1983). Instead, they are based on the revenue-expenditure ratio (3). The first ratio analyzed here is called the Cash Flow Ratio (CFR) being defined in the following way:

$$\text{CFR} = \frac{R_t - E_t}{R_t} = 1 - \frac{E_t}{R_t} = \frac{q(r-g)}{(1+r-q)(1+g)} \tag{10}$$

This ratio is positive if $r > g$, negative if $g > r$, and zero if $r = g$.

The Appendix presents the partial derivatives of the ratio. CFR is a positive function of r and negative in g in the same ways as the revenue-expenditure ratio (3). The ratio does not explicitly depend on the expenditure structure L but it depends on the lag parameter q that is negatively related to L. Let us consider the derivatives in three stages of company life. First, in the growth stage when $g > r$, the derivative is negative and, thus, the effect of L is positive. Second, in the break-even stage when $g = r$, the ratio is independent of q and L. Third, in the profitability stage when $r > g$, the effect of q is positive. Thus, the effect of q (and L) on CFR is conditional to the relation between r and g.

The second ratio analyzed in this framework is Operating Cash Flow Ratio (OCFR) that explicitly depends on the expenditure structure L as follows:

$$\text{OCFR} = \frac{R_t - L \cdot E_t}{R_t} = 1 - \frac{L \cdot E_t}{R_t} = 1 - L \cdot \frac{(1+r)(1+g-q)}{(1+r-q)(1+g)} \tag{11}$$

The formula of OCFR explicitly includes both L and q which means that the chain rule must be applied in differentiation to take account of change simultaneously in both variables. The Appendix shows that OCFR is an increasing function of r and decreasing in g. The derivative of OCFR with respect to L, q (chain rule) is negative for the profitability ($r > g$) and break-even ($r = g$) stages. For the growth stage ($g > r$) the derivative is almost always negative, too. However, for a very high L close to unity in can be positive.

The third ratio considered here is Profit Margin Ratio (PMR) based on the expense concept (6) instead of the expenditure concept (3). This ratio is got when profit is divided by revenue as follows:

$$\text{PMR} = \frac{R_t - D_t}{R_t} = 1 - \frac{D_t}{R_t} = \frac{rq}{1 + r - q} \tag{12}$$

The Appendix shows that PMR is a positive function of r but independent of g. This ratio does not explicitly depend on the expenditure structure L. However, it is a positive function of q which means that it is negatively (implicitly) related to L.

The fourth profitability ratio analyzed here is the Return on Assets Ratio (ROAR) which is the most widely adopted measure of profitability. This ratio is based on the expense concept (6) but also the asset concept (5) and can be presented as:

$$\text{ROAR} = \frac{R_t - D_t}{A_{t-1}} = r \cdot \frac{1 + g}{1 + r} \tag{13}$$

ROAR is a positive function of both r and g but sensitive especially to r. If $r = g$, ROAR = r. This ratio is independent of L and q which is due to the neutral expense concept adopted. If the rate-of-return (realization) expense (accelerated) theory is adopted instead of the neutral concept, ROAR is resulted as the product of r and the revenue-expenditure ratio (3). This concept is very sensitive to r but also affected by q. If the compound interest (annuity) expense (decelerated) theory is adopted, the resulted ROAR = r only depending on IRR (Laitinen, 2018).

The fifth and last profitability ratio analyzed here is the Gross Profit Margin Ratio (GPMR) that is obtained by dividing the gross profit (difference between revenue and current expenses) by revenue in the following way:

$$\text{GPMR} = \frac{R_t - D_t^c}{R_t} = 1 - \frac{D_t^c}{R_t} = 1 - L \cdot \frac{\left(1 - q^c\right)\left(1 + r\right)\left(1 + g - q\right)}{\left(1 + g - q^c\right)\left(1 + r - q\right)} \tag{14}$$

The derivatives of GPMR are presented in The Appendix. This ratio is an increasing function of r and decreasing in g. In the break-even ($r = g$) and profitability ($r > g$) stages L affects through the chain rule negatively the ratio. However, in the growth stage ($g > r$) the effect can be positive if L is very high. This condition is identical with that derived for OCFR.

3. Numerical Analysis of Profitability Ratios

3.1. Basic Assumptions

Theoretical analysis showed that the expenditure structure has a negative effect on the values of the profitability indicators, except for the rare situation where the structure is very high, close to one. However, it is an empirical question how important the effect is. Let us assess the effect using numerical representative values drawn from Finnish firms. Table 1 presents the distribution of the one-year share of current expenditure from 832 large and middle-size Finnish companies for different industries. This table shows that the expenditure structure is associated with industry in a significant way. The one-way ANOVA indicates that the association is statistically significant F-test statistics being 15.6730 (p-value < 0.001). The lowest mean value of the structure (0.7065) is found for companies in Real estate activities (12 L) whereas the highest mean value (0.9777) is got for industries Wholesale and retail trade (7 G) and Arts, entertainment and recreation (18 R). The mean of the average values in the mean column of the table is 0.9185. Thus, referring to the values in the table the following three levels of structure (share of current expenditure) are selected to assess the effect: $L = 0.850$ (low structure), $L = 0.915$ (average structure), and $L = 0.980$ (high structure).

Table 1. Industrial evidence on the expenditure structure from 832 Finnish firms.

		Companies		Current Expenditure/ Total Expenditure	
Class	Industry	Number	Percent	Mean	Median
1	A Agriculture, forestry and fishing 01 - 03	7	0.84	0.8901	0.9477
2	B Mining and quarrying 05 - 09	4	0.48	0.9290	0.9424
3	C Manufacturing 10 - 33	311	37.38	0.9600	0.9743
4	D Electricity, gas, steam and air conditioning supply 35	18	2.16	0.7886	0.8138
5	E Water supply; sewerage, waste management and remediation activities 36 - 39	3	0.36	0.8990	0.9257
6	F Construction 41 - 43	49	5.89	0.9748	0.9891
7	G Wholesale and retail trade; repair of motor vehicles and motorcycles 45 - 47	151	18.15	0.9777	0.9910
8	H Transportation and storage 49 - 53	50	6.01	0.9397	0.9734
9	I Accommodation and food service activities 55 - 56	24	2.88	0.9519	0.9698
10	J Information and communication 58 - 63	52	6.25	0.9530	0.9730
11	K Financial and insurance activities 64 - 66	40	4.81	0.9551	0.9802
12	L Real estate activities 68	7	0.84	0.7065	0.7918
13	M Professional, scientific and technical activities 69 - 75	57	6.85	0.9524	0.9730
14	N Administrative and support service activities 77 - 82	29	3.49	0.9461	0.9686
16	P Education 85	6	0.72	0.9009	0.9512
17	Q Human health and social work activities 86 - 88	10	1.20	0.9322	0.9702
18	R Arts, entertainment and recreation 90 - 93	7	0.84	0.9777	0.9819
19	S Other service activities 94 - 96	7	0.84	0.8987	0.9155
	In all	832	100.00	0.9185	

In the numerical experiments, the association between the expenditure structure L and the lag parameter q must be described, corresponding the chain rule in the theoretical analysis. This association is constructed here using the result for the average time lag as $H = q/(1 - q)$. Let us assume for numerical analyses that the constant average time lag for current expenditure H^c is 0.25 years (three months) and for fixed expenditure H^f correspondingly 2.50 years (compare with Laitinen & Laitinen, 2022a). Then, H is approximated using the following weighted average:

$$H = L \cdot H^c + (1 - L) \cdot H^f = L \cdot 0.25 + (1 - L) \cdot 2.50 \tag{15}$$

It is now possible to use H to get an estimate for q as $q = H/(1 + H)$. In this way, q is estimated for the selected three levels of L. Gross Profit Margin Ratio (GPMR) also includes q^c that is calculated simply by $q^c = 0.25/(1 + 0.25) = 0.20$. In the experiments, three levels (low, average, and high) for both IRR r and growth g are specified as 0.05, 0.10, and 0.15, respectively.

3.2. Profitability Ratios

Table 2 presents the numerical values for the five profitability ratios using the parameter values specified above. The figures are presented in three stages (growth, break-even, and profitability stage) for the three levels of the expenditure structure L (low, average, and high structure). The first cash flow ratio, Cash Flow Ratio (CFR) is always negative in the growth stage, because $g > r$. When the expenditure structure rises during the growth period, CFR improves, but remains negative. In the break-even period, CFR is always zero and, thus, independent of the expenditure structure, because $g = r$. In the profitability period, when $r > g$, an increase in the expenditure structure weakens the ratio. This questions its validity as a profitability measure. In the growth and profitability period, changes in growth and IRR of the same magnitude and in the same direction largely cancel each other out. Therefore, CFR mainly reflects the relation between growth and IRR. If we calculate Spearman's rank correlation from the values in **Table 2** (21 observations), the ratio has a significant correlation (p-value = 0.012) only with IRR r (+0.488) and growth g (−0.488). However, the correlation coefficient between CFR and the expenditure structure (and the lag parameter q) is zero.

Table 2. Numerical values of the model parameters and profitability ratios.

	Parameters:					Profitability ratios:				
Stage	r	g	L	q	q^c	CFR	OCFR	PMR	ROAR	GPMR
1. Low expenditure structure										
Growth	0.0500	0.1000	0.8500	0.3701	0.2000	−0.0247	0.1290	0.0272	0.0524	0.1483
	0.1000	0.1500	0.8500	0.3701	0.2000	−0.0220	0.1313	0.0507	0.1045	0.1587

Continued

Break-even	0.0500	0.0500	0.8500	0.3701	0.2000	0.0000	0.1500	0.0272	0.0500	0.1600
	0.1000	0.1000	0.8500	0.3701	0.2000	0.0000	0.1500	0.0507	0.1000	0.1689
	0.1500	0.1500	0.8500	0.3701	0.2000	0.0000	0.1500	0.0712	0.1500	0.1768
Profitability	0.1000	0.0500	0.8500	0.3701	0.2000	0.0241	0.1705	0.0507	0.0955	0.1803
	0.1500	0.1000	0.8500	0.3701	0.2000	0.0216	0.1683	0.0712	0.1435	0.1868
2. Average expenditure structure										
Growth	0.0500	0.1000	0.9150	0.3062	0.2000	−0.0187	0.0679	0.0206	0.0524	0.0886
	0.1000	0.1500	0.9150	0.3062	0.2000	−0.0168	0.0697	0.0386	0.1045	0.0990
Break-even	0.0500	0.0500	0.9150	0.3062	0.2000	0.0000	0.0850	0.0206	0.0500	0.0958
	0.1000	0.1000	0.9150	0.3062	0.2000	0.0000	0.0850	0.0386	0.1000	0.1053
	0.1500	0.1500	0.9150	0.3062	0.2000	0.0000	0.0850	0.0544	0.1500	0.1139
Profitability	0.1000	0.0500	0.9150	0.3062	0.2000	0.0184	0.1018	0.0386	0.0955	0.1124
	0.1500	0.1000	0.9150	0.3062	0.2000	0.0165	0.1001	0.0544	0.1435	0.1201
3. High expenditure structure										
Growth	0.0500	0.1000	0.9800	0.2278	0.2000	−0.0126	0.0077	0.0139	0.0524	0.0297
	0.1000	0.1500	0.9800	0.2278	0.2000	−0.0114	0.0089	0.0261	0.1045	0.0402
Break-even	0.0500	0.0500	0.9800	0.2278	0.2000	0.0000	0.0200	0.0139	0.0500	0.0315
	0.1000	0.1000	0.9800	0.2278	0.2000	0.0000	0.0200	0.0261	0.1000	0.0418
	0.1500	0.1500	0.9800	0.2278	0.2000	0.0000	0.0200	0.0371	0.1500	0.0509
Profitability	0.1000	0.0500	0.9800	0.2278	0.2000	0.0124	0.0322	0.0261	0.0955	0.0436
	0.1500	0.1000	0.9800	0.2278	0.2000	0.0112	0.0310	0.0371	0.1435	0.0525

Legend: r = IRR; g = Growth rate; L = Expenditure structure; q = Lag parameter for total expenditure; q^c = Lag parameter for current expenditure; 1. CFR = Cash Flow Ratio CFR; 2. OCFR = Operating Cash Flow Ratio OCFR; 3. PMR = Profit Margin Ratio PMR; 4. ROAR = Return on Assets Ratio ROAR; 5. GPMR = Gross Profit Margin Ratio GPMR.

The second cash flow ratio, Operating Cash Flow Ratio (OCFR), is directly related to the expenditure structure in the break-even period, because OCFR = 1 − L, if $g = r$. The OCFR weakens considerably when the expenditure structure rises. When the expenditure structure is high, the value of the indicator approaches zero. In the same way as with the ratio CFR, changes in growth and IRR of the same magnitude and in the same direction mostly cancel each other out. OCFR therefore depends on growth and IRR, but its general level is almost completely determined by the expenditure structure in the form of 1 − L. This is demonstrated by the rank correlation between OCFR and the expenditure structure that is −0.948 (p-value = < 0.001). The first accrual-based profitability ratio, Profit Margin Ratio (PMR), quite directly depends on IRR, but not on growth g, so it works reasonably well, also in the break-even stage, as a profitability indicator in that sense. However, the values of PMR deteriorate considerably when the expenditure structure becomes higher, which weakens its comparability. Its rank correlation is +0.753 (p-value < 0.001) with IRR and −0.620 (p-value = 0.001) with the expenditure structure.

The second accrual-based profitability ratio is Return on Assets Ratio (ROAR), which in this framework is independent of the expenditure structure L. ROAR is particularly sensitive to IRR, but its values are also affected by growth. If $g = r$, then ROAR = r. The effect of growth on the key figure is not strong with the values used here, so the validity of the ratio is good, especially in samples where growth variation is small or where growth values are close to ROAR. In the present numerical sample, its only statistically significant rank correlation (0.945) is with IRR (p-value < 0.001). The third accrual-based profitability ratio Gross Profit Margin Ratio (GPMR) is more sensitive to IRR than to growth. However, the level of the ratio values is largely determined by the expenditure structure, but not as strongly as the OCFR values. Thus, the validity of the ratio is questioned. OCFR does not significantly correlate with IRR and g but has a high rank correlation (−0.944) with the expenditure structure (p-value < 0.001).

The sensitivity of the profitability ratios to the expenditure structure can be assessed by calculating the elasticity of the ratios to the expenditure structure. Table 3 shows the elasticities of the ratios, i.e. the relative change of the ratios of each stage (growth, break-even and profitability) in relation to the relative change of the expenditure structure, first from the low expenditure structure (L = 0.850) to the average expenditure structure (L = 0.915) and from there to the high expenditure structure (L = 0.980). The lowest elasticity is found with ROAR, which does not depend on the expenditure structure at all, in which case the elasticity is 0 in all stages. The second lowest elasticity is found with CFR, whose elasticity is a little over three when the expenditure structure changes from low to average and about 4,5 when the expenditure structure changes from average to high. However, the elasticities of the ratio are zero during the break-even stages. PMR has the same elasticity, with the difference that break-even stages also have similar elasticities as other stages. OCFR clearly has the largest elasticities, which are double those of PMR. In the same way, the ratio GPMR also has very high elasticities, but still lower than OCFR. It is clear that the high elasticities, especially with OCFR and GPMR, which are explicitly affected by the expenditure structure (and not only implicitly through the lag parameter q), weaken the validity of the ratios in profitability assessment very significantly.

4. Empirical Evidence

4.1. Data Description

The present mathematical model is based on assumptions related to a steady state and steady long-term growth which usually only approximately hold for larger firms. Therefore, empirical evidence for this study is gathered from a

Table 3. Elasticities of profitability ratios with respect to the expenditure structure.

Profitability ratios:				
CFR	OCFR	PMR	ROAR	GPMR
1. From low to average expenditure structure				
Growth stage				
3.1883	−6.1941	−3.1883	0.0000	−5.2661
3.1298	−6.1374	−3.1298	0.0000	−4.9161
Break-even stage				
0.0000	−5.6667	−3.1883	0.0000	−5.2500
0.0000	−5.6667	−3.1298	0.0000	−4.9211
0.0000	−5.6667	−3.0782	0.0000	−4.6548
Profitability stage				
−3.1298	−5.2698	−3.1298	0.0000	−4.9259
−3.0782	−5.3015	−3.0782	0.0000	−4.6708
2. From average to high expenditure structure				
Growth stage				
4.6011	−12.4888	−4.6011	0.0000	−9.3563
4.5439	−12.2841	−4.5439	0.0000	−8.3672
Break-even stage				
0.0000	−10.7647	−4.6011	0.0000	−9.4423
0.0000	−10.7647	−4.5439	0.0000	−8.4937
0.0000	−10.7647	−4.4928	0.0000	−7.7800
Profitability stage				
−4.5439	−9.6261	−4.5439	0.0000	−8.6183
−4.4928	−9.7165	−4.4928	0.0000	−7.9185

Legend: 1. CFR = Cash Flow Ratio CFR; 2. OCFR = Operating Cash Flow Ratio OCFR; 3. PMR = Profit Margin Ratio PMR; 4. ROAR = Return on Assets Ratio ROAR; 5. GPMR = Gross Profit Margin Ratio GPMR.

sample of middle-sized and large firm having longer time-series of successive financial statements available. This sample has extracted from the ORBIS database of van Dijk (BvD) (Bureau van Dijk, 2023). ORBIS includes financial and other information on more than 489 million companies across the globe. ORBIS captures and blends data from more than 170 different sources and makes the data standardized and comparable. In extracting the sample, a restriction that the selected company must be Finnish and employ more than 50 employees was applied. In addition, the selected company must have a longer time series of total expenditure and net sales available, to estimate the long-term growth rate. The sample was extracted from a period before the COVID-19 pandemic to avoid its potential effects on the data. In this way, the final sample consisted of 832 companies. This sample consisted of mainly industrial (797) and private companies (728). There are only 79 public companies in the data. The industrial distribution of the firms is presented earlier in Table 1. The size distribution is skewed, since the average number of employees is 832 but the median is only 197 employees.

The variables of the analyses were calculated following the theoretical concepts. Current expenditure was defined as the sum of current expenses and the change in inventories. In the same way, fixed (long-term) expenditure was defined as the sum of fixed investments and the chance in fixed assets. The total revenue concept was measured by net sales. Then, the five profitability ratios were calculated as follows:

CFR = (Total revenue − Total expenditure)/Total revenue;

OCFR = (Total revenue − Current expenditure)/Total revenue;

PMR = (Total revenue − Total expense)/Total revenue = EBIT/Total revenue;

ROAR = (Total revenue − Total expense)/Total assets = EBIT/Total assets;

GPMR = (Total revenue − Current expense)/Total revenue.

In these definitions, total revenue (net sales) does not include other revenue and total expense does not include interest expense or taxes. The expenditure structure parameter (L) was estimated as the ratio of current expenditure and total expenditure.

The long-term growth rate (g) was approximated by the standard method using the regression analysis to explain the logarithmic time-series of total expenditure by the time index and using the coefficient of time, as the estimate of growth rate. This estimate was also calculated using the time-series of total revenue and the final approximation was got calculating the weighted average of the estimates, using the sum of the time-series as weights. In general, the regression analysis indicated a high multiple coefficient of correlation. The estimates for total expenditure and total revenue were generally close to each other. This estimation was based on nine-year time-series. Finally, the internal rate of return (r) was approximated using Equation (13) as follows:

$$\text{ROAR} = \frac{R_t - D_t}{A_{t-1}} = r \cdot \frac{1+g}{1+r} \rightarrow r = \text{ROAR} \cdot \frac{1}{1+g-\text{ROAR}} \tag{16}$$

This transformation gives a simple approximation of the internal rate of return based on the assumed method of expiring expenditures as expenses.

4.2. Empirical Results

Table 4 presents descriptive statistics for the research variables. The expenditure structure parameter L is at the company level on average 0.9530 exceeding

Table 4. Descriptive statistics of the variables (N = 832).

Variable	Mean	Median	Standard Deviation	Skewness	Kurtosis
L	0.9530	0.9773	0.0745	−3.9120	21.0660
CFR	−0.0169	0.0017	0.1319	−4.5910	49.1200
OCFR	0.0375	0.0329	0.0866	−0.3860	9.9970
PMR	0.0374	0.0343	0.0798	−0.8270	12.5810
ROAR	0.0613	0.0555	0.1258	1.1540	11.1550
GPMR	0.0618	0.0485	0.0760	1.3760	8.0880
r	0.1322	0.0575	1.1282	20.9960	465.6630
g	0.0216	0.0197	0.0571	0.4400	3.1360

Legend: 1. CFR = Cash Flow Ratio CFR; 2. OCFR = Operating Cash Flow Ratio OCFR; 3. PMR = Profit Margin Ratio PMR; 4. ROAR = Return on Assets Ratio ROAR; 5. GPMR = Gross Profit Margin Ratio GPMR; 6. r = Approximated internal rate of return; 7. g = Estimated long-term growth rate.

the industrial average in **Table 1**. The mean and median of CFR are both close to zero reflecting low skewness but also the quite equal height of r and g in companies. OCFR, PMR, ROAR, and GPMR all show a quite low skewness but a higher kurtosis. OCFR and PMR have a negatively skewed distribution whereas the skewness of ROAR and GPMR is positive. The approximated estimate of r is on average higher that the average level of profitability in numerical experiments. However, the distribution of r tends to have a very high standard deviation, skewness, and kurtosis. These characteristics are a consequence of the transformation used to extract the estimate. The nine-year time-series estimate of g is very low which is typical for long-term growth estimates. However, in spite of taking account of that, it must be concluded that the growth of large Finnish firms has, before the COVID-19 pandemic, been relatively slow.

Table 5 presents the F-test of the one-way analysis of variance (ANOVA) to test the industry effect for the research variables. In this context, the F-test is used to assess whether the expected values of the research variables within industry groups differ from each other. In all, industry classification used in this test is the same as in **Table 1** including 18 industrial groups. The test indicates that the values of the expenditure structure parameter L statistically in a very significant way differ in industry groups from each other (p-value < 0.001). For this variable, F-statistics has the highest value (15.6730). In the same way, each of the five profitability ratios differs in these groups from each other. The value of the F-statistic for these ratios is highest for GPMR (10.6630) indicating most significant differences. The lowest F-value has got by ROAR (1.8070). The values of the growth g also significantly differ in industry groups. Finally, the approximation of the internal rate of return r shows a similar significance of F as ROAR.

Table 5. One-way ANOVA for industry class and the variables (N = 832).

Variable	Sum of Squares	Mean Square	*F*	p-value
L	1.1380	0.0670	15.6730	<0.001
CFR	1.3120	0.0770	4.7770	<0.001
OCFR	0.5400	0.0320	4.5420	<0.001
PMR	0.4680	0.0280	4.6560	<0.001
ROAR	0.4780	0.0280	1.8070	0.0230
GPMR	0.8730	0.0510	10.6630	<0.001
r	39.1890	2.3050	1.8420	0.0200
g	0.1890	0.0110	3.5810	<0.001

Legend: *F* = *F* statistic. Variables: 1. CFR = Cash Flow Ratio CFR; 2. OCFR = Operating Cash Flow Ratio OCFR; 3. PMR = Profit Margin Ratio PMR; 4. ROAR = Return on Assets Ratio ROAR; 5. GPMR = Gross Profit Margin Ratio GPMR; 6. *r* = Approximated internal rate of return; 7. *g* = Estimated long-term growth rate.

Table 6. Spearman rank correlation coefficients between the variables (N = 832).

		L	CFR	OCFR	PMR	ROAR	GPMR	*r*	*g*
L	Correlation	1.000	0.368	−0.239	−0.294	−0.097	−0.480	−0.093	−0.162
	p-value	.	<0.001	<0.001	<0.001	<0.003	<0.001	<0.004	<0.001
CFR	Correlation	0.368	1.000	0.690	0.563	0.658	0.222	0.662	0.087
	p-value	<0.001	.	<0.001	<0.001	<0.001	<0.001	<0.001	<0.006
OCFR	Correlation	−0.239	0.690	1.000	0.891	0.776	0.614	0.777	0.205
	p-value	<0.001	<0.001	.	<0.001	<0.001	<0.001	<0.001	<0.001
PMR	Correlation	−0.294	0.563	0.891	1.000	0.871	0.702	0.872	0.255
	p-value	<0.001	<0.001	<0.001	.	<0.001	<0.001	<0.001	<0.001
ROAR	Correlation	−0.097	0.658	0.776	0.871	1.000	0.495	0.999	0.289
	p-value	<0.003	<0.001	<0.001	<0.001	.	<0.001	0.000	<0.001
GPMR	Correlation	−0.480	0.222	0.614	0.702	0.495	1.000	0.496	0.179
	p-value	<0.001	<0.001	<0.001	<0.001	<0.001	.	<0.001	<0.001
r	Correlation	−0.093	0.662	0.777	0.872	0.999	0.496	1.000	0.259
	p-value	<0.004	<0.001	<0.001	<0.001	0.000	<0.001	.	<0.001
g	Correlation	−0.162	0.087	0.205	0.255	0.289	0.179	0.259	1.000
	p-value	<0.001	<0.006	<0.001	<0.001	<0.001	<0.001	<0.001	.

Legend: p-value refers to 1-tailed significance. Variables: 1. CFR = Cash Flow Ratio CFR; 2. OCFR = Operating Cash Flow Ratio OCFR; 3. PMR = Profit Margin Ratio PMR; 4. ROAR = Return on Assets Ratio ROAR; 5. GPMR = Gross Profit Margin Ratio GPMR; 6. *r* = Approximated internal rate of return; 7. *g* = Estimated long-term growth rate.

Table 6 presents the Spearman rank correlation coefficients between the research variables. These coefficients indicate that the expenditure structure parameter L has a significant negative correlation with r, g and all profitability ratios, except for CFR (+0.369). These correlations are for the most part in line with the signs of the partial derivatives and the results from the numerical experiment. In these analyses, CFR was the only ratio being positively related to L. However, L is correlated with ROAR (−0.097), although the rank correlation coefficient is relatively low. ROAR was showed theoretically to be independent of L. The correlation is caused by the empirical negative correlation between L and (approximated) r (−0.093). Although profitability ratios are clearly affected by L, they have all a high positive correlation with ROAR and r showing that these ratios anyway reflect the differences in profitability. The long-term growth rate g is also positively related to these ratios but the correlations are clearly lower than for r.

5. Conclusion

Ratio analysis is an old and important method for comparing financial performance of companies. Profitability plays a key role in this performance, as profitability is a basic condition for economic activity. Researchers agree that the ratio analysis should only be applied to comparable companies that are homogeneous in terms of extraneous factors affecting profitability ratios. These factors influencing the ratios have not been studied sufficiently and systematically. This study dealt with one factor, that has got little or no scientific research. This issue is the expenditure structure, which in this context is measured by the share of current expenditure out of all expenditure. The hypothesis of the research is the assumption that the expenditure structure has a significant effect on profitability ratios, which is why it is an important factor affecting comparability across firms and should be used in classifying companies into homogeneous groups.

The research results gave strong support to the presented hypothesis. Based on the theoretical, numerical, and empirical results, the expenditure structure has a strong effect on the values of most profitability indicators. Five ratios were evaluated in the study, two of which are cash flow-based (CFR and OCFR) and three are accrual-based (PMR, ROAR and GPMR) including expense concepts. Cash flow ratios do not include a causal relationship between expenditure and revenue, so both IRR (true profitability) and growth rate affect them relatively equally. However, the expenditure structure has a strong effect, especially on the cash flow indicator OCFR, which is explicitly affected by the expenditure structure. Profit-based margin PMR is sensitive to IRR, but the expenditure structure also affects its values considerably. If you calculate the gross margin GPMR, which is explicitly affected by the expenditure structure, the effect of the structure is decisive. In this framework, ROAR proved to be theoretically independent of the expenditure structure. The indicator is also sensitive to IRR, probably regardless of the depreciation method, so it works well in measuring profitability.

The research results have a clear meaning for the application of profitability indicators. Cash flow ratios do not have a clear connection with profitability except as a pair of growth. In the same way, the expenditure structure confuses the connection of accrual-based ratios with profitability, with the exception of ROAR. In practice, you have to be particularly careful when applying the indicators that are explicitly affected by the expenditure structure (OCFR and GPMR). The values of these indicators are mostly significantly based on the effect of this structure. It is therefore recommended that the expenditure structure be utilized in classifying companies into homogeneous groups. However, the importance of the expenditure structure is not limited to cross-sectional analysis, as it is also essential in time series analysis. The changes in the ratios may be mainly due to the fact that the expenditure structure has changed due to investments, for example. The expenditure structure can also be inappropriately changed with the help of Earnings Management (EM). Both issues have nothing to do with a change in profitability.

In this study, a simple mathematical model has been used in the derivation of the theoretical results, where only one method of expiring expenditures as expenses is applied. In future studies, the model should be more complicated and include other ways of expensing. It is likely that the main results of this study will not change substantially, but more detailed results will be obtained. In this study, simple numerical (experiment) data and a cross-sectional small sample have been used to evaluate the importance of the expenditure structure. In further studies, it is worth using extensive empirical cross-sectional and time-series data and advanced statistical methods for analysis. In this study, only correlations were used to show empirically the effects of the expenditure structure on profitability ratios. In this way, systematic scientific information is obtained on how the expenditure structure affects the values of the ratios and how the material should be homogenized before comparative analyses of profitability. This no doubt provides new information about the limits of comparability. In this study, the usability of the results in practice was not tested. In future studies, these tests should be made and compares with the results obtained from using industry classification.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix. Partial Derivatives of the Profitability Ratios

1. Cash Flow Ratio CFR

$$\text{CFR} = \frac{q(r-g)}{(1+r-q)(1+g)}$$

$$\frac{\partial \text{CFR}}{\partial r} = \frac{q(1+g-q)}{(1+g)(1+r-q)^2} > 0$$

$$\frac{\partial \text{CFR}}{\partial g} = \frac{-q(1+r)}{(1+g)^2(1+r-q)} < 0$$

$$\frac{\partial \text{CFR}}{\partial q} = \frac{(r-g)(1+r)}{(1+g)(1+r-q)^2} > 0 \text{ if } r > g, \ = 0 \text{ if } r = g, \ < 0 \text{ if } r < g$$

2. Operating Cash Flow Ratio OCFR

$$\text{OCFR} = 1 - L \cdot \frac{(1+r)(1+g-q)}{(1+g)(1+r-q)}$$

$$\frac{\partial \text{OCFR}}{\partial r} = \frac{Lq(1+g-q)}{(1+g)(1+r-q)^2} > 0$$

$$\frac{\partial \text{OCFR}}{\partial g} = \frac{-Lq(1+r)}{(1+g)^2(1+r-q)} < 0$$

$$\frac{\partial \text{OCFR}}{\partial L, q} = \frac{-(1+r)(g-r)\left((q^c-q)L + (1+g-q)(1-L)(1+r-q)\right)}{(1-L)(1+g)(1+r-q)^2}$$

< 0 if $r \geq g$ and only > 0 if:

$$L > \frac{(1+g-q)(1+r-q)}{(1+g-q)(1+r-q) - (g-r)(q^c-q)}$$

L must be very close 1 to fulfil this condition.

3. Profit Margin Ratio PMR

$$\text{PMR} = \frac{qr}{1+r-q}$$

$$\frac{\partial \text{PMR}}{\partial r} = \frac{q(1-q)}{(1+r-q)^2} > 0$$

$$\frac{\partial \text{PMR}}{\partial g} = 0$$

$$\frac{\partial \text{PMR}}{\partial q} = \frac{r(1+r)}{(1+r-q)^2} > 0$$

4. Return on Assets Ratio ROAR

$$\text{ROAR} = r \cdot \frac{1+g}{1+r}$$

$$\frac{\partial \text{ROAR}}{\partial r} = \frac{1+g}{(1+r)^2} > 0 \text{ if } r > 0$$

$$\frac{\partial \mathrm{ROAR}}{\partial g} = \frac{r}{1+r} > 0 \text{ if } r > 0$$

$$\frac{\partial \mathrm{ROAR}}{\partial q} = 0$$

5. Gross Profit Margin Ratio GPMR

$$\mathrm{GPMR} = \frac{R_t - D_t^c}{R_t} = 1 - \frac{D_t^c}{R_t} = 1 - L \cdot \frac{\left(1-q^c\right)\left(1+r\right)\left(1+g-q\right)}{\left(1+g-q^c\right)\left(1+r-q\right)}$$

$$\frac{\partial \mathrm{GPMR}}{\partial r} = \frac{Lq\left(1-q^c\right)\left(1+g-q\right)}{\left(1+g-q^c\right)\left(1+r-q\right)^2} > 0$$

$$\frac{\partial \mathrm{GPMR}}{\partial g} = \frac{L\left(q-q^c\right)\left(1+r\right)\left(q^c-1\right)}{\left(1+g-q^c\right)^2\left(1+r-q\right)} < 0$$

$$\frac{\partial \mathrm{GPMR}}{\partial L, q} = \frac{-\left(1-q^c\right)\left(1+r\right)\left(\left(1+g-q\right)\left(1+r-q\right)\left(1-L\right)+L\left(g-r\right)\left(q^c-q\right)\right)}{\left(1-L\right)\left(1+g-q^c\right)\left(1+r-q\right)^2} < 0$$

if $r \geq g$ and only > 0 if:

$$L > \frac{\left(1+g-q\right)\left(1+r-q\right)}{\left(1+g-q\right)\left(1+r-q\right)-\left(g-r\right)\left(q^c-q\right)}$$

L must be very close 1 to fulfil this condition. Condition is same as for Operating Cash Flow Ratio OCFR.