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Simulation Analysis of Transponder Power Based on Target Flight Trajectory

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Abstract

The directional angle of the exterior trajectory measurement equipment in the transponder antenna coordinate system is an important basis for interpreting the transponder antenna gain, analyzing the uplink and downlink power of the transponder, and evaluating the measurement and tracking ability of the equipment. The mathematical model established in this paper deduces the direction angle of the exterior trajectory measurement equipment in the transponder antenna coordinate system according to the track information of the flight target, and then obtains the transponder power received by the exterior trajectory measurement equipment combined with the installation position of the transponder, the antenna pattern and the secondary radar formula. It can effectively evaluate the tracking ability of the equipment in measuring segment and adjust the working state of the equipment according to the actual situation. At the same time, it provides a theoretical basis for the ground measurement equipment to receive the transponder power is too low, resulting in the measurement data accuracy is not up to standard, or even lost.

Keywords

Transponder, Coordinate Transformation, Antenna Gain

1. Introduction

In the space TT & C mission, the ground exterior trajectory measurement equipment and the cooperative target carried by aircraft, namely the transponder (also known as secondary radar), trigger and respond to complete the acquisition, tracking and measurement of the target [1]. During the flight, the position and attitude change constantly, and the gain of the transponder antenna fixed on the missile body changes relative to the fixed ground exterior trajectory measurement equipment, which determines whether the exterior trajectory measurement equipment can complete the measurement task in the segment and the accuracy of the measurement data to a certain extent. According to the statistics of a certain unit, the phenomenon of losing the target is the fluctuation of echo signal, which is considered as the influence of the antenna gain of the transponder, that is, the direction diagram of the antenna and its pointing. Especially in the case of new shot, no previous reference experience, theoretically and scientifically obtain the transponder power that exterior trajectory measurement equipment may receive, which to some extent becomes an important condition for analyzing and solving practical problems. In this paper, a corresponding theoretical mathematical model is established to calculate the response signal receiving power of the exterior trajectory measurement equipment in a given segment. In the following chapters, firstly the corresponding coordinate systems are established according to the actual situation, and then the coordinate system conversion formulas are deduced through the geometric relationship. At the same time, the expression of transponder power received by the exterior trajectory measurement equipment is deduced by using the coordinate system conversion formula combined with flight trajectory information, antenna pattern information and secondary radar formula [2]. Next, the validity of the model is verified by using the simulated flight trajectory data. Finally, the simulation result is discussed and analyzed, and a conclusion is drawn.

2. Establishment and Solution of Mathematical Model [3]

2.1. Establish Coordinate System

In order to make a systematic and accurate analysis of the problem to be solved in this paper, it is necessary to establish corresponding coordinate systems in combination with the actual situation, and put the problem in the appropriate coordinate systems for analysis and derivation. Therefore, firstly, the necessary coordinate systems are systematically defined, and then the conversion formulas of the corresponding coordinate systems are deduced according to the geometric relationship.

1) Ground coordinate system: take the exterior trajectory measurement equipment as the origin, the *y*-axis is the direction of the reverse plumb line, and the x-axis is perpendicular to the *y*-axis and points to the true north. The established right-hand system is called the ground coordinate system (also known as the station center coordinate system), which is represented by $O_r - X_r Y_r Z_r$.

2) Body coordinate system: take the projectile body centroid as the origin, the direction from the origin to the warhead is the positive direction of the *x*-axis, and the normal direction of the plane where the trajectory is located in the *y*-axis. The right-hand system is called the body coordinate system, which is represented by $O_m - X_m Y_m Z_m$.

3) Reference coordinate system: take the origin of the body coordinate system as the origin, the *x*, *y*, *z*-axis of reference coordinate system and the *x*, *y*, *z*-axis of

ground coordinate system are established respectively, which is called the reference coordinate system and represented by $O-UVW$.

4) Transponder coordinate system: the origin and *x*-axis are defined the same as body coordinate system, the direction of the main response axis of the transponder antenna as the positive direction of the *y*-axis. The established right-hand system is called the transponder coordinate system and is represented by $O_b - X_b Y_b Z_b$.

According to the above definition, $O_r - X_r Y_r Z_r$ and $O-UVW$ are translation relations, $O-UVW$ and $O_m - X_m Y_m Z_m$ are rotation relations, $O_r - X_r Y_r Z_r$ and $O_m - X_m Y_m Z_m$ are rotation relations after translation, $O_m - X_m Y_m Z_m$ and $O_b - X_b Y_b Z_b$ are rotation relations. Then, the formulas of translation transformation and rotation transformation of coordinate systems are derived respectively (See **Figure 1** & **Figure 2**).

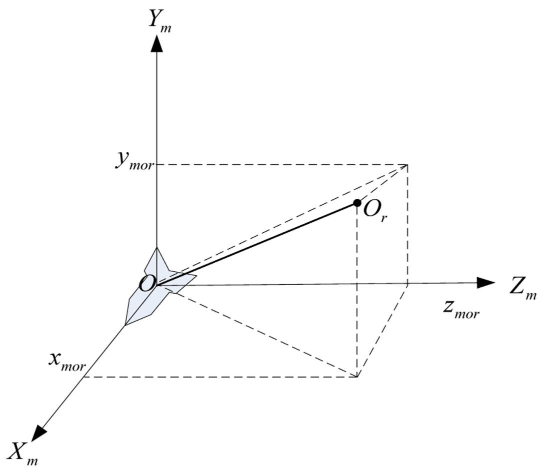


Figure 1. Schematic diagram of body coordinate system.

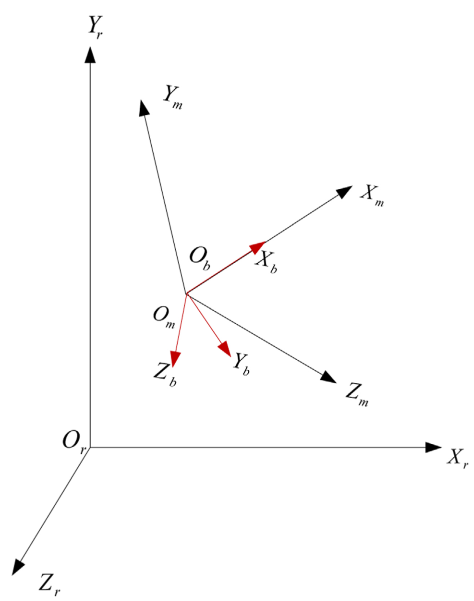


Figure 2. Schematic diagram of coordinate systems relationship.

2.2. Coordinate System Transformation

1) Translation transformation: without losing generality, take the two-dimensional case as shown in Figure 3(a), (x, y) is the original coordinate, (u, v) is the coordinate after translation, and (a, b) is the amount of translation. According to the geometric relationship:

$$\begin{cases} u = x - a \\ v = y - b \end{cases} \tag{1}$$

Similarly, for the three-dimensional case, if the original coordinate is (x, y, z), the translated coordinate is (u, v, w), and the amount of translation is (a, b, c), the following relationship can be obtained:

$$\begin{cases} u = x - a \\ v = y - b \\ w = z - c \end{cases} \tag{2}$$

Written in matrix form:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{3}$$

2) Rotation transformation: the rotation transformation of any coordinate system can be decomposed into the following three simple rotation transformations: the rotation of the x-axis in the y-o-z plane, the rotation of the y-axis in the x-o-z plane, and the rotation of the z-axis in the x-o-y plane. Without losing generality, the expression of coordinate rotation transformation is derived below for the rotation of the x-axis in the y-o-z plane. As shown in Figure 3(b), Let (u, v, w) be the coordinate of any point P before rotation, and (x, y, z) be the coordinate after rotation, α is the counterclockwise rotation angle. According to the geometric relationship:

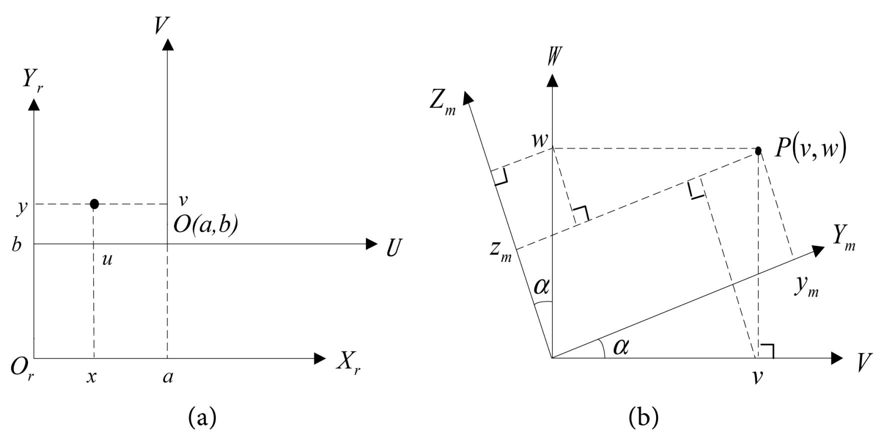


Figure 3. Schematic diagram of translation and rotation of coordinate systems. (a) Translation; (b) Rotation.

$$\begin{cases} x = u \\ y = v\cos(\alpha) + w\sin(\alpha) \\ z = -v\sin(\alpha) + w\cos(\alpha) \end{cases} \tag{4}$$

Written in matrix form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{5}$$

2.3. Model Derivation

In this section, the mathematical model is established to obtain the transponder power at the exterior trajectory measurement equipment by using the flight trajectory information obtained from the exterior trajectory measurement equipment. The derivation of the model is generally divided into the following five steps: 1) Convert the spherical coordinate flight trajectory data obtained by the external measuring equipment into rectangular coordinate data. 2) Convert the flight trajectory data of rectangular coordinates into the body coordinate system. 3) Convert the body coordinate system data into the transponder coordinate system, and obtain the elevation and azimuth. 4) Figure up the antenna gain by introducing the antenna pattern. 5) The flight trajectory data and antenna pattern gain are brought into the radar formula to obtain the received power of the exterior trajectory measurement equipment.

1) Flight trajectory spherical coordinate to rectangular coordinate

Let the spherical coordinate of the projectile in the $O_r - X_r Y_r Z_r$ at a certain time be $(\rho_{rom}, \phi_{rom}, \theta_{rom})$, then its rectangular coordinate $(x_{rom}, y_{rom}, z_{rom})$ is:

$$\begin{cases} x_{rom} = \rho_{rom} \sin\theta_{rom} \cos\phi_{rom} \\ y_{rom} = \rho_{rom} \sin\theta_{rom} \sin\phi_{rom} \\ z_{rom} = \rho_{rom} \cos\theta_{rom} \end{cases} \tag{6}$$

2) Ground coordinate system to body coordinate system

Set the exterior trajectory measurement equipment in the $O_m - X_m Y_m Z_m$ is $(x_{mor}, y_{mor}, z_{mor})$, and the angles of rotation along the *x*, *y*, and *z* axes from the $O-UVW$ coordinate system to the $O_m - X_m Y_m Z_m$ coordinate system are α, β, γ. According to the previous definition, the translation of $O-UVW$ relative to the $O_r - X_r Y_r Z_r$ coordinate system is $(x_{rom}, y_{rom}, z_{rom})$. Because the coordinate of the exterior trajectory measurement equipment in the $O_r - X_r Y_r Z_r$ coordinate system is (0, 0, 0), the expression obtained according to the translation and rotation formula of the coordinate system is:

$$\begin{bmatrix} x_{mor} \\ y_{mor} \\ z_{mor} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \times \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 - x_{rom} \\ 0 - y_{rom} \\ 0 - z_{rom} \end{bmatrix} \tag{7}$$

Substitute the Formula (6):

$$\begin{bmatrix} x_{mor} \\ y_{mor} \\ z_{mor} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \times \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\rho_{rom}\sin\theta_{rom}\cos\phi_{rom} \\ -\rho_{rom}\sin\theta_{rom}\sin\phi_{rom} \\ -\rho_{rom}\cos\theta_{rom} \end{bmatrix} \tag{8}$$

3) Body coordinate system is converted to transponder coordinate system to gain elevation and azimuth

According to the definition of the coordinate system, the relationship between $O_b - X_bY_bZ_b$ and $O_m - X_mY_mZ_m$ is a rotation transformation along the *x*-axis. Let the rotation angle be α_{res} and the coordinate of the exterior trajectory measurement equipment in $O_b - X_bY_bZ_b$ be $(x_{bor}, y_{bor}, z_{bor})$. According to the rotation formula of coordinate system:

$$\begin{bmatrix} x_{bor} \\ y_{bor} \\ z_{bor} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_{res} & \sin\alpha_{res} \\ 0 & -\sin\alpha_{res} & \cos\alpha_{res} \end{bmatrix} \begin{bmatrix} x_{mor} \\ y_{mor} \\ z_{mor} \end{bmatrix} \tag{9}$$

Substitute the Formula (8):

$$\begin{bmatrix} x_{bor} \\ y_{bor} \\ z_{bor} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_{res} & \sin\alpha_{res} \\ 0 & -\sin\alpha_{res} & \cos\alpha_{res} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \times \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\rho_{rom}\sin\theta_{rom}\cos\phi_{rom} \\ -\rho_{rom}\sin\theta_{rom}\sin\phi_{rom} \\ -\rho_{rom}\cos\theta_{rom} \end{bmatrix} \tag{10}$$

According to the geometric relationship, the elevation angle θ_{bor} and azimuth angle ϕ_{bor} of the exterior trajectory measurement equipment in the $O_b - X_bY_bZ_b$ can be expressed as:

$$\begin{cases} \theta_{bor} = \arctan\left(\dfrac{\sqrt{x_{bor}^2 + z_{bor}^2}}{y_{bor}}\right) \\ \phi_{bor} = \arctan\dfrac{z_{bor}}{x_{bor}} \end{cases} \tag{11}$$

4) Calculate the antenna gain of transponder

By substituting (11) into the expression of transponder antenna pattern, the transponder antenna gain G is:

$$G = B(\theta_{bor}, \phi_{bor}) = B\left(\arctan\left(\frac{\sqrt{x_{bor}^2 + z_{bor}^2}}{y_{bor}}\right), \arctan\frac{z_{bor}}{x_{bor}}\right) \tag{12}$$

5) Obtain the received power of exterior trajectory measurement equipment by the formula of radar

Received power of exterior trajectory measurement equipment can be obtained by substituting flight trajectory data and transponder antenna gain into the secondary radar formula:

$$S_{ri} = \frac{P_{res} G A_e}{4\pi R^2} = \frac{P_{res} B\left(\arctan\left(\frac{\sqrt{x_{bor}^2 + z_{bor}^2}}{y_{bor}}\right), \arctan\frac{z_{bor}}{x_{bor}}\right) A_e}{4\pi\left(x_{rom}^2 + y_{rom}^2 + z_{rom}^2\right)} \tag{13}$$

Therein, A_e is effective aperture of antenna of exterior trajectory measurement equipment, P_{res} is transmitting power of the transponder, R is radial distance between transponder and exterior trajectory measurement equipment.

3. Simulation Verification

Assume at will and generate all flight trajectory data including position and attitude angle as shown in Figure 4.

3.1. Model Validation

In the simulation process of this paper, due to the direct generation of data in rectangular coordinate system, there is no conversion process in Formula (6). Bring the flight trajectory data into Formula (7) to obtain the trajectory of the exterior trajectory measurement equipment in the $O_m - X_m Y_m Z_m$ coordinate system, as shown in Figure 5(a).

According to common conditions, let the transponder antenna angle be $\alpha_{res} = -1.5184(\text{rad})$ substituted into Formula (10) to obtain the trajectory of the exterior trajectory measurement equipment in the $O_b - X_b Y_b Z_b$ coordinate system, as shown in Figure 5(b).

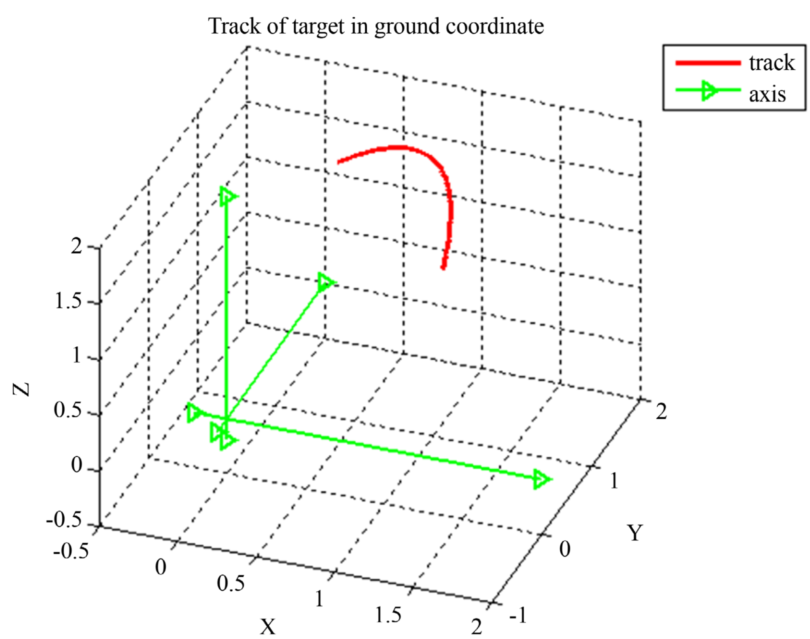
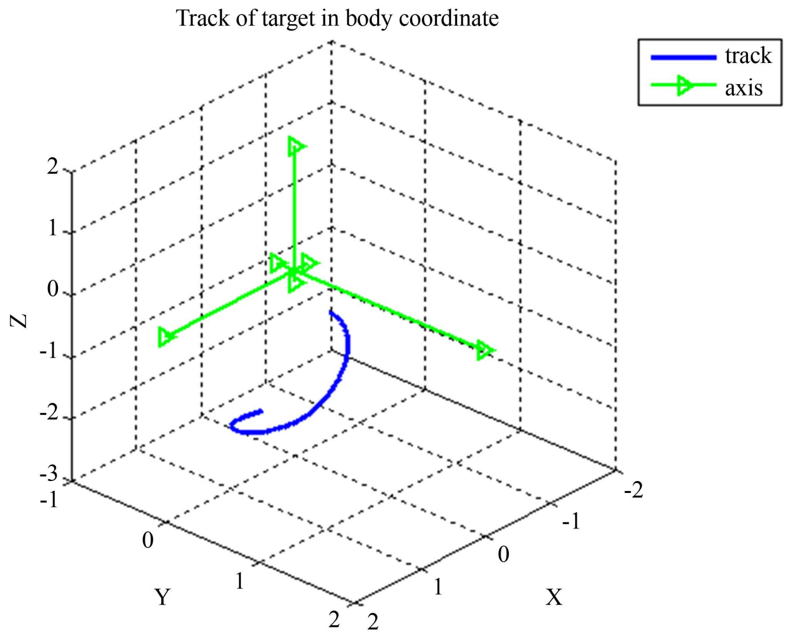
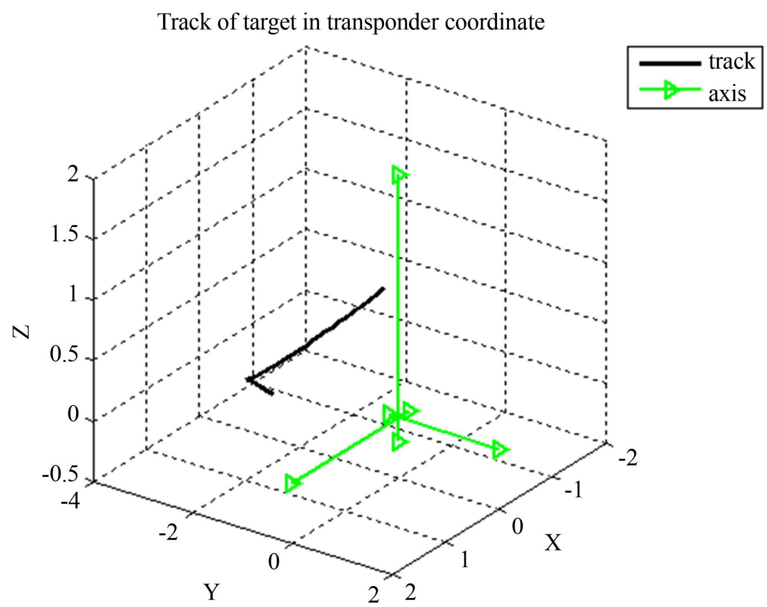


Figure 4. Target flight trajectory.



(a)

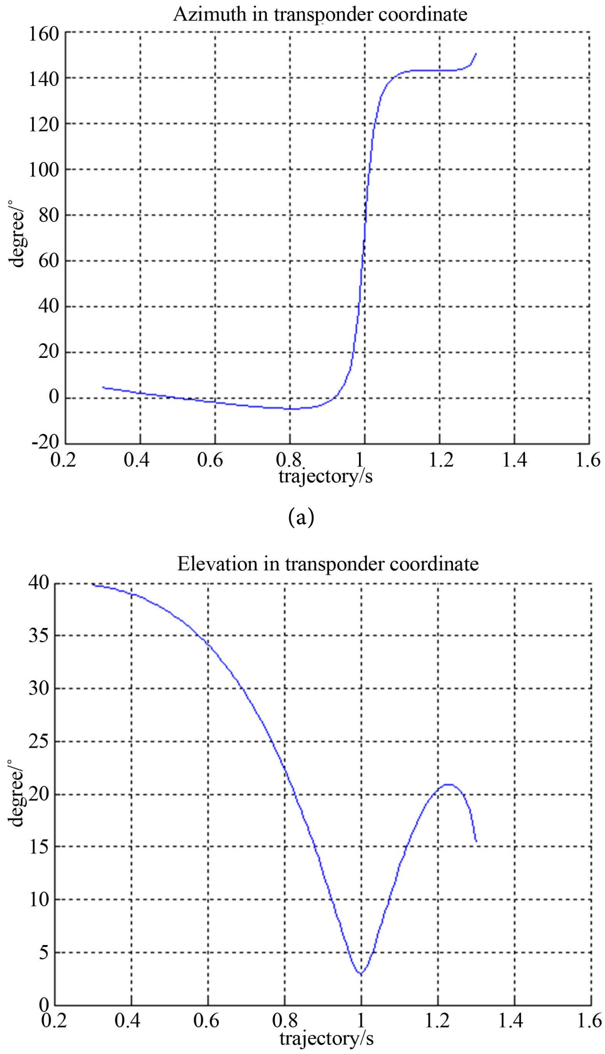


(b)

Figure 5. Target trajectory. (a) Target in body coordinate system; (b) Target in transponder coordinate system.

Call Formula (11) to obtain the elevation and azimuth of the equipment in the $O_b - X_b Y_b Z_b$, as shown in the following **Figure 6**.

It can be seen from **Figure 6** that within all flight trajectories of the target, the variation range of elevation is large. In some measuring segments, the exterior trajectory measurement equipment will leave the main lobe of the transponder antenna, resulting in very low reception or failure to receive the response power.



(a)

(b)

Figure 6. Azimuth and elevation of all flight trajectories. (a) Azimuth (b) Elevation.

In addition, the exterior trajectory measurement equipment practically only observes part of the flight trajectory, so only a given measurement segment in the flight trajectory is analyzed subsequently. As shown in **Figure 7**, the measuring segment marked in blue is the subsequent simulation measuring segment.

Bring the measuring segment data into the above model to obtain the elevation and azimuth of the exterior trajectory measurement equipment in the $O_b - X_b Y_b Z_b$, as shown in the following **Figure 8**.

Assuming the antenna of transponder is a uniformly weighted 7*7 array antenna, its pattern is shown in **Figure 9**. According to Formula (12), the change of transponder antenna gain is shown in **Figure 9**.

Bring the antenna gain and flight trajectory information into Formula (13) to obtain the response power change curve received by the equipment in the measuring segment, as shown in **Figure 10**.

Convert to received power: (**Figure 11**).

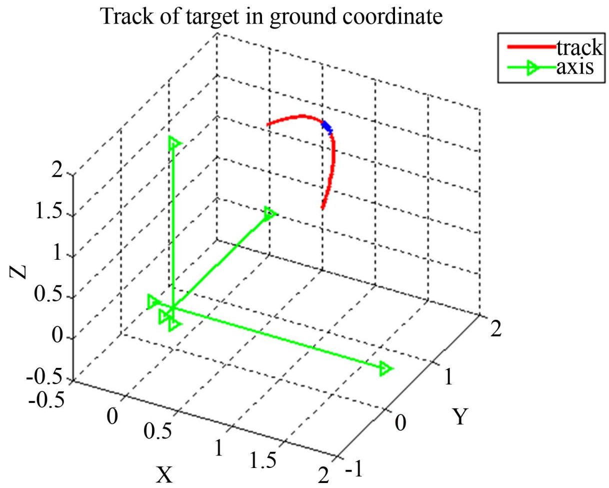
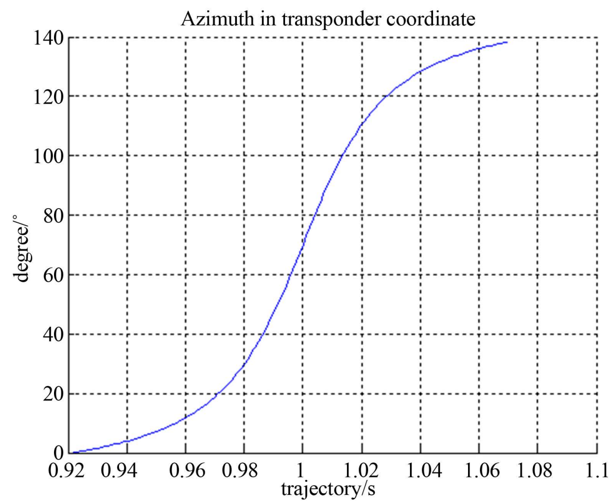
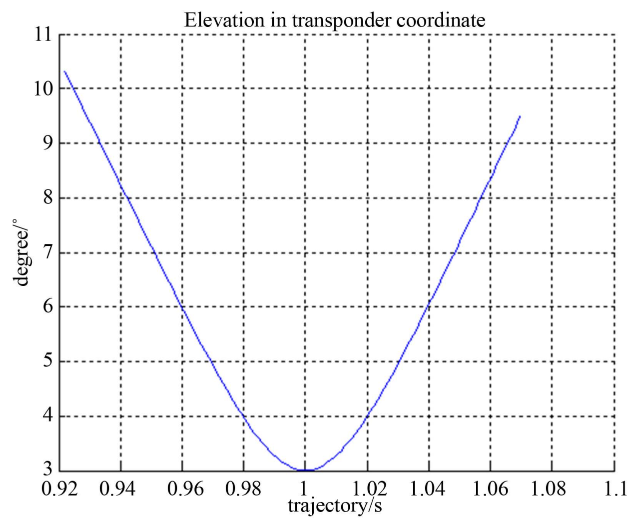


Figure 7. Measuring segment.

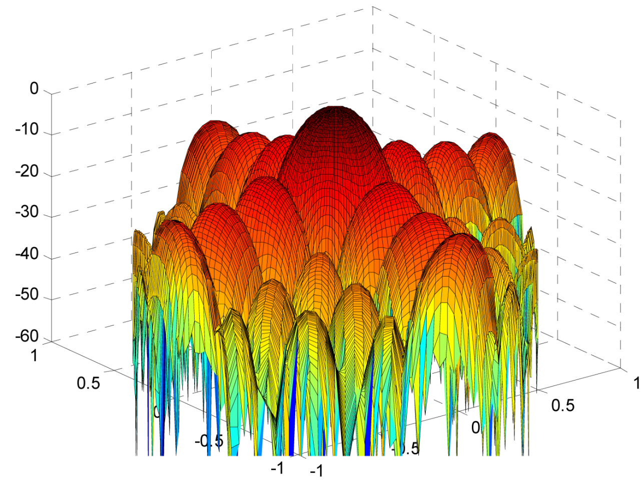


(a)

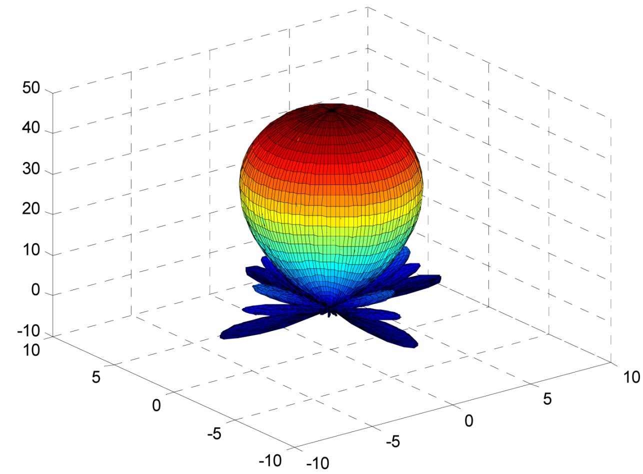


(b)

Figure 8. Azimuth and elevation of measuing segment. (a) Azimuth; (b) Elevation.



(a)



(b)

Figure 9. Transponder's antenna pattern. (a) Directional cosine coordinate system; (b) Spherical coordinate system.

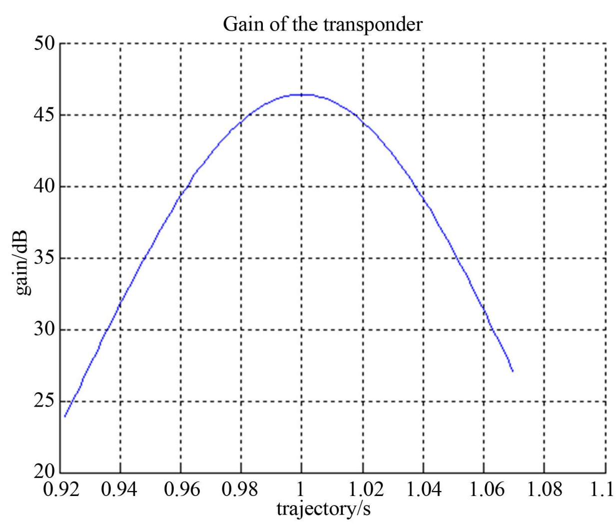


Figure 10. Gain change of transponder in measuring segment.

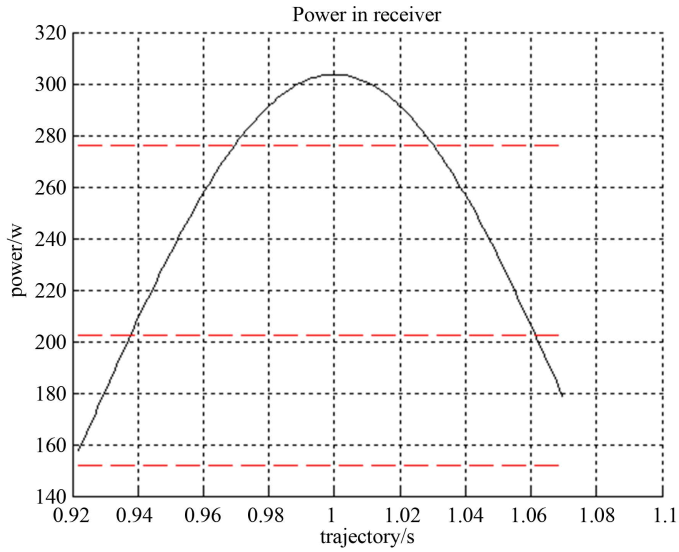


Figure 11. Received power of exterior trajectory measurement equipment in measuring segment.

3.2. Result Analysis

According to the simulation results can be seen that, in the process of flying projectile target position and attitude change constantly, the direction of the transponder antenna mounted on the body changes accordingly, resulting in changes in the receiving power of the exterior trajectory measurement equipment. When the transponder antenna is in an unfavorable position relative to the exterior trajectory measurement equipment, so that the receiving power is lower than the receiving sensitivity, the measurement target will be lost. Therefore, it is necessary to comprehensively consider the flight track and the position and capability of the station on the route. Reasonable arrangement of station relay measurement, selection of adequate power transponder, and installation in an appropriate position are important guarantees for effective control of target flight information.

4. Conclusion

The mathematical model established in this paper, through the derived expression of the received power of the exterior trajectory measurement equipment, brings into the actual exterior trajectory measurement equipment data (including missile target flight trajectory information, attitude change, transponder antenna pattern, transponder power, exterior trajectory measurement equipment antenna effective aperture, etc.), which can guide the equipment status setting state and personnel operation prediction to a certain extent. At the same time, master and understand the respondent signal quality in the measuring segment of the certain equipment. If the influence of rocket tail flame, propagation medium, environment and other practical factors are added, the model can be further optimized and improved.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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