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Parametric Modeling Approach to Covid-19 Pandemic Data

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Abstract

The problem of skewness is common among clinical trials and survival data, which has been the research focus derivation and proposition of different flexible distributions. Thus, a new distribution called Extended Rayleigh Lomax distribution is constructed from Rayleigh Lomax distribution to capture the excessiveness of some survival data. We derive the new distribution by using beta logit function proposed by Jones (2004). Some statistical properties of the distribution such as density, cumulative density, reliability rate, hazard rate, reverse hazard rate, moment generating and likelihood functions; skewness, kurtosis and coefficient of variation are obtained. We also performed the expected estimation of model parameters by maximum likelihood; goodness of fit and model selection criteria, including Anderson Darling, CramerVon Misses, Kolmogorov Smirnov (KS), Akaike Information, Bayesian Information, and Consistent Akaike Information Criterion is employed to select the better distribution from those models considered in the work. The results from the statistics criteria show that the intended distribution performs well and has a good representation of the States in Nigeria's Covid-19 death cases data than other competing models.

Keywords

Anderson Darling, Cramer-von Mises, Covid-19, Kolmogorov Smirnov, Link Function, Survival Analysis

1. Introduction

In survival analysis, problems are encountered in the analysis of clinical data because distributions proposed are not flexible enough to follow the movement of the data to give accurate results. In light of this, there is a need to develop a more flexible parametric model using Covid-19 data for example. In recent times, there was the outbreak of the third wave of Covid-19 pandemic called Delta Variant after the second wave generating a global outcry. Many researches/works have been done by several researchers since the breakup of the pandemic in December 2019 from various fields, such as Medicine, Statistics, Economics, etc., with different ideas, models, methods and approaches in their respective works. These include Badmus *et al.* (2020), Dey *et al.* (2020), WHO, (2020), Yoo, (2020) amongst others [1] [2] [3] [4]. Moharraza *et al.* [5] in their work developed a simple model to assess the spread of global Covid-19 pandemic and tested the validity of their model using the cases of the coronavirus disease compiled by the WHO. Meanwhile, the results from the analysis supported the validity of their developed model.

Zheng and Bobasera [6] compared and contrast the Spanish flu and Covid-19 pandemic using an approach called a "powerful approach" to quantify the degree of mean distance between them and reported according to their results that vaccines from the progress in science and technology have reduced the death probability in some countries. E.g., about 0.0001 United Kingdom about 0.001 in Italy, the United States, Canada, and San Marino considered in their work

The main goal of the extensions of the proposed model is to enhance the robustness and flexibility of the classical model. Also, motivating factors, including the model, are a family of beta generalized models in such that when equating one or more parameter(s) to one (1), the model becomes the baseline distribution. In addition, the model has capability to increase and decrease shapes due to heavily skewed and heavy tail probability distribution that plays an important role in modeling skewed data like clinical data.

Most clinical data are always skewed, thus a new distribution is constructed and generated from a parent distribution called Rayleigh Lomax (RL) distribution by Kawsar *et al.* [7] is generated using beta link function introduced by Jones [8]. This is expected to have different shapes for the survival and hazard rate functions. More parameters are added to the parent distribution, and the flexibility and capability of the distribution to model real life data are established. There is still room for further study from the topic such as "Log-Beta Raleigh Lomax model, Modified Raleigh Lomax distribution and mixture of Rayleigh, Exponentiated Rayleigh Lomax, Lehmann Type II Rayleigh Lomax, Lomax and exponential distribution in modeling real life phenomenon".

The paper is arranged in the following order. Section 2 contains material and methods which involve the properties of the proposed Extended Rayleigh Lomax distribution. In Section 3, we obtain the moments and generating function. Section 4 includes an estimation of parameters, analysis of secondary data obtained from Covid-19 weekly report in Nigeria, results and discussion. Section 5 includes based on the findings.

2. Material and Methods

There are several methods in literature which have been used by many researchers. In this study, we consider beta logit function introduced by Jones [8], which can jointly convolute two or more distributions.

2.1. Properties of Extended Rayleigh Lomax (ERL) Distribution

Density Function

The density function of the above distribution is obtained using the beta link function given as:

$$g(z,p,q)=\frac{c(z)\left[C(z)\right]^{p-1}\left[1-C(z)\right]^{q-1}}{B(p,q)} \tag{1}$$

$$z,p,q,\theta,\lambda,\beta \simeq ERL(z,p,q,\theta,\lambda,\beta>0)$$

$$C(z)=1-\mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}}$$

and

$$c(z)=\frac{\beta\lambda}{\theta}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda+1}\mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}} \quad x\geq-\theta \quad \text{and} \quad \theta,\lambda,\beta>0$$

where, $C(z)$ and $c(z)$ are cdf and pdf of the parent distribution respectively, a and b are additional shape parameters to the parent distribution.

$$g(z,p,q,\theta,\lambda,\beta)=\frac{1}{B(p,q)}\left[C(z)\right]^{p-1}\left(1-C(z)\right)^{q-1}c(z)<z,p,q,\theta,\lambda,\beta>0 \tag{2}$$

where, θ, λ and β are initial scale and shape parameters, while a and b are new parameters called shape, introduced to the distribution. Then (2) becomes extended Rayleigh Lomax (ERL) Distribution and it plots are shown in **Figure 1**.

Some new and existing distributions from the extended Rayleigh Lomax distribution are mentioned below:

1) When $p=1$ in (2), we have the Lehmann Type II Rayleigh Lomax Distribution (New)

2) If $q=1$ in (2), we get the pdf of exponentiated Rayleigh Lomax distribution (New)

3) When $\beta=1$ in (2), this consists of beta Lomax distribution (New)

4) When $\frac{\beta}{\theta}=\beta$, $\lambda=1$ and $\lambda=1$ in (2), it then becomes beta Rayleigh distribution (New)

5) If $p=q=1$ in (2), it yields Rayleigh Lomax distribution which is the parent distribution [7].

6) When $q=\beta=1$ in (2), this consists exponential Lomax distribution by El-Bassiouny *et al.* [9].

7) If $p=q=\lambda=\theta=1$ in (2), it yields Rayleigh Lomax distribution which is the parent distribution [10].

2.2. Distribution Function of BRL Distribution

The associative distribution function cdf in (2) is given as

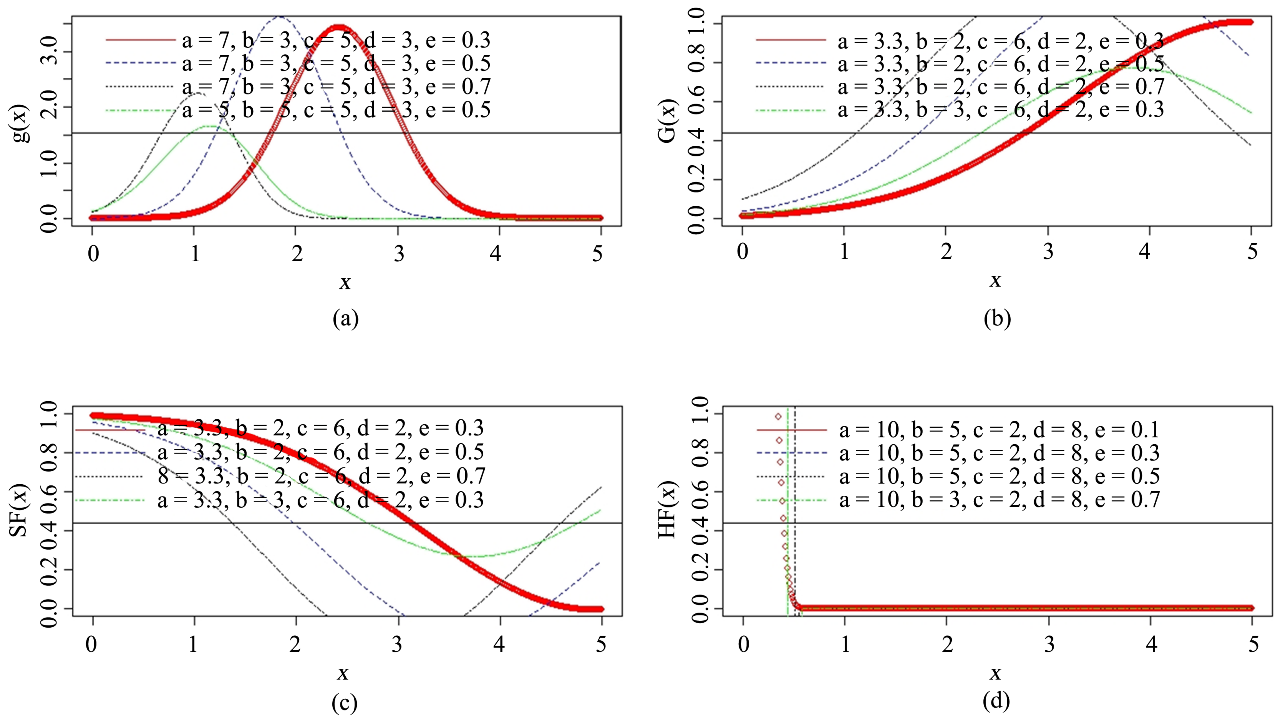

Figure 1. Depicts the: (1) pdf, plot of density function; (b) cdf, plot of distribution function; (c) survival, plot of survival rate function; and (d) hazard function in plots of ERL distribution, plot of hazard rate function.

$$
\begin{aligned}
P(Z \le z) &= \int_0^z f(c)\,\mathrm{d}c = G(z, p, q, \theta, \lambda, \beta) \\
&= \int_0^z \frac{1}{B(p,q)} \left[C(z)\right]^{p-1} \left(1 - C(z)\right)^{q-1} c(z)\,\mathrm{d}c
\end{aligned} \tag{3}
$$

We set

$$
k(x) = 1 - \mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta + x}\right)^{-2\lambda}} \tag{4}
$$

$$
\frac{\mathrm{d}c}{\mathrm{d}z} = \frac{\beta\lambda}{\theta}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda+1} \mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}} \left[1 - \mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}}\right]
$$

$$
\frac{\mathrm{d}c}{\mathrm{d}z} = \frac{\beta\lambda}{\theta}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda+1} \mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}} - \left[\mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}}\right]^2
$$

Also

$$
\mathrm{d}z = \frac{\mathrm{d}c}{\frac{\beta\lambda}{\theta}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda+1} \mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}} \left[1 - \mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}}\right]}
$$

Putting $\mathrm{d}x$ in Equation (2), we realize:

$$
g(z, p, q, \theta, \lambda, \beta) = \frac{1}{B(p,q)} \int_0^\infty \left[1 - \mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}}\right]^{p-1} \left[\mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}}\right]^{q-1} \mathrm{d}c \tag{5}
$$

and c in Equation (4) becomes

$$c(z)=\frac{\mathrm{d}c(z)}{\mathrm{d}z}=\frac{\beta\lambda}{\theta}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda+1}\mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)^{-2\lambda}}$$

Equation (5) can be expressed as

$$g(z,p,q,\theta,\lambda,\beta)=\frac{1}{B(p,q)}C^{a-1}(1-C)^{b-1}\frac{\mathrm{d}c}{\mathrm{d}z} \tag{6}$$

Now, putting (2) in (6), we get

$$G(z,p,q,\theta,\lambda,\beta)=P(Z\le z)=\int_0^z\frac{1}{B(p,q)}C^{p-1}(1-C)^{q-1}\frac{\mathrm{d}c}{\mathrm{d}z}$$

$$=\int_0^z\frac{1}{B(p,q)}C^{p-1}(1-C)^{q-1}\frac{\mathrm{d}c}{\mathrm{d}z}$$

where, $B(z,p,q)=\int_0^z C^{p-1}(1-C)^{q-1}\mathrm{d}c$ and it is called the incomplete beta function.

$$G(z,p,q,\theta,\lambda,\beta)=\int_0^z\frac{C^{p-1}(1-C)^{q-1}\mathrm{d}c}{B(p,q)}=\frac{B(c,p,q)}{B(p,q)} \tag{7}$$

(7) becomes the cumulative distribution function of ERL distribution.

2.3. The Survival Rate/Reliability Function

The reliability function of BRL distribution is given by

$$Sur(z,p,q,\theta,\lambda,\beta)=1-G(z,p,q,\theta,\lambda,\beta)=1-\int_0^z f(c)\mathrm{d}c$$

$$1-\int_0^z\frac{C^{p-1}(1-C)^{q-1}\mathrm{d}c}{B(p,q)}=1-\frac{B(c,p,q)}{B(p,q)}$$

$$Sur(z,p,q,\theta,\lambda,\beta)=\frac{B\left[(p,q)-(z,p,q)\right]}{B(p,q)} \tag{8}$$

2.4. The Hazard Rate Function

$$haz(z,p,q,\theta,\lambda,\beta)=\frac{g(z,p,q,\theta,\lambda,\beta)}{1-G(z,p,q,\theta,\lambda,\beta)}=\frac{\frac{C^{p-1}(1-C)^{q-1}c}{B(p,q)}}{\frac{B\left[(p,q)-(z,p,q)\right]}{B(p,q)}}$$

$$=\frac{C^{p-1}(1-C)^{q-1}c}{B\left[(p,q)-(z,p,q)\right]} \tag{9}$$

2.5. The Reversed Hazard Rate Function

$$Rhaz(z,p,q,\theta,\lambda,\beta)=\frac{g(z,p,q,\theta,\lambda,\beta)}{G(z,p,q,\theta,\lambda,\beta)}=\frac{\frac{C^{p-1}(1-C)^{q-1}c}{B(c,p,q)}}{B(p,q)}$$

$$=\frac{B(p,q)C^{p-1}(1-C)^{q-1}c}{B(c,p,q)} \tag{10}$$

2.6. Testing the Trueness of the PDF of ERL Distribution

The ERL distribution is a probability density function with the use of:

$$\int_0^\infty g_{ERL}(z)\,\mathrm{d}z = 1 \tag{11}$$

Jones (2004) in his generalized beta distribution of first kind is given by:

$$g_z(z,p,q,u) = [B(p,q)]^{-u}[C(y)]^{pu-1}\left[1-C(y)^u\right]^{q-1}K(y) \quad 0<y<1$$

where p,q and $u>0$, therefore differentiating (p) above, we obtain

$$g_{ERLD}(z) = [B(p,q)]^{-u}G^{pu-1}\left(1-G^u\right)^{q-1}\frac{\mathrm{d}G}{\mathrm{d}y}$$

$$\int_{-\infty}^{\infty} g_{ERLD}(z) = \int_{-\infty}^{\infty}\frac{u}{B(p,q)}G^{pu-1}\left(1-G^u\right)^{q-1}\mathrm{d}G$$

Putting $M = G^u$, then differentiating M with respect to G

$$\frac{\mathrm{d}M}{\mathrm{d}G} = UG^{u-1}$$

$$\mathrm{d}G = \frac{\mathrm{d}M}{UG^{u-1}}$$

$$G = M^{\frac{1}{u}}$$

$$\int_{-\infty}^{\infty} g_{ERLD}(z)\,\mathrm{d}z = \int_0^1\frac{u}{B(p,q)}\left(M^{\frac{1}{u}}\right)^{pu-1}(1-M)^{q-1}\frac{\mathrm{d}M}{UG^{u-1}}$$

$$= [B(p,q)]^{-1}\int_0^1\frac{M^{p-\frac{1}{u}}(1-M)^{q-1}}{M^{1-\frac{1}{u}}}\mathrm{d}M$$

$$= [B(p,q)]^{-1}\int_0^1 M^{p-1}(1-M)^{q-1}\,\mathrm{d}M$$

$$\int_0^1 M^{p-1}(1-M)^{q-1}\,\mathrm{d}M = B(p,q)$$

Therefore, $g_{ERLD}(z) = \frac{B(p,q)}{B(p,q)} = 1$.

Hence, the g_{ERL} distribution has a true continuous probability density function.

3. Moments and Generating Function

In this section, we derive and obtain mgf of the distribution $m(t) = E\left(\mathrm{e}^{tz}\right)$ and the general *r*th moment of a beta generated distribution defined by Hosking [11]

$$\mu_r^1 = B(p,q)^{-1}\int_0^1\left[F^{-1}(z)\right]^r z^{p-1}[1-z]^{q-1}\,\mathrm{d}z \tag{12}$$

Cordeiro *et al.* [12] also discussed another mgf for generated beta distribution.

$$m(t) = B(p,q)^{-1}\sum_{j=0}^{\infty}(-1)^j\binom{q-1}{j}e(a,r;pj-1) \tag{13}$$

where,

$$e(a,r)=\int_{-\infty}^{\infty}\mathrm{e}^{tz}\left[F(z)\right]^{m} f(z)\mathrm{d}z$$

then,

$$M_z(t)=B(p,p)^{-1}\sum_{j=0}^{\infty}(-1)^j\binom{q-1}{j}\int_{-\infty}^{\infty}\mathrm{e}^{tz}\left[F(z)\right]^{p(j+1)-1} f(z)\mathrm{d}z \tag{14}$$

Putting pdf and cdf of the Extended Rayleigh Lomax distribution into Equation (14), we get

$$M_{ERLD(z)}(t)=B(p,q)^{-1}\sum_{j=0}^{n}(-1)^j\binom{q-1}{j}\int_{-\infty}^{\infty}\mathrm{e}^{tz}\left[F(z)\right]^{p(j+1)-1} f(z) \tag{15}$$

If $p=q=1$ in Equation (14) that becomes the moment generating function of the baseline distribution.

Hence, the *r*th moment of the ERL distribution is obtained, since the moment generating function of the parent distribution is given by

$$\begin{aligned}M_z(t)&=\sum_{j=0}^{\infty}\frac{t^j}{j!}\int_0^{\infty}z^j f(z,\beta,\lambda,\theta)\mathrm{d}z\sum_{c=0}^{\infty}\frac{t^k}{k!}\\&\cdot\sum_{j=0}^{\infty}(qK_j)\theta^j\left(\frac{2}{\beta}\right)^{\frac{1}{2\lambda}}(-\theta)^{q-j}r\left(\frac{j}{2\lambda}+1\right)\end{aligned} \tag{16}$$

Equation (16) can be re written as

$$\begin{aligned}M_{ERLD(z)}(t)&=B(p,q)^{-1}\sum_{j=0}^{n}(-1)^j\binom{q-1}{j}\int_{-\infty}^{\infty}\mathrm{e}^{tz}\left[F(z)\right]^{p(j+1)-1}\sum_{i=0}^{\infty}\frac{t^i}{i!}\\&\cdot\sum_{h=0}^{\infty}(iK_h)\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{2\lambda}}(-\theta)^{i-h}r\left(\frac{h}{2\lambda}+1\right)\\&=B(p,q)^{-1}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{h=0}^{\infty}(-1)^j\binom{q-1}{i}\frac{t^i}{i!}(iK_h)\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{2\lambda}}\\&\cdot(-\theta)^{i-h}r\left(\frac{h}{2\lambda}+1\right)\cdot\left[F(z)\right]^{p(j+1)-1}\end{aligned} \tag{17}$$

and the *r*th moment of ERL distribution is obtained from Equation (17)

$$\begin{aligned}\mu^{i}_{ERLD(r)}&=E(z^r)=B(p,q)^{-1}\sum_{j=0}^{\infty}\sum_{h=0}^{\infty}(-1)^j\binom{q-1}{i}\cdot\left[F(z)\right]^{p(j+1)-1}\\&\cdot\frac{t^i}{r!}\binom{r}{h}\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{2\lambda}}(-\theta)^{r-h}r\left(\frac{h}{2\lambda}+1\right)\end{aligned} \tag{18}$$

Letting $p=q=1$ in (18) gives the rth moment of the baseline distribution by Kawsar *et al.* [7]

$$\mu^{i}_{r}=E(z^r)=\sum_{h=0}^{\infty}\binom{r}{h}\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{2\lambda}}(-\theta)^{r-h}r\left(\frac{h}{2\lambda}+1\right)$$

Other measures such as the Skewness (SK_{ERLD}) (p, q, β, λ, θ) and Kurtosis (KT_{ERLD}) (p, q, β, λ, θ) are also obtained below:

The *r*th moment of the ERL distribution is written as:

$$\mu_{ERLD(r)}^{i}=\int_{0}^{\infty}z^{r}F_{ERLD}(z)\mathrm{d}z$$

That is,

$$\mu_{ERLD(r)}^{i}=\int_{0}^{\infty}z^{r}\left(\left[B(p,q)\right]^{-1}\left(C(z)\right)^{p-1}\left(1-C(z)\right)^{q-1}\mathrm{d}c(z)\right)$$

where, $C(z)=\left(1-w(z)\right)^{-u}$

I.e. $W(z)=\mathrm{e}^{-\frac{\beta}{2}\left(\frac{\theta}{\theta+z}\right)}$ and $u=2\lambda$

therefore,

$$\mu_{ERLD(r)}^{i}=\frac{(rK_h)\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{u}}(-\theta)^{r-h}r\left(\frac{h}{u}+1\right)}{B(a,b)}\cdot\sum_{j=0}^{\infty}\sum_{h=0}^{\infty}(-1)^j\binom{q-1}{i}\left(\left[\left(1-w(z)\right)^{-u}\right]^{p(j+1)-1}\right)$$
$$=X\left(\binom{r}{h}\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{u}}(-\theta)^{r-h}r\left(\frac{h}{u}+1\right)\right) \tag{19}$$

where,

$$X=\frac{\sum_{j=0}^{\infty}\sum_{h=0}^{\infty}(-1)^j\binom{q-1}{i}\left(\left[\left(1-w(z)\right)^{-u}\right]^{p(j+1)-1}\right)}{B(p,q)}$$

At the same time, the first four central moments $\mu_r^i=1,2,3,4$ are obtained through (17) as:

Furthermore, the mean and second to fourth moments of the ERL distribution are given as follows:

$\mu=\mu_1^{|}$, $\mu_2=\mu_2^{|}-\mu^2$, $\mu_3=\mu_3^{|}-3\mu\mu_2^{|}+2\mu^3$, and

$\mu_4=\mu_4^{|}-4\mu\mu_3^{|}+6\mu^2\mu_2^{|}-3\mu^4$

$$\mu_1^{|}=X\left(\binom{1}{h}\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{u}}(-\theta)^{1-h}r\left(\frac{h}{u}+1\right)\right) \tag{20}$$

$$\mu_2^{|}=X\left(\binom{2}{h}\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{u}}(-\theta)^{2-h}r\left(\frac{h}{u}+1\right)\right) \tag{21}$$

$$\mu_3^{|}=X\left(\binom{3}{h}\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{u}}(-\theta)^{3-h}r\left(\frac{h}{u}+1\right)\right) \tag{22}$$

$$\mu_4^{|}=X\left(\binom{4}{h}\theta^h\left(\frac{2}{\beta}\right)^{\frac{h}{u}}(-\theta)^{4-h}r\left(\frac{h}{u}+1\right)\right) \tag{23}$$

Other measures such as skewness, kurtosis and coefficient of variation of the ERL distribution are given below.

3.1. Skewness of the ERL Distribution

The skewness is a means of measuring non symmetry of the distribution. The skewness is given by:

$$SK_{ERLD} = \frac{\mu_3}{\mu_2^{1.5}} \tag{24}$$

3.2. Kurtosis of the ERL Distribution

The kurtosis is another measure that measures the peak of the distribution. The kurtosis of the BRL distribution is given as:

$$KT_{ERLD} = \frac{\mu_4}{\mu_2^2} - 3 \tag{25}$$

3.3. Coefficient of Variation of the ERL Distribution

This is also a measure of variability of a probability distribution. The CV of the ERL distribution is given as:

$$CV_{ERLD} = \frac{\sqrt{\mu_2}}{\mu} \tag{26}$$

4. Estimation of Parameter

We made attempt to derive the maximum likelihood estimates (MLEs) of the ERL distribution parameters including: θ, λ, β, a and b which are scale and shape parameters. According to Cordeiro *et al.* [12], the log likelihood function is given as:

$$\begin{aligned} L(\varphi) = {} & n\log\left[B(p,q)\right] + \sum_{i=1}^{n}\log\left[C(z,\delta)\right] \\ & + (p-1)\sum_{i=1}^{n}\log\left[1+C(z,\delta)\right] + (p-1)\left[C(z,\delta)\right] \end{aligned} \tag{27}$$

$\varphi = (p,q,k,\delta)$ and $\delta = (\theta,\lambda,\beta)$ are vectors

If $c = 1$, it becomes Equation (27) which leads to

$$\begin{aligned} L(\varphi) = {} & const - n\log\left[B(p,q)\right] + \sum_{i=1}^{n}\log\left[C(z,\delta)\right] \\ & + (p-1)\sum_{i=1}^{n}\log\left[1+C(z,\delta)\right] + (p-1)\left[C(z,\delta)\right] \end{aligned} \tag{28}$$

$c(z,\delta)$ and $C(z,\delta)$ have been stated at the beginning.

The log likelihood function of ERL distribution is given as:

$$\begin{aligned} L_{BRLD}(\varphi) = {} & -n\log\left[B(p,q)\right] + \sum_{i=1}^{n}\left[c(z)\right] + (p-1)\sum_{j=1}^{n}\log\left[C(z)\right] \\ & + (q-1)\sum_{i=1}^{n}\log\left[1-C(z)\right] \end{aligned} \tag{29}$$

Taking the differentiation in respect to p, q, θ, λ and β give the following:

$$\frac{\partial L(\varphi)}{\partial p} = -n\frac{r|(p)}{r(p)} + n\frac{r|(p+q)}{r(p+q)} + \sum_{z=1}^{n}\log\left(1-C(Z,\delta)\right) \tag{30}$$

$$\frac{\partial L(\varphi)}{\partial q} = -n\frac{r|(q)}{r(q)} + n\frac{r|(p+q)}{r(p+q)} + \sum_{z=1}^{n}\log\left(C(Z,\delta)\right) \tag{31}$$

$$\frac{\partial L(\varphi)}{\partial\theta}=\sum_{j=1}^{n}\left[\frac{\frac{\partial\left[C(Z,\delta)\right]}{\partial\theta}}{C(Z,\delta)}\right]+(p-1)\sum_{z=1}^{n}\left[\frac{\frac{\partial\left[1-C(Z,\delta)\right]}{\partial\theta}}{1-C(Z,\delta)}\right]+(q-1)\sum_{z=1}^{n}\left[\frac{\frac{\partial\left[C(Z,\delta)\right]}{\partial\theta}}{(Z,\delta)}\right] \tag{32}$$

$$\frac{\partial L(\varphi)}{\partial\lambda}=\sum_{j=1}^{n}\left[\frac{\frac{\partial\left[C(Z,\delta)\right]}{\partial\lambda}}{C(Z,\delta)}\right]+(p-1)\sum_{z=1}^{n}\left[\frac{\frac{\partial\left[1-C(Z,\delta)\right]}{\partial\lambda}}{1-C(Z,\delta)}\right]+(q-1)\sum_{z=1}^{n}\left[\frac{\frac{\partial\left[C(Z,\delta)\right]}{\partial\lambda}}{(Z,\delta)}\right] \tag{33}$$

$$\frac{\partial L(\varphi)}{\partial\beta}=\sum_{j=1}^{n}\left[\frac{\frac{\partial\left[C(Z,\delta)\right]}{\partial\beta}}{C(Z,\delta)}\right]+(p-1)\sum_{z=1}^{n}\left[\frac{\frac{\partial\left[1-C(Z,\delta)\right]}{\partial\beta}}{1-C(Z,\delta)}\right]+(q-1)\sum_{z=1}^{n}\left[\frac{\frac{\partial\left[C(Z,\delta)\right]}{\partial\beta}}{(Z,\delta)}\right] \tag{34}$$

4.1. Analysis of Data

The data used for the analysis is a secondary data obtained from Covid-19 situation weekly epidemiological report 39; 5th-11th July, 2021 (NCDC website state the website) [13]: Thirty-six (36) States including Federal Capital Territory (FCT) with reported laboratory-confirmed Covid-19 cases, recoveries, deaths, samples tested and active cases (37 data points); and was accessed on Thursday 22nd July, 2021 put date accesses at reference not here. Only the death cases from all states of the federation are used for the analysis.

4.2. Result and Discussion

The summary of goodness of fit statistics is used to check for normality of the data; skewness, kurtosis, Anderson Darling (AD), Kolmogorov Smirnov (KS) and Cramer-Von-Mises (CVM) shown in **Table 1** with their values clearly indicate that the data does not follow normal distribution since p-values less than 5%, skewness greater than 0 (zero) and kurtosis also greater than 3 [14] [15]. While, graphs from **Figure 2** show the nature of the data, the scatter, theoretical quantiles, boxplot, histogram, density and empirical cumulative distribution

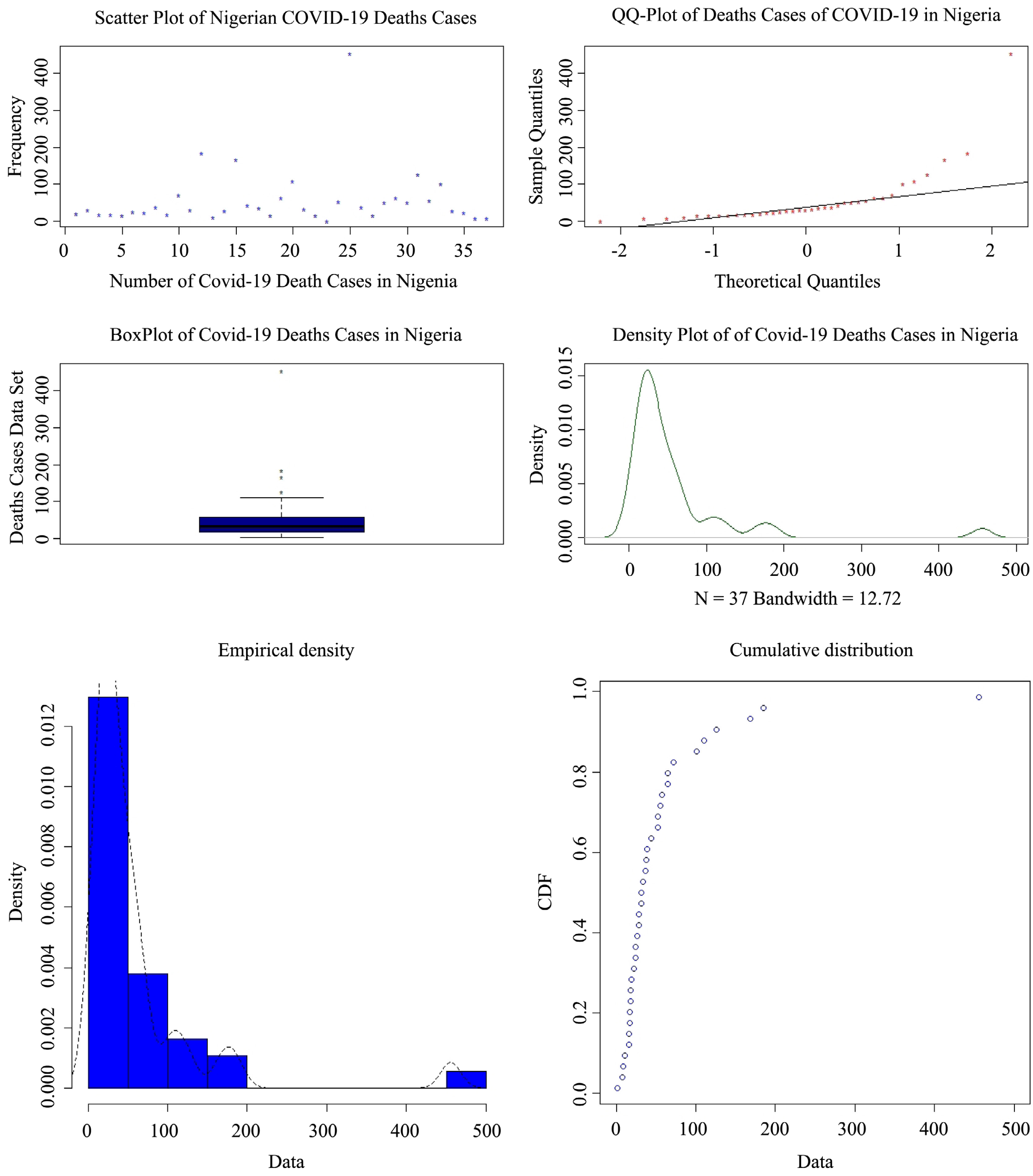

Figure 2. The scatter, theoretical quantiles, boxplot, histogram, density and distribution plot.

function (ecdf) plot show the data is skewed. For instance, non-linearity by scatter and quantiles plots, outliers by boxplot and skewness by histogram and density plots. The minimum and maximum values in the data set are inclusive.

The results obtained in **Table 2** are based on parameter estimates by method of maximum likelihood estimation (MLEs). The standard error values are in

Table 1. Summary of goodness of fit statistics of death cases data.

Mini	Maxi	Skewness	Kurt	AD	KS	CVM
2.00	456.00	3.55	14.39	3.883e−12	< 2.2e−16	3.591e−09

Table 2. Contains the MLE, standard error (in parenthesis) and model selection criteria.

Model-Par/Model Sel	ERLD*	ExpLD	LRLD	BRD	RLD	ExpRLD	BLD
a	**0.801 (0.015)**	2.203 (0.044)	-	1.801 (0.038)	-	1.802 (0.040)	0.801 (0.015)
b	**5.500 (0.132)**	-	2.500 (0.056)	0.999 (0.019)	-	-	2.501 (0.057)
θ	**0.025 (NA)**	0.002 (NA)	0.008 (NA)	0.048 (NA)	0.000 (NA)	0.016 (NA)	0.010 (NA)
λ	**0.113 (0.003)**	0.109 (0.002)	0.117 (0.003)	-	0.116 (0.003)	0.110 (0.002)	0.117 (0.003)
β	**0.501 (0.014)**	-	0.500 (0.014)	2.500 (0.063)	2.900 (0.078)	1.500 (0.037)	-
$-2\text{Log}L$	**1298.939**	1629.850	2003.402	2201.424	2391.883	2498.683	2775.822
AIC	**2607.878**	3265.700	4014.804	4410.848	4789.766	5005.366	5559.644
CAIC	**2609.813**	3266.427	4016.054	4412.098	4790.493	5006.616	5560.916
HQIC	**2610.718**	3267.404	4017.076	4413.120	4791.470	5007.638	5561.916
BIC	**2615.933**	3270.533	4021.248	4417.292	4794.599	5011.810	5566.088

bracket for all the models. The model ERLD is compared with other six models: ExpLD, LRLD, BRD, RLD, ExpRLD and BLD. Also, model selection criterion is performed on all models considered in the study. From the results, ERLD has the smallest values in all as we can see bold and starred where AIC = 2607.878, CAIC = 2609.813, HQIC = 2610.718 and BIC = 2615.933, which indicates that it is a robust and flexible model.

5. Conclusion

Despite the level of Nigerian Covid-19 death cases data set, the ERL distribution follows the movement of the data and has better representation of the data than any of the other existing distributions. The proposed distribution, being flexible and versatile, can accommodate increasing, decreasing, bathtub and unimodal shape hazard function. It is therefore useful and effective in the analysis of clinical and survival data.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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