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Moments of Inertia, Magnetic Dipole Moments, and Electric Quadrupole Moments of the Lithium Isotopes

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Abstract

The single-particle Schrödinger fluid model is designed mainly to calculate the moments of inertia of the axially symmetric deformed nuclei by assuming that each nucleon in the nucleus is moving in a single-particle potential which is deformed with time *t*, through its parametric dependence on a classical shape variable $\alpha(t)$. Also, the Nilsson model is designed for the calculations of the single-particle energy levels, the magnetic dipole moments, and the electric quadrupole moments of axially symmetric deformed nuclei by assuming that all the nucleons are moving in the field of an anisotropic oscillator potential. On the other hand, the nuclear superfluidity model is designed for the calculations of the nuclear moments of inertia and the electric quadrupole moments of deformed nuclei which have no axes of symmetry by assuming that the nucleons are moving in a quadruple deformed potential. Furthermore, the cranked Nilsson model is designed for the calculations of the total nuclear energy and the quadrupole moments of deformed nuclei which have no axes of symmetry by modifying the Nilsson potential to include second and fourth order oscillations. Accordingly, to investigate whether the six p-shell isotopes ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li, and ^{11}Li have axes of symmetry or not, we applied the four mentioned models to each nucleus by calculating their moments of inertia, their magnetic dipole moments, and their electric quadrupole moments by varying the deformation parameter β and the non-axiality parameter γ in wide ranges of values for this reason. Hence for the assumption that these isotopes are deformed and have axes of symmetry, we applied the single-particle Schrödinger fluid model and the Nilsson model. On the other hand, for the assumption that these isotopes are deformed and have no axes of symmetry, we applied the cranked Nilsson model and the nuclear super fluidity model. As a result of our calculations, we can conclude that the nucleus ^{6}Li may be assumed to be deformed and has an axis of symmetry.

Keywords

Single-Particle Schrödinger Fluid Model, Nilsson Model, Cranked Nilsson Model, Nuclear Superfluidity Model, Moments of Inertia, Magnetic Dipole Moments, Electric Quadrupole Moments

1. Introduction

In the shell model, there is a core made up of paired nucleons. This core may be spherically symmetric in which case it gives rise to the spherical symmetric of the independent particle model or axially symmetric, as in the Nilsson model [1] which also be referred to as the deformed independent particle model. The analysis of nuclear spectra within the framework of the shell model is feasible only for relatively few nuclei: those which are close to magic numbers. The shell model has a limited range of useful applicability, covering nuclei in the vicinity of closed shells only. The complexity of nuclear spectra increases very rapidly as we go farther and farther away from closed shell [2]. The basic ideas concerning non spherical nuclei have been most completely described by A. Bohr and B. Mottelson [2]. A non-spherical nucleus is characterized by the moment of inertia about the axis perpendicular to the symmetry axis of the nucleus, its magnetic dipole moment, and its electric quadrupole moment. The elongation of the nucleus is related to the interaction between the surface and the nucleons outside closed shells.

A description of deformed nuclei has been given by a model proposed and developed by A. Bohr and B. Mottelson [2]. The success of the independent-particle approximation for spherical nuclei near closed shells naturally suggests adopting a similar procedure for deformed nuclei. Thus, as a first guess for the deformed nucleus internal wave function, one must take an independent-particle wave function, generated from a deformed potential. One of the most successful models for generating realistic intrinsic single particle states of deformed potentials is that first proposed by Nilsson [1]. This model was limited to nuclei with axially symmetric quadruple deformations. Positive values of the deformation parameter correspond to prolate deformation and negative values to oblate deformation. The success of the description of many nuclei by means of deformed potential can be taken as an indication that by distorting a spherical potential in this manner we automatically obtain the right combination of spherical eigenfunctions that makes the corresponding Slater determinant a better approximation to the real nuclear wave function [2]. From this point of view, the deformed potential is a definite prescription for a convenient mixing of various configurations of the spherical potential. Considerable evidence has accumulated for rotational structure for the p and s-d shell nuclei. The absolute values of the rotational energies or equivalently the moments of inertia require a knowledge of the fine details of the intrinsic nuclear structure. Consequently, the investigation of the nuclear moments of inertia is a sensitive check for the validity of the nuclear structure theories [3]-[11].

The investigation of the different characteristics of the deformed nuclei, whether having axes of symmetry or not, is a very interesting subject in the theory of nuclear structure. Many authors have used several models to deal with this problem [12]-[19]. A common feature of systems that have rotational spectra is the existence of a deformation, by which is implied a feature of anisotropy that makes it possible to specify an orientation of the system. The moments of inertia require knowledge of the fine details of the intrinsic nuclear structure.

The large quadrupole moments observed in some nuclei, which do not belong to closed shells, implied a collective deformation and thereby a rotational degree of freedom. The most central parameter of collective rotation is the quadrupole moment and the moment of inertia of deformed nuclei [4] [13] [14]. The study of the velocity fields for the rotational motion of the axially symmetric deformed nuclei led to the formulation of the Schrödinger fluid [4] [5] [6] [7] [8] [14] [15]. Since the Schrödinger-fluid theory is an independent particle model, the cranking model approximation for the velocity fields and the moments of inertia play the dominant role in this theory.

The single particle Schrödinger fluid is one of the very interesting models which is created directly from the time-dependent Schrödinger wave equation by a suitably chosen type of complex wave functions [14] [15]. The single particle Schrödinger fluid is a concept which is used to describe the collective motions of the nucleons in an axially symmetric deformed nucleus. This concept can be applied to study the rotational motion of a deformed nucleus. This model makes it possible to formulate the well-known equation of continuity, Euler's equation, and Navier-Stokes equations of fluid mechanics [20] as results from the separation of the real and the imaginary parts of the time-dependent Schrödinger wave equation. The single-particle potential that represents the residual interparticle interaction inside the nucleus is taken in the form of a three-dimensional anisotropic oscillator.

Until now, the best description of the nuclear moments of inertia can be obtained within the framework of the so-called nuclear superfluidity model [16] [17] [18] [19]. The method adopted for the calculation of the eigenvalues and the eigenfunctions of the non-axial Hamiltonian makes it therefore possible to use the Belyaev formula for the determination of the nuclear moments of inertia.

It is well known that the nucleons inside the nucleus occupy approximately 1/50 of the volume of the nucleus. It is not surprising to find that nucleon properties are maintained inside the nucleus. In particular, this situation is responsible for the fact that the magnetic dipole moments of nucleons inside nuclei are the same as for free nucleons. In accordance with the above, we describe the motion of each nucleon individually in a common field. Because the surface of deformed nuclei is distorted at some moment, the potential felt by the nucleons is not spherically symmetric [18] [19].

The next step was to include the effect of deformation on the single particle motion. Hartree-Fock calculations revealed that ground states of alpha-like nuclei in the p-shell are axially deformed [2]. The Nilsson model was limited to nuclei with axially symmetric quadrupole deformations, where the deformation is measured by the deformation parameter β.

In treating the internal motion in the nucleus, it is assumed that the individual nucleons move independently in a certain fixed non-spherical field of the nucleus. The Hamiltonian of the internal motion can then be represented, as in the ordinary model, in the form of a sum of one-particle Hamiltonians. According to Nilsson's model [1] the nucleons inside the nucleus are moving independently in an averaging field in the form of anisotropic oscillator, with $\omega_x = \omega_y \neq \omega_z$, added to it a spin-orbit term and a term proportional to the square of the orbital angular momentum of the nucleon. The nucleon energy eigenvalues and eigenfunctions are then obtained by solving the time-independent Schrödinger wave equation in spherical polar coordinates and applying the method of diagonalizing the matrices [9].

Because the surface is distorted at some moment, the potential felt by particles is not spherically symmetric, the particles will move in orbits appropriate to an aspherical shell-model potential. In the case of deformed nuclei, the theoretical question to be settled first is whether the nucleus has an axis of symmetry. Historically, several applications of the theory were made [5] [9] [11] on the assumption that the deformed nucleus does have such an axis of symmetry. Most of the work in the heavier nuclei is made on this assumption. In a field with axial symmetry, only the component of the angular momentum along the axis of symmetry is conserved. Historically, much after the application of the axially symmetric-rotor model, a systematic attempt was made by several authors, especially Davydov and his collaborators [12] [19] to check the consequences of the general rotor Hamiltonian that has no axis of symmetry (usually called an asymmetric rotor). The total motion of the nucleons is thus composed of two parts: an internal motion with respect to the body-fixed reference frame, described by an internal wave function, and the motion of the body-fixed reference frame itself. The nuclear superfluidity model [17] [18] [19] and the cranked Nilsson model [13] provide us with powerful methods for calculating the nuclear moment of inertia, the electric quadrupole moment, and other characteristics of deformed nuclei which have no axes of symmetry.

Accordingly, it is important to know whether the nucleus is deformed and has an axis of symmetry or not in order that we can choose the suitable model which produces correct values for its deformation characteristics, such as its moment of inertia, magnetic dipole moment, and electric quadrupole moment, which are in good agreement with the corresponding well-known experimental values.

In this paper, we carry out the derivations of the four models: the Nilsson model, the single-particle Schrödinger fluid model, the Cranked Nilsson model, and the nuclear superfluidity model, which are different in their structures and therefore in their applications, and accordingly clarify how the moments of inertia, the magnetic dipole moment, and the electric quadrupole moment of a deformed nucleus can be obtained in framework of these models. Our choice of the six lithium isotopes; ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li, and ^{11}Li, is due to the fact that their complete deformation structures are not yet completely known. Accordingly, we applied the above mentioned four models to the six lithium isotopes in order to calculate their moments of inertia, their magnetic dipole moments and their electric quadrupole moments. The variations of these moments with respect to the deformation parameter β, which describes the deviation from the spherical case, the non-axiality parameter γ, which shows that the nucleus does not have an axis of symmetry, and the non-deformed oscillator parameter $\hbar\omega_0^0$, are also given in this paper.

2. The Nilsson Model

2.1. Formulation of the Model

In the Nilsson model [1], all nucleons are assumed to move in the field of the following potential:

$$V = \frac{M}{2}\left(\omega_x^2 x'^2 + \omega_y^2 y'^2 + \omega_z^2 z'^2\right) + C\boldsymbol{\ell}\cdot\boldsymbol{s} + D\boldsymbol{\ell}^2 \tag{2.1}$$

where:

$$\omega_x^2 = \omega_y^2 = \omega_0^2\left(1 + \frac{2}{3}\delta\right), \tag{2.2}$$

$$\omega_z^2 = \omega_0^2\left(1 - \frac{4}{3}\delta\right) \tag{2.3}$$

In Equation (2.1), M is the nucleon mass, $\boldsymbol{\ell}$ is its orbital angular momentum vector, and $\boldsymbol{s}$ is its spin vector. Hence, the single particle Hamiltonian in the Nilsson model is given by:

$$H = H^{(0)} + C\boldsymbol{\ell}\cdot\boldsymbol{s} + D\boldsymbol{\ell}^2 \tag{2.4}$$

where:

$$H^{(0)} = -\frac{\hbar^2}{2M}\nabla'^2 + \frac{M}{2}\left(\omega_x^2 x'^2 + \omega_y^2 y'^2 + \omega_z^2 z'^2\right), \tag{2.5}$$

In Equations (2.2) and (2.3) ω_0 is given by

$$\omega_0^2 = \omega_0^2(\delta) = \omega_0^0\left[1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right]^{-\frac{1}{6}} \tag{2.6}$$

where δ is the deformation parameter. The parameter δ is related to well-known deformation parameter β by the relation:

$$\delta = \frac{3}{2}\sqrt{\frac{5}{4\pi}}\beta \cong 0.95\beta \tag{2.7}$$

Transforming to the dimensionless variables

$$x=\sqrt{\frac{M\omega_0}{\hbar}}x';\ y=\sqrt{\frac{M\omega_0}{\hbar}}y';\ z=\sqrt{\frac{M\omega_0}{\hbar}}z' \tag{2.8}$$

we get:

$$H^{(0)}=-\frac{\hbar\omega_0}{2}\nabla^2+\frac{\hbar\omega_0}{2}r^2\left[1+\frac{2}{3}\delta\left(1-3\cos^2\theta\right)\right] \tag{2.9}$$

We rewrite Equation (2.9) as follows:

$$H^{(0)}=H_0^{(0)}+H_\delta^{(0)} \tag{2.10}$$

where:

$$H_0^{(0)}=-\frac{\hbar\omega_0}{2}\left[\nabla^2-r^2\right], \tag{2.11}$$

and

$$H_\delta^{(0)}=-\frac{\hbar\omega_0}{3}\delta r^2\left(1-3\cos^2\theta\right)=-\frac{1}{3}\hbar\omega_0\delta r^2\sqrt{\frac{16\pi}{5}}Y_{2,0}\left(0,\phi\right). \tag{2.12}$$

Finally, we have

$$H=H_0^{(0)}+H_\delta^{(0)}+C\boldsymbol{\ell}\cdot\boldsymbol{s}+D\boldsymbol{\ell}^2 \tag{2.13}$$

Schrödinger wave equation for the unperturbed Hamiltonian $H_0^{(0)}$ is given by:

$$H_0^{(0)}\psi_N^o=E_N^{(0)}\psi_N^0, \tag{2.14}$$

with solutions [4] [10]

$$\psi_N^0=\left|N\ell\Lambda\right\rangle=R_{N\ell}\left(r\right)Y_{\ell,\Lambda}\left(\theta,\phi\right), \tag{2.15}$$

$$E_N^{(0)}=\left(N+\frac{3}{2}\right)\hbar\omega_0 \tag{2.16}$$

where:

$$N=0,1,2,\cdots,\quad \ell=N,N-2,N-4,\cdots=\begin{cases}0 & N\text{ is even}\\ 1 & N\text{ is odd}\end{cases},$$
$$\Lambda=-\ell,-\ell+1,\cdots,0,1,\cdots,\ell-1,\ell \tag{2.17}$$

From the state $\left|N\ell\Lambda\right\rangle$ we construct the function $\left|N\ell\Lambda\Sigma\right\rangle$, as usual, in the form $\left|N\ell\Lambda\Sigma\right\rangle=R_{Nl}\left(r\right)Y_{l\Lambda}\left(\theta,\phi\right)\chi_{s,\Sigma}$. For Fermions, like nucleons, $s=\frac{1}{2},\Sigma=\pm\frac{1}{2}$. The nuclear state has definite values of the parity π, even or odd value of ℓ, and Ω, where:

$$\Omega=\Lambda+\Sigma \tag{2.18}$$

For the ground state, $m_j=\Omega=j$, which is positive so that Ω is not a negative value. Hence, the least possible value of this quantum number is $\Omega=\frac{1}{2}$. Finally, applying the variational method to the perturbed Hamiltonian $H_\delta^{(0)}$, with respect to the nuclear state $\left|N\ell\Lambda\Sigma\right\rangle$ we obtain the final energy eigenvalues and eigenfunctions.

2.2. The Magnetic Dipole Moment

Using the total spin vector $\boldsymbol{j} = \boldsymbol{\ell} + \boldsymbol{s}$ and the total rotational vector $\boldsymbol{I} = \boldsymbol{j} + \boldsymbol{R}$ we can write the magnetic dipole moment as [1]

$$\mu = \frac{1}{I+1}\left[\left(g_s - g_\ell\right)\langle \boldsymbol{s}\cdot\boldsymbol{I}\rangle + \left(g_\ell - g_R\right)\langle \boldsymbol{J}\cdot\boldsymbol{I}\rangle + g_R\left\langle I^2\right\rangle\right] \tag{2.19}$$

The complete method of calculating the magnetic dipole moment is given by Nilsson [1].

2.3. The Electric Quadrupole Moment

Assuming a charge distribution in accordance with the Thomas-Fermi statistical model applied to the oscillator potential, one obtains, for the case of the axially symmetric nuclei, the intrinsic quadrupole moment, to the second order in the deformation parameter δ [1]

$$Q_0 = 0.8ZeR^2\delta\left(1+\frac{2\delta}{3}\right), \tag{2.20}$$

where Z is the number of protons and R is to be taken equal to the radius of charge of the nucleus. The relation between the measured quadrupole moment, denoted by Q_S, and Q_0 is given by:

$$Q_S = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}Q_0, \tag{2.21}$$

where I is the spin-quantum number of the specified nuclear state and K is its component along the body-fixed z-axis. Calculating the charge radius of the nucleus, the measured quadrupole moment for a nucleus with an axis of symmetry is then obtained as function of the deformation parameter δ.

3. The Single-Particle Schrödinger Fluid

3.1. Formulation of the Model

Let us consider a nucleus consisting of A nucleons. We assume that each nucleon in this nucleus (proton or neutron) has mass M and is moving in a single-particle potential $V(\boldsymbol{r}, a(t))$, which is deformed with time t, through its parametric dependence on a classical shape variable $\alpha(t)$. Here, $\alpha(t)$ is assumed to be an externally prescribed function of t. Thus, the Hamiltonian for the present problem is given by [4] [7] [14] [15]

$$H(\boldsymbol{r}, \boldsymbol{v}, a(t)) = -\frac{\hbar^2}{2M}\nabla^2 + V(\boldsymbol{r}, a(t)). \tag{3.1}$$

The single-particle time-dependent wave function $\Psi(\boldsymbol{r}, \alpha(t), t)$ which satisfies the time-dependent Schrödinger wave equation that describes the motion of a nucleon, is defined as

$$H(\boldsymbol{r}, \boldsymbol{v}, \alpha(t))\Psi(\boldsymbol{r}, \alpha(t), t) = i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r}, \alpha(t), t). \tag{3.2}$$

To obtain a fluid dynamical description of the wave function $\Psi(\boldsymbol{r},\alpha(t),t)$, we use the polar form of the wave function. We first isolate the explicit time dependence in the form:

$$\Psi(\boldsymbol{r},\alpha(t),t)=\psi(\boldsymbol{r},\alpha(t))\exp\left\{-\frac{i}{\hbar}\int_0^t \epsilon(\alpha(t'))\mathrm{d}t'\right\}, \tag{3.3}$$

where ϵ is the energy density which depends on the time through the parameter $\alpha(t)$. Then, we write the complex wave function $\psi(\boldsymbol{r},\alpha(t))$ in the following polar form:

$$\psi(\boldsymbol{r},\alpha(t))=\Phi(\boldsymbol{r},\alpha(t))\exp\left\{-\frac{iM}{\hbar}S(\boldsymbol{r},\alpha(t))\right\}, \tag{3.4}$$

where $\Phi(\boldsymbol{r},\alpha(t))$ and $S(\boldsymbol{r},\alpha(t))$ are assumed to be real functions of $\boldsymbol{r}$ and $\alpha(t)$. Finally, we assume that the function $\Phi(\boldsymbol{r},\alpha(t))$ is positive definite. In the case of rotation, the parameter $\alpha(t)$ becomes the angle of rotation, $\theta=\Omega t$, where Ω is the angular velocity. Substituting Equations (3.1), (3.3) and (3.4) into (3.2) and separating the real and the imaginary parts we get, when multiplying by 2Φ, a pair of coupled equations for Φ and S as follows:

$$\left[H-M\left(\frac{\partial S}{\partial t}-\frac{1}{2}\nabla S\cdot\nabla S\right)\right]\Phi=\epsilon\Phi. \tag{3.5}$$

and

$$\Phi^2\nabla^2 S+\nabla\Phi^2\cdot\nabla S=\frac{\partial\Phi^2}{\partial t}, \tag{3.6}$$

We may call Equation (3.5) a modified Schrödinger equation because it differs from the usual time-independent Schrödinger equation $H\Phi=\epsilon\Phi$ by an added term which we refer to as the "dynamical modification potential":

$$V_{dyn}=-M\left[\frac{\partial S}{\partial t}-\frac{1}{2}(\nabla S)\cdot(\nabla S)\right]. \tag{3.7}$$

Hence, we obtain two equations the first is:

$$\rho\nabla\cdot\boldsymbol{v}+\boldsymbol{v}\cdot\nabla\rho=-\frac{\partial\rho}{\partial t}, \tag{3.8}$$

where $\boldsymbol{v}$ is the irrotational velocity and ρ is the density. This equation is the well-known equation of continuity in fluid mechanics [20]. It can be rewritten in the form:

$$\nabla\cdot(\rho\boldsymbol{v})=-\frac{\partial\rho}{\partial t}, \tag{3.9}$$

where $\rho=\Phi^2$ and $\boldsymbol{v}=-\nabla S$.

The second equation is:

$$(H+V_{dyn})\Phi=\epsilon\Phi, \tag{3.10}$$

which is a modified Schrödinger equation with:

$$V_{dyn}=-M\left(\frac{\partial S}{\partial t}-\frac{1}{2}v^2\right). \tag{3.11}$$

Equation (3.4) can be written simply as $\psi = \Phi \exp\left\{-i\frac{MS}{\hbar}\right\}$, so that:

$$S = \frac{i\hbar}{2M}\ln\left(\frac{\psi}{\psi^*}\right). \tag{3.12}$$

and

$$\boldsymbol{v} = -\nabla S = \frac{i\hbar}{2M}\left[\frac{\nabla\psi^*}{\psi^*} - \frac{\nabla\psi}{\psi}\right].$$

Therefore,

$$\boldsymbol{v} = \frac{i\hbar}{2M|\psi|^2}\left[\psi\nabla\psi^* - \psi^*\nabla\psi\right]. \tag{3.13}$$

The current of the single particle state is defined by $\boldsymbol{j} = \rho\boldsymbol{v}$ [14], so that:

$$\boldsymbol{j} = \frac{i\hbar}{2M}\frac{|\Phi|^2}{|\psi|^2}\left[\psi\nabla\psi^* - \psi^*\nabla\psi\right], \tag{3.14}$$

where $\rho = |\Phi|^2$, and we finally get the current density:

$$\boldsymbol{j} = \frac{i\hbar}{2M}\left[\psi\nabla\psi^* - \psi^*\nabla\psi\right]. \tag{3.15}$$

Euler's equation for the non-viscous fluid flow is given by [20]

$$\frac{\partial\boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{\nabla p}{\rho}, \tag{3.16}$$

where p is the pressure on the fluid at a point $P(\boldsymbol{r})$ at an instant of time t. For an ideal fluid, ∇p is related to the enthalpy per unit mass, w, of the fluid by the following manner:

$$\frac{\nabla p}{\rho} = \nabla w. \tag{3.17}$$

Therefore, Euler's equation can be rewritten as

$$\frac{\partial\boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla w. \tag{3.18}$$

After integration and using $\boldsymbol{v} = -\nabla s$ we get

$$\frac{\partial S}{\partial t} - \frac{1}{2}v^2 = w, \tag{3.19}$$

so that

$$\frac{\partial S}{\partial t} - \frac{1}{2}(\nabla S)^2 = w, \tag{3.20}$$

where S is the velocity potential for v, and we can write:

$$V_{dyn} = -M\left[\frac{\partial S}{\partial t} - \frac{1}{2}(\nabla S)^2\right] = -Mw, \tag{3.21}$$

So that the modified Schrödinger equation takes the form

$$(H - Mw)\Phi = \epsilon\Phi, \tag{3.22}$$

where w is now the "enthalpy" of the single-particle Schrödinger fluid [14] [15]. Hence, we have a set of fluid dynamical equations completely analogous to those which describe a classical fluid.

For the modified potential we take the anisotropic oscillator, as proposed by Nilsson [1] with angular frequencies given by:

$$\omega_z^2 = \omega_0^2\left(1-\frac{4}{3}\delta\right), \tag{3.23}$$

$$\omega_x^2 = \omega_y^2 = \omega_0^2\left(1+\frac{2}{3}\delta\right). \tag{3.24}$$

The parameter ω_0 depends on the deformation parameter δ in the following way [1] [9]

$$\omega_0 = \omega_0(\delta) = \omega_0^0\left\{1-\frac{12}{9}\delta^2-\frac{16}{27}\delta^3\right\}^{-\frac{1}{6}}, \tag{3.25}$$

where ω_0^0 is the value of $\omega_0(\delta)$ for $\delta = 0$. The deformation parameter δ is related to the well-known deformation parameter β by (2.7).

The parameter β can vary in the range $-0.5 \le \beta \le 0.5$.

Also, we assume that the adiabatic approximation is valid for this fluid, that is the angle of rotation θ is constant of time. Hence, the collective kinetic energy T of the nucleus is given by [14] [15]

$$T = \frac{1}{2}M\int \rho_T \boldsymbol{v}_T \cdot (\boldsymbol{\Omega}\times\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}, \tag{3.26}$$

where ρ_T is the total density distribution of the nucleus and $\boldsymbol{v}_T$ is the total velocity field:

$$\boldsymbol{v}_T = \frac{\sum_{occ}\rho_K \boldsymbol{v}_K}{\sum_{occ}\rho_K}. \tag{3.27}$$

3.2. Moments of Inertia from Fluid Dynamical Viewpoint

We now examine the cranking moment of inertia in terms of the velocity fields. Bohr and Mottelson [2] showed that for harmonic oscillator case at the equilibrium deformation, where:

$$\frac{\mathrm{d}}{\mathrm{d}\delta}\sum_{i=1}\left(E_{n_x n_y n_z}\right)_i = 0, \tag{3.28}$$

the cranking moment of inertia is identically equal to the rigid moment of inertia:

$$\Im_{cr} = \Im_{rig} = \sum_{i=1} M\left\langle y_i^2 + z_i^2\right\rangle. \tag{3.29}$$

In terms of expression (3.19) involving the velocity fields, this result asserts the equality of the collective kinetic energy of the Schrödinger fluid and that of rigidly rotating classical fluid:

$$\frac{M}{2}\int \rho_T \boldsymbol{v}_T \cdot (\boldsymbol{\Omega}\times\boldsymbol{r})\,\mathrm{d}\tau = \frac{1}{2}\Im_{rig}\Omega^2 = \frac{M}{2}\int \rho_T (\boldsymbol{\Omega}\times\boldsymbol{r})^2\,\mathrm{d}\tau, \tag{3.30}$$

at the equilibrium deformation. We emphasize that the above equations hold for any number of nucleons occupying any set of single particle harmonic oscillator states at the deformation defined by equilibrium condition (3.28). It holds for a one particle state. For this case, Equation (3.30) becomes:

$$\frac{M}{2}\int \rho_K \boldsymbol{v}_K \cdot (\boldsymbol{\Omega}\times\boldsymbol{r})\,\mathrm{d}\tau = \frac{M}{2}\int \rho_K (\boldsymbol{\Omega}\times\boldsymbol{r})^2\,\mathrm{d}\tau, \tag{3.31}$$

at the equilibrium deformation of the single particle state

$$|i\rangle \equiv |n_x n_y n_z\rangle. \tag{3.32}$$

Equation (3.31) is a remarkable identity. The scalar product of v_K and $(\Omega\times r)$ which occurs on the left side is replaced on the right side, by the absolute square of $(\Omega\times r)$. It forces one to inquire whether the irrotational field $\boldsymbol{v}_K$ is equal to $(\boldsymbol{\Omega}\times\boldsymbol{r})$. The answer, of course, is no. For, $\boldsymbol{v}_K$ posses compressible line vortices. It could be impossible to equal the velocity field for rigid rotation $\boldsymbol{v}_{rig} = \boldsymbol{\Omega}\times\boldsymbol{r}$, which has no singularity and is everywhere incompressible and rotational. Despite this qualitative difference between $\boldsymbol{v}_K$ and the other velocity in Equation (3.31), this shows that, as regards their effects under the integral upon the overall kinetic energy (or the internal parameter), these two velocity fields are equivalent at the equilibrium deformation. We note that the cranking moment of inertia $\Im_{cr}$ and the rigid moment of inertia $\Im_{rig}$ are equal only when the harmonic oscillator is at the equilibrium deformation. At other deformations, they can, and do, deviate substantially from one another [14].

The following-expressions for the cranking moment of inertia, $\Im_{cr}$, and the rigid-body moment of inertia, $\Im_{rig}$, hold [14] [15]:

$$\Im_{cr} = \frac{E}{\omega_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[\sigma^2(1+q)+\frac{1}{\sigma}(1-q)\right], \tag{3.33}$$

$$\Im_{rig} = \frac{E}{\omega_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[(1+q)+\sigma(1-q)\right], \tag{3.34}$$

where E is the total nuclear ground-state energy defined by

$$E = \sum\nolimits_{occ}\left[\hbar\omega_x(n_x+n_y+1)+\hbar\omega_z\left(n_z+\frac{1}{2}\right)\right], \tag{3.35}$$

and q is the ratio of the summed single particle quanta in the y-and z-directions:

$$q = \frac{\sum_{occ}\left(n_y+\frac{1}{2}\right)}{\sum_{occ}\left(n_z+\frac{1}{2}\right)}. \tag{3.36}$$

The quantity q is known as the anisotropy of the configuration.

σ is the deformation of the potential [14] [15]. Concerning the magnetic dipole moment of an axially deformed nucleus, we can apply the same method as given by [21].

4. The Cranked Nilsson Model

It is well known that nearly all fully microscopic theories of nuclear rotation are based on or related in some way to the cranking model, which was introduced by Inglis [18] [19] in a semi classical way, but it can be derived fully quantum mechanically, at least in the limit of large deformations, and not too strong K-admixtures ($K \ll I$). The cranking model has the following advantages [13]:

1) In principle, it provides a fully microscopic description of the rotating nucleus. There is no introduction of redundant variables, therefore, we can calculate the rotational inertia parameters microscopically within this model and get a deeper insight into the dynamics of rotational motion.

2) It describes the collective angular momentum as a sum of single-particle angular momentum. Therefore, collective rotation as well as single-particle rotation, and all transitions in between such as decoupling processes, are handled on the same footing.

3) It is correct for very large angular momenta, where classical arguments apply.

A simple and widely used way to describe the change of the single-particle structure with rotation is given by the cranked Nilsson model (CNM) [11] [13]. It is the method of calculating the shell correction energy that made it possible to do large-scale calculations where the nuclear potential-energy surface was explored in detail as a function of different deformation degrees of freedom. Important achievements in this field include the prediction of super deformed high-spin states and terminating bands.

4.1. The Single-Particle Hamiltonian

The single particle Hamiltonian in this model assumes the form [13]

$$H = H^{(0)} + H^{(1)} - \omega j_x , \tag{4.1}$$

where:

$$H^{(0)} = \frac{p^2}{2M} + \frac{1}{2} M \left\{ \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right\} . \tag{4.2}$$

Here, the oscillator parameters ω_x, ω_y and ω_z assume the form [13]

$$\begin{aligned} \omega_x &= \omega_0 (\beta, \gamma) \left[1 - \left(\sqrt{\frac{5}{4\pi}} \beta \right) \cos \left(\gamma - \frac{2\pi}{3} \right) \right], \\ \omega_y &= \omega_0 (\beta, \gamma) \left[1 - \left(\sqrt{\frac{5}{4\pi}} \beta \right) \cos \left(\gamma + \frac{2\pi}{3} \right) \right], \\ \omega_z &= \omega_0 (\beta, \gamma) \left[1 - \left(\sqrt{\frac{5}{4\pi}} \beta \right) \cos (\gamma) \right], \end{aligned} \tag{4.3}$$

where β (or ε) and γ are the quadrupole deformation degrees of freedom. The second term in the right-hand side of Equation (4.1) is given by:

$$H^{(1)} = 2\hbar \omega_0 \sqrt{\frac{4\pi}{9}} \rho^2 \varepsilon_4 V_4 + V' , \tag{4.4}$$

where the stretched square radius ρ^2 is written in the form

$$\rho^2 = \frac{M}{\hbar}\left\{\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right\}, \tag{4.5}$$

The hexadecapole potential is defined to obtain a smooth variation in the γ-plane so that the axial symmetry is not broken for $\gamma = -120^\circ, -60^\circ, 0^\circ$ and 60°. It is of the form [13]:

$$V_4 = a_{40}Y_{4,0} + a_{42}\left(Y_{4,2} + Y_{4,-2}\right) + a_{44}\left(Y_{4,4} + Y_{4,-4}\right), \tag{4.6}$$

where the a_{4i} parameters are chosen as

$$a_{40} = \frac{1}{6}\left(5\cos^2\gamma + 1\right),\quad a_{42} = -\frac{1}{12}\sqrt{30}\sin 2\gamma,$$

$$a_{44} = \frac{1}{12}\sqrt{70}\sin^2\gamma.$$

and

$$V' = -\kappa(N)\hbar\omega_0^o\left\{2\ell_t \cdot s + \mu(N)\left(\ell_t^2 - \left\langle \ell_t^2\right\rangle_N\right)\right\}. \tag{4.7}$$

In Equation (4.7) t refers to the stretched coordinates $\xi = x\sqrt{M\omega_x/\hbar}$, etc., and ε_4 in Equation (4.4) refers to the hexadecapole deformations degree of freedom.

4.2. Derivations

4.2.1. The Hamiltonian $H^{(0)}$

The angular frequencies, Equations (4.3), can be simplified to

$$\begin{aligned}\omega_x &= \omega_0\left[1 - \frac{2}{3}\varepsilon\left(\cos\gamma\cos 120 + \sin\gamma\sin 120\right)\right] \\ &= \left[1 + \frac{\varepsilon}{3}\left(\cos\gamma - \sqrt{3}\sin\gamma\right)\right]\end{aligned}, \tag{4.8}$$

$$\begin{aligned}\omega_y &= \omega_0\left[1 - \frac{2}{3}\varepsilon\left(\cos\gamma\cos 120 - \sin\gamma\sin 120\right)\right] \\ &= \omega_0\left[1 + \frac{\varepsilon}{3}\left(\cos\gamma + \sqrt{3}\sin\gamma\right)\right]\end{aligned}, \tag{4.9}$$

$$\omega_z = \omega_0\left[1 - \frac{2}{3}\varepsilon\cos\gamma\right]. \tag{4.10}$$

Taking the squares of these equations and adding we get

$$\begin{aligned}&\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \\ &= \omega_0^2 r^2 + \frac{2}{3}\varepsilon\cos\gamma\,\omega_0^2 r^2\left(1 - 3\cos^2\theta\right) - \frac{2}{3}\varepsilon\sqrt{3}\sin\gamma\,\omega_0^2 r^2\sin^2\theta\cos 2\varphi \\ &\quad + \frac{\varepsilon^2}{9}\cos^2\gamma\,\omega_0^2 r^2\left(1 + 3\cos^2\theta\right) - \frac{\varepsilon^2}{9}\sqrt{3}\sin^2 2\gamma\,\omega_0^2 r^2\sin^2\theta\cos 2\varphi \\ &\quad + \frac{\varepsilon^2}{9}3\sin^2\gamma\,\omega_0^2 r^2\left(1 - \cos^2\theta\right).\end{aligned} \tag{4.11}$$

Accordingly, the Hamiltonian $H^{(0)}$ takes the form

$$
\begin{aligned}
H^{(0)} = & -\frac{\hbar^2}{2M}\nabla^2 + \frac{1}{2}M\omega_0^2 r^2 - \frac{M}{2}\left(\frac{2}{3}\varepsilon\cos\gamma\,\omega_0^2 r^2\sqrt{\frac{16\pi}{5}}Y_{2,0}\right) \\
& -\frac{M}{2}\left(\frac{2}{3}\varepsilon\sqrt{3}\sin\gamma\,\omega_0^2 r^2\sqrt{\frac{8\pi}{15}}\left(Y_{2,2}+Y_{2,-2}\right)\right) \\
& +\frac{M}{2}\left(\frac{\varepsilon^2}{9}\cos^2\gamma\,\omega_0^2 r^2\left(\sqrt{\frac{16\pi}{5}}Y_{2,0}+2\right)\right) \\
& -\frac{M}{2}\left(\frac{\varepsilon^2}{9}\sqrt{3}\sin^2 2\gamma\,\omega_0^2 r^2\sqrt{\frac{8\pi}{15}}\left(Y_{2,2}+Y_{2,-2}\right)\right) \\
& +\frac{M}{2}\left(\frac{\varepsilon^2}{9}3\sin^2\gamma\,\omega_0^2 r^2\left(\sqrt{\frac{16\pi}{5}}Y_{2,0}-2\right)\right).
\end{aligned} \tag{4.12}
$$

The two deformation parameters ε and δ are equal and they are related to the well-known deformation parameter β by the following relation:

$$
\varepsilon = \delta = \frac{3}{2}\sqrt{\frac{5}{4\pi}}\beta \cong \beta. \tag{4.13}
$$

Hence, the Hamiltonian $H^{(0)}$ takes to the first order in β the form:

$$
H^{(0)} = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega_0^2 r^2 - \beta m\omega_0^2 r^2 Y_{2,0}\cos\gamma - \frac{\sqrt{2}}{2}\beta m\omega_0^2 r^2\left(Y_{2,2}+Y_{2,-2}\right)\sin\gamma. \tag{4.14}
$$

4.2.2. The Hamiltonian $H^{(1)}$

Direct substitution for the different quantities in the operator ρ^2 gives:

$$
\begin{aligned}
\rho^2 = & \frac{M\omega_0}{\hbar}\left[\left(1+\frac{\varepsilon}{3}\left(\cos\gamma-\sqrt{3}\sin\gamma\right)\right)x^2\right. \\
& \left. +\left(1+\frac{\varepsilon}{3}\left(\cos\gamma+\sqrt{3}\sin\gamma\right)\right)y^2 + \left[1-\frac{2}{3}\varepsilon\cos\gamma\right]z^2\right] \\
= & \frac{M\omega_0}{\hbar}r^2\left[1-\frac{\varepsilon}{3}\cos\gamma\sqrt{\frac{16\pi}{5}}Y_{2,0}-\frac{\varepsilon}{3}\sqrt{3}\sin\gamma\sqrt{\frac{8\pi}{15}}\left(Y_{2,2}+Y_{2,-2}\right)\right].
\end{aligned} \tag{4.15}
$$

4.2.3. The Term $2\hbar\omega_0\sqrt{\frac{4\pi}{9}}\rho^2\varepsilon_4 V_4$

Substituting for ρ^2 and V_4 in this term, we obtain

$$
\begin{aligned}
& 2\hbar\omega_0\sqrt{\frac{4\pi}{9}}\rho^2\varepsilon_4 V_4 \\
& = \sqrt{\frac{16\pi}{9}}M\omega_0^2\varepsilon_4 r^2\left[\frac{1}{6}\left(5\cos^2\gamma+1\right)Y_{4,0}\pm\frac{1}{12}\sqrt{30}\sin 2\gamma\left(Y_{4,2}+Y_{4,-2}\right)\right. \\
& \left.+\frac{1}{12}\sqrt{70}\sin^2\gamma\left(Y_{4,4}+Y_{4,-4}\right)\right]\times\left[-\frac{\varepsilon}{3}\cos\gamma\sqrt{\frac{16\pi}{5}}Y_{2,0}\times\left[\frac{1}{6}\left(5\cos^2\gamma+1\right)Y_{4,0}\right.\right. \\
& \left.\left.\pm\frac{1}{12}\sqrt{30}\sin 2\gamma\left(Y_{4,2}+Y_{4,-2}\right)+\frac{1}{12}\sqrt{70}\sin^2\gamma\left(Y_{4,4}+Y_{4,-4}\right)\right]\right] \\
& -\frac{\varepsilon}{3}\sqrt{3}\sin\gamma\sqrt{\frac{8\pi}{15}}\left(Y_{2,2}+Y_{2,-2}\right)\times\left[\frac{1}{6}\left(5\cos^2\gamma+1\right)Y_{4,0}\right.
\end{aligned}
$$

$$\pm\frac{1}{12}\sqrt{30}\sin 2\gamma\left(Y_{4,2}+Y_{4,-2}\right)+\frac{1}{12}\sqrt{70}\sin^2\gamma\left(Y_{4,4}+Y_{4,-4}\right)\bigg]. \tag{4.16}$$

4.3. The Single Particle Energy Eigenvalues and Eigenfunctions

The method of finding the energy eigenvalues and eigenfunctions of the Hamiltonian H can be summarized as follows:

(i) Solving the Schrödinger's equation

$$H_0^{(0)}\psi_i^{(0)} = E_i^{(0)}\psi_i^{(0)}, \tag{4.17}$$

exactly.

(ii) Modifying the functions $\psi_i^{(0)}$ to become eigenfunctions for the solutions of the corresponding equation for $H_0^{(0)} + V'$.

(iii) Using the functions obtained in step (ii) to construct the complete function ψ, the eigenfunction of the Hamiltonian H, in the form of linear combinations of the above functions, as basis functions, with given total angular momentum j and parity π.

(iv) Constructing the Hamiltonian matrix H by calculating its matrix elements with respect to the basis functions defined in step (iii).

(v) Diagonalizing the Hamiltonian matrix H to find the energy eigenvalues E_n and eigenfunctions ψ_n as functions of the non-deformed oscillator parameter $\hbar\omega_0^0$ and the parameters of the potentials.

4.3.1. The Solutions of Equation (4.17)

The solutions of the equation $H_0^{(0)}\psi_i^{(0)} = E_i^{(0)}\psi_i^{(0)}$, are given, with the usual notations, by [8] [9] [10] [11]

$$\psi_i^{(0)} \equiv \left|N\ell\Lambda\right\rangle = R_{N\ell}(r)Y_{\ell\Lambda}(\theta,\varphi), \tag{4.18}$$

$$E_i^{(0)} = \varepsilon_N^0 = \left(N+\frac{3}{2}\right)\hbar\omega_0(\delta), \tag{4.19}$$

where $Y_{\ell\Lambda}(\theta,\varphi)$ are the normalized spherical harmonics with $\Lambda = -\ell, -\ell+1, \cdots, 0, \cdots, \ell-1, \ell$ and ℓ is the nucleon orbital angular momentum quantum number.

The radial wave functions $R_{N\ell}(r)$ are given by

$$R_{N\ell}(r) = a_0^{-\frac{3}{2}}\sqrt{\frac{2\Gamma\left(\frac{N-\ell+2}{2}\right)}{\Gamma\left(\frac{N+\ell+3}{2}\right)}}\mathrm{e}^{-\frac{\rho^2}{2}}\rho^{\ell}L_{\frac{N-\ell}{2}}^{\ell+\frac{1}{2}}\left(\rho^2\right), \tag{4.20}$$

where $\rho = \frac{r}{a_0}$, $a_0 = \sqrt{\frac{\hbar}{M\omega_0(\delta)}}$ and the number of quanta of excitation N is related to the orbital angular momentum quantum number ℓ by $\ell = N, N-2, \cdots, 0$ or 1.

The last function in the right-hand side of Equation (4.20) is the associated Laguerre polynomial. Since the nucleon has spin $\frac{1}{2}$ and intrinsic spin wave functions $\chi_{s\Sigma}$, where $\Sigma = \pm\frac{1}{2}$, the single particle wave functions of the Hamiltonian $H^{(0)}$ are, then, given by

$$\psi_i^{(0)} \equiv \left|N\ell\Lambda\Sigma\right\rangle = R_{N\ell}\left(r\right)Y_{\ell\Lambda}\left(\theta,\varphi\right)\chi_{s\Sigma}\,. \tag{4.21}$$

4.3.2. The Eigenfunctions of the Hamiltonian $H_0^{(0)} + V'$

Wave functions with given values of the number of quanta of excitations *N*, the orbital angular momentum quantum number ℓ, the total angular momentum quantum number *J*, and the parity π can be constructed from the functions (4.21), in the usual manner [8] [9] [10] [11], as follows:

$$\left|N\ell J\pi\right\rangle = \sum\nolimits_{\Lambda+\Sigma=\Omega}\left(\ell\Lambda,\frac{1}{2}\Sigma\,|\,J\Omega\right)\left|N\ell\Lambda\Sigma\right\rangle. \tag{4.22}$$

The functions $\left|N\ell J\pi\right\rangle$ are used as basis functions for the construction of the single particle nuclear wave functions with given total angular momentum *J* and parity π, in the usual manner, as follows:

$$\left|J\pi\right\rangle = \sum\nolimits_{N\ell}\sum\nolimits_{\Lambda+\Sigma=\Omega}C_{N\ell}\left(\ell\Lambda,\frac{1}{2}\Sigma\,|\,J\Omega\right)\left|N\ell\Lambda\Sigma\right\rangle. \tag{4.23}$$

Accordingly, we obtain 15 wave functions, states, namely:

$$\left|\frac{1}{2}^+\right\rangle,\left|\frac{3}{2}^+\right\rangle,\left|\frac{5}{2}^+\right\rangle,\left|\frac{7}{2}^+\right\rangle,\left|\frac{9}{2}^+\right\rangle,\left|\frac{11}{2}^+\right\rangle,\left|\frac{13}{2}^+\right\rangle,\left|\frac{1}{2}^-\right\rangle,\left|\frac{3}{2}^-\right\rangle,\left|\frac{5}{2}^-\right\rangle,\left|\frac{7}{2}^-\right\rangle,\left|\frac{9}{2}^-\right\rangle,\left|\frac{11}{2}^-\right\rangle,\left|\frac{13}{2}^-\right\rangle$$

and $\left|\frac{15}{2}^-\right\rangle$.

The matrix elements of the Hamiltonian $H_0^{(0)} + V'$ with respect to the functions (4.23) are given by:

$$\begin{aligned}
\left\langle J^\pi\left|H_0^{(0)} + V'\right|J^\pi\right\rangle &= \sum\nolimits_{N\ell N'\Lambda,\Sigma,\Lambda',\Sigma'}\left(\ell\Lambda,\frac{1}{2}\Sigma\,|\,J\Omega\right)\left(\ell\Lambda',\frac{1}{2}\Sigma'\,|\,J\Omega'\right)\\
&\times C_{N\ell}C_{N'\ell}\left[\left(N+\frac{3}{2}\right)\hbar\omega_0\left(\delta\right)\delta_{N,N'}\delta_{\Lambda,\Lambda'}\delta_{\Sigma,\Sigma'}\delta_{\Omega,\Omega'}\right.\\
&-\chi\hbar\omega_0^0\left[\left(2\Lambda\Sigma+\mu\ell\left(\ell+1\right)\right)\delta_{\Lambda,\Lambda'}\delta_{\Sigma,\Sigma'}+\sqrt{\left(\ell-\Lambda\right)\left(\ell+\Lambda+1\right)}\delta_{\Lambda+1,\Lambda'}\delta_{\Sigma-1,\Sigma'}\right.\\
&\left.\left.+\sqrt{\left(\ell+\Lambda\right)\left(\ell-\Lambda+1\right)}\delta_{\Lambda-1,\Lambda'}\delta_{\Sigma+1,\Sigma'}\right]\delta_{N,N'}\delta_{\Omega,\Omega'}\right].
\end{aligned} \tag{4.24}$$

4.4. The Hamiltonian Matrix Elements

4.4.1. The Matrix Elements of the Operator r^2

The matrix elements of the operator r^2 with respect to the basis functions $\left|N\ell\Lambda\Sigma\right\rangle$ are given, with the usual notations [8] [9] [10] [11], by

$$\begin{aligned}
\left\langle N\ell\Lambda\Sigma\left|r^2\right|N'\ell\Lambda'\Sigma'\right\rangle = a_0^2&\left[\left(N+\frac{3}{2}\right)\delta_{N,N'}+\sqrt{n\left(n+\ell+\frac{1}{2}\right)}\delta_{N-Z,N'}\right.\\
&\left.+\sqrt{\left(n+1\right)\left(n+\ell+\frac{3}{2}\right)}\delta_{N+Z,N'}\right]\delta_{\Lambda,\Lambda'}\delta_{\Sigma,\Sigma'},
\end{aligned} \tag{4.25}$$

where $a_0^2 = \dfrac{\hbar}{m\omega_0(\delta)}$ and $N = 2n + \ell$.

4.4.2. The Matrix Elements of the Operator $Y_{LM}(\theta,\varphi)$

The matrix elements of the spherical harmonics $Y_{LM}(\theta,\varphi)$ with respect to the basis functions $|N\ell\Lambda\Sigma\rangle$ are given by [9]

$$\langle N\ell\Lambda\Sigma|Y_{LM}|N'\ell\Lambda'\Sigma'\rangle = (-1)^{2\ell+1}\sqrt{\frac{2L+1}{4\pi}}\begin{pmatrix}\ell & L & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell & L & \ell\\ -\Lambda & M & \Lambda'\end{pmatrix}\delta_{\Sigma,\Sigma'}. \quad (4.26)$$

From the condition that $m_1 + m_2 = -m_3$ to be satisfied by the 3*j*-symbols in Equation (4.26), $\begin{pmatrix}j_1 & j_2 & j_3\\ m_1 & m_2 & m_3\end{pmatrix}$, we get:

$$\langle \ell\Lambda|Y_{2,0}|\ell\Lambda'\rangle = (-1)^{2\ell+1}\sqrt{\frac{5}{4\pi}}\begin{pmatrix}\ell & 2 & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell & 2 & \ell\\ -\Lambda & 0 & \Lambda'\end{pmatrix}\delta_{\Lambda,\Lambda'}, \quad (4.27)$$

$$\langle \ell\Lambda|Y_{2,2}|\ell\Lambda'\rangle = (-1)^{2\ell+1}\sqrt{\frac{5}{4\pi}}\begin{pmatrix}\ell & 2 & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell & 2 & \ell\\ -\Lambda & 2 & \Lambda'\end{pmatrix}\delta_{\Lambda-2,\Lambda'}, \quad (4.28)$$

$$\langle \ell\Lambda|Y_{2,-2}|\ell\Lambda'\rangle = (-1)^{2\ell+1}\sqrt{\frac{5}{4\pi}}\begin{pmatrix}\ell & 2 & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell & 2 & \ell\\ -\Lambda & -2 & \Lambda'\end{pmatrix}\delta_{\Lambda+2,\Lambda'}. \quad (4.29)$$

4.5. Total Nuclear Quantities

We define the total quantities [13]

$$E_{sp} = \sum\nolimits_{occ} e_i = \sum\nolimits_{occ} e_i^{\omega} + \hbar\omega\sum\nolimits_{occ} m_i, \quad (4.30)$$

$$I = \sum\nolimits_{occ} m_i, \quad (4.31)$$

with the summation over the occupied orbitals in a specific configuration of the nucleus. The shell energy is now calculated from [13]

$$E_{shell}(I) = E_{sp}(I) - \langle E_{sp}(I)\rangle, \quad (4.32)$$

where $\langle E_{sp}(I)\rangle$ is the smoothed single-particle sum evaluated according to the Strutinsky prescription. The detailed formulas for $\langle E_{sp}(I)\rangle$ are discussed for $I = 0$ and $I \neq 0$ in [13].

The pairing energy is an important correction that should decrease with increasing spin and become essentially unimportant at very high spins. To obtain an ($I = 0$) average pairing gap Δ, which varies as $A^{-\frac{1}{2}}$, the pairing strength G is chosen as [13]

$$G_{p,n} = \frac{1}{A}\left(g_0 \pm g_1\frac{N-Z}{A}\right)(\mathrm{MeV}), \quad (4.33)$$

with $g_1/g_0 \approx 1/3$. Furthermore, the number of orbitals included in the pairing calculation should vary as $\sqrt{Z}$ and $\sqrt{N}$ for protons (p) and neutrons (n), respectively.

The total nuclear energy is now calculated by replacing the smoothed single-particle sum by the rotating-liquid-drop energy and adding the pairing correction,

$$E_{tot}\left(\overline{\varepsilon},I\right)=E_{shell}\left(\overline{\varepsilon},I\right)+E_{RLD}\left(\overline{\varepsilon},I\right)+E_{pair}\left(\overline{\varepsilon},I\right), \tag{4.34}$$

or

$$E_{tot}\left(\overline{\varepsilon},I\right)=E_{sp}-A+E_{LD}-BI^2+\frac{\hbar^2}{2\Im_{rig}} \tag{4.35}$$

where $\overline{\varepsilon}=\left(\varepsilon,\gamma,\varepsilon_4\right)$, E_{LD} is liquid drop energy, $A=\left\langle E_{sp}\left(I\right)\right\rangle$ and B is the smooth moment of inertia factor, $B=\frac{\hbar^2}{2\Im_{strut}}$. The shell and pairing energies are evaluated separately for protons and neutrons at $I=0$, while the renormalization of the moment of inertia introduces a coupling when evaluating E_{shell} for $I>0$. In the computer program, E_{pair} is included only for $I=0$. The protons and neutrons are also coupled through the requirement that the shape of the respective potentials and the rotational frequencies are identical.

In the liquid drop model [3] [13], the nuclear mass is given by:

$$E_{L.D.}=-a_V\left(1-\kappa_V\left(\frac{N-Z}{A}\right)^2\right)A+\frac{3}{5}\frac{e^2Z^2}{R_c}\left[B_c\left(\overline{\varepsilon}\right)-\frac{5\pi^2}{6}\left(\frac{d}{R_c}\right)^2\right] \tag{4.36}$$

$$+a_s\left(1-\kappa_s\left(\frac{N-Z}{A}\right)^2\right)A^{2/3}B_s\left(\overline{\varepsilon}\right)+\begin{cases}+12/\sqrt{A} & \text{odd}-\text{odd nuclei}\\ 0 & \text{odd}-\text{even nuclei}\\ -12/\sqrt{A} & \text{even}-\text{even nuclei}\end{cases} \tag{4.37}$$

In this formula, $B_c\left(\overline{\varepsilon}\right)=B_{coul}\left(\overline{\varepsilon}\right)/B_{coul}\left(\overline{\varepsilon}=0\right)$ and $B_s\left(\overline{\varepsilon}\right)=B_{surf}\left(\overline{\varepsilon}\right)/B_{surf}\left(\overline{\varepsilon}=0\right)$, are the surface and Coulomb energies of a nucleus with a sharp surface in units of their corresponding values for spherical shape. The second term in the Coulomb energy is a (shape-independent) diffuseness correction with d being the diffuseness. The Coulomb energy constant is often defined as $a_c=\left(3/5\right)\left(e^2/R_c\right)$. When calculating the nuclear mass, one should note that the average pairing energy should be subtracted from E_{pair}.

The calculation of the Coulomb correction in particular, is somewhat involved: the original six-dimensional integral can be simplified only to four dimensions [13]. Furthermore, the use of stretched coordinates leads to complicated expressions. The radius expressed as a function of the angles in the stretched-coordinate system is obtained by requiring a constant value for the potential in (4.1) (neglecting the $V'-\omega j_x$ term):

$$\rho^2\propto\frac{1}{1-\frac{2}{3}\varepsilon\sqrt{\frac{4\pi}{5}}\left(\cos\gamma Y_{20}-\frac{1}{\sqrt{2}}\sin\gamma\left(Y_{22}+Y_{2-2}\right)\right)+\varepsilon_4\sqrt{\frac{4\pi}{9}}V_4\left(\gamma\right)}, \tag{4.38}$$

where the spherical harmonics are functions of the angles θ_t and φ_t and the harmonic-oscillator part of the potential is expressed in ε and γ. From the definition of the stretched coordinates, it is straightforward to express the angles θ_t and φ_t in the corresponding angles in the spherical system, θ and φ.

Because of the incompressibility of nuclear matter, the nuclear volume is kept constant when the nucleus is deformed. This is achieved by varying the frequency $\omega_0(\varepsilon,\gamma,\varepsilon_4)$ from its value for a spherical shape, ω_0^0. The integration of the nuclear volume is most easily performed in the stretched-coordinate system and then multiplied with the corresponding Jacobian, a constant proportional to $\sqrt{\omega_x\omega_y\omega_z/\omega_0^3}$. From the single-particle wave functions, the electric (or mass) quadrupole moment may be calculated as

$$Q_2 = \sum\nolimits_{occ} p_i \left\langle \chi_i^{\omega} \middle| q_2 \middle| \chi_i^{\omega} \right\rangle, \tag{4.39}$$

where $p_i = 1$ for protons and 0 (or 1) for neutrons. However, as we go further away from closed shells some new, very simple and systematic features start to show up for some nuclei. This is true for nuclei with mass number A in the range $155 < A < 185$, for A > 225, for nuclei in the s-d shell $19 \le A \le 25$ and for p shell nuclei in $9 \le A \le 14$. Odd-A nuclei in these regions are characterized by exceptionally large positive quadruple moments, even-even nuclei in the same region all have a rather low-lying first excited state with $J = 2^+$ and electric quadruple radiation are strongly enhanced.

5. The Nuclear Superfluidity Model

5.1. Single Nucleon in a Deformed Nucleus

For a nucleus with quadruple deformation, one can write the nuclear radius as [2] [3]

$$R = R_0\left[1 + \sum\nolimits_{\mu} \alpha_{2\mu} Y_{2\mu}(\theta,\phi)\right] \tag{5.1}$$

where R_0 is the radius of the sphere having the same volume and $Y_{2\mu}$ are the spherical harmonics. Since the body-centered frame was selected as the principal axes, we have for the set of $\alpha_{2\mu}$ in this frame $\alpha_{2,1} = \alpha_{2,-1} = 0$ and $\alpha_{2,2} = \alpha_{2,-2}$.

The Hamiltonian of single nucleon in this average field is given by [18] [19]

$$H = T + V(\beta,\gamma,r) \tag{5.2}$$

where:

$$V(\beta,\gamma,r) = \frac{1}{2}M\omega^2 r^2 - M\omega^2 r^2\left\{\alpha_{20}Y_0^2 + \alpha_{22}\left[Y_2^2 + Y_{-2}^2\right]\right\} \tag{5.3}$$

$$T = -\frac{\hbar^2}{2M}\nabla^2 \tag{5.4}$$

The single-particle Hamiltonian then becomes:

$$H = -\frac{\hbar^2}{2M}\nabla^2 + \frac{1}{2}M\omega^2 r^2 - M\omega^2 r^2\left\{\alpha_{20}Y_0^2 + \alpha_{22}\left[Y_2^2 + Y_{-2}^2\right]\right\} \tag{5.5}$$

The coefficients $\alpha_{2\mu}$ can be expressed, for $\omega_x \ne \omega_y \ne \omega_z$, as

$$\alpha_0 = \beta\cos\gamma, \quad \alpha_{2,2} = \alpha_{2,-2} = \frac{1}{\sqrt{2}}\beta\sin\gamma \tag{5.6}$$

The frequencies ω_x, ω_y and ω_z are connected with the deformation parameters β and γ by [9]

$$\omega_x = \omega_0 \exp\left\{\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma+\frac{\pi}{3}\right)\right\}, \tag{5.7}$$

$$\omega_y = \omega_0 \exp\left\{\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma-\frac{\pi}{3}\right)\right\}, \tag{5.8}$$

$$\omega_z = \omega_0 \exp\left\{-\sqrt{\frac{5}{4\pi}}\beta\cos\gamma\right\}. \tag{5.9}$$

The single particle Hamiltonian then becomes:

$$\begin{aligned} H &= -\frac{\hbar^2}{2M}\nabla^2 + \frac{M}{2}\omega_0^2 r^2 - M\omega_0^2\beta\cos\gamma r^2 Y_{2,0}(\theta,\varphi) \\ &= -\frac{\sqrt{2}}{2}M\omega_0^2\beta\sin\gamma r^2\left(Y_{2,2}(\theta,\varphi)+Y_{2,-2}(\theta,\varphi)\right) \end{aligned} \tag{5.10}$$

This Hamiltonian does not produce the experimental single-particle energy levels so that we add to it two terms, suggested by Nilsson [1], proportional to the spin-orbit term and the square of the orbital-angular momentum vector of the nucleon as follows:

$$\begin{aligned} H &= -\frac{\hbar^2}{2M}\nabla^2 + \frac{M}{2}\omega_0^2 r^2 - M\omega_0^2\beta\cos\gamma r^2 Y_{2,0}(\theta,\varphi) \\ &= -\frac{\sqrt{2}}{2}m\omega_0^2\beta\sin\gamma r^2\left(Y_{2,2}(\theta,\varphi)+Y_{2,-2}(\theta,\varphi)\right) + C\ell\cdot s + D\ell^2 \end{aligned} \tag{5.11}$$

The constants C and D are given by [1]

$$C = -2\chi\hbar\omega_0^0, \quad D = -\mu\chi\hbar\omega_0^0, \tag{5.12}$$

The Hamiltonian (5.10) then becomes:

$$H = H^0 + H^1 + H' \tag{5.13}$$

with

$$H^0 = -\frac{\hbar^2}{2M}\nabla^2 + \frac{1}{2}M\omega_0^2 r^2 \tag{5.14}$$

$$H^1 = -m\omega_0^2\beta\cos\gamma r^2 Y_{2,0}(\theta,\varphi) - 2\chi\hbar\omega_0^0\ell\cdot s - \mu\chi\hbar\omega_0^0\ell^2. \tag{5.15}$$

$$H' = -\frac{\sqrt{2}}{2}m\omega_0^2\beta\sin\gamma r^2\left(Y_{2,2}(\theta,\varphi)+Y_{2,-2}(\theta,\varphi)\right) \tag{5.16}$$

The solutions of the Schrödinger equation corresponding to the Hamiltonian H^0 are given, with the usual notations, by:

$$|N\ell\Lambda\Sigma\rangle \equiv R_{N\ell}(r)Y_{\ell\Lambda}(\theta,\varphi)X_{s\Sigma}, \tag{5.17}$$

$$\varepsilon_N^0 = \left(N+\frac{3}{2}\right)\hbar\omega_0. \tag{5.18}$$

In Equation (5.17), $X_{s\Sigma}$ are the single-particle spin eigenfunctions with $\Sigma = \pm\frac{1}{2}$, $Y_{\ell\Lambda}(\theta,\varphi)$ are the normalized spherical harmonic functions with

$\Lambda = -\ell, -\ell+1, \cdots, 0, \cdots, \ell-1, \ell$, and the radial wave functions $R_{N\ell}(r)$ are given by (4.20).

The Schrödinger wave equation

$$H\varphi_i = \varsigma_i \varphi_i, \tag{5.19}$$

can then be solved by applying the perturbation method [9] to obtain the perturbed energy eigenvalues ς_i and the corresponding eigenfunctions φ_i, to the second order of the approximation for H' as follows:

$$\varsigma_i = E_i + \sum_{j\neq i} \frac{\left|H'_{ij}\right|^2}{E_i - E_j}, \tag{5.20}$$

$$\varphi_i = \psi_i + \sum_{j\neq i} \frac{H'_{ij}}{E_i - E_j}\psi_j + \sum_{j\neq i}\sum_{s\neq i} \frac{H'_{is}H'_{js}}{(E_i - E_s)(E_i - E_j)}\psi_j \tag{5.21}$$

5.2. Calculations of the Moment of Inertia

5.2.1. Even-Even Nuclei

In the calculations, the chemical Potentials λ_n and λ_p are determined from the relation [9]

$$\sum_i \left\{1 - \frac{\varsigma_i - \lambda_{p,N}}{\sqrt{(\varsigma_i - \lambda_{P,N})^2 + \Delta^2}}\right\} = N_{p,N} \tag{5.22}$$

The expression for the moment of an even-even nucleus is given by [9]:

$$\mathfrak{H}_{s.f.} = \hbar^2 \sum_{i,k} \frac{\left|\langle i|J_x|k\rangle\right|^2}{E_i + E_k}\left\{1 - \frac{(\varsigma_i - \lambda)(\varsigma_k - \lambda) + \Delta^2}{E_i E_k}\right\} \tag{5.23}$$

where E_i is given by the relation:

$$E_i = \sqrt{(\varsigma_i - \lambda)^2 + \Delta^2} \tag{5.24}$$

The only non-vanishing matrix elements $\langle i|J_x|k\rangle$ are:

$$\langle N\ell\Lambda\Sigma|J_x|N\ell\Lambda - 1\Sigma\rangle = \langle N\ell\Lambda - 1\Sigma|J_x|N\ell\Lambda\Sigma\rangle = \frac{1}{2}\sqrt{(\ell+\Lambda)(\ell-\Lambda+1)} \tag{5.25}$$

5.2.1. Odd-Mass Nuclei

It is known that moments of inertia for the odd-mass nuclei are almost larger than that of neighboring doubly even nuclei. The odd-mass nucleus may be represented as a superfluid doubly even core plus the residual unpaired nucleons. In the present work, the moment of inertia □$\mathfrak{H}_{s.f.}$ of the odd-mass nucleus is obtained as:

$$\mathfrak{H}_{s.f.} = \mathfrak{H}_1 + \mathfrak{H}_2 \tag{5.26}$$

$\mathfrak{H}_1$ is the superfluid core contribution and $\mathfrak{H}_2$ is the unpaired nucleon contribution. Of course, the contribution of even-even nuclei is not identical with that of the equivalent doubly even nucleus. This is clear since the unpaired particle will occupy one of the available levels [9]. Such partially filled levels correspond to unfilled ones with respect to the doubly even equivalent nucleus:

$$\mathfrak{H}_1 = J_{EE.} = \frac{1}{2}\hbar^2 \sum\nolimits_{i,k} \frac{\left|\langle i|J_x|k\rangle\right|^2}{E_i + E_k}\left\{1 - \frac{(\varsigma_i - \lambda)(\varsigma_k - \lambda) + \Delta^2}{E_i E_k}\right\} \tag{5.27}$$

The unpaired nucleon contribution $\mathfrak{H}_2$ was calculated with the help of the cranking model [9]

$$\mathfrak{H}_2 = 2\hbar^2 \sum\nolimits_i \frac{\langle i|J_x|P\rangle^2}{E(i) - E(P)} \tag{5.28}$$

5.3. Calculations of the Electric Quadrupole Moment

For the non-axial case, the intrinsic quadrupole moment of a nucleus consisting of Z protons is given by [11]

$$Q_0 = \sum\nolimits_{i=1}^{Z} Q_i, \tag{5.29}$$

where the single-particle operator Q_i is given by

$$Q_i = e\sqrt{\frac{16\pi}{5}} \int \left(\Psi^i_{\Omega^\pi}\right)^2 r_i^2 Y_{2,0}(\theta_i, \phi_i)\,\mathrm{d}\tau. \tag{5.30}$$

Carrying out the integration in Equation (5.30) with respect to the wave functions $|\Omega^\pi\rangle$ which is evaluated in terms of the functions $|Nl\Lambda\Sigma\rangle$, one then obtains

$$Q_i = e\sqrt{\frac{16\pi}{5}} \sum\nolimits_{\alpha,\beta} C^i_\alpha C^i_\beta \langle N_\alpha l_\alpha|r^2|N_\beta l_\beta\rangle \langle l_\alpha \Lambda_\alpha|Y_{2,0}|l_\beta \Lambda_\beta\rangle. \tag{5.31}$$

Filling the single-particle wave functions $|\Omega^\pi\rangle$ for the given nucleus in its ground-state it is then possible to calculate the quadrupole moment by calculating the necessary matrix elements of Equation (5.31) and evaluating the expansion coefficients of the functions $|\Omega^\pi\rangle$ in terms of the functions $|Nl\Lambda\Sigma\rangle$ as obtained from the variational and the perturbation methods.

6. Results and Discussions

According to previous works [9] [10] [11], the Nilsson model parameters χ, μ and β are allowed to take on the values $\chi = 0.05, 0.06, 0.07$ and 0.08, $\mu = 0$, for $N = 0,1$ and 2; and $\mu = 0.35$ for $N = 3$, β takes values in the interval $-0.50 \le \beta \le 0.50$ with a step 0.01. Accordingly, the magnetic dipole moments and the electric quadrupole moments for the six p-shell nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li are calculated.

By assigning suitable values for the quantum numbers n_x, n_y and n_z, in accordance with the Nilsson model, we constructed the ground states of the six p-shell nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li by filling their ground states with successive single-particle states as given by Equation (3.32). For more details concerning this filing, see Appendix-1 in ref. [8]. Accordingly, the ground state in each nucleus is filled with the corresponding wave functions. As a first result, $\mathfrak{I}_{cr}$ and $\mathfrak{I}_{rig}$ are calculated for each nucleus. The corresponding reciprocal moments of inertia $\frac{\hbar^2}{2\Im_{crank}}$ and $\frac{\hbar^2}{2\Im_{rigid}}$ are calculated.

In **Figures 1-6**, we present the calculated values of the reciprocal moments of inertia according to the cranking model and the rigid-body model as functions of the deformation parameter β for the nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li, respectively.

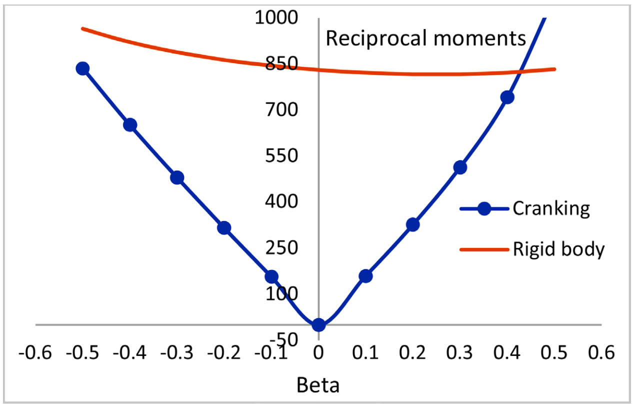

Figure 1. Reciprocal moments of inertia of the nucleus ^{6}Li.

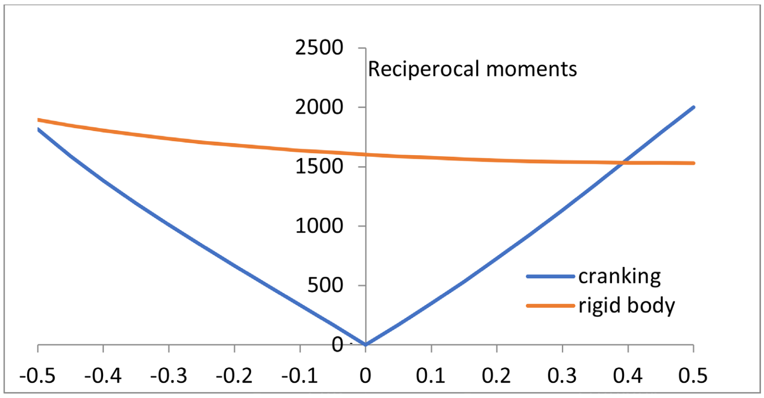

Figure 2. Reciprocal moments of inertia of the nucleus ^{7}Li.

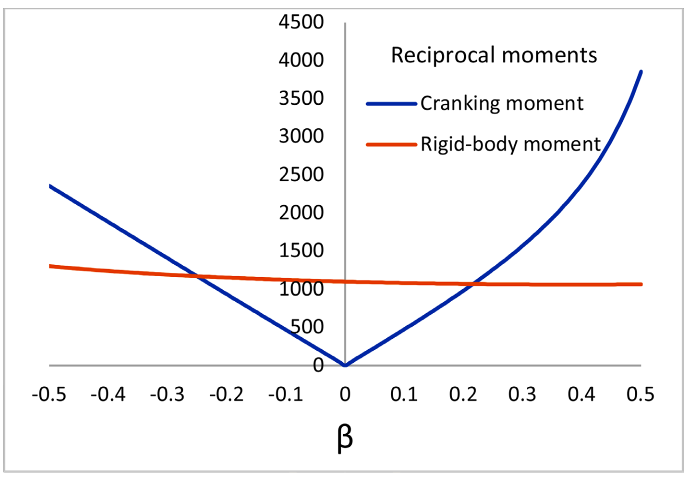

Figure 3. Reciprocal moments of inertia of the nucleus ^{8}Li.

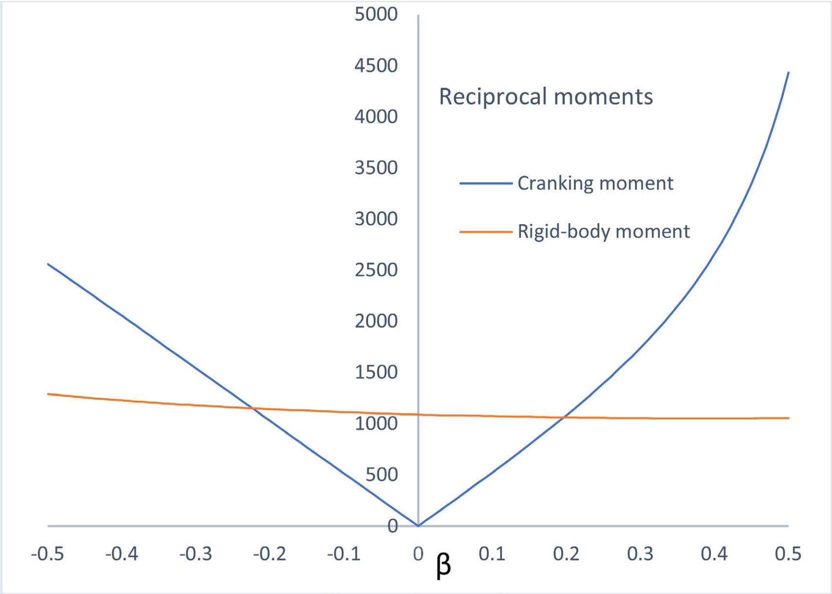

Figure 4. Reciprocal moments of inertia of the nucleus ^{9}Li.

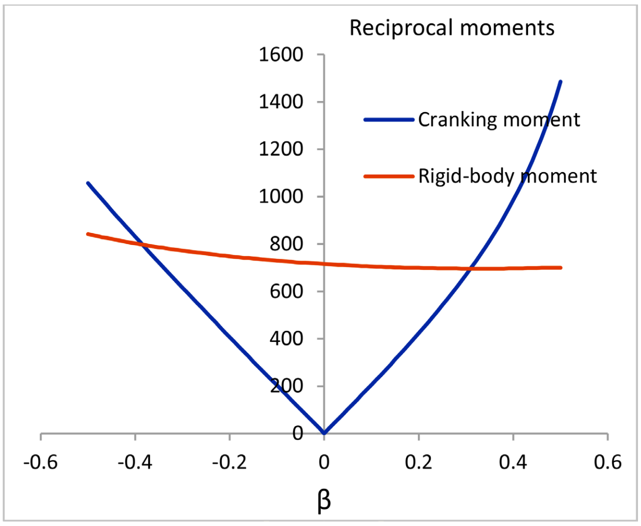

Figure 5. Reciprocal moments of inertia of the nucleus ^{10}Li.

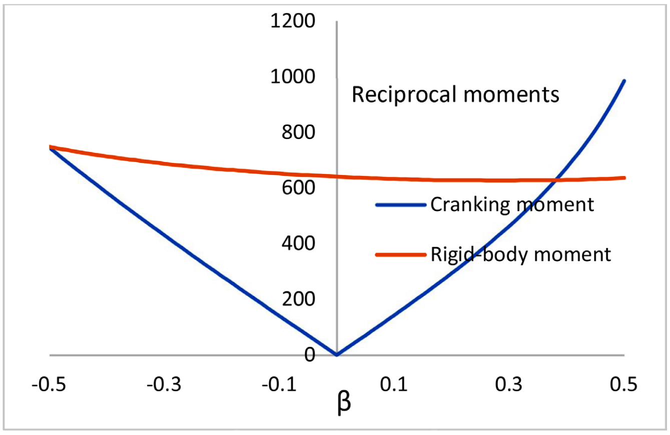

Figure 6. Reciprocal moments of inertia of the nucleus ^{11}Li.

In Table 1, we present the calculated values of the reciprocal moments of inertia of the nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li, which are in good agreement with the corresponding experimental values [22]. The experimental moments of inertia are also given in this table. The values of the nondeformed oscillator parameter $\hbar\omega_0^0$, for the six nuclei, are given. The values of the deformation parameter β are also given in this table.

It is seen from Table 1 that the calculated values of the cranking-model reciprocal moments of inertia are in better agreement with the corresponding experimental values rather than the other values. The disagreement between the values of the rigid-body reciprocal moments of inertia and the corresponding experimental values are due to the fact that the pairing correlation is not taken in concern in this model [9] [11].

For the comparison between the calculated values of the magnetic dipole moments for the cases of axial symmetry and nonaxial (asymmetrical case), we only consider one nucleus as a sample case. For this purpose, we present in Table 2, the calculated values of the magnetic dipole moment of the nucleus ^{6}Li, for both symmetric and asymmetric cases. Experimental values are also given.

For the electric quadrupole moments, we also did the same as for the magnetic

Table 1. The calculated values of the reciprocal moments of inertia of the nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li which are in good agreement with the corresponding experimental values are given. The values of $\hbar\omega_0^0$ and β are also given in this table.

Nucleus	β	$\hbar\omega_0^0$ MeV	$\frac{\hbar^2}{2\Im_{crank}}$ KeV	$\frac{\hbar^2}{2\Im_{rigid}}$ KeV	$\frac{\hbar^2}{2\Im_{exper}}$ KeV [22]
^{6}Li	−0.29 0.26	9.594	489.34 493.44	843.12 716.04	500
^{7}Li	−0.18 0.21	11.543	712.22 687.14	1621.34 1588.34	650.00
^{8}Li	−0.16 0.16	13.21	747.17 773.01	1140.84 1075.15	750.00
^{9}Li	−0.17 0.17	14.14	872.85 909.16	1133.23 1064.51	900.00
^{10}Li	−0.35 0.31	12.02	722.63 697.41	786.12 695.55	715.72
^{11}Li	−0.21 0.20	12.77	298.20 293.90	668.94 627.87	297.71

Table 2. The magnetic dipole moment of ^{6}Li.

Case	γ	β	$\hbar\omega_0^0$ (MeV)	μ (NM)
Axially Symmetric	0˚	0.26	9.594	0.826
Asymmetric	30˚	0.28	9.594	0.939
Experimental	---	0.20 - 0.26	---	0.822 [23]

dipole moments. Accordingly, the calculated values of the electric quadrupole moment of ^{6}Li (in e m barns) are given in Table 3 for the axially symmetric case ($\gamma = 0^\circ$) and for the non-axial case corresponding to $\gamma = 30^\circ$.

In Table 4, we present the calculated values of the reciprocal moment of inertia of the nucleus ^{6}Li by using the concept of the single-particle Schrödinger fluid, for axially symmetric case, for both cranking-and rigid-body models, and the nuclear superfluidity model for the non-axial case. Also, we present in Table 4 the corresponding experimental value. The values of the deformation parameter β, the non-axiality parameter γ and the oscillator parameter $\hbar\omega_0^0$ are also given in Table 4.

In Table 5, we present the calculated values of the magnetic dipole moments of the six nuclei by applying the Nilsson model.

In Table 6, we present the calculated values of the electric quadrupole moments of the six nuclei by applying the cranked Nilsson model.

It is seen from Figures 1-6, and Table 1 that each nucleus has two values of the deformation parameter β which produce good agreement between the calculated and the corresponding experimental reciprocal moments of inertia for

Table 3. The electric quadrupole moment of ^{6}Li.

Case	γ	β	$\hbar\omega_0^0$ (MeV)	Q_S (e m barns)	Q_{exp} (e m barns) [23]
Symmetric	0°	−0.06	9.594	−0.078	−0.083
Asymmetric	30°	−0.12	9.594	−0.074	−0.083

Table 4. Reciprocal moment of inertia of ^{6}Li.

Case	γ	β	$\hbar\omega_0^0$ (MeV)	$\frac{\hbar^2}{2\Im}$ (KeV)
Cranking	0°	0.27	9.594	493.44
Rigid body	0°	0.24	9.594	716.04
Superfluidity	25°	0.28	9.594	548.58
Experimental	---	0.20 - 0.26	---	500.0 [22]

Table 5. Calculated values of the magnetic dipole moments of the six nuclei by applying Nilsson model. The calculated values are also given [23] [24].

Nucleus	μ_{calc} (NM)	μ_{exp} (NM) [23] [24]
^{6}Li	0.826	0.822
^{7}Li	3.323	3.2565
^{8}Li	1.683	1.6534
^{9}Li	3.465	3.4391
^{10}Li	2.555	2.5345
^{11}Li	3.744	3.668

Table 6. Calculated values of the electric quadrupole moments of the six nuclei by applying the cranked Nilsson model. The calculated values are also given [23] [24].

Nucleus	Q_S (e m barns)	Q_{exp} (e m barns)
^{6}Li	−0.082	−0.083
^{7}Li	−3.964	−4.06
^{8}Li	3.023	3.17
^{9}Li	2.452	2.53
^{10}Li	3.211	3.25
^{11}Li	−3.332	−3.1

the cranking model. Concerning the rigid-body model, the calculated values are not in good agreement with the corresponding experimental values, a result which always occurs with this model for most of the deformed nuclei [9].

Finally, we see that the single particle Schrödinger fluid has been applied successfully to the six p-shell nuclei ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li with a suitable choice and filling of the single-particle anisotropic harmonic oscillator states especially for the cranking-model moments of inertia of deformed p-shell nuclei.

Concerning the calculation of the magnetic dipole moments of the six nuclei, it was found that the Nilsson model is an accurate framework for calculating these moments. The Cranking Nilsson model is the right model for the calculation of the electric quadrupole moments of the six nuclei. Furthermore, the nuclear superfluidity model also produced good results for the calculation of the nuclear moments of inertia for the nuclei which do not possess axes of symmetry.

According to the obtained results for the deformation characteristics, the assumption that the nucleus ^{6}Li is deformed and has an axis of symmetry seems to be the more reliable assumption, a result which has been also obtained for some nuclei in the s-d shell [6] [11].

7. Conclusion

To show that whether a nucleus is deformed and has an axis of symmetry, it is important to calculate its deformation properties, such as its moment of inertia, its magnetic dipole moment, and its electric quadrupole moment. These properties depend on the correct choice of the nuclear model which upon its application provides us with results in good agreement with the corresponding experimental values. For this reason, we choose in the present paper the well-known four models of deformed nuclei; two of them assume that the nucleus does have an axis of symmetry and the others assume that the nucleus does not possess an axis of symmetry. The six lithium isotopes ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li are not known to have axes of symmetry or not. Accordingly, it was our goal to choose these isotopes and apply the four well-known models of deformed nuclei: the single-particle Schrödinger fluid model, the Nilsson model, the cranked Nilsson model, and the nuclear superfluidity model to decide whether these nuclei have axes of symmetry or not. According to our calculations, the nucleus ^{6}Li is the only nucleus which has an axis of symmetry.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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