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Simulation of 2D Nonlinear Manifestations in a Layered-Block Medium with Hierarchical Inclusions

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Abstract

The geological environment is an open system that can be influenced by external and internal factors. They can lead it to an unstable state, which, as a rule, manifests itself locally in the form of zones called dynamically active elements, by which it is possible to identify, on the one hand, potential catastrophic sources that disrupt the technological process when mining the rock massif; on the other hand, when producing oil from wells, this process can contribute to improving the movement of oil inclusions in the plastic environment of the reservoir. These objects, both in the rock massif and in the oil reservoir, differ from the host geological environment in their structural forms, which are often hierarchical forms. The process of their nonlinear activation can be observed using borehole monitoring of acoustic longitudinal and transversal waves, for the mathematical support of which new 2D modeling algorithms have been developed in that paper using the method of integral and integro-differential equations with the inclusion of nonlinear terms in the dependence of the wave parameter on frequency. When interpreting the results of acoustic monitoring, it is necessary to use the data of such observation systems that are configured to study the hierarchical structure of the environment.

Keywords

New Algorithms, Nonlinear Terms, Hierarchical Inclusions, Acoustic Waves

1. Introduction

In most cases, in specific geological systems, oscillatory processes are nonlinear. Therefore, the development of the theory of such processes and new modeling methods are of great practical importance [1] [2], especially for the formulation of criteria for the stability of mountain ranges in the process of mining. In the book [3], the theory and methods of studying the nature of instability of physical systems of various nature described by partial differential equations are presented from a unified point of view. To substantiate these methods, the theory of wave processes in linear systems is used, since the development of instability at the initial stage can be described using the concept of small perturbations of the state under study. It turned out that to establish the threshold of instability and its nature, it is sufficient to know only the dispersion equation connecting the wave vector and the frequency of a given wave process. This made it possible to develop a unified approach to the study of instabilities in various systems, regardless of their nature and frequency range. By their nature, all types of instability can be divided into two types: absolute and convective [4]. Therefore, the study of the conditions under which the considered system is unstable and the determination of the nature of this instability are the starting points for theoretical and experimental studies of the generation and amplification of waves of any type. As is known from the theory of stability of systems with a finite number of degrees of freedom [5], the problem of stability of any of its states is reduced to solving systems of linear differential equations, while solving the problem of the nature of the steady state in the general case requires solving a system of nonlinear differential equations. In this case, in the general case, an implicit dependence between frequency ω and wave number k is obtained, which is determined from the dispersion equation $D(\omega, k) = 0$. In this paper, we propose two algorithms for nonlinear acoustic 2D modeling of a layered block medium with hierarchical inclusions.

2. Algorithm for 2D Propagation of Acoustic Longitudinal Waves in a Layered Block Medium with Hierarchical Inclusions of a Rock Massif, Taking into Account Nonlinear Manifestations in Them

To consider the behavior of the elastic rock massif within the framework of a hierarchical medium model of arbitrary rank, an algorithm has been developed for solving a direct two-dimensional problem for the acoustic field in a dynamic version. In this case, the model of local hierarchical heterogeneity of the L-th rank is represented by a crack-like inclusion. The model of all ranks is represented in the approximation when the Lamé parameter $\mu = 0$, both in the inclusions and in the surrounding medium. In this case, the dynamic seismic problem can be considered independently for the case of P and S wave propagation. Here we will consider the first case for the proposed model. The idea, presented in [6] for solving the direct problem for the two-dimensional case of longitudinal wave propagation through a local elastic heterogeneity with a hierarchical structure located in the J-th layer of an N-layered medium is extended to the case when a crack-like turning exists.

$$
\begin{aligned}
&\frac{k_{1jil}^2-k_{1j}^2}{2\pi}\iint_{S_{Cl}}\varphi_l\left(M\right)G_{Sp,j}\left(M,M^0\right)\mathrm{d}\tau_M+\frac{\sigma_{ja}}{\sigma_{jil}}\varphi_{l-1}^0\left(M^0\right)\\
&-\frac{\sigma_{ja}-\sigma_{jil}}{\sigma_{jil}2\pi}\oint_{Cl}G_{Sp,j}\frac{\partial\varphi_l}{\partial n}\mathrm{d}c=\varphi_l\left(M^0\right),M^0\in S_{Cl}\\
&\frac{\sigma_{jil}\left(k_{1jil}^2-k_{1j}^2\right)}{\sigma\left(M^0\right)2\pi}\iint_{S_{Cl}}\varphi_l\left(M\right)G_{Sp,j}\left(M,M^0\right)\mathrm{d}\tau_M+\varphi_{l-1}^0\left(M^0\right)\\
&-\frac{\sigma_{ja}-\sigma_{jil}}{\sigma\left(M^0\right)2\pi}\oint_{Cl}G_{Sp,j}\frac{\partial\varphi_l}{\partial n}\mathrm{d}c=\varphi_l\left(M^0\right),M^0\notin S_{Cl}
\end{aligned}
\tag{1}
$$

where $G_{Sp,i}\left(M,M^0\right)$ is the function of the source of the seismic field, it coincides with the function of the expression [6], $k_{1jil}^2=\omega^2\left(\sigma_{jil}/\lambda_{jil}\right)$ is the wave number for the longitudinal wave, in the given expression, the index *ji* denotes the belonging of the properties of the medium inside the heterogeneity, *ja*—outside the heterogeneity, $l=1,\cdots,L-1$ is the number of the hierarchical level, $\boldsymbol{u}_l=grad\varphi_l$, φ_l^0 is the potential of a normal seismic field in a layered medium in the absence of heterogeneity of the previous rank, if $l=2,\cdots,L$, $\varphi_l^0=\varphi_{l-1}$, if $l=1$, $\varphi_l^0=\varphi^0$, which coincides with the corresponding expression from [6]. If, when moving to the next hierarchical level, the two-dimensional axis does not change, but only the cross-sectional geometries of the nested structures change, then similarly to [6], we can write out an iterative process of modeling the seismic field (the case of formation of only a longitudinal wave). The iterative process refers to the modeling of the displacement vector when moving from the previous hierarchical level to the next level. Within each hierarchical level, an integro-differential equation and an integro-differential representation are calculated using algorithms (1). If at a certain hierarchical level the structure of a local heterogeneity splits into several heterogeneities, then the double and contour integrals in expressions (1) are taken over all heterogeneities. If *l* = *L*, then within the heterogeneities of this hierarchical level there is a fractured heterogeneity. In this case, system (1), taking into account [7], is rewritten in the form that takes into account the propagation of the main wave from an acting source and two envelope waves:

$$
k_{1jiL}^2=\omega^2\left(\sigma_{jiL}/\lambda_{jiL}\right)+\frac{-b_{1jL}\pm\sqrt{b_{1jL}^2+4b_{2jL}\left(\omega_{1jL}+\gamma_{1jL}a_{1jL}^2\right)}}{2b_{2jL}}
\tag{2}
$$

$$
\begin{aligned}
&\frac{k_{1jiL}^2-k_{1j}^2}{2\pi}\iint_{S_{0L}}\varphi_L\left(M\right)G_{Sp,j}\left(M,M^0\right)\mathrm{d}\tau_M+\frac{\sigma_{ja}}{\sigma_{jiL}}\varphi_{L-1}^0\left(M^0\right)\\
&-\frac{\sigma_{ja}-\sigma_{jiL}}{\sigma_{jiL}2\pi}\oint_{CL}G_{Sp,j}\frac{\partial\varphi_L}{\partial n}\mathrm{d}c=\varphi_L\left(M^0\right),M^0\in S_{0L}\\
&\frac{\sigma_{jiL}\left(k_{1jiL}^2-k_{1j}^2\right)}{\sigma\left(M^0\right)2\pi}\iint_{S_{0L}}\varphi_L\left(M\right)G_{Sp,j}\left(M,M^0\right)\mathrm{d}\tau_M+\varphi_{L-1}^0\left(M^0\right)\\
&-\frac{\sigma_{ja}-\sigma_{jiL}}{\sigma\left(M^0\right)2\pi}\oint_{CL}G_{Sp,j}\frac{\partial\varphi_L}{\partial n}\mathrm{d}c=\varphi_L\left(M^0\right),M^0\notin S_{0L}
\end{aligned}
\tag{3}
$$

according to the nonlinear dispersion relation for the nonlinear Schrödinger equation, which describes the propagation of an envelope wave moving over the carrier wave [7]. The coefficients $b_{1jL}, b_{2jL}, \gamma_{1jL}$ in the layer j are constant, real quantities, the first two correspond to the dispersion terms, the third corresponds to the nonlinear term, a_{1jL} is the maximum amplitude of the envelope. If this wave process also propagates to layer j-1 at $l = L$. Expressions (2)-(3) will be rewritten as:

$$\begin{aligned}
&\frac{k_{1(j-1)iL}^2 - k_{1(j-1)}^2}{2\pi} \iint_{S_{0L}} \varphi_L(M) G_{Sp,j-1}(M, M^0) \mathrm{d}\tau_M + \frac{\sigma_{(j-1)a}}{\sigma_{(j-1)iL}} \varphi_{L-1}^0(M^0) \\
&- \frac{\sigma_{(j-1)a} - \sigma_{(j-1)iL}}{\sigma_{(j-1)iL} 2\pi} \oint_{CL} G_{Sp,j-1} \frac{\partial \varphi_L}{\partial n} \mathrm{d}c = \varphi_L(M^0), M^0 \in S_{0L} \\
&\frac{\sigma_{(j-1)iL}\left(k_{1(j-1)iL}^2 - k_{1(j-1)}^2\right)}{\sigma(M^0) 2\pi} \iint_{S_{0L}} \varphi_L(M) G_{Sp,(j-1)}(M, M^0) \mathrm{d}\tau_M + \varphi_{L-1}^0(M^0) \\
&- \frac{\sigma_{(j-1)a} - \sigma_{(j-1)iL}}{\sigma(M^0) 2\pi} \oint_{CL} G_{Sp,(j-1)} \frac{\partial \varphi_L}{\partial n} \mathrm{d}c = \varphi_L(M^0), M^0 \notin S_{0L}
\end{aligned} \tag{4}$$

$$\begin{aligned}
k_{1(j-1)iL}^2 &= \omega^2 \left(\sigma_{(j-1)iL} / \lambda_{(j-1)iL}\right) \\
&+ \frac{-b_{1(j-1)L} \pm \sqrt{b_{1(j-1)L}^2 + 4b_{2(j-1)L}\left(\omega_{1(j-1)L} + \gamma_{(j-1)L} a_{1(j-1)L}^2\right)}}{2b_{2(j-1)L}}
\end{aligned} \tag{5}$$

This process can be continued, and hierarchical inclusions with nonlinear characteristics can approach the surface of a multilevel block system, which will allow warning of a possible catastrophic event.

3. Algorithm for 2D Propagation of Acoustic Transversal Waves in a Layered Block Medium with Hierarchical Inclusions of a Rock Massif, Taking into Account Nonlinear Manifestations in Them

Similarly to (1), the same process is written for modeling the propagation of an elastic shear wave in an N-layer medium with a two-dimensional hierarchical structure of an arbitrary cross-section morphology using integral differential relations written out in [8].

$$\begin{aligned}
&\frac{k_{2jil}^2 - k_{2j}^2}{2\pi} \iint_{S_{Cl}} u_{xl}(M) G_{Ss,j}(M, M^0) \mathrm{d}\tau_M + \frac{\mu_{ja}}{\mu_{jil}} u_{x(l-1)}^0(M^0) \\
&+ \frac{\mu_{ja} - \mu_{jil}}{\mu_{jil} 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{Ss,j}}{\partial n} \mathrm{d}c = u_{xl}(M^0), M^0 \in S_{Cl} \\
&\frac{\mu_{jil}\left(k_{2jil}^2 - k_{2j}^2\right)}{\mu(M^0) 2\pi} \iint_{S_{Cl}} u_{xl}(M) G_{Ss,j}(M, M^0) \mathrm{d}\tau_M + u_{x(l-1)}^0(M^0) \\
&+ \frac{\mu_{ja} - \mu_{jil}}{\mu(M^0) 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{Ss,j}}{\partial n} \mathrm{d}c = u_{xl}(M^0), M^0 \notin S_{Cl}
\end{aligned} \tag{6}$$

where $G_{Ss,j}\left(M,M^0\right)$ is the function of the source of the seismic field of the problem under consideration, it coincides with the Green's function written in [8] for the corresponding problem, $k_{2jil}^2=\omega^2\left(\sigma_{jil}/\mu_{jil}\right)$ is the wave number for the shear wave, μ is the Lamé constant, u_{xl} is the component of the displacement vector, $l=1,\cdots,L$ is the hierarchical number level, u_{xl}^0 is a component of the vector of displacements of the seismic field in a layered medium in the absence of heterogeneity of the previous rank, if $l=2,\cdots,L$, $u_{xl}^0=u_{x(l-1)}$, if $l=1$, $u_{xl}^0=u_x^0$ that coincides with the corresponding expression for the normal field in [8].

If, when moving to the next hierarchical level, the two-dimensional axis does not change, but only the cross-sectional geometries of the nested structures change, then, similarly to [6], we can write out an iterative process of modeling the seismic field (the case of only shear wave formation). The iterative process refers to the modeling of the displacement vector when moving from the previous hierarchical level to the next level. Within each hierarchical level, an integro-differential equation and an integro-differential representation are calculated using algorithms (6). If at a certain hierarchical level the structure of a local heterogeneity splits into several heterogeneities, then the double and contour integrals in expressions (6) are taken over all heterogeneities. If $l=L$, then within the heterogeneities of this hierarchical level there is a fractured heterogeneity. In this case, system (6), taking into account [7], is rewritten in the form that takes into account the propagation of the fundamental wave from the acting source and two envelope waves:

$$\begin{aligned}
&\frac{k_{2jiL}^2-k_{2j}^2}{2\pi}\iint\limits_{S_{CL}} u_{xL}(M)G_{Ss,j}\left(M,M^0\right)\mathrm{d}\tau_M+\frac{\mu_{ja}}{\mu_{jiL}}u_{x(L-1)}^0\left(M^0\right)\\
&+\frac{\mu_{ja}-\mu_{jiL}}{\mu_{jiL}2\pi}\oint\limits_{CL} u_{xL}(M)\frac{\partial G_{Ss,j}}{\partial n}\mathrm{d}c=u_{xL}\left(M^0\right),\ M^0\in S_{CL}\\
&\frac{\mu_{jiL}\left(k_{2jiL}^2-k_{2j}^2\right)}{\mu\left(M^0\right)2\pi}\iint\limits_{S_{CL}} u_{xL}(M)G_{Ss,j}\left(M,M^0\right)\mathrm{d}\tau_M+u_{x(L-1)}^0\left(M^0\right)\\
&+\frac{(\mu_{ja}-\mu_{jiL})}{\mu\left(M^0\right)2\pi}\oint\limits_{CL} u_{xL}(M)\frac{\partial G_{Ss,j}}{\partial n}\mathrm{d}c=u_{xL}\left(M^0\right),M^0\notin S_{CL}
\end{aligned}\tag{7}$$

$$k_{2jiL}^2=\omega^2\left(\sigma_{jiL}/\mu_{jiL}\right)+\frac{-b_{3jL}\pm\sqrt{b_{3jL}^2+4b_{4jL}\left(\omega_{2jL}+\gamma_{2jL}a_{2jL}^2\right)}}{2b_{4jL}}\tag{8}$$

according to the nonlinear dispersion relation for the nonlinear Schrödinger equation, which describes the propagation of an envelope wave moving over the carrier wave [7]. The coefficients $b_{3jL},b_{4jL},\gamma_{2jL}$ in the layer j are constant, real quantities, the first two correspond to the dispersion terms, the third corresponds to the nonlinear term, a_{2jL} is the maximum amplitude of the envelope.

If this wave process also propagates to layer j-1 at $l=L$, Expressions (7)-(8) will be rewritten as:

$$\begin{aligned}
&\frac{k^2_{2(j-1)iL}-k^2_{2(j-1)}}{2\pi}\iint_{S_{CL}} u_{xL}(M)G_{Ss,(j-1)}(M,M^0)\,\mathrm{d}\tau_M+\frac{\mu_{(j-1)a}}{\mu_{(j-1)iL}}u^0_{x(L-1)}(M^0)\\
&+\frac{\mu_{(j-1)a}-\mu_{(j-1)iL}}{\mu_{(j-1)iL}2\pi}\oint_{CL} u_{xL}(M)\frac{\partial G_{Ss,(j-1)}}{\partial n}\mathrm{d}c=u_{xL}(M^0),\ M^0\in S_{CL}\\
&\frac{\mu_{(j-1)iL}\left(k^2_{2(j-1)iL}-k^2_{2(j-1)}\right)}{\mu(M^0)2\pi}\iint_{S_{CL}} u_{xL}(M)G_{Ss,(j-1)}(M,M^0)\,\mathrm{d}\tau_M+u^0_{x(L-1)}(M^0)\\
&+\frac{\mu_{(j-1)a}-\mu_{(j-1)iL}}{\mu(M^0)2\pi}\oint_{CL} u_{xL}(M)\frac{\partial G_{Ss,(j-1)}}{\partial n}\mathrm{d}c=u_{xL}(M^0),\ M^0\notin S_{CL}
\end{aligned}\tag{9}$$

$$\begin{aligned}
k^2_{2(j-1)iL}&=\omega^2\left(\sigma_{(j-1)iL}/\mu_{(j-1)iL}\right)\\
&+\frac{-b_{3(j-1)L}\pm\sqrt{b^2_{3(j-1)L}+4b_{4(j-1)L}\left(\omega_{2(j-1)L}+\gamma_{2jL}a^2_{2(j-1)L}\right)}}{2b_{4(j-1)L}}
\end{aligned}\tag{10}$$

This process can be continued, and hierarchical inclusions with nonlinear characteristics can approach the surface of a block multilevel system.

4. Discussion and Conclusion

From the obtained results, it follows that, according to the data of acoustic monitoring, it is possible to determine the spatial distribution of fractured heterogeneities within hierarchical inclusions located in specific layers of a block massif. Comparing the values of the amplitudes of the longitudinal and transverse waves for a set of exciting frequencies for given physical mechanical characteristics of the enclosing medium and anomalous inclusions, taking into account the dispersion relations, it is possible to estimate the degree of nucleation of the fluctuation process leading to the instability of the layered lateral massif. It is of interest to analyze this process when its maximum manifestations in the amplitudes of the longitudinal and transverse waves will affect different frequencies.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Bykov, V.G. (2015) Nonlinear Waves and Soliton in Models of Fault-Block Geological Environments. *Geology and Geophysics*, **56**, 1008-1024. https://doi.org/10.1016/j.rgg.2015.04.010

[2] Blakier, O. (1969) Analysis of Nonlinear Systems. Mir, Moscow.

[3] Fedorchenko, A.M. and Kotsenko, N.Y. (1981) Absolute and Convective Instability in Plasma and Solids. Nauka. Main Edition of Physical and Mathematical Literature, Moscow.

[4] Lifshits, E.M. and Pitaevsky, L.P. (2002) Physical Kinetics. Fizmatlit, Moscow.

[5] Malkin, I.G. (1966) Theory of Stability of Motion. Nauka, Moscow.

[6] Hachay, O.A. and Khachay, A.Y. (2013) Modeling the Electromagnetic and Seismic Fields in Hierarchically Heterogeneous Media. *Bulletin of SURGU*, *Series* "*Computational Mathematics and Informatics*", **2**, 48-55. https://doi.org/10.14529/cmse130204

[7] Scott, E. (2007) Nonlinear Science. The Birth and Development of Coherent Structures. Fizmatlit, Moscow.

[8] Hachay, O.A. and Khachay, A.Y. (2011) On the Integration of Seismic and Electromagnetic Active Methods for Mapping and Monitoring the State of Two-Dimensional Heterogeneities in an N-Layered Medium. *Bulletin of SURGU. Series* "*Computer Technologies*, *Control*, *Radio Electronics*", **2**, 49-56.