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Incommensurability, Abstraction, and Idealization: A Conceptualist Approach

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Abstract

The main purpose of this essay is to explore the relationship between the incommensurability of paradigms or general theories, a thesis due to Kuhn and Feyerabend, and the idealizations and abstractions that permeate concepts and fundamental laws of physical theories. One can find some unrealistic or idealizational suppositions underlying the semantic level of the differences between the conceptual vocabularies of incommensurable theories, on which these theories rest. This kind of suppositions relates to the idealizations and abstractions involved in forming concepts and formulating laws. Generally, the fundamental laws of physical theories have an abstract and idealized character. Physical theories contain conceptual networks of kind concepts and quantitative concepts, which are interconnected by several sorts of relationships. The discrepancies between such conceptual network, and their underlying assumptions, of alternative paradigms or theories make their comparison at a theoretical level, that is, their commensurability implausible. That at once strengthens the incommensurability thesis and further supports it. In addition, it follows that Kuhn's philosophy of science indeed involves a version of conceptual or epistemological relativism about science.

Keywords

Idealizational Assumption, Conceptual Incongruency, Divergent World Views, Conceptual Holism, Epistemological Relativism

1. Introduction

The most controversial concept of Kuhn's and Feyerabend's philosophies of science is that of incommensurable paradigms or global theories, respectively. Both Kuhn (1962, 1970) and Feyerabend (1962, 1978) reject the dichotomy between theoretical and observational concepts of logical empiricism, dominant in the sixty's, the thesis that there is an observation language which could be neutral and impartial between two alternative and rival theories and adopt a holistic conception of meaning or concepts. Besides, they held the thesis, advanced by Hanson (1958), that scientific observation is theory-laden. That is part of the philosophical background of the thesis about incommensurable paradigms.

Initially, Kuhn held that a paradigm includes a view of the world, the world which is subject matter of a scientific discipline, a claim which becomes crucial for his conception of scientific revolution: "The transition from Newtonian to Einsteinian mechanics illustrates with particular clarity the scientific revolution as a displacement of the conceptual network through which scientists view the world." (Kuhn, 1970: p. 102)

Thus, two different and alternative paradigms provide two mutually exclusive world views. The conceptual networks or vocabularies of two diverse paradigms differ such way that the world views they provide are not only dissimilar but also conflicting. Nobody could hold two rival paradigms during a revolutionary transition because, among other reasons, the world views of those paradigms are divergent and exclude each other. During a revolutionary transition, the paradigms involved are conceptual rivals since the ways that both conceptualize the world are incompatible, not in a logical sense but in the sense that nobody could adopt both world views congruently at the same time.

Later, Kuhn (1983) claims that the partial lack of translating between the respective conceptual vocabularies of two historically alternative paradigms results in their incommensurability. Conceptual diversity, mutually excluding world views, and the lack of complete translation between them, among other reasons, make the paradigms involved in a scientific revolution incommensurable. Kuhn's main argument in favour of his thesis of incommensurability consists in the assertion that the conceptual vocabularies, or lexicons, of alternative paradigms are entirely different, and their mutual translation is only partial with significant loss of content. We think that there is a further reason related to that semantic claim favouring the incommensurability of paradigms: the conceptual network of theories, and hence their fundamental laws are permeated of idealizations and abstractions. This circumstance reinforces the incommensurability thesis, because it reveals the conceptual discrepancy, variance, divergency or disparity between incompatible paradigms at a deeper level involving unrealistic suppositions.

Kuhn's and Feyerabend's notion of incommensurability at the semantic or conceptual level, as described, is against the absolutist theses of the realist philosophers and entails certain conceptual or epistemological relativism, in the sense that anything that we may claim about how the physical word is, is relative to and depends on the paradigm or global theory that we maintain or construct; in particular, its conceptual vocabulary and world view. This paper, adopting a conceptual relativist approach, aims to explore the relationship between the idealizations and abstractions, which permeate concepts and fundamental laws of physical theories, and the incommensurability of paradigms.

After exposing some considerations on the sociohistorical character of paradigms rivalry (Section 2), we outline the central concepts of our conceptualist approach (Section 3). Then, we discuss the main issue of incommensurability between paradigms from the former approach (Section 4). The relevant concepts, under discussion, of Kuhn's philosophy of science—his central concepts of paradigm and incommensurable—are widely developed in several of his publications, specially (Kuhn, 1962, 1970, 1974, 1981, 1983).[1] We close this essay with some final remarks (section 5).

2. On the Sociohistorical Character of the Rivalry between Paradigms

It should be noted that Kuhn presents a sociohistorical approach to some issues within the philosophy of science and that several of the thesis he holds are limited to specific periods in the history of a given scientific discipline—normal science and scientific revolution—and they do not have a general and abstract dimension apart from sociohistorical contexts, as understood by many philosophers. By Kuhn's account, two paradigms are alternatives and rivals in the stage of a revolution as a sociohistorical phenomenon. After a revolutionary change, the paradigms involved are no longer rivals, though they are still incommensurable. The revolutionary paradigm displaces the old paradigm and takes its place as the dominant view of the world. Their rivalry ceases, but not their incongruency and mutual exclusion at a conceptual level.

The issue of rational choice between alternative and rival paradigms is a question that is present only for scientists during the stages of paradigm transition of scientific revolutions. For example, we may say that Galileo understood both Ptolemean and Copernican conceptions of our planetary system, although they persist as incommensurable paradigms in the Kuhnian sense that their world views, conceptual vocabularies, and classifications of the celestial bodies are all quite different, and total mutual translations of their theoretical claims are not possible. Galileo understood both former paradigms perfectly well without the need for any translation between Ptolemean and Copernican claims about the system of the world. Though such paradigms were different and incompatible, for Galileo they were not alternative rivals in the sense that he would have been confronted with the scientific predicament of making a rational choice between them—although he deals with criticisms from detractors.

Likewise, more than one century ago, the need of chose between Newtonian mechanics and relativity theory was present for the scientists at that time. Nowadays, physicists have not need to choose between such theories as well as between classical physics and quantum mechanics. These three theories are alternatives in the weak sense that a physicist may apply any one of them in function of the problem in question. Since many decades ago, physicists have realized that the domains of application of such pairs of theories, with possible

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[1] See Bird (2022).

overlaps, are different but not conflicting. Today we do not have to answer Eddington's question about which table is real: the classical or the quantum table.

The previous order of ideas is compatible with Kuhn's thesis about the existence of incommensurable paradigms as a sociohistorical phenomenon. Questions about the rational choice between paradigms and the lack of communication among scientists have a historical dimension, limited to a period of a revolutionary change of paradigms. Nevertheless, the issue about the incommensurability of paradigms or theories in the epistemological sense, referring to the differences in their conceptual networks, in their classifications of the objects under study and, mainly, in their world views, surpasses the sociohistorical context because of the philosophical consequences of these differences; particularly, concerning the realist/antirealist debate.

As for that debate, it should be remarked that some realistically inclined philosophers who agree in general terms with some of Kuhn's theses in the philosophy of science have tried, at least, to make Kuhn's thesis about the change of world view during a revolution compatible with the realist thesis that the existence of the world, the *real* world, is independent of our minds, languages or theories. Sankey (1994) intends to hold that, although there is a conceptual disparity between the lexicons of the paradigms involved, following the approach of a causal theory of reference it becomes plausible to think that the theories contained in these paradigms share the world about which they talk to a significant extent. However, as we may see, the issue of incommensurability overpass semantics proposes about meaning and reference. Moreover, regarding this problem, Hoyningen-Huene provides a Kantian interpretation of the image of world change:

> In his book, *Reconstructing Scientific Revolutions*, Hoyningen-Huene argues that Kuhn's metaphysical stance is, in fact, a dynamic Kantian position, which is based on a distinction between an unknown "world-in-itself" and a "phenomenal world" that is jointly constituted from inputs of "the world-in-itself" and the conceptual contributions of the human subject. Kuhn differs from Kant, however, in allowing that the human conceptual contributions vary with the theory change (Hoyningen-Huene & Sankey, 2001: p. 17).

On such a Kantian interpretation of Kuhn's philosophy, to which Kuhn sometimes expresses himself sympathy, Hoyningen-Huene (1993, p. 32) says that the world that changes when a scientific revolution takes place is the phenomenal world in which scientists live but not the world-in-itself. Hoyningen-Huene's thesis has as antecedent a distinction between an absolute *a priori*, that of Kant's philosophy, and a relative *a priori* concerning diverse cultures and historical periods, due to Reichenbach. In contrast to Kant's *a priori*, which is fixed, universal and necessary, Reichenbach's relative *a priori* is variable, local and contingent. This concept does not have an *a priori* character indeed because it is only *a priori* in a weak sense, that sense which connotes beforehand, that is, that experience requires a given conceptual framework in advance to take place, which does not justify a postulation of an ineffable world. In Hoyningen-Huene's interpretation of Kuhn's philosophy of science, instead of the Kantian absolute categories, there are Kuhn's paradigm lexicons, which provide the conceptual contribution to the cognitive experience. This last idea only *relativizes* the possible cognitive experience to a given paradigm, in opposition to the epistemological theses held by most realist philosophers.

There is a vast literature on Kuhn's philosophical work[2]. Numerous authors, from a realistic vein, exposed critical studies against that work from Popper (1970) to Devitt (1979) and Burian (1984). Our interpretation of Kuhn's thesis about incommensurable paradigms comes from epistemological relativism, which is congenial, for example, with Doppelt's (1978, p. 34) claim: "Kuhn's relativism hinges on his key argument that competing and historically successive scientific theories are 'incommensurable' with one another: that they are *in some sense* sufficiently different, disparate, incongruous relative to one another to block the possibility of comparative evaluation on the same scale of criteria.". However, we do not have space here to review even the congenial studies.[3]

Returning to our subject, we think that from a conceptualist approach, Kuhn's philosophy of science makes good sense in the responses to questions as to how Planck could provide a solution to the black body problem, Einstein a solution to the photoelectric effect problem, and Bohr propose a solution to the problem about the spectrum of energy emission of the hydrogen atom. It is well known that the first and key step in finding solutions to those anomalies of classical physics was a conceptual improvement: conceive the energy as a discontinuous magnitude and incorporate the concept of quantum of energy. This conceptual change allowed these great physicists to conceive of those phenomena in an innovative mode, understanding them in a novel way. Our point about this is that Planck, Einstein and Bohr were perfectly able to understand the classical concept of energy as a continuous magnitude and the quantum concept of energy as a discrete magnitude without the need to translate the nomic statements which involve these concepts; moreover, despite the impossibility of translation because the concepts of energy involved exclude each other. The lesson to learn about this is that understanding different and alternative theories, which involve mutually exclusive concepts, and which provide diverse world views, do not require that one could translate their respective nomic statements, as Kuhn has held. That is a conceptual issue, not an ontological one: the *conceptualizations* of the world that the theories in question hold are different, and on pain of incoherence, nobody can embrace both.

The previous is only one case, a revolutionary one, that shows that comprehending a new conceptualization of a phenomenon, which departs from a traditional one, does not need a translation between the statements of the theories

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[2] See, for example, Incommensurability Bibliography, in Hoyningen-Huene and Sankey (2001).

[3] Foremost books about Kuhn's philosophy of science are Horwich (1993), Sankey (1994) and Bird (2000). For relativism see Kusch (2020).

involved. There are undoubtedly other examples, and Kuhn has provided several. However, our aim here is not to push that topic but to briefly explore the relationship between the abstract and idealized character of physical concepts and laws on the one hand and the incommensurability thesis on the other.

3. Conceptual System, Abstraction, and Idealization

Before continuing this essay, let us briefly expound on how we understand the concepts of idealization and abstraction from the perspective of conceptual holism, drawing this expression from Brown (2007). In the first place, we find a semantic level. Furthermore, there is a higher level of physical concepts, abstract and idealized in character.

At the semantic level, we can distinguish between class or kind concepts and quantitative or metric concepts (Hempel, 1952). Connotation is the general relation between concepts. More as a terminological convention than a conceptual definition, we shall say that a given concept connotes another concept when the former entails the latter—but does not imply, as Brown (2007) claims: implying is a logical relation that holds between statements, not between concepts[4].

A physical theory contains a network of both types of concepts, which are linked among them by several different sorts of relationships. The main relation among class concepts is subsumption, whereas connotation is the main relation among quantitative concepts. Every physical theory has its particular system of concepts, some being special and exclusive to it, while other concepts might be shared by other physical theories. If, for a moment, we think about a physical theory as a system of laws, we might want to say that the concepts distinctive of a given theory are an integral part of its fundamental laws.

What is central for us here is the sorts of relationships that may obtain between: 1) class concepts, 2) quantitative concepts, and 3) class concepts and quantitative concepts. Nevertheless, we shall assume the semantic concept of denotation to be able to talk about extra-conceptual entities—particularly, physical entities and magnitudes, using appropriate concepts. For class concepts (1), we point out the connotation relation and the subsumption relation that is fulfilled between a pair of concepts when the class of entities denoted by one concept is a subclass of the class denoted by the other concept. Thus, connotation may occur between kind concepts, whereas subsumption may happen between classes denoted by concepts. Mainly, the subsumption relation allows us to make classifications of large sets of entities under study or even an entire taxonomy of the universe postulated by a theory (as is the case of the standard model of elementary particles) employing the connotations that exist between the kind concepts (taxon) involved, which, besides establishing the subsuming relations, indicate other relations such as that of overlap and disjunction. As for (2), the relations between quantitative concepts, we point out the connotations that could occur between a pair of concepts of this type and the relation between a quantitative

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[4] In logical contexts, the terms "entail" and "imply" are often used without distinction. However, they are not synonymous. In other contexts, these terms can express different concepts.

concept and the magnitude that it denotes. In addition, we shall emphasize a quite special connection of nomic character that arises when some quantitative concepts occur in a physical equation, as in the cases of pressure, temperature and volume, as well as force, mass and acceleration. A nomic relation among certain quantitative concepts holds, then, when an equation is formulated in terms of them. This sort of nomic relation among quantitative concepts, and the measurements of the magnitudes denoted by them, allow physicists to calculate predictions with the aid of mathematics and to try to validate them via measurements, observations, and experiments. Finally, regarding (3), the relation between class concepts and quantitative concepts, we can say that class concepts are often characterized by quantitative concepts.

Class and quantitative concepts provide physicists with an appropriate conceptual arsenal for describing states of physical systems as well as changes of states or processes that such systems undergo, which is required to understand what there is and what happens in the physical world postulated by a theory. Then, physical theories embody conceptual systems that simultaneously enable and limit our conceptualizations of how the world is and our cognitive access to what happens in it. From the point of view of conceptual holism, we may say that because we think about the world and understand what occurs in it in virtue of our conceptual systems, they acquire significant relevance to a philosophical stance about scientific knowledge. Nevertheless, generally, the descriptions of the state of a physical system and its possible evolutions are provided by a theory concerning idealized systems in terms of abstract concepts.

At an underlying level, which seems to be overlooked by many philosophers, we have mainly the concepts of abstraction and idealization (along with their inverse concepts, namely, concretization and des-idealization). *Abstraction* is an Aristotelian concept which may be characterized as the intellectual procedure of selecting a few parameters, factors, magnitudes, or variables as relevant, subtracting others, from a type of physical system or process for a specific purpose, such as an explanation or prediction (see Chakravarty, 2001: p. 328). This procedure involves not only the omission of some factors or magnitudes from the systems and processes under study but also the generalization of the factors or magnitudes selected as relevant to an entire class of a certain kind. *Idealization* is a concept that originated with Galileo as an intellectual procedure which consists in the deliberate simplification, the intentional distortion, of physical systems and processes with the intention, among others, of formulating laws and constructing models which allow the quantification of the factors and magnitudes selected as relevant for the mathematical manipulation (see McMullin, 1985: p. 248).

Examples of idealized systems include the frictionless pendulum, material bodies as masses concentrated at extensionless points, inertial bodies free of acceleration, rigid bodies, perfectly elastic spheres, and subsystems like earth-moon or sun-earth as well as systems of elementary particles free of gravitational interactions, the harmonic oscillator, the Coulomb interaction, the hydrogen atom, the square-well potential and the central potential scattering.

Both such procedures combined entail that the laws formulated, and the models constructed, simplify and distort the physical systems and processes under consideration. A result is that the nomic statements of a theory are fulfilled only in idealized systems, which rest on unrealistic assumptions, and that the models constructed cannot represent physical systems in a non-metaphorical sense.

The procedures just mentioned, abstraction and idealization, which are distinctive of intellectual theorization, have the inverse procedures, concretization (see Nowak, 1992) and des-idealization (see McMullin, 1985). These two intellectual procedures allow the establishment of less idealized laws and less abstract models (see Rolleri, 2013). As an inverse procedure to abstraction, we may consider *concretization* as the inclusion of factors or magnitudes previously omitted either in the formulation of laws or the construction of models. The *des-idealization* consists of diminishing the simplifications and distortions of the physical systems and processes in consideration via eliminating some counter-actual or counterfactual assumptions. To speak of counterfactual assumptions might imply that one knows the relevant facts; in that case, it may be better to speak of idealizational assumptions instead, borrowing the notion from Nowak (1992) as a statement which contains an idealizing condition. Lastly, we can add *approximation* as the procedure of including some additional parameters or magnitudes and make the values of the parameters and magnitudes which intervened in the physical system and process under study more accurate. That eventually permits a better match of a theory's predictions with the available experimental data.

All the former intellectual procedures, by and large, allow physicists to attain approximative claims, extracted from des-idealized theoretical models, about the systems and processes within the domain of application of a theory, claims that fit models of data (Suppes, 1962) with some inexactness obtained by empirical means, models which are instantiated in some processes undergone in physical systems.

To close this section, we underline that generally kind concepts are achieved by abstraction, whereas quantitative concepts often embrace idealizations of physical systems and processes to formulate nomic statements about them and eventually measure the magnitudes denoted.

4. Incommensurability

According to Feyerabend (1978: p. 68, fn. 118), the relation between incommensurable theories is not the logical relation of inconsistency; for him, it is instead a semantical issue which involves more than a discrepancy amongst concepts but a species of conceptual incongruity: "I emphasised that mere *difference* of concepts does not suffice to make theories incommensurable in my sense. The situation must be rigged in such a way that the conditions of concept formation in one theory forbid the formation of the basic concepts of the other…". If we consider that the conditions of concept formation include procedures of idealization of the entities postulated and abstraction of some of the quantitative properties or magnitudes attributed to them by a theory, then something which precludes a congruency concept formation in an alternative theory consists in that the respective idealization and abstraction are quite different. That difference between the idealizations and abstractions of two alternative theories could be viewed as an obstacle to comparing their basic principles, thus a source of incommensurability.

Let us take a significant example from Kuhn that shows the lack not only of conceptual comparison but also of conceptual congruence: Given Newton mechanics' absolute concepts of space, time and mass, the formation of the relativist concepts of space, time and mass becomes incongruent with them because it involves different idealizations and abstractions. Concerning the idea that under certain conditions, Newton physics is a special case of Einstein physics, Kuhn points out that:

> The physical referents of these Einsteinian concepts are by no means identical with those of the Newtonian concepts that bear the same name (Newton mass is conserved; Einsteinian is convertible with energy. Only at low relative velocities may the two be measured in the same way, and even then they must not be conceived to be the same). Unless we change the definitions of the variables [in the Einsteinian version of the laws], the statements we have derived are not Newtonian […] It has not, that is, shown Newton's Laws to be the limiting case of Einstein's. For in the passage to the limit it is not only the forms of the laws that have changed. Simultaneously we have had to alter the fundamental structural elements of which the universe to which they apply is composed (Kuhn, 1970: p. 102).

Underlying the last remark of this quotation, we find the idea that the elements that compose Newtonian and Einsteinian universes rest on entirely different idealized assumptions.[5] Newton's idealizations of space and time as separate and absolute entities, gravitation as an attraction force at a distance between bodies, and mass as a conserved amount of matter, is not only discrepant but incongruent with the relativistic conception of space-time, which rests on the idealizations and abstractions of a continued tetra-dimensional entity, where gravitation is a consequence of the geometry of the universe that could be curved in certain regions as an effect of near masses, as well as mass and energy interconvertible. When we keep those diverse underlying suppositions in mind, we come to realize that Newton's laws and Einstein's equations not only differ in their meanings and their predictions but also significatively in their idealized suppositions of the structure of the universe and their abstractions about magnitudes as mass and gravitation. The differences or, better, discrepancies and incongruencies of these two theories are deeper than the level of meaning, trans-

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[5] Sankey consider that such failure of derivation of Newton's laws from Einstein's is due to the conceptual disparity between the theories (1994, p. 22). We may add that there is underlying an incongruence between the idealizations and abstractions involved in these theories to that conceptual disparity.

lation and language, *pace* Davidson. They diverge at the level of their underlying idealizational assumptions, and their respective abstractions related to their different systems of concepts.

The discrepancy and incongruity between classical physics and quantum theory are still more radical than that. These physical theories differ in nature, structure, content, and subject matter.

Let us take as a starting point Kuhn's thesis about the lexicon or conceptual vocabulary that every theory owns, which permits constructing a taxonomy of the whole universe of a theory. The classifications of elementary particles are obtained by abstraction of a magnitude that the theory attributed to them, such as the spin or some other criterion. If one chooses the type of spin, integer or semi-integer, one gets a taxonomy with two main categories: bosons and fermions. In contrast, if one chooses the modes of interaction among particles, one obtains a different taxonomy with two main categories: leptons and hadrons. Our point is that abstractions are required and performed to make such classifications or taxonomies of the entities of the model of elementary particles: the concepts of boson and fermion as well as of lepton and hadron are abstract concepts. As we can see, these abstractions involve subtracting other magnitudes like mass or charge. Of course, classical mechanics does not make those sorts of abstractions to characterize the classical bodies. Instead, in classical physics, we find the idealization that conceives the material bodies as particles with the mass concentrated in extensionless points. That shows that because the theoretical procedures of abstraction involved are quite different, there is no way to compare the classical and quantum concepts of particles. It is not just a question of a difference between meanings but of the results of the abstractions performed to acquire them. This issue is overlooked by some critics like Davidson (1974).

Another significative divergence, brought out by Lorentz, results from the assumption that in quantum theory, systems 'move' in a Hilbert space. The contrast resides in that the representation of the evolutions of classical systems in Euclidean space has a physical counterpart, whereas the representation of the evolutions of quantum systems in a Hilbert space, as Lorenz objected, lacks such a counterpart. Perhaps, we could say that quantum depictions of vectors state in a Hilbert space is just an abstract mathematical devices unprovided of a physical correlate, and that the idea of "movement" in such space is an unrealistic supposition.

A major difference between classical and quantum mechanics resides in the principle of superposition. The Schrödinger equation, which depicts the time-evolution of the wave functions, is unitary and of deterministic character. However, under Born's probabilistic interpretation of such an equation, one can obtain probability distributions that assign numerical values to possible alternative future states of quantum systems. The solutions of the Schrödinger equation, given specific physical situations, provides certain wave functions ψ which under Born's interpretation, the square modulus of ψ, $|\psi|^2$, express probability amplitudes to the superposed states conforming with the superposition principle. In

1930 Dirac formulated that principle, which is a consequence of the fact that the Schrödinger equation is homogenous and linear[6]; it asserts that: "If a QM system *S* can be in a state $|b_1>$ and also in a state $|b_2>$, then it can be in each linear combination of both $|\psi> = c_1|b_1> + c_2|b_2>$ (where c_1 and c_2 are complex numbers)." (Auletta, 2001: p. 39). In general, for *n* vectors state, we have: $|\psi> = \Sigma_j^n c_j|b_j>$. Hughes (1989: p. 92) points out that Dirac finds the most significant difference between classical and quantum mechanics in the role played by the principle of superposition. A non-formal version of such a principle is the following:

> The superposition principle says that a new state of a system may be composed of two or more states in such a way that the new state shares some of the properties of each of the combined states. If A and B ascribe two different properties to a particle, such as being at two different places, then the superposition of states, written as A + B, has something in common with both states A and B. In particular, the particle will have non-zero probabilities for being in each of the two states but not elsewhere if the position of the particle is to be observed (Aczel, 2001: p. 25).

Although the Schrödinger equation allows a deterministic image of the universe of the quantum domain, the principle of superposition impedes a realistic interpretation of such an image because we cannot think that a quantum system is at the same time in two different states, represented by the vectors $|b_1>$ and $|b_2>$, which are alternative and exclude each other. Instead, we may say that the image of superposed states is unrealistic and that such a principle is of abstract character. Nevertheless, Born's probabilistic interpretation of the alternative possible states that quantum systems could adopt gives the principle of superposition plenty of sense. Thus, under the probabilistic interpretation of the wave functions ψ, the principle of superposition admits a modal interpretation in terms of possible physical states of quantum systems, which contrasts with a realistic interpretation of it.

By abstracting what one could observe in experiments, one could support the random character of quantum processes involved in agreement with the modal content of the superposition principle, considering probability as a measure of chance. According to von Neumann's projective postulate, one of the superposed states is observed randomly when a quantum system and a measurement apparatus interact. We may insist that in contrast with the initial and final states, the status of the superposition of states is not physical. The concept of a set of superposed states is an idealization implicit in quantum mathematical formalism without a factual counterpart. We think the novel concept of superposed states is an exemplary case of a highly abstract and idealized concept of major interest involved in the formalism of quantum mechanics.

The former point could be amplified considering that the very wave function is just a mathematical device, in contrast to the waves themselves, as Born sustained at the time of the birth of the old quantum theory. Auletta remarks on

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[6] See Auletta, 2001: p. 39.

this question, referring to Born's probabilistic interpretation, in the following terms: "The wave function determines only the probability that a particle—which brings with itself energy and momentum—takes a place; but no energy and no momentum pertains to the wave." (2001, p. 104).

The high abstract and idealized character of quantum theory is perhaps revealed by von Neumann's projection postulate, which according to Margenau, gives origin to the Einstein, Podolsky, Rosen paradox[7].

Von Neumann's postulate refers to measurement processes in which an appropriate apparatus *A* and a quantum system *S* are coupled, producing a change of state in each other, which realizes the measurement of a quantum observable Ô. This projection postulate can be expressed as follows: "If the observable Ô is measured on *S* in an arbitrary state |ψ>, then the latter is projected after the measurement onto one of the basis vectors $|b_j>$ (i.e., P_{bj}) of the representation in which Ô is diagonal, i.e. in an eigenstate of Ô for which the probability is $|<bj|\psi>|^2$." (Auletta, 2001: p. 223). This postulate involves the so-called "collapse" of the wave function ψ since the interaction between *A* and *S* reduces randomly the prior superposed states of system *S* to one of the eigenstates of the observable Ô.

The projective operator *P* involves the reduction of |ψ>, representing the superposition of states $\Sigma_j^n c_j|b_j>$ of the system *S*, to a vector state $|b_j>$ in particular. Thus, from a formal point of view, the reduction involved refers only to a mathematical operation, represented by the operator *P*, which projects |ψ> onto $|b_j>$. What is the physical import of such a reduction? Von Neumann's postulate has an operational interpretation in physical situations of measurements, and only there; beyond those situations, it has no meaning at all. According to the Copenhagen interpretation, which is the pertinent interpretation in this context, nothing can be said about actual states, evolutions, or changes of quantum systems unless we perform measurements of the relevant observables.

Then, apart from the probabilistic sense attributed to both wave functions and collapse of the wave functions, we have, in the first place, Born's objection to the physical content of wave functions and, in the second place, a lack of physical correlates either to superposition principle or to projective postulate. Our point about that consists in that the concepts of wave function, superposed states, collapse of the wave function as well as the representation of all that in Hilbert space have a high abstract and idealized status. Since quantum theory and its models are permeated by all the previous, we may say that it rests on several idealizational suppositions.

The conceptual systems of classical and quantum mechanics are so radically different that it is hard even to try to point out some convergences. Indeed, these theories share some terms like position, momentum, energy, and time, but the concepts they express are not comparable due to Heisenberg's indeterminacy principles. The nomic statements of quantum theory involving those pairs of concepts are, if not counterfactual, at least counter-actual, in contrast with clas-

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[7] As Auletta points out (2001, p. 223).

sical nomic statements, which involve such Newton's concepts that are, to some extent, factual. As we indicate in the preceding section, a quite special connection of nomic character is obtained when some quantitative concepts occur in physical equations. This sort of nomic relation among quantitative concepts, and the measurements of the magnitudes denoted by them, are in those pairs of concepts so divergent that they impede any comparison.

The comparison at a theoretical level of classical and quantum mechanics is so implausible that we shall not attempt it. As we have seen, at a theoretical level, the quantum theory involves some principles which rest on idealized assumptions since they have no physical counterpart. The Newton mechanics lack principles which could be in correlation, or something like that, with the principle of superposition of states and the projective postulate. It seems that, at a general level, the comparison of the systems of concepts of these two theories is not feasible. However, this does not exclude the possibility of a partial comparison at the level of predictions (but this last topic is beyond our purposes here). The former remarks suit Kuhn's concept of incommensurable theories.

5. Final Remarks

In one of his last papers, Kuhn (1993) said that the dominant topics of his work were incommensurability and the nature of the conceptual change in paradigm transitions. About the former theme, he explains that:

> Concerned from the start with the *development* of knowledge, I have seen each stage in the evolution of a given field as built—not quite squarely—upon its predecessors, the earlier stage providing the problems, the data, and most of the concepts prerequisite to the emergence of the stage that followed. In addition, I have insisted that some changes in conceptual vocabulary are required for the assimilation and development of the observations, laws, and theories deployed in the later stage (whence the phrase "not quite squarely" above). (1993, p. 314).

Thus, a paradigm offers some conceptual material and mainly *problems* for the development of an alternative paradigm which intends to resolve the anomalous problems of the previous paradigm. This sort of consideration enlightens the subject that we mentioned earlier about the original contributions by Planck, Einstein, and Bohr to the birth of quantum theory, as well as the known case of the account of Mercury's perihelium by general relativity, which was an anomaly in classical mechanics. So, to resolve anomalous problems, conceptual innovation is needed, often bringing a change in the world view.

One of our theses in this essay is that both conceptual innovation and world view change generate modifications in the idealizations and abstractions of the theories involved. Sometimes, some idealized suppositions are eliminated when a world view change happens, as was the case in the transition from geocentric cosmology to heliocentric cosmology at the time of Newton, where the idealized circular trajectories and spherical forms of the celestial bodies were removed. At times, the idealized assumptions change simultaneously with conceptual innovation and world view variation, as in the transition from Newtonian physics to relativity theory. Other times, the conceptual innovations, together with a world view change, incorporate novel, original idealized suppositions without antecedents in the earlier theory, as was the case of the transition from classical to quantum mechanics, where a new world, to use Hacking's expression, was revealed. It seems implausible to generalize the types of changes of the idealized assumptions involved in paradigm transitions.

We could perhaps say that incommensurability becomes in degrees, that the dimension of incommensurability is not always the same. In the cosmology revolution, even if the world view change was radical, in some sense, the ontology does not change; the individual objects were preserved. However, the new kind concepts classify them differently, according to the nominalist solution to the problem of world change due to Hacking (1993). The Einsteinian revolution suits well with the order of ideas expressed by the former quotation from Kuhn; it was built (but not quite squarely, of course) from Newtonian physics, changing the view of the universe—yielding the world change problem as Kuhn saw it. The quantum revolution, in contrast, presents incommensurability to a major extent. The theoretical change was radical, the innovation of concepts was enormous, the level of abstraction and idealization became higher at the level of principles and postulates, and the world change problem presented another face, because a new world was discovered or, better, a novel ontology was postulated, where neither Kuhn's taxonomic solution nor Hacking's nominalist solution to the world change problem fit.

It seems that corresponding to the amount, so to speak, of incommensurability of some paradigm transitions, the extent of idealization or abstraction varies by virtue of concept innovation, ontology postulation and world view change involved in such revolutionary transitions. Generally, a scientific revolution brings with it changes such as novel idealizations and abstractions, which become another source of incommensurability between paradigms.

Let us conclude that, if we are right about the issue of this essay, in addition to the semantic impossibility of total translations between the theories of alternative paradigms, the comparison becomes implausible at a conceptual level of their nomic statements because of the discrepancies between both the abstract and idealized concepts and the related underlying idealizational assumptions. Therefore, the theoretical procedures of idealization and abstraction are another route to incommensurability. Besides, this shows that Kuhn's philosophy of science is indeed a version of conceptual or epistemological relativism, because it displays that even at the level of idealizational suppositions, our *conceptualization* of the world depends on, and is relative to, a paradigm.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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