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Math Modules Training Improves Math Achievement & Associated Cognitive Processing

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Abstract

Background: Previous research point to a correlation between mathematical skills and cognitive processes involved in planning and simultaneous processing. Consistent with multicomponent models of mathematical achievement (domain-general and domain-specific skills), PASS theory appears to be very useful as a multifactorial framework that provides specific tests to monitor the development of mathematical competence and to direct intervention procedures and improve mathematical skills. **Objective**: This study was conducted to assess the impact of the Math Modules Cognitive Training Program on the mathematical competence of typical 2nd-grade students in calculation, problem-solving, and underlying mental functions, compared to a control group. The program was designed to optimize the Planning/FE, Attention, Simultaneous, and Successive cognitive processes through a series of tasks. **Participants**: The study involved 60 students aged between 6 and 8 years (*Mdn* = 7 years and 7 months), who were in the second grade of two urban public schools. **Method**: The program focused on mathematical skill tasks related to fluent calculation and mathematical problem solving that requires PASS cognitive processes for successful completion. The intervention group received the Math Modules program, and the control group followed their usual classroom program. Students were evaluated in calculation, problem-solving, and PASS cognitive processes. **Results:** Our results showed that the Math Modules Cognitive Training Program focused on calculation and problem solving skills were effective in improving children's mathematical performance and their PASS cognitive processes, generating gains not achieved by the control group. **Conclusions:** Our study suggests that fluid calculation and problem-solving math tasks, based on planning and simultaneous processing, could foster curricular math competency.

Keywords

Math Proficiency, Math Modules, PASS Processes, Cognitive Training Program, Calculation, Problem-Solving

1. Introduction

Math Competence

Math competence (MC) refers to conceptual comprehension, procedural fluency, strategic competence, reasoning, and productivity, and it manifests in the contexts of people's lives. Geary (2011) has emphasized the consequences of poor MC in the professional context. Duncan et al. (2007) have indicated the high risk of poor MA at the end of the educational stage in students who begin with poor competence. Early intervention for schoolchildren at risk of long-term mathematical difficulty is critical to their personal, social, and professional well-being (Parsons & Bynner, 2005). Mathematical achievement (MA) in kindergarten predicts MA in subsequent schooling (Duncan et al., 2007). Intervention in mathematical knowledge, a predictor of long-term risk in arithmetic competence, has the potential to produce considerable social and personal benefits (Hudson, Price, & Gross, 2009).

Research has identified domain-specific cognitive skills such as calculus and numerical comparison skills and the underlying numerical representations that are important for MA. Domain-general skills involved in many areas of learning and mathematical learning, such as language or spatial ability, as well as executive functions (EFs), have been identified. Both types of specific and general skills contribute to MA. Among the MA's domain-general skills, current research has paid attention to the EFs necessary to monitor and control thoughts, actions, and mathematical learning and achievement (Bull & Lee, 2014; Gilmore, Keeble, Richardson, & Cragg, 2015). Three types of EFs have been considered: flexible thinking or shifting, inhibition of irrelevant responses and interference, and updating (a variation of the executive component of the working memory) (see detailed discussion of EF in Das & Misra, 2015). The relationship, specifically between shifting and mathematics, has been suggested by Bull and Lee (2014). Shifting contributes to MA as it supports the alternation between strategies and solving multistep math problems (Andersson, 2008; Van der Sluis, De Jong, & Van der Leij, 2007). Updating as related to working memory, is also a predictor of counting skills (Bull, Espy, & Wiebe, 2008). De Smedt et al. (2009) and Van der Ven, Kroesbergen, Boom, and Leseman (2012) found that the increase in working memory is related to the increase in mathematics in 1st- and 2nd-grade children.

PASS model of mathematical competence

According to multicomponent models of mathematical achievement (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017; Geary, 2013; Geary & Hoard, 2005; LeFevre et al., 2010), the PASS theory seems to be very useful as a multi-factorial framework that provides specific tests (Das-Naglieri: Cognitive Assessment System) for monitoring the development of MC and for directing intervention procedures for enhancement of math skills. In fact, it is reasonable to consider that the four PASS processes (planning, attention, simultaneous processing, and subsequent processing) can essentially intervene in the learning of the foundational skills that make up math proficiency (Das & Misra, 2015).

Planning/EF is the predominant cognitive process for math. Similarly, simultaneous processing is required to understand the proposed problems. Attentional control (Geary, 2013) is proposed to be included in planning/EF. Simultaneous processing involves logical-grammatical relations (following Luria in the PASS theory). At the last level are the two components of EF: inhibition and shifting. Non-verbal matrix-type tests and verbal-simultaneous tests are both used to assess logical and grammatical relations in word problems (**Figure 1**). Working Memory is not shown as a central component.

Working memory involves two activities that must be carried out simultaneously: the storage and processing of information. It becomes especially difficult to do both when children are asked to solve a problem in their heads. As shown in Table the Modules for Mathematics provide cognitive training in both Working Memory and Attention Control

Research suggests that planning/EF is the most important, if not the only, requirements for the learning of mathematics.

Geary (2013) summarizes the role of EF as attentional control and explains it as follows:

> "Children with better ability to maintain effortful attentional control and focus, including a better ability to ignore irrelevant internal and external

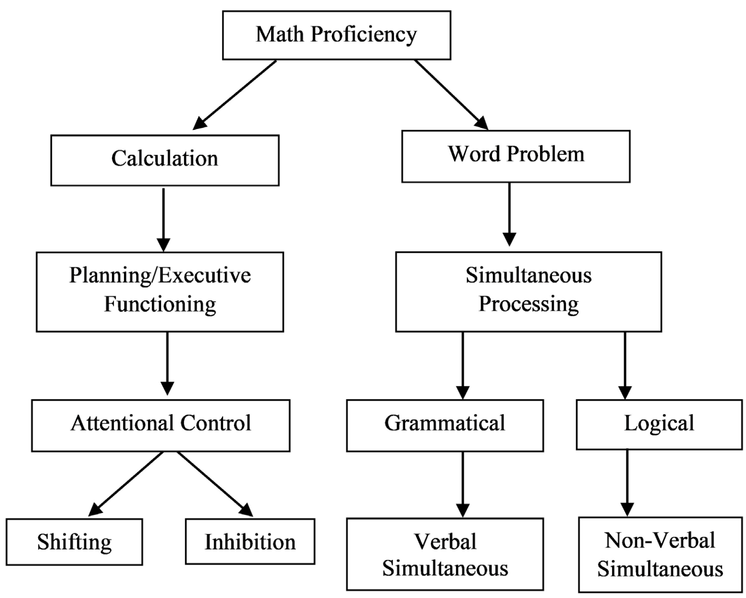

Figure 1. Representation of mathematical competence based on the PASS theory.

distractions (e.g., another child), learn more quickly than their less attentive peers [...] maintain goal-relevant information in mind while processing other information, as measured by working memory" (p. 24).

In Geary's model, the cognitive process that drives all other cognitive processes is attentional control, which is equivalent to planning in the PASS theory (Das & Misra, 2015) rather than attention.

Math Modules Cognitive Training

Modules for Math (Das, 2014a, 2014b) constitute the intervention instruments (**Table 1**). Das' work on PASS theory and its relation to academic performance

Table 1. Math Modules, abilities, and cognitive focus of cognitive training program.

Math Modules	Abilities	Cognitive Focus
Shifting Patterns. The tasks require the participant to calculate (add, subtract, multiply, or divide) according to the task and the letter presented.	Size and value are promoted with shifting, one of the three skills that make up executive functioning (or planning), which is required to apply the rule of shifting between addition and subtraction or between multiplication and division, as well as to do simple arithmetic.	Planning/Executive Functioning Simultaneous processing Processing speed Attention
Learning about Number Line aims to introduce the mathematical concept of the number line using images. Each image shows different animals or numbers. The images can be congruent (the image of the larger animal is larger than the smaller animal) or incongruent (the image of the larger animal is smaller than the smaller animal).	Number Line, in the global task, requires the participant to look at two pictures simultaneously and say whether the animal in the second picture is big or small compared to the first picture. The formal school task contains numbers. The underlying skill that promotes it is inhibition.	Planning/Executive Functioning Simultaneous processing Processing speed Attention
Let us Count contains the animals of Noah's Ark. In some boxes, there are three animals and in others, seven.	Numerosity, counting the number of digits in a box. If there are seven, the child, after counting them, should say big; if there are three, small. The numbers inside the box can be congruent or incongruent. This task allows us to determine the understanding of the value of the number. Shifting is the skill that supports the development of this skill.	Planning/Executive Functioning Simultaneous processing Processing speed Attention
Mapping, Meaning, and Estimating comprise nonverbal tasks, corresponding to logical relations, and verbal tasks, corresponding to grammatical relations. Figures Memory, Copy a Picture and Trace a Path are memory tasks that assess simultaneous nonverbal processing. Draw a Picture requires simultaneous verbal processing.	Verbal and nonverbal simultaneous. The one relating to the logical relationship is closely related to "abstract reasoning", in contrast, translating verbal sentences to construct a picture requires grammatical relations or verbal simultaneous processing. These types of tasks facilitate problem-solving and planning.	Simultaneous processing Planning/Executive Functioning Attention
Working with Memory for Numbers. It contains tasks that are presented visually and then reproduced verbally in an auditory form. Other tasks in contrast are presented aurally to be reproduced visually and spatially.	Working Memory is required for learning mathematics in conjunction with inhibition and shifting skills.	Planning/Executive Functioning Attention

(Das, Naglieri, & Kirby, 1994; Das & Misra, 2015; Georgiou, Guo, Naveevkumar, Vieira, & Das, 2020; Naglieri, 2015; Naglieri & Otero, 2018; Kroesbergen, Van Luit, Naglieri, Tadei, & Franchi, 2010), on the specific links between PASS and Mathematics (Cai, Georgiou, Wen, & Das, 2016; Iglesias-Sarmiento & Deaño, 2016; Iseman & Naglieri, 2011; Kirby & Ashman, 1984; Kroesbergen et al., 2010), studies with typically developing children (Das, 2021; Georgiou, Manolitsis, & Tziraki, 2015; Kroesbergen et al., 2010; Naglieri & Das, 1987; Naglieri & Rojahn. 2004), with mathematical learning difficulties, (Cai, Li, & Deng, 2013; Iglesias-Sarmiento & Deaño, 2011, Kroesbergen, VanLuit, & Naglieri, 2003) or with developmental disorders (Deaño, Alfonso, & Das, 2015; Iglesias-Sarmiento, Deaño, Alfonso, & Conde, 2017, Iseman & Naglieri, 2011) and intervention studies (Clark, Pritchard, & Woodward, 2010; Deaño, Alfonso, & Das, 2015; Iseman & Naglieri, 2011; Naglieri & Gottling, 1997; Naglieri & Johnson, 2000) confirm the predictive and causal role of the relationship of planning and simultaneous processing in mathematical skills, suggesting that they are trainable skills.

Math Modules Cognitive training contains some skills identified as most relevant at the beginning of early school mathematical learning (Das, 2014a, 2014b), built on the mathematical knowledge observed in infants and from which arithmetic facts are derived. These are skills that depend not only on mathematical domain-specific knowledge but also on domain-general cognitive skills such as planning, attention, and simultaneous and successive processing. Some modules focus more on improving executive functioning with tasks that fit their construct. Two types of processing have been identified in the EF construct: response inhibition and shifting/cognitive flexibility (Chan, Shum, Toulopoulou, & Chen, 2008). Working memory, or updating, is the third type. Other modules look more at simultaneous verbal and nonverbal processing.

Cognitive skills involved in goal-directed activities include planning or some of its components such as inhibition, shifting, and working memory (e.g., Miyake et al., 2012; Lehto et al., 2003), which are significant correlates of mathematical skills (Bull et al., 2008; Bull & Lee, 2014; Cragg et al., 2017; Lan et al., 2011; Swanson, 2006).

When active planning is required in step-by-step operations, rather than accessing long-term memory, the frontal lobes must be involved (Das & Misra, 2015). Shifting is a component of planning (Das, 2014a, 2014b; Das & Misra 2015) that facilitates participants' ability to extract rules and relevant information from long-term memory without a step-by-step operation. The development of this skill enables the realization required of the learner in the Shifting Patterns module: to add and subtract, multiply, and divide, depending on the task and the letter presented.

Estimation of the number line does not depend only on numerical knowledge but also on other domain-general skills (Zhu, Cai, & Leung, 2017). The process of attention has been shown to be related to initial numerical knowledge in first grade, but with increasing grade level, planning and problem-solving skills have a greater predictive effect than attention on mathematical computation (Naglieri & Rojahn, 2004; Kroesbergen et al., 2010). The number line estimation module involves multiple cognitive processes, including problem representation and execution. Problem representation requires students to understand and integrate relevant information, maintain the psychological representation of the problem in working memory, find the path to problem-solving, and ultimately form a solution to the problem and, in its execution, proceed to monitor and debug the process (Li & Wang, 2010).

Discovering the cardinal value of a number word is determined by its order in a list, and successive numbers are related by the successor function, learned inductively on the basis of the meaning of cardinals one through four (Sarnecka & Carey, 2008). At this point, children are proficient in the successor function and have discovered how verbal numerals represent natural numbers (Noël & Rousselle, 2011). Fluency in apprehending the number of small sets of elements in Arabic numerals and their combinations (Geary et al., 2009) may be a critical aspect of early competence with numerals. The ability to map Arabic numerals into corresponding quantities may be a related critical skill (Rousselle & Noël, 2007). In addition, the recognition and naming of numbers, the apprehension of their value, and the ability to compose and decompose quantities in relation to task demands are also important. Processing strategies are obviously required for the development of this mathematical competence, which requires shifting ability, which is a central component of planning (Das, 2014a; Das & Janzen, 2004; Das & Misra, 2015).

Non-verbal simultaneous processing skills are required to solve non-verbal tasks corresponding to logical relationships related to abstract thinking. The development of nonverbal simultaneous skills facilitates a crucial step that is explicit understanding of the logical structure of the number system (Das & Misra, 2015). One of the earliest indicators that children are coming to understand numerical values and their relationships, is their ability to explicitly order relative magnitudes. That is, knowing that 9 is one more than 8, for example, and that these magnitudes can be systematically ordered on a number line. Grammatical relations serve as a representation of the verbal simultaneous processing skill, which is relevant to verbal word problem-solving tasks. Both processing skills are required in the Mapping, Meaning, and Estimating module tasks (Das, 2014).

Working memory (WM) is a skill necessary for the successful elaboration of a step-by-step procedure (Das & Janzen, 2004). WM is essential for mathematics because it reduces the material to be remembered into smaller pieces (chunks) and then recalls them. To lighten the WM load, one can use notes or write down the calculations that children progress through step-by-step in solving a division or multiplication (e.g., carry). General features of the domain include short-term memory for sequences that can be reformulated as part of successive processing, such as forming step-by-step sequences, integrating stimuli in a certain order, and making serial chain progressions of sequences that may not be verbal. Long-term memory is essentially full of stored knowledge (Das & Janzen, 2004). Mathematics must build on this knowledge base. Without prior knowledge of basic numbers and operations, mathematical problems cannot be solved (Das & Misra, 2015).

With all this, a program was designed to aim to foster inferential learning and internalization of strategies tailored to individual students, promoting generalization and the transfer of knowledge. To achieve this, students engaged in discovery-based learning, guided independent practice, small group collaboration, and verbalization of their learning.

This study was conducted to assess the impact of the Math Modules Cognitive Training Program on the mathematical competence of typical 2nd-grade students in calculation, problem-solving, and underlying mental functions compared to a control group. The program was designed to optimize the planning/FE, attention, simultaneous, and subsequent cognitive processes through a series of tasks.

2. Method

2.1. Participants

The study involved 60 students aged between 6 and 8 years (*Mdn* = 7 years and 7 months) who were in the second grade of two urban public schools in the province of Ourense, Spain. The corresponding informed consents were obtained from the heads of the educational institutions where the students were attending school. Recognizing the importance of the development of the research, informed consent was requested from the parents or legal guardians of the children. The participants whose parents gave their consent were assigned to either the intervention or the comparison group using a multistage probability sampling method (see **Table 2** for details). Data collection and implementation of the intervention program was done accordingly with the guidelines contained in the Declaration of Helsinki.

2.2. Materials

Effect on cognitive processes. To measure the pretest-posttest cognitive processes, the subtests of the Cognitive Assessment System Battery (D.N.:CAS; Naglieri & Das, 1997) were employed to assess the four PASS (Planning, Attention, Simultaneous, and Successive) processes.

Effect on mathematics performance. To assess pretest-posttest measures, the

Table 2. Participants distribution by sex and group.

Groups	Boys	Girls	Total
"Modules" intervention	14	16	30
Control	17	13	30
TOTAL	**31**	**29**	**60**

WIAT III (Wechsler Individual Achievement Test, 3rd Edition; Wechsler, 2009) was employed. The WIAT III is a standardized, individually administered test that evaluates the academic achievement of students aged between 4 years and 0 months and 19 years and 11 months. The test comprises 16 subtests that assess strengths and weaknesses in the processes involved in learning. Mathematical performance was evaluated using the subtests of mathematical competence, which include written calculation fluency and mathematical problem solving.

2.3. Study Design and Analyses

The study utilized a pre-intervention-post design to compare the intervention and control groups. Multivariate analyses of variance (MANOVA) 2 × 2 [group (2; math vs. comparison)] x [measure (2; pre, post)] were done with the repeated measures GLM method for cognitive processes and their subtests (D.N.: CAS) as well as for mathematical performance in calculation and problem solving (WIAT). The Greenhouse-Geisser correction was applied to adjust for degrees of freedom (gl) in cases of sphericity violations. To demonstrate the intervention's effect (pre/post) and address the study's objectives, the analysis was presented by examining the impact of the intervention type on the dependent variables. The data were analyzed using the Statistical Package for Social Sciences (SPSS) version 18.0.

3. Results

3.1. Effect of the Math Modules Intervention on Cognitive Processes

For the cognitive processes subtests, significant main effects of measure [F (6,199,719,120) = 3.264, p = .003, η^2 *partial* = .027] and group [F (6,199,719,120) = 5.859, p = .001, η^2 *partial* = .048] were found. Univariate contrasts showed statistically significant differences in cognitive process subtest scores from pretest to posttest measure in number matching [F (1,116) = 12.469, p = .001, η^2 *partial* = .097], code planning [F (1,116) = 6.685, p = .011, η^2 *partial* = .054], connection planning [F (1,116) = 9.481, p = .003, η^2 *partial* = .076], nonverbal matrices [F (1,116) = 4.983, p = .028, η^2 *partial* = .041], expressive attention [F (1,116) = 22.091, p = .001, η^2 *partial* = .160], receptive attention [F (1,116) = 23.859, p = .001, η^2 *partial* = . 171] and in speaking speed/sentence questioning [F (1,116) = 7.199, p = .008, η^2 *partial* = .058], participants obtained significantly higher mean scores on the posttest measure than on the pretest. The comparison group obtained overall higher mean scores than the intervention group on the subtests of number matching [F (1,116) = 6.905, p = .010, η^2 *partial* = .056], spatial-verbal relations [F (1,116) = 22.049, p = .001, η^2 *partial* = .160], receptive attention [F (1,116) = 11.873, p = .001, η^2 *partial* = .093], word series [F (1,116) = 10.908, p = .001, η^2 *partial* = .086], sentence repetition [F (1,116) = 16.411, p = .001, η^2 *partial* = .124] and speaking speed/sentence questions[F (1,116) = 27.336, p = .001, η^2 *partial* = .191].

There was a significant interaction between the measured variables and groups for the cognitive processes subtests [F (6,199,719,120) = 2.206, p = .039, η^2 *partial* = .019]. Univariate contrasts based on the measure and group revealed statistically significant differences in the subtest scores for cognitive processes. In terms of groups, the comparison group had significantly higher mean scores than the intervention group in number matching, spatial-verbal relations, receptive attention, word strings, and sentence repetition in the pretest measure (Table 3). In the posttest measure, the intervention group performed equally to the comparison group in all subtests, but still demonstrated differences in spatial-verbal relations and sentence repetition (Table 3).

Comparing the scores of each group from pretest to posttest measures showed that the intervention group made significant gains in scores on four of the cognitive process subtests. Effect sizes associated with the gains were large for number matching [F (1,116) = 19.475, p = .001, η^2 *partial* = .144], expressive attention [F (1,116) = 23.208, p = .001, η^2 *partial* = .167] and receptive attention [F (1,116) = 20.233, p = .001, η^2 *partial* = .149] and small for nonverbal matrix relations [F (1,116) = 5.493, p = .021, η^2 *partial* = .045]. The comparison group obtained significant medium-size gains from one measure to another in speaking speed/sentence questions [F (1,116) = 13.668, p = .001, η^2 *partial* = .105] and small-size gains in connection planning [F (1,116) = 6.673, p = .011, η^2 *partial* = .054] and receptive attention [F (1,116) = 5.807, p = .018, η^2 *partial* = .048].

3.2. Effect of the Math Modules Intervention on Mathematics Achievement

The main effect of group [F (1,116) = 8.230, p = .005, η^2 *partial* = .066] but not of measure [F (1,116) = 3.717, p = .056, η^2 *partial* = .031] on calculus and problem solving was obtained. Univariate contrasts showed statistically significant differences in calculation and problem solving scores from pretest to posttest measure on calculation [F (1,116) = 8.798, p = .004, η^2 *partial* = .070] and problem solving [F (1,116) = 14.138, p = .001, η^2 *partial* = .109], where participants obtained significantly higher mean scores on the posttest measure than on the pretest. The intervention group obtained overall higher mean scores than the comparison group in calculation [F (1,116) = 16.637, p = .001, η^2 *partial* = .105] and problem solving (F (1,116) = 12.500, p = .001, η^2 *partial* = .097].

The analysis of scores from the intervention group and the comparison group revealed that in the pretest measure, the intervention group had significantly higher mean scores than the comparison group in calculation and problem solving (Table 4). In the posttest measure, the intervention group maintained the difference in calculation, but they were equal in problem solving (Table 4). These results suggest that the Math Modules cognitive training program may have a positive impact on the mathematical performance of 2nd-grade students in both calculation and problem solving.

When contrasting the scores of each group from the pretest to the posttest, it was found that both groups gained significantly in calculation and problem-solving

Table 3. Descriptive statistics and results of analysis of variance for the Math-Modules intervention group and comparison group in the pretest and posttest measures of cognitive processes (D.N.:CAS).

Cognitive process	Measure	Group: Intervention Math Modules $n = 30$	Group: Comparison $n = 30$	Measure × Group $N = 60$		
		M (DT)	*M (DT)*	*F (gl)*[1]	*p*	η^2 *partial*[2]
Planning	Pretest	109.13 (8.40)	114.47 (12.99)	3.105	.081	.026
	Posttest	118.77 (14.26)	120.27 (10.35)	.246	.621	.002
Numerical matching	Pretest	9.43 (2.08)	11.60 (2.16)	14.245	<.001	.109
	Posttest	11.97 (2.50)	11.93 (2.13)	.003	.954	.000
Code planning	Pretest	12.87 (2.47)	13.63 (3.05)	1.133	.289	.010
	Posttest	14.23 (2.94)	14.90 (2.66)	.857	.357	.007
Connection planning	Pretest	12.17 (1.49)	11.70 (1.97)	1.068	.304	.009
	Posttest	12.97 (2.08)	12.87 (1.36)	.049	.825	.000
Simultaneous	Pretest	110.67 (11.85)	116.07 (11.35)	3.006	.086	.025
	Posttest	115.27 (13.66)	116.33 (11.24)	.117	.733	.001
Nonverbal matrices	Pretest	12.47 (2.91)	13.07 (2.45)	.741	.391	.006
	Posttest	14.10 (3.11)	13.63 (2.24)	.448	.504	.004
Spatial-verbal relations	Pretest	10.50 (2.33)	13.20 (2.71)	16.860	<.001	.127
	Posttest	11.23 (2.96)	12.90 (2.11)	6.424	.013	.052
Figure memory	Pretest	12.60 (3.50)	12.20 (2.78)	.270	.604	.002
	Posttest	12.77 (2.34)	12.03 (3.17)	.908	.343	.008
Attention	Pretest	108.73 (9.96)	115.60 (10.30)	4.955	.028	.041
	Posttest	118.90 (15.24)	120.67 (11.54)	.328	.568	.003
Expressive attention	Pretest	10.47 (2.03)	11.27 (1.64)	2.142	.146	.018
	Posttest	13.10 (2.92)	12.27 (1.62)	2.324	.130	.020
Number	Pretest	13.07 (3.15)	13.47 (2.60)	.304	.583	.003
	Posttest	12.73 (2.82)	13.48 (2.65)	1.021	.314	.009
Receptive attention	Pretest	11.03 (2.30)	13.20 (2.36)	12.115	.001	.095
	Posttest	13.83 (2.81)	14.70 (2.14)	1.938	.167	.016
Successive	Pretest	103.90 (12.31)	114.47 (12.31)	10.654	.001	.084
	Posttest	106.57 (14.17)	119.97 (11.17)	17.134	<.001	.129
Word series	Pretest	9.87 (2.87)	11.93 (2.00)	9.908	.002	.079
	Posttest	10.77 (3.09)	11.77 (2.01)	2.320	.130	.020
Sentence repetition	Pretest	10.67 (2.07)	12.33 (2.26)	9.287	.003	.074
	Posttest	11.17 (2.04)	12.63 (2.09)	7.191	.008	.058

Continued

Speech rate/SentenceQuestion	Pretest	11.33 (2.70)	12.63 (2.85)	3.599	.060	.030
	Posttest	11.40 (2.54)	15.17 (2.52)	30.216	<.001	.207
Complete scale (Composite)	Pretest	111.90 (9.73)	121.50 (11.49)	11.034	.001	.087
	Posttest	123.27 (11.71)	127.53 (11.72)	2.179	.143	.018

[1]df = 1116. [2]Effect size small .01, medium .06, large .14 (Cohen, 1988).

Table 4. Descriptive statistics and results of the analysis of variance for the Math-Modules intervention and comparison groups in pretest and posttest measures of mathematical performance (WIAT).

Mathematical performance	Measure	Group				
		Intervention Math Modules $n = 30$	Comparison $n = 30$	Measure × Group $N = 60$		
		M (DT)	*M (DT)*	*F (gl)*[1]	*p*	η^2 *partial*[2]
Calculation	Pretest	29.87 (9.00)	24.07 (9.49)	5.841	.017	.048
	Posttest	35.37 (9.79)	28.63 (8.87)	7.872	.006	.064
Problem Solving	Pretest	36.90 (3.32)	34.10 (2.81)	11.111	.001	.087
	Posttest	38.43 (3.58)	37.03 (3.26)	2.778	.098	.023

[1]df = 1,116. [2]Effect size small .01, medium .06, large .14 (Cohen, 1988).

scores. In the intervention group, the score increase was small in calculation [$F(1,116) = 5.252$, $p = .024$, η^2 *partial* = .043] and medium in problem solving [$F(1,116) = 12.195$, $p = .001$, η^2 *partial* = .095].

4. Discussion

The Math Modules cognitive training program resulted in notable improvements in both computational fluency and underlying PASS mental functions, specifically in the cognitive functions of number matching (NPS), expressive attention (EA), receptive attention (RA), and nonverbal matrices (simultaneous processing). These improvements were attributed to the intervention effect and were evidenced by a significant group-measured interaction, with large effect sizes associated with NPS, EA, and RA and a small effect size associated with nonverbal matrices. The improvements in planning (number matching), attention and inhibitory control (expressive attention), and simultaneous processing (nonverbal matrices) were observed in the post-test measure, specifically for the group that followed the Math Modules intervention program.

Having followed the Math Modules program produced a significant effect on the scores of the planning/FEs and simultaneous processes, which are the processes underlying the model of mathematical competence proposed by the PASS theory.

The improvements produced are consistent with the PASS theory model of mathematical competence (Das & Misra, 2015), as well as with PASS theory-based training studies, both predictive (Cai, Li, & Deng, 2013; Cai, Georgiou, Wen, & Das, 2016); meta-analysis (Georgiou et al., 2020); and intervention (Clark, Pritchard, & Woodward, 2010; Deaño, Alfonso, & Das, 2015; Iseman & Naglieri, 2011; Naglieri & Gottling, 1997; Naglieri & Johnson, 2000) and in typical and atypical learning groups.

The findings of the study suggest that the Math Modules Cognitive Training Program is effective in improving children's mathematical achievement in fluent calculation as well as enhancing the functioning of their planning (matching numbers), simultaneous (nonverbal matrices), and attention (expressive attention and receptive attention) processes. These improvements were observed in comparison to a control group that did not show any notable changes in their performance from pre- to post-tests in fluid calculation, cognitive processes, or executive functions.

Why should the math module improve PASS? Verbalization boosts Planning because it allows one to formulate strategies for solving a similar problem and regulate activity through one's own overt or covert speech PASS Theory that comprises three major components: planning, attention, and information processing (Simultaneous & Successive).

Planning is a cognitive process that involves using available information from simultaneous and successive processing as well as allocating attentional resources during problem solving. Additionally, planning, which includes executive processes, is responsible for controlling and organizing behavior, selecting and constructing strategies, and monitoring performance.

The PASS theory also includes the component of "information processing". The information-processing model, initially introduced by Das, Kirby, and Jarman in 1975 and further discussed in their book, aligns with Luria's understanding of the brain and the three hypothesized functional units that are crucial for cognitive functioning and information processing (Luria, 1966; 1973; 1980) (see Appendix for Luria's neuropsychological reports). Since these three systems interact with each other, it is essential to consider their roles in information processing and cognitive behavior as interconnected. Consequently, it is reasonable to conclude that all four PASS processes are integral to the learning of mathematics.

Limitations

The study has some limitations. The participants present differences in their initial scores. In addition, only one subtest is used to measure a function, as is the case with executive functions Das-Naglieri Cognitive Assessment System does not have extended tests of executive functions or working memory. Despite these limitations, the study shows that the intervention group improves its scores from the initial to the final measure, while the comparison group does not show any significant change. The improvement in selected CAS subtests has a certain importance because cognitive measures of processing in CAS are not impervious to training. However, this may be open to other interpretations.

In this study, we have used a comparison of two groups: one that was subjected to math module intervention and another that was an untreated control group that continued with usual classroom instruction. However the children in the treated group received special attention from the teacher during the intervention. Ideally, a third should be an attention-control group. They should be engaged in an unrelated activity, like looking at interesting pictures shown while talking about it with the teacher. Such a procedure is recommended for an intervention study.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix

Three Cortical areas and Four PASS functions (Luria, 1966, 1973; 1980) (Das, Kirby, & Jarman, 1975; Das, Naglieri, & Kirby, 1994)

Luria's model of cortical areas focuses on three major regions: the occipital, parietal, and fronto-temporal regions. These regions are associated with different higher cortical functions in the human brain.

1) Occipital Region: The occipital region, located at the back of the brain, is primarily involved in visual processing. It contains the primary visual cortex, which receives and processes visual information from the eyes.

2) Parietal Region: The parietal region, situated towards the top and back of the brain, plays a crucial role in sensory integration, spatial perception, and attention.

3) Fronto-Temporal Region: The fronto-temporal region encompasses the frontal and temporal lobes of the brain. The frontal lobe is associated with executive functions, such as planning, decision-making, and cognitive control. It is also involved in personality, social behavior, and motor control. The temporal lobe is involved in Luria's model highlights the importance of these three cortical regions in higher cognitive functions and their interplay in complex cognitive processes (Luria, 1980; 1973).

Luria's work emphasizes the interconnectedness of different cortical areas and their contributions to human cognition. He presents a holistic approach to understanding brain function, emphasizing that higher mental processes arise from the integration of multiple brain regions working together (Das, Kirby & Jarman, 1975).

In regard to children learning mathematics a major concern of the current study, all regions of the brain are working in unison. Cognitive functions as proposed by Luria include Planning. Arousal-attention, and the two information processing -simultaneous & successive processing. The introduction to Math Modules presents a simple view of PASS theory. Looking back at the earlier origin of PASS theory (Das & Kirby, 2022), each of these functions involve Perceptual, Memory (mnestic) and Conceptual (gnostic) components. All mathematical learning obviously requires the use of three psychological functions. Above all, they need knowledge base. Major math skills such as Size &Value, Number Line, Numerosity are the object of cognitive learning in Math Modules.