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Differential Equation Model of Carbon, Nitrogen and Zinc Components in Growing Tomatoes

Weijie Lin[1], Hailing Xian[1*], Wenwen Wang[2], Caiyun Huang[1], Zhenkun Jiang[1], Xiaoliang Zhou[1]

[1]School of Mathematics and Statistics, Lingnan Normal University, Zhanjiang, China
[2]Zhanjiang Mechanical and Electrical School, Zhanjiang, China
Email: *zxlmath@163.com, *710149570@qq.com

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Abstract

Tomato is a common food on the human table. Up to now, the research on the growth and development model of tomato has been about 50 years. There are many researches on the main nutrients of tomato, such as carbon and nitrogen, but few on the trace element zinc. In this paper, taking plant nutrient C, N and Z_n as variables, the differential equation model of C, N and Z_n in tomato growth and development was established. According to the research of tomato as a whole and divided into root and leaf, the one-compartment and two-compartment models of tomato growth and development were established. The model was analyzed by Matlab program, and the existing experimental data was used to test the numerical simulation results, which proves that the model conforms to the facts.

Keywords

Carbon, Nitrogen and Zinc, Differential Equation Model, Tomato Growth and Development

1. Introduction

The widely cultivated tomato is a common vegetarian dish on people's tables. However, the long-term excessive use of chemical fertilizer has led to many serious problems in the taste and quality of tomato [1]. Therefore, how to improve the quantity and quality of tomato production by precise fertilization is a topic of great research significance.

In recent decades, there have been many achievements on this subject. In 1986, Spitters *et al.* calculated the masses of dry matter of the organs of the tomato based on the dry matter classification method and established a dynamic simulation model of tomato [2]. In 1996, Heuvelink analyzed tomato growth in greenhouses and constructed TOMSIM model based on existing crop models [3]. In 1997, Zhou, *et al.* proposed a growth and development model of tomato by observing the change of dry matter in tomato growth stages when the total solar radiation and temperature were changed in a polyvinyl chloride film covered the test field [4]. In 1998, Zhang put forward a differential equation about the relationship between nitrogen and vegetable yield [5]. In 2012, Wang added the effects of oxygen and phosphorus concentration in the establishment of plant growth model [6]. In 2014, Ji *et al.* studied and established the root growth of tomato model [7]. In 2019, when studying the growth and development models of plants, Li established single compartment, two-compartment and three-compartment models of tomatoes to optimize plant growth [8]. In 2020, Zhang established differential equation model for the requirements proportion of carbon, nitrogen and potassium under the influence of audio frequency [9]. For more relevant works of literature, we may refer to [10] [11].

2. Some Basic Facts and Assumptions about the Subject

2.1. Basic Facts

(F1) The nutrients needed by tomato are mainly provided by the transformation of carbon and nitrogen.

(F2) Leaf organs fix carbon elements in the chloroplast and root organs fix nitrogen elements, and these elements have a fixed proportion in the body of tomato during its growth.

(F3) Carbon is transported from leaf to root, while nitrogen is in reverse.

(F4) In the whole process of plant growth, the amount of carbon fixed is related to the leaf area, while the amount of nitrogen is related to the root surface area, and the root area has a fixed proportion to the leaf area.

(F5) Chloroplasts in the leaves, with the help of light, convert carbon from carbon dioxide into organic matter through photosynthesis. And this organism provides energy for the plant body. Nitrogen absorbed by root organs can be converted into amino acids, which can be converted into proteins in cells. Protein is an important component of tomato structure, and can also participate in the chemical reaction of tomato cells and the composition of cell membrane.

(F6) The main consumption ways of organic matter formed by photosynthesis are:

1) For active transport and other work energy, transport of the two main nutrients above.

2) Conversion energy used for material transformation, such as photosynthesis, dehydration condensation and other material transformation process. It is used to convert small molecules into large molecules for storage.

3) Maintenance energy used to maintain some easily decomposed protein structures, and binding energy consumed during tissue formation.

2.2. Basic Hypotheses

(H1) Plant nutrients are provided only by carbon and nitrogen.

(H2) The use of carbon requires the involvement of proteins or nitrogen.

(H3) In plants, the energy provided by organic matter for macromolecular conversion increases, while the energy for the maintenance of unstable protein structures decreases.

(H4) The ratio of carbon to nitrogen in a tomato is independent of the aging of the tissue, *i.e.*, it is the same in all tissues.

3. One-Compartment Differential Equation Model

The basic idea of one-compartment model is studying tomato as a whole, providing nutrients and required trace elements of tomato by external sunlight, carbon dioxide, nitrogen fertilizer and zinc fertilizer, ignoring the specific function and morphological differences of root and leaf organs and building a simple one-compartment model of tomato growth and development. The idea is shown in **Figure 1**.

At time t, the concentration of carbon, nitrogen and zinc elements in tomato were assumed to be $C(t), N(t)$ and $Z_n(t)$ functions. The nutrient consumption during tomato growth should be a function of $C(t), N(t)$ and $Z_n(t)$, denoted as the tomato nutrient consumption rate function $f(C, N, Z_n)$ which satisfies:

(D1) When any one of the three elements of carbon, nitrogen or zinc fails to meet the requirements of tomato growth, the consumption rate is correspondingly slower.

(D2) When the supply of carbon, nitrogen and zinc exceeds demand, the consumption rate is a definite value that depends only on the variety and heredity of the tomato.

According to hypothesis (H2), the rate of carbon consumption in tomato body is defined as $Vf(C, N, Z_n)$, where V is the volume. According to the hypothesis of (H4), let the ratio of carbon, nitrogen and zinc in tomato be $1:\mu:\gamma$, that is to say, $\mu Vf(C, N, Z_n)$ and $\gamma Vf(C, N, Z_n)$ functions express the rates of nitrogen and zinc consumption respectively in tomato. Based on hypothesis (H3), in order to avoid the infinite growth of tomato, W is expressed as the total mass of tomato, and both a and b are positive numbers. The ratio of binding

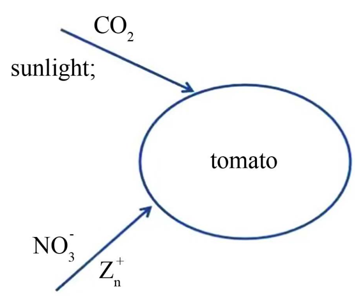

Figure 1. Idea diagram of carbon, nitrogen and zinc model.

energy to total energy in tomato is set as a monotonically decreasing function $R_1 = a - bW$. Then $R_1 V(t) f(C, N, Z_n)$ represents the amount of binding energy in tomato at time *t*. The conversion coefficient of tomato dry tissue from 1kmol carbon to measurable tomato mass (kg) was set as r (the conversion rate of binding energy to mass). Then the differential equation model of tomato growth can be expressed as:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = r(a - bW)V(t) f(C, N, Z_n). \tag{1}$$

According to the relation between volume, mass and density, $V(t) = \frac{W(t)}{\rho}$, Equation (1) can be arranged as

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{r(a - bW)}{\rho} W f(C, N, Z_n) \tag{2}$$

where ρ represents the density of tomato. To meet the conditions (D1) and (D2), $f(C, N, Z_n)$ can be selected a fractional linear form as follows

$$f(C, N, Z_n) = \frac{\alpha CNZ_n}{1 + \beta CNZ_n} \tag{3}$$

where α, β are positive numbers. According to the law of conservation of mass, the three equation models of $C(t), N(t)$ and $Z_n(t)$ are now explored.

The mass of carbon at time ($t + \Delta t$) in the tomato should be equal to the amount of carbon at time t in the tomato adding the input and then minus the output in time interval $[t, t + \Delta t]$, that is,

$$W_C(t + \Delta t) = W_C(t) + R_3 W(t)\Delta t - Vf(C, N, Z_n)\Delta t$$

where $W_C(t) = V(t)C(t)$, R_3 is ratio coefficient of the amount of carbon formed by tomato per unit time to plant mass and $V(t) = \frac{W(t)}{\rho}$. Let $\Delta t \to 0$, then

$$\frac{\mathrm{d}(W_C)}{\mathrm{d}t} = \rho R_3 W(t) - Wf(C, N, Z_n). \tag{4}$$

Similarly, the nitrogen mass of the tomato at time ($t + \Delta t$) should be equal to the nitrogen mass at time t plus the input and then minus the output in $[t, t + \Delta t]$, that is,

$$W_N(t + \Delta t) = W_N(t) + R_5 W(t)\Delta t - \mu Vf(C, N, Z_n)\Delta t$$

where $W_N(t) = V(t)N(t)$, R_5 is ratio coefficient of nitrogen absorbed by the root organ in Δt time and μ is ratio coefficient of nitrogen consumption in Δt time according to (H4), and

$$W_{Z_n}(t + \Delta t) = W_{Z_n}(t) + R_7 W(t)\Delta t - \tau Vf(C, N, Z_n)\Delta t$$

where $W_{Z_n}(t) = V(t)Z_n(t)$, R_7 is ratio coefficient of zinc absorbed by the root organ in Δt time and τ is ratio coefficient of zinc consumption in Δt time according to (H4). Let $\Delta t \to 0$, we get

$$\frac{\mathrm{d}(W_N)}{\mathrm{d}t} = \rho R_5 W(t) - \mu W f(C, N, Z_n), \tag{5}$$

$$\frac{\mathrm{d}(W_{Z_n})}{\mathrm{d}t} = \rho R_7 W(t) - \tau W f(C, N, Z_n). \tag{6}$$

By combining (2), (4), (5) and (6), we get the one-compartment ordinary differential equation model of carbon, nitrogen and zinc content in growing tomato

$$\begin{cases} \dfrac{\mathrm{d}W}{\mathrm{d}t} = \dfrac{r(a-bW)}{\rho} W f(C, N, Z_n), \\ \dfrac{\mathrm{d}(W_C)}{\mathrm{d}t} = \rho R_3 W - W f(C, N, Z_n), \\ \dfrac{\mathrm{d}(W_N)}{\mathrm{d}t} = \rho R_5 W - \mu W f(C, N, Z_n), \\ \dfrac{\mathrm{d}(W_{Z_n})}{\mathrm{d}t} = \rho R_7 W - \tau W f(C, N, Z_n) \end{cases} \tag{7}$$

where $r, \rho, \mu, \tau, R_3, R_5$ and R_7 are all positive constants.

In the thorough experiment on cultivated tomato [12], Song, *et al.* proved that dynamic liquid level method can promote the growth and development of tomato, and obtain the experimental results as in **Table 1**.

In order to make better use of Matlab simulation, we introduce new transformation,

$$y_1(t) = W(t),\ \ y_2(t) = W_C(t),\ \ y_3(t) = W_N(t),\ \ y_4(t) = W_{Z_n}(t).$$

Then, (7) can be transformed into

$$\begin{cases} \dfrac{\mathrm{d}y_1}{\mathrm{d}t} = \dfrac{r(a-by_1)}{\rho} \dfrac{\alpha y_1 y_2 y_3 y_4}{y_1^3 + \beta y_1 y_2 y_3 y_4}, \\ \dfrac{\mathrm{d}y_2}{\mathrm{d}t} = \rho R_3 y_1 - \dfrac{\alpha y_1 y_2 y_3 y_4}{y_1^3 + \beta y_1 y_2 y_3 y_4}, \\ \dfrac{\mathrm{d}y_3}{\mathrm{d}t} = \rho R_5 y_1 - \dfrac{\mu \alpha y_1 y_2 y_3 y_4}{y_1^3 + \beta y_1 y_2 y_3 y_4}, \\ \dfrac{\mathrm{d}y_4}{\mathrm{d}t} = \rho R_7 y_1 - \dfrac{\tau \alpha y_1 y_2 y_3 y_4}{y_1^3 + \beta y_1 y_2 y_3 y_4}. \end{cases} \tag{8}$$

With the measured data of tomato experiment **Table 1**, we choose the initial conditions $y_1(0) = 5,\ y_2(0) = 0.01,\ y_3(0) = 2,\ y_4(0) = 2$, and parameter values in the model (8)

$$a = 200,\ b = 1,\ r = 30,\ \rho = 100,\ \mu = 0.22,\ \tau = 0.29,\ \alpha = 0.08,\ \beta = 1.6,$$
$$R_3 = 0.0002,\ R_5 = 0.00002,\ R_7 = 0.00004.$$

Table 1. Experimental data of tomato growth.

Time	3.19	3.26	4.2	4.9	4.18	4.27
Total dry plant weight	3.86	4.65	8.366	13.284	19.421	39.53
The root dry weight	0.243	0.354	0.783	1.31	1.901	2.770

Numerical solution of model (8) calculated by Matlab is shown in **Figure 2**, which reveals that the tomato after the 15th day enters into rapid growth, at this time tomato requires a large amount of fertilizer and the absorption of nutrients by tomato accelerates accordingly. By comparing the curve with the "o" data and simple computation in **Figure 2**, we get the relative error $\varepsilon = 0.0023$ which shows that the model has a good fitting effect on the measured data. Therefore, model (8) can be used as a simulation model of tomato growth and development.

4. Two-Compartment Differential Equation Model

According to the facts (F1) and (F3), we consider a two-compartment model, that is, the tomato is made up of roots and leaves and the two parts can transport nutrients to each other (see **Figure 3**).

Let $W_s, C_s, N_s, Z_{ns}, W_r, C_r, N_r, Z_{nr}$ represent the mass of leaf, carbon concentration, nitrogen concentration, zinc concentration and the mass of root, carbon concentration, nitrogen concentration, zinc concentration respectively. A two-compartment model with four equations is established by using one-compartment modeling method. The form of tomato nutrient consumption rate function $f(C, N, Z_n)$ selected here is still fractional linear, which is the same as (3) in the previous model. We give the equations of leaf weight W_s and root weight W_r as follows

$$\frac{\mathrm{d}W_s}{\mathrm{d}t} = \frac{r(a - bW_s)W_s}{\rho_s} f(C_s, N_s, Z_{ns}), \tag{9}$$

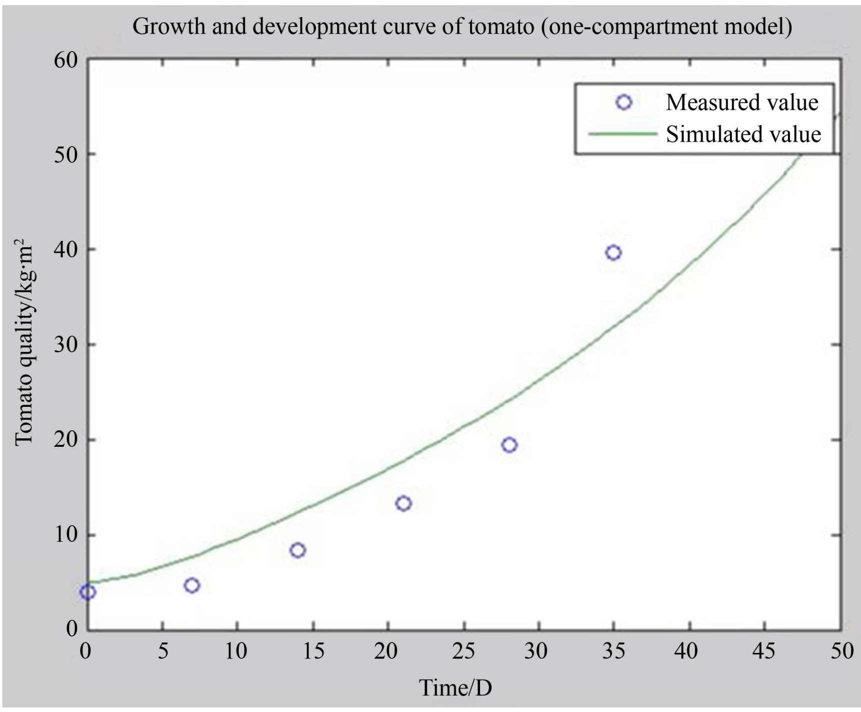

Figure 2. Simulated data and measured values of model (8).

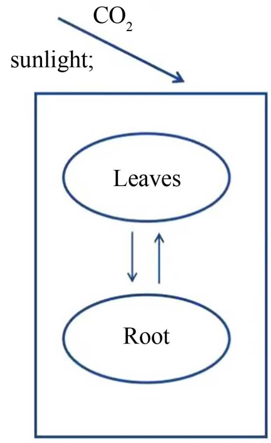

Figure 3. Basic ideas of the two-room model 2.

$$\frac{\mathrm{d}W_r}{\mathrm{d}t}=\frac{r\left(a-bW_r\right)W_r}{\rho_r}f\left(C_r,N_r,Z_{nr}\right). \tag{10}$$

Suppose that the rate of the carbon being transported from leaf to root is proportional to the difference in carbon concentrations between the two parts, and let the ratio be R_2. According to the mass conservation of carbon element, the mass model of carbon element can be obtained,

$$\frac{\mathrm{d}\left(W_{sC_s}\right)}{\mathrm{d}t}=\rho_s R_3 W_s\left(t\right)-\rho_s R_2\left(C_s-C_r\right)-W_s f\left(C_s,N_s,Z_{ns}\right). \tag{11}$$

Assuming the coefficients R_4 and R_6 similarly, the mass equation models of nitrogen and zinc are obtained,

$$\frac{\mathrm{d}\left(W_{rN_r}\right)}{\mathrm{d}t}=\rho_r R_5 W_r\left(t\right)-\rho_r R_4\left(N_r-N_s\right)-\mu W_r f\left(C_r,N_r,Z_{nr}\right). \tag{12}$$

$$\frac{\mathrm{d}\left(W_{rZ_{nr}}\right)}{\mathrm{d}t}=\rho_r R_7 W_r\left(t\right)-\rho_r R_6\left(Z_{nr}-Z_{ns}\right)-\tau W_r f\left(C_r,N_r,Z_{nr}\right). \tag{13}$$

And the model of nitrogen in leaf organs is as follows

$$\frac{\mathrm{d}\left(W_{sN_s}\right)}{\mathrm{d}t}=\rho_s R_4\left(N_r-N_s\right)-\mu W_s f\left(C_s,N_s,Z_{ns}\right). \tag{14}$$

The quality model of zinc in leaf organs is as follows

$$\frac{\mathrm{d}\left(W_{sZ_{ns}}\right)}{\mathrm{d}t}=\rho_s R_6\left(Z_{nr}-Z_{ns}\right)-\tau W_s f\left(C_s,N_s,Z_{ns}\right). \tag{15}$$

The mass model of carbon element in root organs can be obtained from mass conservation

$$\frac{\mathrm{d}\left(W_{rC_r}\right)}{\mathrm{d}t}=\rho_r R_2\left(C_s-C_r\right)-W_r f\left(C_r,N_r,Z_{nr}\right). \tag{16}$$

Combining (11)-(16), we can obtain a two-compartment differential equation model of carbon, nitrogen and zinc components in growing tomatoes

$$
\begin{cases}
\dfrac{\mathrm{d}W_s}{\mathrm{d}t}=\dfrac{r(a_s-bW_s)W_s}{\rho_s}f(C_s,N_s,Z_{ns}),\\
\dfrac{\mathrm{d}W_r}{\mathrm{d}t}=\dfrac{r(a_r-bW_r)W_r}{\rho_r}f(C_r,N_r,Z_{nr}),\\
\dfrac{\mathrm{d}\left(W_{sC_s}\right)}{\mathrm{d}t}=\rho_sR_3W_s(t)-\rho_sR_2(C_s-C_r)-W_sf(C_s,N_s,Z_{ns}),\\
\dfrac{\mathrm{d}\left(W_{rC_r}\right)}{\mathrm{d}t}=\rho_rR_2(C_s-C_r)-W_rf(C_r,N_r,Z_{nr}),\\
\dfrac{\mathrm{d}\left(W_{sN_s}\right)}{\mathrm{d}t}=\rho_sR_4(N_r-N_s)-\mu W_sf(C_s,N_s,Z_{ns}),\\
\dfrac{\mathrm{d}\left(W_{rN_r}\right)}{\mathrm{d}t}=\rho_rR_5W_r(t)-\rho_rR_4(N_r-N_s)-\mu W_rf(C_r,N_r,Z_{nr}),\\
\dfrac{\mathrm{d}\left(W_{sZ_{ns}}\right)}{\mathrm{d}t}=\rho_sR_6(Z_{nr}-Z_{ns})-\tau W_sf(C_s,N_s,Z_{ns}),\\
\dfrac{\mathrm{d}\left(W_{rZ_{nr}}\right)}{\mathrm{d}t}=\rho_rR_7W_r(t)-\rho_rR_6(Z_{nr}-Z_{ns})-\tau W_rf(C_r,N_r,Z_{nr}).
\end{cases}
\tag{17}
$$

Let

$$
\begin{aligned}
&y_1(t)=W_s(t),\ y_3(t)=W_{sC_s}(t),\ y_5(t)=W_{sN_s}(t),\ y_7(t)=W_{sZ_{ns}}(t),\\
&y_2(t)=W_r(t),\ y_4(t)=W_{rC_r}(t),\ y_6(t)=W_{rN_r}(t),\ y_8(t)=W_{rZ_{nr}}(t).
\end{aligned}
$$

Then (17) can be converted into the following ordinary differential equations

$$
\begin{cases}
\dfrac{\mathrm{d}y_1}{\mathrm{d}t}=\dfrac{r(a_s-by_1)}{\rho_s}\cdot\dfrac{\alpha y_1y_3y_5y_7}{y_1^3+\beta y_3y_5y_7},\\
\dfrac{\mathrm{d}y_2}{\mathrm{d}t}=\dfrac{r(a_r-by_2)}{\rho_r}\cdot\dfrac{\alpha y_2y_4y_6y_8}{y_2^3+\beta y_4y_6y_8},\\
\dfrac{\mathrm{d}y_3}{\mathrm{d}t}=\rho_sR_3y_1-\rho_sR_2\left(\dfrac{y_3}{y_1}-\dfrac{y_4}{y_2}\right)-\dfrac{\alpha y_1y_3y_5y_7}{y_1^3+\beta y_3y_5y_7},\\
\dfrac{\mathrm{d}y_4}{\mathrm{d}t}=\rho_rR_2\left(\dfrac{y_3}{y_1}-\dfrac{y_4}{y_2}\right)-\dfrac{\alpha y_2y_4y_6y_8}{y_2^3+\beta y_4y_6y_8},\\
\dfrac{\mathrm{d}y_5}{\mathrm{d}t}=\rho_sR_4\left(\dfrac{y_6}{y_2}-\dfrac{y_5}{y_1}\right)-\mu\dfrac{\alpha y_1y_3y_5y_7}{y_1^3+\beta y_3y_5y_7},\\
\dfrac{\mathrm{d}y_6}{\mathrm{d}t}=\rho_rR_5y_2-\rho_rR_4\left(\dfrac{y_6}{y_2}-\dfrac{y_5}{y_1}\right)-\mu\dfrac{\alpha y_2y_4y_6y_8}{y_2^3+\beta y_4y_6y_8},\\
\dfrac{\mathrm{d}y_7}{\mathrm{d}t}=\rho_sR_6\left(\dfrac{y_8}{y_2}-\dfrac{y_7}{y_1}\right)-\tau\dfrac{\alpha y_1y_3y_5y_7}{y_1^3+\beta y_3y_5y_7},\\
\dfrac{\mathrm{d}y_8}{\mathrm{d}t}=\rho_rR_7y_2-\rho_rR_6\left(\dfrac{y_8}{y_2}-\dfrac{y_7}{y_1}\right)-\tau\dfrac{\alpha y_2y_4y_6y_8}{y_2^3+\beta y_4y_6y_8}.
\end{cases}
\tag{18}
$$

Based on Table 1, for the programming calculation of (18) the initial conditions are taken as follows

$$
\begin{aligned}
&y_1(0)=0.11,\ y_2(0)=0.00243,\ y_3(0)=0.35,\ y_4(0)=0.035,\\
&y_5(0)=0.008,\ y_6(0)=0.0378,\ y_7(0)=0.05,\ y_8(0)=0.058.
\end{aligned}
$$

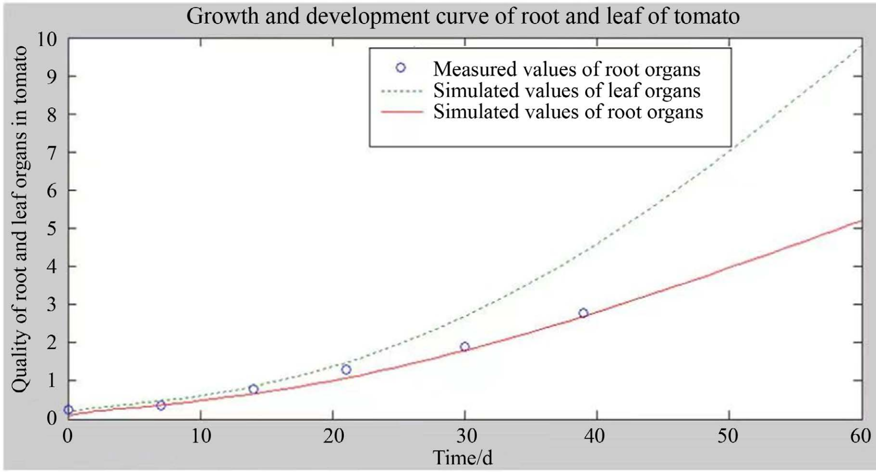

Figure 4. Growth and development curve of tomato two-compartment model.

Take the parameter values

$$a_s = 320,\ a_r = 80,\ b = 1,\ R_2 = R_4 = 0.0003,\ R_6 = 0.0006,$$
$$R_3 = R_5 = 0.002,\ R_7 = 0.00004,\ r = 30,\ \rho_s = \rho_r = 100,$$
$$\mu = 0.22,\ \tau = 0.29,\ \alpha = 0.08,\ \beta = 1.6.$$

By using Matlab program, the results are obtained in **Figure 4**, which show that the mass growth of tomato root and leaf organs is slow at the beginning and then becomes faster and faster. In general, the growth of leaf organs is faster than that of root organs, but there was little difference between them in the early stage. The demarcation point is about 15 days, after which the leaf organs begin to grow faster than the root organs. Compared with one-compartment model, two-compartment model shows not only root growth but also leaf growth. So the simulated data values are more consistent with the facts.

5. Conclusion

In this paper, the growth and development of tomato were studied and verified by modeling the differential equation based on nutrients. Based on reasonable assumptions, the law of tomato growth and development and the law of conservation of element mass, the one-compartment model and two-compartment model were constructed, then Matlab was used to simulate the model and the growth curve was obtained. From model (8) and its numerical solution, *i.e.* **Figure 2**, we know that the tomato after the 15th day enters into rapid growth, at this time tomato requires a large amount of fertilizer and the absorption of nutrients by tomato accelerates accordingly. From model (18) and its numerical solution, *i.e.* **Figure 4**, we know that the mass growth of tomato root and leaf organs is slow at the beginning and then becomes faster and faster. In general, the growth of leaf organs is faster than that of root organs, but there was little difference between them in the early stage. The demarcation point is about 15 days, after which the leaf organs begin to grow faster than the root organs. Through the modeling and analyzing (8) and (18), we have a clear understanding of tomato growth mechanism by using plant nutrients C, N and Zn as variables. This is significant to improve the quantity and quality of tomato production by precise fertilization.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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