

# An Epistemological and Ontological View of Statistical Mechanics

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## Abstract

In this article, an epistemological and ontological view of Statistical Mechanics is presented. The author believes that this is a clear-cut example that can prove to be useful as a resource material for teachers, and also, a clear pedagogical material for science students when taking courses of Philosophy of Science. Statistical Mechanics is a regular course on most science and engineering programs, we believe that this fact facilitate the understanding of the philosophical discussion here presented.

## **Keywords**

Philosophy of Science, Epistemology, Ontology, Statistical Mechanics

# **1. Introduction**

The American philosopher Tim Maudlin wisely states that: "All physical theory must be clear and discuss two fundamental questions: What is there and what is doing. The answer to the first question is given by the ontology of the theory, whereas the answer to the second by its dynamics. Ontology must have a clear mathematical description whereas the dynamics must be stated through precise equations describing how the ontology should o could evolve" (Maudlin, 2019: 37), this is a statement all scientists would agree on. On the other hand, epistemology (from the Greek  $\dot{\epsilon}\pi_{I0}\tau\eta\mu\eta$  (episteme), "knowledge", and  $\lambda \dot{0}\gamma 0 \zeta$  (logos), "theory") has knowledge as its object of study. As a theory of knowledge, it is concerned with analysis, its obtainment and the criteria through which it is justified or invalidated. Furthermore, ontology (from the Greek  $0v\tau 0\zeta$ , genitive case of the participle of the verb  $\varepsilon \mu \mu$ , to be; and  $\lambda \dot{0}\gamma 0\zeta$ , science, study, theory) is the investigation of being as being, or of being in general, beyond anything in particular; it is the study of being insofar as it exists. To confuse an ontological

problem with an epistemological problem is a mistake in academic philosophy. An ontic answer to an epistemic question, or an epistemic answer to an ontic question, is considered a category error. In spite of this, many problems in physics and mathematics necessarily involve the ontological and epistemological analysis of them. As an example, we have the philosophical problems associated with quantum mechanics and, in particular, with Heisenberg's Uncertainty Principle; these are problems which are both epistemic and ontological. The observables of a quantum system are an epistemological problem, while the quantum states of a quantum system are an ontological problem. Throughout the debate between Einstein and Bohr, it can be noted that while Einstein holds an ontological point of view, i.e. independent of measurements or observers, Bohr holds an epistemic point of view, i.e. about what can be known about quantum systems. The question of whether nature can be described independently of who describes it, i.e. "when no one is looking", has been widely debated without general agreement. We can distinguish ontological questions as being those referring to the structure and reality of a system as such, while epistemological questions refer to the knowledge and information that can be obtained from such systems.

Often, when taking courses of Philosophy of Science –or Philosophy, in general-, it is mainly -but not exclusively- science students who struggle in order to clearly distinguish what ontological and epistemological problems are about. Based on this observation, an example that may be easily followed by science students, and which has been taken from Statistical Mechanics, is offered in this article.

The case we present is that of a complex system that is in a state of equilibrium formed by N molecules (which will be treated as point masses of mass m, where N is a very large number -of the order of Avogadro number  $6.022 \times 10^{23}$  molecules/mol-) is discussed from the point of view of Statistical Mechanics, where it is clear that the adjective "complex" is fully justified. In particular, the case of systems formed by distinguishable particles is presented. This is done by closely following the physical-mathematical presentation of these systems which is found in standard undergraduate scientific texts such as Reif (2009), García-Colín Scherer (1995) and Kestin & Dorfman (1975). In particular, it is shown that these systems allow ontological and epistemological problems to be discussed with a clear physical interpretation. This is in contrast to similar discussions carried out by other authors (e.g. Atmanspacher (2001) and Primas (1990)) who, in spite of also analyzing physical systems, focus the ontological and epistemic discussion on the analysis of information flow.

## 2. Ontology of a Complex System

The ontological approach arises from the assumption that the particles are found in a container with rigid and insulating walls, and therefore, the volume V and the total energy U of the system are known. Likewise, each particle will require for its description three coordinates of position,  $r_i = (x_p \ y_p \ z_i)$  and three coordinates of momentum,  $p_i = m (v_{x^p} \ v_{y^p} \ v_{zi}) = (p_{x^p} \ p_{y^p} \ p_{zi})$ , where the subscript *i* refers to each of the *N* particles (i.e. the *i*-th particle), and *v* denotes the velocity component along each axis. A Cartesian space of 6 coordinates is required to specify the dynamic state of the system; this is a 6-dimensional space that we will denote as  $\mu$ . Each point in this space where a particle is located, *i*, is:  $w_i (x_p \ y_p \ z_p \ p_{x^p} \ p_{y^p} \ p_{zi})$ . The points  $w_i \in \mu$  define a dynamical state, and each particle corresponds, at a given instant, to the point  $w_i$  in this space. The points  $w_i \in \mu$  describe in their entirety the ontic state of the system, where "in its totality" indicates that an ontic state is precisely the way it is, without reference to epistemic knowledge or ignorance of it. This is what there is, what exists, independently of the rest of the universe. Given the complexity of the system, it is practically impossible to know the points  $w_i (x_p \ y_p \ z_p \ p_{x^p} \ p_{x^p} \ p_{z^j}) \in \mu$  for i = 1, 2, ..., N, excepting an omnipotent mind.

In  $\mu$  space, each point corresponds to a possible dynamical state of a particle in the system, and each particle corresponds to a point in this space at each given instant; therefore, the totality of particles will be represented by a cloud of Npoints in space. Each of the points  $w_i \in \mu$  does not remain fixed in time, but each of them describes trajectories determined by the laws of classical mechanics called Newtonian physics; in particular, by Newton's first three laws: the law of inertia, the law of force, mass and acceleration, F = ma, and the law of action and reaction. Given the initial conditions at an arbitrary time t = 0, denoted by  $r_i^o = (x_i^o, y_i^o, z_i^o)$  and  $p_i^o = (p_{xi}^o, p_{yi}^o, p_{zi}^o)$ , the coordinates and momentum of the N particles at any other time t will be given by functions g and h, such that:

$$r_i(t) = g\left(x_i^o, y_i^o, z_i^o\right) \tag{1}$$

$$p_i(t) = h\left(p_{xi}^o, p_{yi}^o, p_{zi}^o\right) \tag{2}$$

for  $i = 1, 2, 3, \dots, N$ .

However, despite the fact that the above formulation is in principle the complete solution of the ontological problem, it would not be possible to obtain an epistemic solution from this information, end even less so, a solution of practical value, such as the specific heat of the complex system. We can see that given the enormous value of *N*, it is not possible for a limited mind like the human mind to have access to the ontology of a complex system of this nature.

## 3. Epistemology of a Complex System

To obtain information, and therefore, an epistemic solution of a complex system such as the one described above, scientists have applied ideas developed by the Austrian physicist Ludwig Boltzmann at the end of the nineteenth century. The fundamental idea of Boltzman's method is to ignore the individual dynamics of each particle and concentrate on the dynamics of clusters or groups of particles called "microstates", each of which is characterized by a number of particles  $n_j$ and an energy  $\varepsilon_r$ . This description is epistemically the most detailed to which one can aspire. With this approach, the impossibility of any attempt to solve the original ontological problem from the individual knowledge of the position and momentum of each particle is being conceded. All efforts will now be concentrated on solving the epistemic problem; that is, on answering what it is that we can know from the formulation by microstates of the given complex system. For each instant t, we have in each cell j the following:

- $n_1$  particles (o points in space  $\mu$ ) in cell  $\varepsilon_1$
- $n_2$  particles (o points in space  $\mu$ ) in cell  $\varepsilon_2$
- .
   .
   *n<sub>j</sub>* particles (o points in space μ) in cell ε<sub>j</sub>
   .
   .
   .

#### And so on.

From each microstate, we will know not only how many particles (o points in space  $\mu$ ) are in the cell, but also which ones they are. Therefore, for a number of particles *N*, and knowing the macroscopic epistemic characteristics of the system, such as its volume *V* and energy *E*, where evidently;

$$n_1 + n_2 + \dots + n_j + \dots = \sum_i n_i = N$$
 (3)

and

$$\varepsilon_1 n_1 + \varepsilon_2 n_2 + \dots + \varepsilon_j n_j + \dots = \sum_i \varepsilon_i n_i = E$$
 (4)

There will be an immensity of microstates at time *t* that will be consistent with this information. The total number of microstates corresponding to the distribution  $(n_1, n_2, ..., n_b, ...)$  will be denoted by *W*, and is given by:

$$W = \frac{N!}{\prod_{j} n_{j}!} \tag{5}$$

Since there is an enormous number of conceivable distributions, the total number of them, which will be denoted by  $\Omega(E, V, N)$ , is obtained by adding all the W's for all these distributions. That is,

$$\Omega(N, E, V) = \sum_{\{n_i\}} \frac{N!}{\prod_j n_j!}$$
(6)

This expression implies a fundamental hypothesis: that all the microstates that appear in it are equally likely. Assuming that in equilibrium there will be a distribution  $(n_1, n_2, ..., n_p, ...)$  that will appear with an overwhelmingly higher frequency than any other distribution, then, this distribution is the one that will effectively contribute to the final value of  $\Omega$  (*E*, *V*, *N*). This distribution is known as the Maxwell-Boltzmann distribution at time *t*, and will be denoted by f(r, p, t). Note that given the dimensions of a cell in a six-dimensional  $\mu$  space as *drdp*,

then f(r, p, t) drdp, is the number of particles in that cell. The question of how to relate the observable epistemic properties of the system to the total number of distributions  $\Omega$  (*E*, *V*, *N*) is solved by using the concept of entropy S = S(U, V,*N*), where *U* is the internal energy of the system, and *V* and *N* are its volume and number of particles. In thermodynamics, it is established that the change in entropy of a system  $\delta S$  is related to the change of heat  $\delta Q$  at temperature *T* by;

$$\delta S = \frac{\delta Q}{T} \tag{7}$$

On the other hand, since for every process that occurs in an isolated, closed system, entropy never decreases, the equilibrium state is the one for which *S* has a maximum value. That is, both *S* and  $\Omega$  have a maximum value at equilibrium. In 1872, By identifying *U* with *E* in *S* (*U*, *V*, *N*) and in  $\Omega$  (*E*, *V*, *N*), Boltzmann found the relationship between *S* and  $\Omega$ , which is given by:

$$S = k \ln \Omega \tag{8}$$

where *k* is Boltzmann's constant. In the words of García-Colín Scherer (1995): "This equation is the greatest result of statistical mechanics for systems in equilibrium. The entropy of an isolated, closed-ended system in equilibrium is proportional to the logarithm of the total number of microstates in the system that correspond to a distribution or macrostate that overwhelmingly prevails over any other possible one."

By now making use of the so-called "fundamental relation of thermodynamics" that relates the temperature T and pressure P of a system to its changes in energy dE, entropy dS and volume dV,

$$dE = TdS - PdV \tag{9}$$

the epistemic solution of the original complex system is obtained. Relation (8) establishes the link between the total number of distributions  $\Omega$  and entropy *S*, while relation (9) establishes the link between entropy and the macroscopic epistemic properties of the original complex system, such as its temperature *T*, pressure *P* and volume *V*, as well as many others, such as density, heat capacity, compressibility and thermal expansion, among others.

It is important to note that relations (5) and (6) are equations of abstract mathematical statistics and have an ontological value of their own, regardless of the universe in which we are, while (8) and (9) are equations with evident empirical content that have an epistemic value exclusive and are inevitably linked to our empirical universe.

## 4. From Epistemology to Ontology

It is interesting that the epistemic solution of this problem allows the ontological problem to be fed back. That is, from the initial ontological premise that supposes the existence of a system of N point particles of mass m, epistemological results are obtained on its macroscopic phenomenology. This is despite the fact that the initial ontological premise, i.e. the existence of N point particles of mass m, was originally only an assumption. Boltzmann's kinetic theory presupposes

the reality of atoms and molecules; yet, in his time, almost all philosophers and most scientists, such as Ernst Mach and Wilhelm Ostwald, did not accept their existence. At the end of the 19th century, Boltzmann attempted to formulate an intermediate position that would allow atomists and non-atomists to do physics without arguing about the existence of atoms; that is, a position in which the initial ontology was irrelevant. His solution purported to use Hertz's concept that atoms are "images" (Bilder); Atomists might think that these images were real atoms, while anti-atomists might think that these images were just a useful but unreal representation. However, this stance did not satisfy either group. Proponents of "pure thermodynamics," such as Ostwald, sought to disprove kinetic theory and statistical mechanics because of the assumption about the existence of atoms and molecules in these theories as well as because of their interpretation of the second law of thermodynamics. A further philosophical objection was raised by physicist Gustav Jaumann, a student of Mach, who interpreted Hertz's results to imply that all electromagnetic behavior was continuous, suggesting the non-existence of atoms and molecules, in fact suggesting that ultimate ontological reality was exclusively electromagnetic in nature. In the late 19th and early 20th centuries, the discrete nature of matter and energy was firmly established with quantum theory and experimental evidence such as Brownian motion, Avogadro number measurements, the photoelectric effect, the Compton effect, Rutherford's scattering experiments and, finally, Bohr's model of the atom in which the latter synthesizes Rutherford's atomic ideas with Planck's quantum ideas. As a result of all this empirical evidence, it became clear that the initial ontological premise, the assumption of the existence of N particles of mass m, must be correct; Boltzmann's approach provides an epistemic answer to this ontological model. The argument that the ontology of the model could be different and still provide the same epistemology is only an interesting historical curiosity, since an acceptable ontology must be compatible with the totality of the available theoretical and empirical epistemic evidence (Quine, 1951).

## 5. From Ontology to Epistemology

Since f(r, p, t) is the Maxwell-Boltzmann distribution at time t mentioned above, this distribution represents the ontological state of the system; that is, the reality of what is there, i.e. N particles at time t with a certain distribution of position r and momentum p in space  $\mu$ . Regardless of the complex molecular dynamics, molecules are distributed in phase space according to f(r, p, t), as we have seen this is a statistical result. It can be shown that the dynamics of this system is given by the Boltzmann equation,

$$\frac{\partial f}{\partial t} + \frac{p}{m} \cdot \nabla f + F \cdot \frac{\partial f}{\partial p} = \frac{\partial f}{\partial t} \Big|_{Colision}$$
(10)

On the left side, the first term represents the explicit temporal variation of the distribution function, while the second term provides its spatial variation (in the coordinates; x, y, z), and the third term describes the effect of any force F acting

on the particles; finally, the right side of the equation represents the temporal change of the distribution function due to the effect of collisions between the particles, and assuming that there are no collisions, the right side is zero. This is one of the most important equations in statistical physics because it is applicable to out-of-equilibrium systems. This equation is associated with an entire ontology and its corresponding epistemology. The ontology of the system is mathematically given by the function f(r, p, t), i.e. the Maxwell Boltzmann distribution. From this equation, we can describe, for example, the transport of heat and properties such as the thermal conductivity and viscosity of the system, among many others; that is, the entire phenomenology of the system, which is the epistemology that is accessible to us.

## **6.** Conclusions

In this article, a clear example of an epistemological and ontological view taken from Statistical Mechanics is presented. The author believes that it may be useful to beginner students taking Philosophy courses, or more specifically, Philosophy of Science, courses. This is important in order to avoid confusing ontological and epistemological problems.

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## **Conflicts of Interest**

The author declares no conflicts of interest regarding the publication of this paper.

#### References

- Atmanspacher, H. (2001). Determinism Is Ontic, Determinability Is Epistemic. In H. Atmanspacher, & R. Bishop (Eds.), *Between Chance and Choice* (pp. 49-74). Imprint Academic.
- García-Colín Scherer, L. (1995). Termodinámica Estadística. UAM-Iztapalapa.
- Kestin, J. & Dorfman, J. R. (1975). *A First Course in Statistical Thermodynamics.* Academic Press.
- Maudlin, L. (2019). *Philosophy of Physics*. Princeton University Press. https://doi.org/10.1515/9780691190679
- Primas, H. (1990). Mathematical and Philosophical Questions in the Theory of Open and Macroscopic Quantum Systems. In A.I. Miller (Ed.), *Sixty-Two Years of Uncertainty* (pp. 233-257). Springer.
- Quine, W. V. O. (1951). From a Logical Point of View. Harvard University Press.
- Reif, F. (2009). Fundamentals of Statistical and Thermal Physics. Mc. Graw-Hill.