Scientific Research Publishing

Open Journal of Fluid Dynamics, 2023, 13, 226-231
https://www.scirp.org/journal/ojfd
ISSN Online: 2165-3860
ISSN Print: 2165-3852

Method of Analytical Resolution of the Navier-Stokes Equations

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How to cite this paper: Tougma, J.L.W. (2023) Method of Analytical Resolution of the Navier-Stokes Equations. *Open Journal of Fluid Dynamics*, **13**, 226-231.
https://doi.org/10.4236/ojfd.2023.135017

Received: May 17, 2023
Accepted: December 2, 2023
Published: December 5, 2023

Abstract

In this article, we present a method for solving the Navier-Stokes equations. They started by finding an analytical solution of the nonlinear convective term $\boldsymbol{v}\nabla\boldsymbol{v}$. They solved the Navier Stokes equations as a differential equation. Finally they made a numerical and experimental verification which shows that the two solutions converge, after having found the analytical solution. Underlying principles study, those various phenomena in universe are interconnected logic for the development of new technologies as an example: news engines, applied fluids mechanics. This study's applications are exceptionally wide such as External aerodynamics: airplane, glider, missile, launcher, space probe, automobile, flying insects, buildings and bridges; Hydraulics: pipes, open channels, waves, rivers, blood circulation; meteodynamics: meteorology, climatology.

Keywords

Navier Stokes, Differential Equation, Fluids Mechanics

1. Introduction

Navier-Stokes equations are partial differential equations governing the incompressible fluids motion [1]. These equations constitute the basic equations of fluid mechanics [2]. They are named after Claude Navier and George Stokes, two 19th century physicists. These equations are described as below read [3] [4] [5]:

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = \boldsymbol{f} + \alpha\nabla^2\boldsymbol{v} - \frac{1}{\rho}\nabla P \tag{1}$$

It is difficult to model a fluid as a single-phase continuous medium at the resolution of these equations. The mathematical existence of Navier-Stokes equations solutions was not demonstrated because, it is such a difficult problem to solve and so important (because its equations govern the water flow in a pipe, the ocean currents, the air movements in the atmosphere) [6] [7]. The complication comes from $\boldsymbol{v}\nabla\boldsymbol{v}$ term. This term varies as velocity field square, and it has done the equation mathematically inextricable. We approach this by trying to find an analytical way of solving $\boldsymbol{v}\nabla\boldsymbol{v}$. After numerical verification, the solution and $\boldsymbol{v}\nabla\boldsymbol{v}$ converge. Then we finish by resolving by this solution the Navier-Stokes equations as a differential equation. Finally we find an analytical solution that can help to resolve many fluid mechanics problems.

2. Methods

After a series of numerical and experimental studies [8] [9] [10], we have found that $\boldsymbol{v}\nabla\boldsymbol{v}$ converge as:

$$F(v) = (N-10)\sum_{n=1}^{+\infty}\frac{v^{500n+2}}{n!} - 10^{-2}\sum_{p=0}^{N-10} pv \quad (2)$$

After simulation we obtain this Area 1 for $F(v)$ with v random **Figure 1**: To compare with the Area 2 generates numerically by $\boldsymbol{v}\nabla\boldsymbol{v}$ with v random, we simulate it with python **Figure 2**: We can see that the two areas are the same, which confirms that:

$$\boldsymbol{v}\nabla\boldsymbol{v} = (N-10)\sum_{n=1}^{+\infty}\frac{v^{500n+2}}{n!} - 10^{-2}\sum_{p=0}^{N-10} pv \quad (3)$$

3. Results and Discussion

Then Navier Stokes equations become with $v=\alpha$ the viscosity constant:

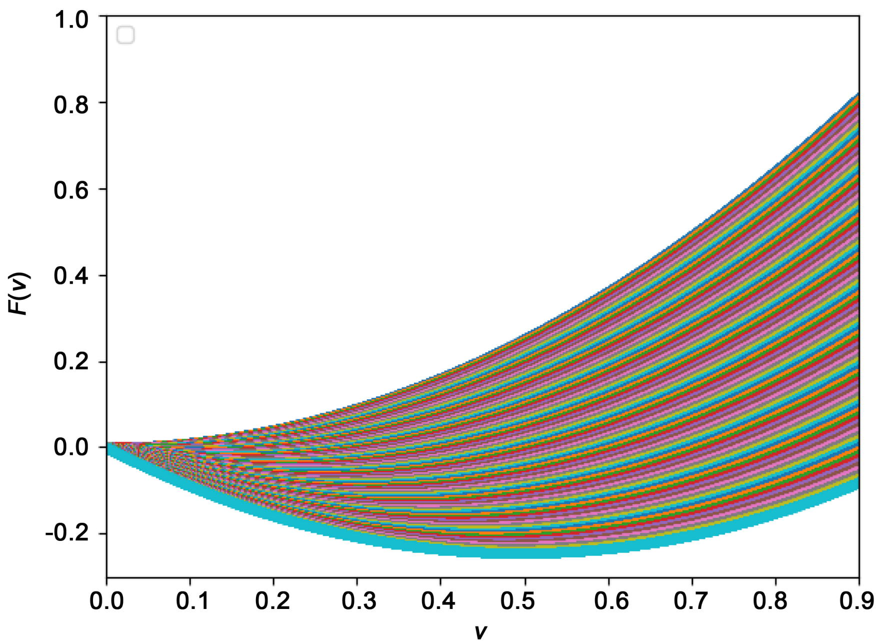

Figure 1. Area 1.

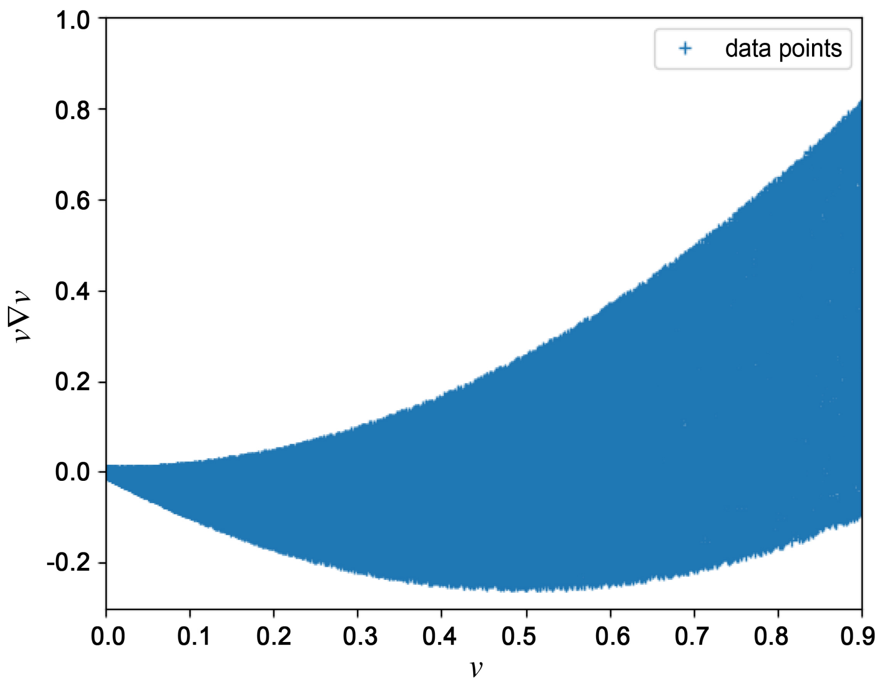

Figure 2. Area 2.

$$\frac{\partial \boldsymbol{v}}{\partial t}+(N-10)\sum_{n=1}^{+\infty}\frac{v^{500n+2}}{n!}-10^{-2}\sum_{p=0}^{N-10}pv=\boldsymbol{f}+\alpha\nabla^2\boldsymbol{v}-\frac{1}{\rho}\nabla P \quad (4)$$

$$\frac{\partial \boldsymbol{v}}{\partial t}+(N-10)\sum_{n=1}^{+\infty}\frac{v^{500n+2}}{n!}-\frac{(N-10)(N-9)}{2\times10^2}v=\boldsymbol{f}+\alpha\nabla^2\boldsymbol{v}-\frac{1}{\rho}\nabla P \quad (5)$$

Let $v=h(t)g(r)$ with $r\in\{x,y,z\}$ a coordinate:

$$\frac{1}{h}\frac{\partial \boldsymbol{h}}{\partial t}+(N-10)\sum_{n=1}^{+\infty}\frac{v^{500n+1}}{n!}-\frac{(N-10)(N-9)}{2\times10^2}=\frac{\boldsymbol{f}}{v}+\alpha\frac{1}{g}\nabla^2\boldsymbol{g}-\frac{1}{v\rho}\nabla P \quad (6)$$

with $\frac{1}{h}\frac{\partial \boldsymbol{h}}{\partial t}=-K^2$; $K^2=\frac{1}{\tau_c}$ the temporal decay, we obtain:

$$h=h_0\exp\left(-\frac{t}{\tau_c}\right) \quad (7)$$

Let's take $\omega^2=\frac{1}{\tau_c}+\frac{(N-10)(N-9)}{2\times10^2}$ we get:

$$\iint f\mathrm{d}r^2+\alpha\iint \mathrm{d}^2v-\frac{1}{\rho}\iint\nabla P\mathrm{d}r^2-(N-10)\sum_{n=1}^{+\infty}\iint\frac{v^{500n+2}}{n!}\mathrm{d}r^2+\omega^2\iint v\mathrm{d}r^2=0 \quad (8)$$

$$\frac{fr^2}{2}+\alpha\frac{v^2}{2}-\frac{Pr}{\rho}-(N-10)\sum_{n=1}^{+\infty}\frac{1}{n!}\iint v^{500n+2}\mathrm{d}r^2+\omega^2\iint v\mathrm{d}r^2=0 \quad (9)$$

with $g=\exp(ar)$ we have $\mathrm{d}v=av\mathrm{d}r$ that implies $\mathrm{d}r=\frac{1}{a}\frac{\mathrm{d}v}{v}$, then we get:

$$a^2\frac{fr^2}{2}+a^2\alpha\frac{v^2}{2}-a^2\frac{Pr}{\rho}-(N-10)\sum_{n=1}^{+\infty}\frac{1}{n!}\iint v^{500n}\mathrm{d}v^2+\omega^2\iint\frac{1}{v}\mathrm{d}v^2=0 \quad (10)$$

$$a^2\frac{fr^2}{2}+a^2\alpha\frac{v^2}{2}-a^2\frac{Pr}{\rho}-(N-10)\sum_{n=1}^{+\infty}\frac{1}{n!(500n+1)(500n+2)}v^{500n+2}$$
$$-\frac{\omega^2 vt}{\tau_c}+a^2\omega^2\int rv\mathrm{d}r=0 \tag{11}$$

$$a^2\frac{fr^2}{2}+a^2\alpha\frac{v^2}{2}-a^2\frac{Pr}{\rho}-(N-10)\sum_{n=1}^{+\infty}\frac{1}{n!(500n+1)(500n+2)}v^{500n+2}$$
$$+\omega^2\left[-\frac{vt}{\tau_c}+a^2\int rv\mathrm{d}r\right]=0 \tag{12}$$

$$\frac{1}{\omega^2}\left[a^2\frac{fr^2}{2}+a^2\alpha\frac{v^2}{2}-a^2\frac{Pr}{\rho}-(N-10)\sum_{n=1}^{+\infty}\frac{1}{n!(500n+1)(500n+2)}v^{500n+2}\right]$$
$$-\frac{vt}{\tau_c}+a^2\int rv\mathrm{d}r=0 \tag{13}$$

$$\frac{D\rho}{Dt}=-av \tag{14}$$

$$\left(\frac{\partial}{\partial t}+v.\nabla\right)\rho=-av \tag{15}$$

$$\left(\frac{\partial}{\partial t}+v.\nabla\right)\frac{\rho}{v}=-a \tag{16}$$

$$-\left(\frac{\partial}{\partial t}+v.\nabla\right)\frac{\rho}{v}=a \tag{17}$$

with $\lambda=a$, we get:

$$\begin{cases}\lambda_{liquid}=-\left(\frac{\partial}{\partial t}+v.\nabla\right)\rho & \text{for a liquid } \rho_{liquid}\gg v_{fluid}\\ \lambda_{gaz}=-\nabla\rho & \text{for a gaz } \rho_{gaz}\le v_{gaz}\end{cases}$$

τ_c must be determined experimentally according the type fluid.

$$v(r\in\{x,y,z\})=\left[\lambda^2\alpha\sum_{n=1}^{+\infty}\frac{\left[2\left(\lambda r-\frac{t}{\tau_c}\right)\right]^n}{2n!}-(N-10)\sum_{n=1}^{+\infty}\sum_{j=1}^{+\infty}\frac{(500n+2)^{j-1}}{n!j!(500n+1)}\left(\lambda r-\frac{t}{\tau_c}\right)^j+\lambda^2\frac{fr^2}{2}-\lambda^2\frac{Pr}{\rho}\right]\frac{\tau_c}{\omega^2 t}$$
$$+\frac{\lambda^2\tau_c}{t}\int\sum_{n=1}^{+\infty}\frac{\left(\lambda r^{1-n}-\frac{tr^{-n}}{\tau_c}\right)^n}{n!}\mathrm{d}r \tag{18}$$

The analytical method only compensates solutions in which non-linear and complex structures in the Navier-Stokes equations are ignored within several assumptions [11] [12] [13]. Our analytical solution is only valid for fundamental cases such as fluids dynamics under an evenly distributed force and constant Volumic mass.

Scope of Future Work

The TOUGMA's solution is crucial to understand the fluid properties. The range of applications is extraordinarily broad [14] [15]:

1) External aerodynamics: airplane, glider, missile, launcher, space probe, automobile, flying insects, buildings and bridges;

2) Enternal aerodynamics: aircraft engines, gas turbines, rocket engines, air conditioning;

3) Hydrodynamics: boat, submarine, marine propulsion;

4) Hydraulics: pipes, open channels, waves, rivers, blood circulation; meteodynamics: meteorology, climatology;

5) Astrophysics: formation of stars, stellar systems, galaxies, the universe, stellar jets;

6) Armament: explosions;

7) Safety: table fires, forest fires and confined fires;

The present investigation will be very helpful to the researchers who are engaged for those area research works in earth and in Universe.

Acknowledgements

The present study received no financial aid from any government/non-government agency.

Conflicts of Interest

There are no conflicts of interests for this study.

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List of Symbols

P: Position

v: Velocity

α : is the fluid coefficient viscosity

τ_c : Experimental time

K^2 : Temporal decay

λ : Flux wave length

l_c : Experimental critical length

ρ : Volumic mass

$\frac{D}{Dt}$: is a material derivative, stated as $\frac{\partial}{\partial t} + v \cdot \nabla$

∇ : Divergence