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Study of Differential Cross-Section and S-Matrix Using Volkov Function and Taylor Series Expansion for Elastic Scattering

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Abstract

This paper deals with S-matrix, born first approximation, amplitude, and differential cross-section (DCS), using Volkov function and Taylor series expansion in laser field, scattering. Equation (30) copes-with DCS and Equation (36) deals with S-matrix, with different parameters, moreover, both equations contain real and imaginary parts. The DCS increases with increasing angle and polarizabilities, constant with dipole distance for both emission and absorption of single-photon. The DCS for both emission and absorption is responded to low incidence energy (30 eV - 60 eV) and photon energy (15 eV) while at high energy only emission and absorption are responded for DCS. The DCS between absorption and emission of a photon with angle variation, dipole distance, and atomic polarizabilities was found 1.098 a.u.^{2} and at high incidence, energies were found 0.1 a.u.^{2}.

Keywords

S-Matrix, Born First Approximation, DCS, Emission and Absorption of Photon, Incidence Energy

1. Introduction

Mjjller in June 1947 give birth to S-matrix will make it possible to distinguish between true and false singularities. Nowadays false singularities are used for all potentials with exponential fall-off. The crossed-channel singularities are obtained from relativistic field theories. Heisenberg summarized a finite S-matrix theory of elementary particles, in 1946. The S-matrix theory was still a very general scheme that had rather limited predictive power [1]. The S-matrix for interacting electromagnetic field and half-spin particle field is considered. Particle field quantization is carried out according to a scheme suggested in the works of (Green, 1953) and (Volkov, 1959). The conventional theory of S-matrices allows a simple generalization of the quantization [2]. Hamdolahi *et al.* study the impact of excess pion inside the nucleus on electron scattering and found the DCS variable with presence of pion inside the nuclues [3].

Dipole Approximation is valid, with the wavelength of the laser field being large compared with the size of the atomic system. The Gordon-Volkov wave function is an exact solution of the TDSE for describing the motion of a free, charged particle in a plane-wave electromagnetic field. Volkov wave function has plenty of applications in calculating ionization of atoms, excitation in band-gap semiconductors, and scattering of a charged particle. The appropriate corrections to the Volkov solution were done in presence of the atomic potential besides the plane-wave field [4] [5] [6] [7]. The wave functions of a non-relativistic free electron moving in a homogeneous external field (dipole approximation) were used as the basis for the perturbation method to calculate the cross-section of inverse as well as induced multiphoton bremsstrahlung process (Bergou, 1980). Similar problems were touched on earlier (Denisov and Fedorov, 1967; Brehme, 1971; Ehlotzky, 1978) for non-relativistic [8].

Born approximation (Strutt/Born/Neumann etc. series) is valid for a small perturbation to any linear operator for linear scattering theory and Jacobian's/radiative perturbations. The cross-sectional area of a nucleon is about 10^{-30} fm^2 or 10^{-26} cm^2, the unit 1 barn ≡ 10^{-24} cm^2 = 100 fm^2 which is 100 times the area of a nucleon and quite large to nuclear scale. A very simple radiating element study is an ideal dipole called Hertzian dipole and infinitesimal dipole by Sean Victor Hum in 202.

Calculate of energy for dipole oriented with θ in the electric field at equilibrium position is equal to the work done in bringing the dipole from infinity to the equilibrium position. If the negative charge is displaced along the field by an additional distance *a*, the work done is equal to $-qEa = -pE$, which is the potential energy of the dipole in equilibrium. An infinitesimally small current element is called the Hertz Dipole reported by Kumar in 2019. The electric dipole of two equal magnitudes, oppositely charged particles separated by distance *d*, is given as $E = \frac{2kqd}{z^3}$ and electric dipole moment is defined as $\boldsymbol{p} = q\boldsymbol{d}$, and for large distance ($z \gg d$), $E = \frac{2kp}{z^3}$. A neutral atom has no dipole moment, to begin with, but some molecules (polar molecules) have a permanent dipole moment, even without the external electric field. In one dimension the Born approximation for the scattering amplitude is given by [9],

$$f_B^{1D}(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}r\, V(r)\, \mathrm{e}^{2i\omega r} \quad (1)$$

In three dimensions with a potential depending only on *r*, the Born integral $\int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}} V(r)\,\mathrm{d}\boldsymbol{r}$ is equal to $\left(-\frac{4\pi}{q}\right)\int_0^{\infty}\left(\frac{\sin(qr)}{r}\right)V(r)\,r^2\mathrm{d}r$ and is hence real [10].

Strong-field approximation (SFA) in nonhomogeneous, evolution equation with the inhomogeneous term which determines the departure of the approximate by the exact *ab initio* time-dependent Schrödinger Equation (TDSE). A modification of the nonhomogeneous evolution equation making the inhomogeneous term smaller produces results for the photoelectron spectra which agree quantitatively well with the TDSE results for a system with the Coulomb interaction [11]. In 1994 Reiss and Krainov give an approximation solution of Coulomb-Volkov in strong fields which is well-known as Volkov wave function (Gordon, 1926 and Volkov 1935). The most significant contribution to the transition matrix element should be given by the region of small electron-nucleus distances because the atomic ground state is located there [12].

Series representation of a function. The main purpose of a series is to write a given complicated quantity as an infinite sum of simple terms; and since the terms get smaller and smaller, we can approximate the original quantity by taking only the first few terms of the series. In this section, we finally develop the tool that lets us do this in most cases: a way to write any reasonable function as an explicit power series.

2. Method and Material

Let $\boldsymbol{\alpha}(t)$ representing classical oscillation of electron in field $\boldsymbol{E}(t)$ and $\alpha_0 = E_0/\omega^2$, then Volkov wave function,

$$\chi(\boldsymbol{r},t) = (2\pi)^{-\frac{3}{2}} \exp\left(i\boldsymbol{k}\cdot\boldsymbol{r} - i\boldsymbol{k}\cdot\boldsymbol{\alpha}(t) - \frac{iE_k}{\hbar}t - \frac{i}{\hbar}\int V(\boldsymbol{r},t)\,\mathrm{d}t \right) \tag{2}$$

Here, k is momentum, $\alpha(t) = \alpha_0\left\{\hat{e}_i \sin(\omega t) - e_j \cos(\omega t)\right\}$ and $\alpha_0 = \frac{eA_0}{mc\omega} = \frac{1}{\sqrt{2}}\frac{eE_0}{m\omega^2} = \frac{E_0}{\omega^2}$. For first-order born approximation, the S-matrix element corresponding to the scattering of the electron is

$$S = -\frac{i}{\hbar}\int_{-\infty}^{+\infty}\mathrm{d}t \left\langle \chi_{\boldsymbol{k}_f} \left| V(\boldsymbol{r},t) \right| \chi_{\boldsymbol{k}_i} \right\rangle \tag{3}$$

where $\chi_{\boldsymbol{k}_f}$ and $\chi_{\boldsymbol{k}_i}$ Volkov solution on subsuming in (3)

$$\begin{aligned} S = &-\frac{i}{\hbar(2\pi)^3}\int_{-\infty}^{+\infty}\int \exp\left(i\boldsymbol{k}_f\cdot\boldsymbol{r} - i\boldsymbol{k}_f\cdot\boldsymbol{\alpha}(t) - \frac{iE_{\boldsymbol{k}_f}}{\hbar}t \right)\left\{ V(\boldsymbol{r}) + \alpha_3\frac{\boldsymbol{r}\cdot\boldsymbol{E}}{r^3} \right\} \\ &\times \exp\left(i\boldsymbol{k}_i\cdot\boldsymbol{r} - i\boldsymbol{k}_i\cdot\boldsymbol{\alpha}(t) - \frac{i\boldsymbol{E}_{\boldsymbol{k}_i}}{\hbar}t \right)\mathrm{d}^3r\mathrm{d}t \end{aligned} \tag{4}$$

$$\begin{aligned} S = &-\frac{i}{\hbar(2\pi)^3}\int_{-\infty}^{+\infty}\int \exp\left\{ -i\left(\boldsymbol{k}_f - \boldsymbol{k}_i\right)\cdot\boldsymbol{r} \right\}\exp\left\{ i\left(\boldsymbol{k}_f - \boldsymbol{k}_i\right)\cdot\boldsymbol{\alpha}(t) \right\} \\ &\times \exp\left\{ i\left(E_{\boldsymbol{k}_f} - E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar} \right\}\times V(\boldsymbol{r})\,\mathrm{d}^3r\mathrm{d}t \\ &+\frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{+\infty}\int \exp\left\{ -i\left(\boldsymbol{k}_f - \boldsymbol{k}_i\right)\cdot\boldsymbol{r} \right\}\exp\left\{ i\left(\boldsymbol{k}_f - \boldsymbol{k}_i\right)\cdot\boldsymbol{\alpha}(t) \right\} \\ &\times \exp\left\{ i\left(E_{\boldsymbol{k}_f} - E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar} \right\}\times\left\{ \alpha_3\frac{\boldsymbol{r}\cdot\boldsymbol{E}}{r^3} \right\}\mathrm{d}^3r\mathrm{d}t \end{aligned} \tag{5}$$

Let the momentum transfer of the scattered electron be $q = k_i - k_f$ then

$$S = -\frac{i}{\hbar(2\pi)^3}\int_{-\infty}^{+\infty}\int \exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\}\times\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}$$
$$\times V(r)\mathrm{d}^3r\mathrm{d}t + \frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{+\infty}\int \exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\} \tag{6}$$
$$\times\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}\times\left\{\alpha_3\frac{\boldsymbol{r}\cdot\boldsymbol{E}}{r^3}\right\}\mathrm{d}^3r\mathrm{d}t$$

The first part of Equation (6) is

$$S_1 = -\frac{i}{\hbar(2\pi)^3}\int_{-\infty}^{+\infty}\int \exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\}$$
$$\times\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}V(\boldsymbol{r})\mathrm{d}^3r\mathrm{d}t \tag{7}$$

$$S_1 = f_1\frac{i}{\hbar(2\pi)^2}\int_{-\infty}^{+\infty}\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\}\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}\mathrm{d}t \tag{8}$$

Taking dot product, $\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\} = \mathrm{e}^{-i\alpha\cos\theta q}$ and on using Taylor series expansion we have,

$$\mathrm{e}^{-i\alpha\cos\theta q} = 1 - i\alpha\cos\theta q - \frac{(\alpha\cos\theta q)^2}{2!} - \frac{i(\alpha\cos\theta q)^3}{3!}$$
$$+\frac{(\alpha\cos\theta q)^4}{4!} - \frac{i(\alpha\cos\theta q)^5}{5!} + \cdots \tag{9}$$

On taking the first two-term for the Taylor series we have,
$\mathrm{e}^{-i\alpha\cos\theta q} \approx 1 - i\alpha\cos\theta q$ we have S_1

$$S_1 = f_1\frac{i}{\hbar(2\pi)^2}\int_{-\infty}^{+\infty}(1-i\alpha\cos\theta q)\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}\mathrm{d}t \tag{10}$$

On solving we get,

$$S_1 = f_1\frac{i(1-i\alpha\cos\theta q)}{(2\pi)^2}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right) \tag{11}$$

where $f_1 = -\frac{1}{2\pi}\int\exp(i\boldsymbol{q}\cdot\boldsymbol{r})V(\boldsymbol{r})\mathrm{d}^3r$ and the value is equal to $-\frac{\alpha_p\mathrm{e}^{-qd}}{4d}$ for $q>0$ that is $f_1 = -\frac{\alpha_p\mathrm{e}^{-qd}}{4d}$ [13]. Since we have, differential cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{m^2}{(2\pi)^4}\frac{k_f}{k_i}|f|^2 \tag{12}$$

where f is amplitude therefore with the help of f_1 the differential cross-section become

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 = \frac{m^2}{(2\pi)^4}\frac{k_f}{k_i}\left|-\frac{\alpha_p\mathrm{e}^{-qd}}{4d}\right|^2 \tag{13}$$

Using Taylor series expansion on the exponential term we have $\mathrm{e}^{-2qd} \approx 1-2qd$, therefore the cross-section becomes,

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 = \frac{m^2}{(2\pi)^4}\frac{k_f}{k_i}\frac{\alpha_p^2}{16d^2}(1-2qd) \tag{14}$$

Again second part of Equation (6) is

$$\begin{aligned} S_2 = \frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{t}\int \exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\} \\ \times\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}\left\{\alpha_3\frac{\boldsymbol{r}\cdot\boldsymbol{E}}{r^3}\right\}\mathrm{d}^3r\mathrm{d}t \end{aligned} \tag{15}$$

$$\begin{aligned} S_2 = \frac{-i}{\hbar(2\pi)^3} \propto \alpha E_0\int_{-\infty}^{t}\int \exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\}\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\} \\ \times\left\{\exp(-i\omega t)\frac{\boldsymbol{r}}{r^3}\cdot\boldsymbol{\epsilon}-\exp(i\omega t)\frac{\boldsymbol{r}}{r^3}\cdot\boldsymbol{\epsilon}^*\right\}\mathrm{d}^3r\mathrm{d}t \end{aligned} \tag{16}$$

Here, $\alpha_3\frac{\boldsymbol{r}\cdot\boldsymbol{E}}{r^3} = \alpha_3\frac{E_0}{r^3}\boldsymbol{r}\cdot\left[\exp(-i\omega t)\boldsymbol{\epsilon}-\exp(i\omega t)\boldsymbol{\epsilon}^*\right]$, now on taking the first integral term of Equation (16) as

$$\begin{aligned} I_1 = \alpha_3 E_0\int\int_{-\infty}^{t}\exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\}\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\} \\ \times\left\{\exp(-i\omega t)\frac{\boldsymbol{r}}{r^3}\cdot\boldsymbol{\epsilon}\right\}\mathrm{d}^3r\mathrm{d}t \end{aligned} \tag{17}$$

$$\begin{aligned} I_1 = \alpha_3 E_0\int\exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\frac{\boldsymbol{r}}{r^3}\cdot\boldsymbol{\epsilon}\mathrm{d}^3r\times\int_{-\infty}^{t}(1-i\alpha\cos\theta q) \\ \times\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}\exp(-i\omega t) \end{aligned} \tag{18}$$

Using Taylor series expansion similar as above that $\exp\{-i\boldsymbol{q}\cdot\boldsymbol{\alpha}(t)\} = 1-i\alpha\cos\theta q$ and in the presence of radiation field, the scattered electron gain or lose energy equal to $N\omega$, such that $E_f = E_i + N\omega$. N is the net number of photons exchanged (absorbed or emitted) by the colliding system and the CP field. Therefore, the integration becomes,

$$\begin{aligned} I_1 = \alpha_3 E_0\int\exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\frac{\boldsymbol{r}}{r^3}\cdot\boldsymbol{\epsilon}\mathrm{d}^3r\times(1-i\alpha\cos\theta q) \\ \times\hbar 2\pi\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}-(N+1)\hbar\omega\right) \end{aligned} \tag{19}$$

Replacing N by $N-1$ and using integral value,
$\int\exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\frac{\boldsymbol{r}\cdot\boldsymbol{\epsilon}}{r^3}\mathrm{d}^3r = \frac{2\pi\epsilon\cos\theta}{iq} = -2\pi\frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}}{iq^2}$, the integration becomes,

$$I_1 = -(2\pi)^2\hbar\alpha_3 E_0\frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}}{q^2}(1-i\alpha\cos\theta q)\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}-N\hbar\omega\right) \tag{20}$$

Also for the second integral part of the Equation (18),

$$\begin{aligned} I_2 = -\alpha_3 E_0\int_{-\infty}^{t}\int\exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\times(1-i\alpha\cos\theta q) \\ \times\exp\left\{i\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{t}{\hbar}\right\}\left\{\exp(i\omega t)\frac{r}{r^3}\cdot\boldsymbol{\epsilon}^*\right\}\mathrm{d}^3r\mathrm{d}t \end{aligned} \tag{21}$$

Become,

$$I_2 = (2\pi)^2 \hbar\alpha_3 E_0 \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}}{iq^2} \times (1 - i\alpha\cos\theta q) \times \delta\left(E_{\boldsymbol{k}_f} - E_{\boldsymbol{k}_i} - (N-1)\omega\right) \quad (22)$$

Using

$$\int \frac{\boldsymbol{r}\cdot\boldsymbol{\epsilon}^*}{r^3} \exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\} \mathrm{d}^3 r = \iiint \frac{r\epsilon\cos\theta}{r^3} \exp\{iqr\cos\theta\} r^2 \mathrm{d}r \sin\theta \mathrm{d}\theta \mathrm{d}\phi = -2\pi \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}}{iq^2},$$

therefore

$$I_2 = (2\pi)^2 \hbar\alpha_3 E_0 \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}}{iq^2} \times (1 - i\alpha\cos\theta q) \times \delta\left(E_{\boldsymbol{k}_f} - E_{\boldsymbol{k}_i} - N\hbar\omega\right) \quad (23)$$

Solve similar as I_1 and replacing N by $N+1$, now on substituting I_1 and I_2 in S_2 and solving we get,

$$S_2 = \frac{1}{2\pi} \delta\left(E_{\boldsymbol{k}_f} - E_{\boldsymbol{k}_i} - N\hbar\omega\right) f_2 \quad (24)$$

where, $f_2 = \alpha_3 E_0 \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}}{q^2}(1 + \alpha\cos\theta q + i - i\alpha\cos\theta q)$, since we have,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{m^2}{(2\pi)^4} \frac{k_f}{k_i} |f|^2 \quad (25)$$

where f is amplitude, therefore, scattering becomes

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_2 = \frac{m^2}{(2\pi)^4} \frac{k_f}{k_i} \left| \frac{\alpha_3 E_0 \epsilon\cos\theta}{q} (1 + q\alpha\cos\theta + i - qi\alpha\cos\theta) \right|^2 \quad (26)$$

Now total differential cross-section is represented as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_2 \quad (27)$$

On substituting the value from above we get,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{m^2}{(2\pi)^4} \frac{k_f}{k_i} \left[\left| \frac{\alpha_3 E_0 \epsilon\cos\theta}{q} (1 + q\alpha\cos\theta + i - qi\alpha\cos\theta) \right|^2 + \frac{\alpha_p^2}{16d^2}(1 - 2qd) \right] \quad (28)$$

Neglecting the imaginary part of the Equation (28) and putting $\epsilon = 1$ (unity) solving we get,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{m^2}{(2\pi)^4} \frac{k_f}{k_i} \left[\frac{\alpha_3^2 E_0^2 \cos^2\theta}{q^2} \left(1 + 2q\alpha\cos\theta + q^2\alpha^2\cos^2\theta\right) + \frac{\alpha_p^2}{16d^2}(1 - 2qd) \right] \quad (29a)$$

Taking the imaginary part of the Equation (28) and putting $\epsilon = 1$ (unity) solving we get,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{m^2}{(2\pi)^4} \frac{k_f}{k_i} \left[\frac{\alpha_3^2 E_0^2 \cos^2\theta}{q} \left\{4\alpha\cos\theta + 2i(1 + q\alpha\cos\theta)(1 - q\alpha\cos\theta)\right\} + \frac{\alpha_p^2}{16d^2}(1 - 2qd) \right] \quad (29b)$$

For elastic scattering, we have $\frac{k_f}{k_i} = \left(1 - \frac{l\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}}$,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{m^2\alpha_p^2}{(2\pi)^4}\left(1-\frac{l\hbar\omega}{E_{k_i}}\right)^{\frac{1}{2}}\left[\frac{E_0^2\cos^2\theta}{q}\{4\alpha\cos\theta\right.$$
$$\left.+2i(1+q\alpha\cos\theta)(1-q\alpha\cos\theta)\}+\frac{1}{16d^2}(1-2qd)\right] \tag{30}$$

In atomic unit $m=\hbar=1$, $\omega=15\text{ a.u.}$, $\theta=10^\circ$, $d=20\text{ a.u.}$,

$\alpha_p=\alpha_3=4.5\text{ a.u.}$, $E_0=0.1\text{ a.u.}$, $\alpha=\frac{E_0}{\omega^2}=\frac{1}{15^2}=0.004(\text{a.u})^{-1}$, $q=2\text{ a.u.}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\alpha_p^2}{(2\pi)^4}\left(1-\frac{l\omega}{E_{k_i}}\right)^{\frac{1}{2}}\left[\frac{E_0^2\cos^2\theta}{q}\{4\alpha\cos\theta\right.$$
$$\left.+2i(1+q\alpha\cos\theta)(1-q\alpha\cos\theta)\}+\frac{1}{16d^2}(1-2qd)\right] \tag{31}$$

Also for S-matric from Equation (6)

$$S=S_1+S_2 \tag{32}$$

$$S=f_1\frac{i(1-i\alpha\cos\theta q)}{(2\pi)^2}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)+\frac{1}{2\pi}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}-N\hbar\omega\right)f_2 \tag{33}$$

On substituting the value of the amplitude f_1 and f_2, we get

$$S=-\frac{i(1-i\alpha\cos\theta q)}{(2\pi)^2}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)\frac{\alpha_p\mathrm{e}^{-qd}}{4d}$$
$$+\frac{1}{2\pi}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}-N\hbar\omega\right)\alpha_3E_0\frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}}{q^2}(1+\alpha\cos\theta q+i-i\alpha\cos\theta q) \tag{34}$$

$$S=-\frac{(i+\alpha\cos\theta q)}{(2\pi)^2}\frac{\alpha_p\mathrm{e}^{-qd}}{4d}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)$$
$$+\frac{1}{2\pi}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}-N\hbar\omega\right)\alpha_3E_0\frac{\cos\theta}{q}(1+\alpha\cos\theta q+i-i\alpha\cos\theta q) \tag{35}$$

Unity for $\epsilon=1$, Neglecting imaginary term we have from Equation (30),

$$S=-\frac{\alpha\alpha_p\cos\theta q\mathrm{e}^{-qd}}{16d\pi^2}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)$$
$$+\frac{\alpha_3E_0\cos\theta}{2\pi q}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}-N\hbar\omega\right)(1+\alpha\cos\theta q) \tag{36}$$

Using $\delta(t-t_1)*\delta(t-t_2)=\delta[t-t_1-t_2]$, $(f*g)(t)=(g*f)(t)$, convolution delta properties. The matching feature of the convolution is related to the concept of an internal product between two real functions [14].

$$S=-\frac{\alpha\alpha_p\cos\theta q\mathrm{e}^{-qd}}{16d\pi^2}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)+\frac{\alpha_3E_0\cos\theta}{2\pi q}\delta\left(E_{k_f}-E_{k_i}\right)$$
$$*\delta\left(E_{\boldsymbol{k}_f}-N\hbar\omega\right)(1+\alpha\cos\theta q) \tag{37}$$

Since inner product and dot product is the same at finite dimension, also convolution is an inner product, therefore,

$\delta(t-t_1)*\delta(t-t_2)=\delta(t-t_1)\delta(t-t_2)=\delta[t-t_1-t_2]$, this also follows commutative properties [15].

$$S=-\frac{\alpha\alpha_p\cos\theta q\mathrm{e}^{-qd}}{16d\pi^2}\delta\left(E_{\boldsymbol{k}_f}-E_{\boldsymbol{k}_i}\right)+\frac{\alpha_3E_0\cos\theta}{2\pi q}\delta\left(E_{k_f}-E_{k_i}\right)\delta\left(E_{\boldsymbol{k}_f}-N\hbar\omega\right)(1+\alpha\cos\theta q) \tag{38}$$

Now for elastic scattering S-Matrix, we have $E_{k_f}=E_{k_i}$ therefore,

$$S=-\frac{\alpha\alpha_p\cos\theta q\mathrm{e}^{-qd}}{16d\pi^2}+\frac{\alpha_3E_0\cos\theta}{2\pi q}\delta\left(E_{\boldsymbol{k}_f}-N\hbar\omega\right)(1+\alpha\cos\theta q) \tag{39}$$

An imaginary and real part of the developed equation

The representation and discussion in the result and discussion section are based on the real part of the Equation (31), which is calculated for elastic scattering in the laser field. In this work, the imaginary part is a neglected equation is developed based on the Volkov function and Taylor series with born first approximation. The differential cross-section and S-matric are both calculated to study and both contain real and imaginary parts. For more simple calculations we also used the convolution delta function for the calculation of the S-matrix.

3. Result and Discussion

Differential Cross-Section vs Scattering Angle

The differential cross-section (DCS) with absorption and emission of a photon in leaser field with energy greater (15 eV) than the threshold of hydrogen ground state atom. The DCS and S-matrix are calculated using Taylor series expansion with Volkov function in the material and method section. The nature of DCS with different angles of scattering is given below with absorption and emission of single-photon during scattering. The angle is taken in radian and DCS is taken in log term. DCS calculated has asymptotic for both emission and absorption of a photon in the field and from the field, respectively with the incidence of the electron with 30 eV.

DCS is minimum at scattering angle 0.017 radian is about 9.66 a.u.^{2} when the photon is absorption from field and minimum when photon emitted to the field is about 10.76 a.u.^{2} at scattering angle. DCS is greater when a photon is absorbed by an electron than a photon emitted, the DCS is shifting during the absorption and emission of a photon. The shifting of DCS during emission and absorption is 1.099 a.u.^{2}, which is quite constant. As the photon is absorbed by an electron, the electron goes on oscillation therefore these oscillations because the DCS are increased during the absorption phenomena. But when a photon is emitted to the field the energy of the electron goes decreases and hence the oscillation decreases which causes the decrease in DCS with increasing the angle of scattering. Dhobi et al study the differential cross-section in the presence of a weak laser field (visible and UV) in the case of inelastic scattering and found that DCS initially decreases to a minimum and finally takes a maximum value, when the target emits the energy of 5 eV, 10 eV, 13 eV, 16 eV, 20 eV, 25 eV, and 30 eV. In addition, the differential cross-section also increases with the scattering angle [16]. Also for the scattering of DCS in weak field Dhobi *et al.* design the hamitonian to study the DCS around the proton exchange membrane which is futre work for this team and work is in progress [17] Yadav *et al.* study the DCS with elliptically polarized beam e-H scattering in high intensity with target energy below eV in presence of coulomb potential. The DCS found increases with wavelength and decreases with electron energy within an elliptically polarized beam [18].

Differential cross-section vs incidence energy of electron beam

DCS is high at low incidence electron energy with absorption energy from the field. At low incidence energy, a photon from the field has a high probability to absorb the electron, and hence the oscillation is high which causes an increase in DCS. Therefore here the DCS is high at low energy of incidence of the photon, as the incidence energy of electron increases the probabilities of the interaction of the electron with the absorption of photon from the field is almost zero that means no interaction therefore the DCS at high energy (above 60 eV) the DCS goes with almost constantly with a small difference with increased incidence energy. The maximum DCS is at 30 eV is about 1.44 $a.u^2$.

DCS goes increases with increasing sharply with increasing the incidence of energy electron in between 30 eV to 60 eV and then goes to constant beyond it with emission of a photon in the field. The DCS is less at low energy of incidence of the electron because the emission of energy causes the less oscillation and further increasing the increase of electrons interaction probabilities is almost zero therefore only emission and absorption play an important role for DSC.

Hence in both cases, absorption and emission at low energy of incidence, both photon and incidence energy of electron play an important role to DCS but at high energy of incidence only emission and absorption play an important role because the electron has high energy and interaction between them is negligible (no interaction). So, the DCS for both absorption and emission at high energy is almost constant but the difference is about 0.1 $a.u.^2$ at high incidence energy of the electron.

Differential cross-section vs dipole distance

The DCS for both absorption and emission with incidence energy of 30 eV is shown with dipole distance separation. Since both the incidence and photon energy is constant and the study was done corresponding to a dipole. The observation shows the DCS is independent of dipole separation that is constant for any dipole separation distance. The DCS in the emission case is higher than the absorption case, the difference between the DCS is 1.098 $a.u.^2$.

Differential cross-section vs polarizability

The DCS for both absorption and emission with incidence energy of 30 eV is shown with Polarizabilities in the below **Figure 4**. The DCS increases with Polarizabilities goes increases, but DCS for photon absorption case is higher than emission cases because after the absorption of the electron goes on oscillation which causes stretch bond and hence his stretch bond increase in the DCS. This nature of polarizabilities with DCS for both cases is the same but the difference in DCS for both cases is 1.098 a.u.^{2}. This is how the polarizabilities depend upon the DCS in presence of laser field with incidence electron energy.

The observation of DCS with polarizability, dipole distance, energy of project partcles and scattering angle doesn't shows destructive interference but shows superpostion of projected particles with laser field. Therefore, no intersection in Figures 1-4 was observed.

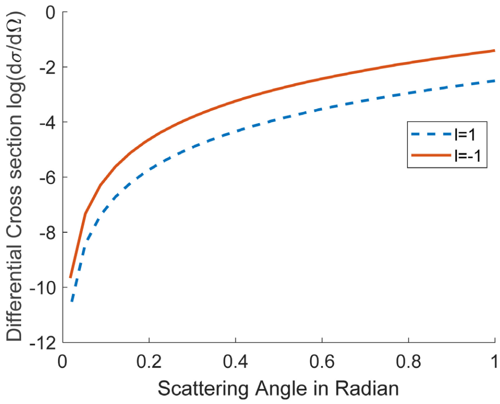

Figure 1. Differential cross-section with scattering angle.

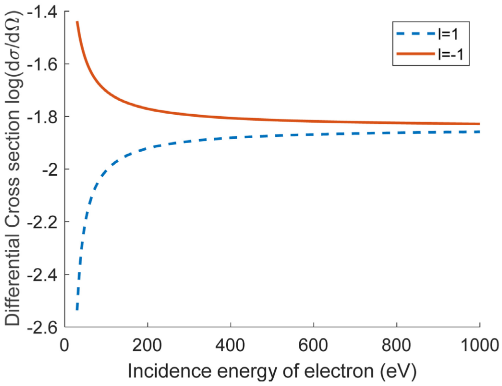

Figure 2. Differential cross-section with incidence energy.

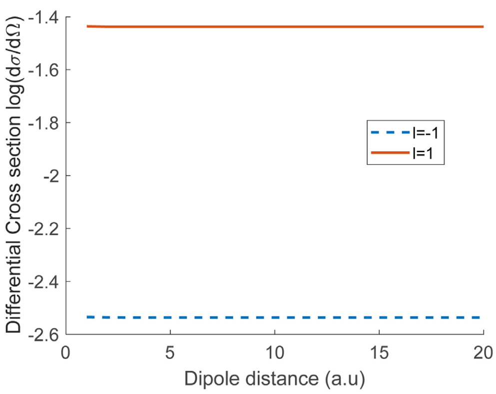

Figure 3. Differential cross-section at 30 eV of incidence energy with dipole separation.

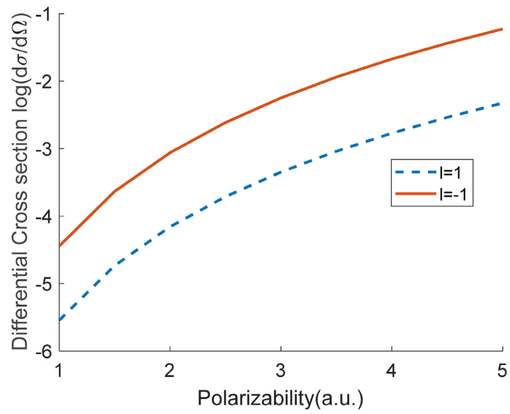

Figure 4. Differential cross-section with polarizabilities.

4. Conclusion

The development of the equation above equation in the material and method section for S-matrix and differential cross-section is studied with different parameters like field energy (photon), incidence energy of electrons, scattering angles, polarizabilities, dipole separation, change in momentum. These parameters are calculated and interconnected using Taylor series expansion, born first Approximation, and Volkov function. The observation is shown in **Figures 1-4**. The observation shows that DCS increase with the increase in scattering angle and constant polarizabilities, and constant dipole separation, for both emission and absorption of a photon. Also, the DCS decreases with the absorption of photons from the field and interaction with low incidence energy while increasing with emission of a photon with low incidence energy, sharply and asymptotic. With large energy of incidence, the DCS for both cases is constant. The S-matrix calculated in Equation (36) and DCS in (30) has both imaginary and real parts with different parameters mentioned above. In this work, we only study the real part which is detail discussed in the result and discussion section.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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