

An Effective Approach to Determine an Initial Basic Feasible Solution: A TOCM-MEDM Approach

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Abstract

Transportation of products from sources to destinations with minimal total cost plays an important role in logistics and supply chain management. In this article, a new and effective algorithm is introduced for finding an initial basic feasible solution of a balanced transportation problem. Number of numerical illustration is introduced and optimality of the result is also checked. Comparison of findings obtained by the new heuristic and the existing heuristics show that the method presented herein gives a better result.

Keywords

Initial Basic Feasible Solution, Total Opportunity Cost Matrix, Pointer Cost, Optimum Solution

1. Introduction

Transportation Problem (TP) is a special type of Linear Programming Problem (LPP). Transportation plan is mainly used to minimize transportation cost. Hitchcock [1] in 1941 presented a study entitled “The Distribution of a Product from Several Sources to Numerous Localities” which was the first formal contribution to the transportation problems. In 1947, Koopmans [2] presented a study called “Optimum Utilization of the Transportation System”. Systematic procedure for finding solution for TPs was developed, primarily by Dantzig [3] in 1951, and then by Charnes *et al.* [4] in 1953. The problem of minimizing transportation cost has been studied since long and is well known by Shenoy *et al.* (1991) [5], Kasana and Kumar (2005) [6], Hamdy (2007) [7], Pandian & Na-

tarajan (2010) [8], Aminur Rahman Khan (2011; 2012) [9] [10], Sharif Uddin *et al.* (2011) [11], Md. Amirul Islam *et al.* (2012) [12], Sayedul Anam *et al.* (2012) [13], Md. Main Uddin *et al.* (2013b) [14], Mollah Mesbahuddin Ahmed *et al.* (2014) [15] [16], Utpal Kanti Das *et al.* (2014a; 2014b) [17] [18]. Several researchers have developed alternative methods for determining an initial basic feasible solution which takes costs into account. Well-known heuristics methods are North West Corner Method (NWCM), Matrix Minima Method (MMM), Vogel's Approximation Method (VAM), Highest Cost Difference Method (HCDM), Extremum Difference Method (EDM), TOCM-MMM Approach, TOCM-VAM Approach, TOCM-EDM Approach, TOCM-HCDM Approach etc.

Reinfeld and Vogel (1958) [19] introduced VAM by describing penalty as the difference of lowest and next to lowest cost in each row and column of a transportation table and allocate to the lowest cost cell corresponding to the highest penalty. Kasana and Kumar (2005) [6] proposed EDM where they define the penalty as the difference of highest and lowest unit transportation cost in each row and column and allocate as like as the VAM procedure. Aminur Rahman Khan (2012) [10] presented HCDM by defining pointer cost as the difference of highest and next to highest cost in each row and column of a transportation table and allocate to the minimum cost cell corresponding to the highest three pointer cost. Sayedul Anam *et al.* (2012) [13] determine the impact of transportation cost on potato distribution in Bangladesh.

Kirca and Satir (1990) [20] first transform the cost matrix to the Total Opportunity Cost Matrix (TOCM). The TOCM is formed by adding the row opportunity cost matrix (ROCM) and the column opportunity cost matrix (COCM) where, for each row in the initial transportation cost matrix, the ROCM is generated by subtracting the lowest cost in the row from the other cost elements in that row and for each column in the initial transportation cost matrix, the COCM is generated by subtracting the lowest cost in the column from the other cost elements in that column. Kirca and Satir then essentially use the MMM with some tie-breaking rules on the TOCM to generate a feasible solution to the transportation problem.

Mathirajan and Meenakshi (2004) [21] applied VAM on the TOCM whereas Md. Amirul Islam *et al.* applied EDM on TOCM (2012) [22] and allocate to the minimum cost cell corresponding to the highest distribution indicator and again HCDM on TOCM (2012) and allocate to the minimum cost cell corresponding to the highest two distribution indicator. Aminur Rahman Khan *et al.* (2015) [23] obtain the pointer cost for each row and column of the TOCM by taking sum of all entries in the respective row or column and make maximum possible allocation to the lowest cost cell corresponding to the highest pointer cost.

Here in this article, the pointer cost has been calculated only one time by taking the difference between the highest and lowest cell cost for each row and column of the TOCM and make maximum possible allocation to the lowest cost cell corresponding to the highest pointer cost. It is to be mentioned that other

allocation is obtained by Modified Extremum Difference Method without calculating the pointer cost time and again.

2. Mathematical Formulation of Transportation Problem

The Transportation Problem can be specified as an allocation problem in which there are m sources (suppliers) and n destinations (customers). Each of the m sources can allocate to any of the n destinations at a per unit carrying cost c_{ij} (unit transportation cost from source i to destination j). Each sources has a supply of a_i units, $1 \leq i \leq m$ and each destination has a demand of b_j units, $1 \leq j \leq n$. The objective is to determine which routes are to be opened and the size of the shipment on those routes, so that the total transportation cost of meeting demand, given the supply constraints, is minimized.

A transportation problem is a complete specification of how many units of the product should be transported from each source to each destination. So, the decision variables are:

x_{ij} = The amount of the shipment from source i to destination j , where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$.

Therefore we get, Minimize: $Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$.

3. Algorithm of Proposed Approach to Find an Initial Basic Feasible Solution

The proposed method named as "TOCM-MEDM Approach" comprises with the following steps:

Proposed Algorithm	
Step 1:	Subtract the smallest cost (c_{ik} where $k = 1, 2, \dots, n$) from each of the cost along the first row ($c_{i1}, c_{i2}, \dots, c_{in}$, where $i = 1, 2, \dots, m$) of the transportation table and write those on the right top corner of the corresponding cost. Similar operation is applicable for rest of the rows.
Step 2:	Applying the same process on each of the column and write the result on the left lower corner of the corresponding cost.
Step 3:	By adding the digit of right top corner and left lower corner construct the TOCM.
Step 4:	Determine the penalty cost for each row of the TOCM by taking difference between the highest and the lowest cell cost in the same row and put it on the right of the corresponding rows of the cost matrix. These numbers are called Row Penalties (RP). In a similar fashion, calculate the Column Penalties (CP) for each of the columns and write them in the bottom of the cost matrix below corresponding columns.
Step 5:	Choose the highest penalty costs and observe the row or column along which it appears. If a tie occurs, choose the row/column along which lowest-cost appears. If it is also same then choose any of them.
Step 6:	Allocate maximum to the cell having lowest unit transportation cost in the row or column along which the highest penalty cost appears. If more than one cell contain lowest-cost, we allocate to the cell where allocation is maximum.
Step 7:	Determination of rest of the allocation:

Continued

Adjust the supply and demand requirements in the respective rows and columns. Then following cases arise:

- Case 1: If the allocation $X_{ij} = a_i$, i -th row is to be crossed out and b_j is reduced to $(b_j - a_i)$. Now complete the allocation along j -th column by making the allocation/allocations in the smallest cost cell/cells continuously. Consider that j -th column is exhausted for the allocation X_{ij} at the cell (k, j) . Now, follow the same procedure to complete the allocation along k -th row and continue this process until entire rows and columns are exhausted. Again if the allocation $X_{ij} = b_j$, just reverse the process for $X_{ij} = a_i$.
- Case 2: If the allocation $X_{ij} = a_i = b_j$, find the next smallest cost cell (i, k) from the rest of the cost cells along i -th row and j -th column. Assign a zero in the cell (i, k) and cross out i -th row and j -th column. After that complete the allocation along k -th row/column following the process described in Case-1 to complete the allocations.
- Step 8: Compute the total transportation cost using the original transportation cost matrix and allocations obtained in Step 6 and Step 7.
- Step 9: Finally calculate the total transportation cost from the cost table. This calculation is the sum of the product of cost and corresponding allocated value of the cost table.

4. Numerical Example with Illustration**4.1. Example-1**

A company manufacture television and it has four factories F_1 , F_2 , F_3 and F_4 whose daily production capacities are 30, 25, 20 and 15 pieces of television respectively. The company supplies televisions to its four showrooms located at D_1 , D_2 , D_3 and D_4 whose daily demands are 30, 30, 20 and 10 pieces of television respectively. The transportation costs per piece of televisions are given in the transportation **Table 1**. Find out the schedule of shifting of television from factories to showroom with minimum cost.

4.2. Solution of Example-1

Iteration 1: In the first row 5 is the minimum element, so we subtract 5 from each element of the first row. Similarly, we subtract 3, 3 and 2 from each element of the 2nd, 3rd and 4th row respectively and place all the differences on the right-top of the corresponding elements in **Table 1**.

Iteration 2: In the similar way, we subtract 2, 3, 7 and 3 from each element of the 1st, 2nd, 3rd and 4th column respectively and place the result on the left-bottom of the corresponding elements in **Table 1**.

Iteration 3: The right-top and left-bottom entry of each element of the transportation table obtained in Iteration 1 and Iteration 2 is added and forms the TOCM as in **Table 2**.

Iteration 4: We determine the pointer cost for each row of the TOCM (**Table 3**) by taking the difference between the highest and the lowest cell cost in the respective row and write them in front of the row on the right [e.g. $(14 - 2) = 12$, $(6 - 0) = 6$, $(10 - 1) = 9$ and $(7 - 0) = 7$].

Table 1. Data of the Example-1.

		Showrooms				Production Capacity
		D_1	D_2	D_3	D_4	
Factories	F_1	7	5	9	11	30
	F_2	4	3	8	6	25
	F_3	3	8	10	5	20
	F_4	2	6	7	3	15
Demand		30	30	20	10	

Table 2. Formation of total opportunity cost matrix.

		Showrooms				Production Capacity
		D_1	D_2	D_3	D_4	
Factories	F_1	${}_57^2$	${}_25^0$	${}_29^4$	${}_811^6$	30
	F_2	${}_24^1$	${}_03^0$	${}_18^5$	${}_36^3$	25
	F_3	${}_13^0$	${}_85^5$	${}_310^7$	${}_25^2$	20
	F_4	${}_02^0$	${}_36^4$	${}_07^5$	${}_03^1$	15
Demand		30	30	20	10	

Table 3. Total opportunity cost matrix (TOCM).

		Showrooms				Production Capacity
		D_1	D_2	D_3	D_4	
Factories	F_1	7	2	6	14	30
	F_2	3	0	6	6	25
	F_3	1	10	10	4	20
	F_4	0	7	5	1	15
Demand		30	30	20	10	

Do the same for each column and place them in the bottom of the cost matrix below the corresponding columns [e.g. $(7 - 0) = 7$, $(10 - 0) = 10$, $(10 - 5) = 5$ and $(14 - 1) = 13$].

Iteration 5: In **Table 4**, maximum pointer cost is 13 and minimum transportation cost corresponding to this is 1 in the cell (4, 4). So we allocate $\min(15, 10) = 10$ units to the cell (4, 4). We adjust the production capacity and demand requirements corresponding to the cell (4, 4) and since the demand is satisfied for the cell (4, 4), we crossed out 4th column.

Iteration 6: Now complete the allocation along 4th row by making the allocation in the smallest cost (0) in the cell (4, 1). Here 4th row is exhausted for the allocation $\min(30, 5) = 5$ units at the cell (4, 1).

Iteration 7: In this stage, the allocation along 1st column will be in the smallest cost (1) which is at the cell (3, 1). By making the allocation of $\min(25, 20) =$

20 units in the cell (3, 1) the 3rd row is crossed out but 1st column is yet to exhaust. Then we will go for next smallest cost along the 1st column corresponding to this is 3 in the cell (2, 1). Therefore 1st column is exhausted for the allocation $\min(25, 5) = 5$ units at the cell (2, 1).

Iteration 8: In this case, we allocate $\min(30, 20) = 20$ units along 2nd row in the smallest cost corresponding to this is 0 in the cell (2, 2). Since the production capacity is satisfied by the cells (2, 1) and (2, 2) then we crossed out 2nd row.

Iteration 9: Now complete the allocation along 2nd column by making the allocation in the smallest cost (2) in the cell (1, 2). Here 2nd column is exhausted for the allocation $\min(10, 30) = 10$ units at the cell (1, 2).

Iteration 10: Since only the 1st row is remaining with one unallocated cell (1, 3), so we allocate rest 20 units to the cell (1, 3). Thereby in **Table 5** we can see that all production capacity and demand values are exhausted.

Iteration 11: Now according to algorithm of Step 8, all these allocations are transferred to the original Transportation **Table 1**, which is shown in the **Table 6**. In this table it is observe that the numbers of basic cells are 7 *i.e.* $(4 + 4 - 1)$ which represents the initial basic feasible solution according to the proposed algorithm.

Table 4. Initial basic feasible solution using TOCM-MEDM approach.

		Showrooms				Production Capacity	Row Pointer
		D_1	D_2	D_3	D_4		
Factories	F_1	7	2	6	14	30	(12)
	F_2	3	0	6	6	25	(6)
	F_3	1	10	10	4	20	(9)
	F_4	0	7	5	1	15	(7)
Demand		30	30	20	10		
Column Pointer		(7)	(10)	(5)	(13)		

Table 5. Allocation on TOCM.

		Showrooms				Production Capacity	Row Pointer
		D_1	D_2	D_3	D_4		
Factories	F_1		10	20		30/20	(12)
		7	2	6	14		
	F_2	5	20			25/20	(6)
		3	0	6	6		
F_3	20				20	(9)	
	1	10	10	4			
F_4	5			10		15/5	(7)
	0	7	5	1			
Demand		30/25/5	30/10	20	10		
Column Pointer		(7)	(10)	(5)	(13)		

Table 6. Final allocation on original cost **Table 1**.

	Showrooms				Production Capacity	
	D_1	D_2	D_3	D_4		
Factories	F_1	10	20			30
		7	5	9	11	
	F_2	5	20			25
		4	3	8	6	
F_3	20					20
	3	8	10	5		
F_4	5			10		15
	2	6	7	3		
Demand	30	30	20	10		90

Iteration 12: Finally according to Step 9, for a flow of 90 units, total transportation cost is

$$(10 \times 5) + (20 \times 9) + (5 \times 4) + (20 \times 3) + (20 \times 3) + (5 \times 2) + (10 \times 3) = 410$$

5. Numerical Example without Illustration

5.1. Example-2

Consider the following transportation problem (Transportation Cost **Table 7**) comprising with three sources and five destinations. The cell entries represent the cost of transportation per unit. Obtain an initial basic feasible solution.

5.2. Solution of Example-2

After formulation and allocation on TOCM, we allocate the quantity on original cost table (*i.e.* **Table 7**) which is showed in **Table 8**.

Hence for the flow of 135 units, the total transportation cost is

$$(45 \times 1) + (15 \times 2) + (18 \times 2) + (17 \times 2) + (22 \times 3) + (5 \times 2) + (13 \times 4) = 273$$

5.3. Example-3

Consider that four products are produce in three machines and their per unit production costs are given in the following cost **Table 9**. Obtain an initial basic feasible solution for a suitable production plan which minimizes the total production cost.

5.4. Solution of Example-3

After formulation and allocation on TOCM, we allocate the quantity on original cost table (*i.e.* **Table 9**) which is showed in **Table 10**.

Hence for the flow of 1200 units, the total production cost is

$$(300 \times 1) + (250 \times 2) + (150 \times 5) + (50 \times 3) + (250 \times 3) + (200 \times 2) = 2850$$

Table 7. Data of the Example-2.

		Showrooms					Supply
		D_1	D_2	D_3	D_4	D_5	
Factories	F_1	4	1	2	4	4	60
	F_2	2	3	2	2	2	35
	F_3	3	5	2	4	4	40
Demand		22	45	20	18	30	

Table 8. Final allocation on original cost **Table 7.**

		Showrooms					Supply
		D_1	D_2	D_3	D_4	D_5	
Factories	F_1		45	15			60
	F_2				18	17	35
	F_3	22		5		13	40
Demand		22	45	20	18	30	

Table 9. Data of the Example-3.

		Products				Capacity
		P_1	P_2	P_3	P_4	
Machines	M_1	3	1	7	4	300
	M_2	2	6	5	9	400
	M_3	8	3	3	2	500
		250	350	400	200	

Table 10. Final allocation on original cost **Table 9.**

		Products				Capacity
		P_1	P_2	P_3	P_4	
Machines	M_1		300			300
	M_2	250		150		400
	M_3		50	250	200	500
Demand		250	350	400	200	

6. Result Analysis

The following **Table 11** shows that the obtained result by our proposed TOCM-MEDM approach is compared with the results obtained by other existing methods and also with the optimal solution through the above three examples. The proposed method provides an initial basic feasible solution either optimal or too close to optimal for balanced transportation problem within less iteration, without making the calculation lengthy and time consuming. Comparative study shows that the proposed method gives better result in comparison to the other existing heuristics available in the literature.

7. Conclusions

The objective of the transportation problem is to determine the shipping schedule or supply route that minimizes the total shipping cost while satisfying the demand and supply limit. The proposed method can be one of the solution procedures to select this route.

Here in this article, we have used mainly two methods, the Extremum Difference Method (EDM) and Modified Extremum Difference Method (MEDM). Basically the proposed algorithm is set up by applying MEDM on TOCM. It is observed that the proposed method performs either same or better than EDM and MEDM. The uniqueness of this method is the computational procedure which is easier (less iteration) than the existing VAM, EDM, HCDM as the

Table 11. Comparison of the results.

Method	Total Transportation/Production Cost		
	Ex 1	Ex 2	Ex 3
North West Corner Method	540	363	4400
Row Minimum Method	470	278	2850
Column Minimum Method	435	295	3600
Least Cost Method	435	278	2900
Vogel's Approximation Method	415	273	2850
Extremum Difference Method	415	273	2850
Modified Extremum Difference Method	410	273	2850
Highest Cost Difference Method	415	273	2900
Average Cost Method	455	273	2900
TOCM-MMM Approach	435	278	2900
TOCM-VAM Approach	430	273	2850
TOCM-EDM Approach	415	278	2850
TOCM-HCDM Approach	415	273	2900
TOCM-SUM Approach	455	275	2850
TOCM-MEDM Approach (Proposed)	410	273	2850
Optimum Solution	410	273	2850

penalty is required to find only for the first allocation. But in case of VAM, EDM and HCDM the penalty is needed to bring out for each and every allocation. By using the proposed method we conclude that we obtain an efficient and modified algorithm for finding an initial basic feasible solution which is optimal or close to optimal as compared to existing methods. It will help to calculate cost related to transportation which plays a vital role to minimize cost or maximize profit.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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