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Rheology of a Viscous-Plastic Liquid in a Porous Medium

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Abstract

The hydrodynamics of the capillary flow of a viscous-plastic liquid in cylindrical rectilinear pores is considered, as a result of which the structural velocity distribution over the pore cross section is obtained. Analytical solutions are proposed for the equations of hydraulic diffusion and nonlinear filtration for a non-Newtonian fluid in a cylindrical porous medium. It is noted that when a non-Newtonian fluid flows in a porous medium, the filtration equations take a nonlinear form due to the effective viscosity, shear, and yield stresses taken into account in its structure. The proposed solutions make it possible to evaluate the state of the porous medium and its main parameters (permeability, hydraulic diffusion, and effective viscosity coefficients). The obtained solutions are compared with existing experimental data for non-Newtonian oils.

Keywords

Stress and Shear Rate, Rheology, Structure and Structural Flow, Hydraulic Diffusion, Filtration, Porous Media, Effective Viscosity, Permeability

1. Introduction

The flow of non-Newtonian fluids in porous media is accompanied by disordered structure formation in the volume. A special case of structure formation is the formation of disordered structures in non-Newtonian oil due to the presence in it of various particles of natural properties and asphalt-resinous substances [1] [2] [3] [4] [5], leading to changes in the rheological properties of the liquid.

The development and construction of deterministic models of non-stationary phenomena in porous media with more complex internal geometry of pores and channels are limited by significant mathematical difficulties associated with the complex structure and structure of a porous medium characterized by structural anisotropy. The extremely small size of the pore channels, their irregular shape, their random coordination, and dispersion in the volume of the reservoir, and the large surface of the rough walls determine that the geometry of the pore space is not among the measured characteristics of a porous medium obtained by accumulating the main types of local information [1]. The structure (granular, fractured, mixed) and the qualitative and quantitative composition of the reservoir (clay, sand, limestone, dolomites, hard rock, etc.) are associated with the conditions of rock formation, their random distribution, and location in the volume do not allow to unambiguously determine or evaluate the main reservoir parameters (porosity, hydraulic diffusion, and permeability coefficients), on which all hydrodynamic parameters and well productivity depend. In this regard, the properties of the reservoirs of various fields are characterized by a change in porosity and permeability in all directions over a wide range. The hydrodynamic theory of transport in solid porous formations is a quasi-continuum theory, the object of which is continuous media. For each considered real medium, by averaging the characteristics of a given transfer process over a set of discrete elements, average values are introduced that characterize the local continuous medium. Thus, continuous media are not real bodies themselves, but their mathematical models. Given the above, it is advisable to consider a homogeneous isotropic porous medium that does not have a set of directions internally related to the geometry of the pores.

It is known that the motion of a viscous fluid in a porous medium is described by the Navier-Stokes equation. However, due to the inhomogeneity of the porous medium, the use of this equation is difficult. Considering an isotropic porous medium, the authors [6] [7] present the Darcy-Forchheimer filtration equation in the form

$$\left(\nabla P-\rho_c \boldsymbol{f}\right)=\eta_{eff} k_p^{-1}\boldsymbol{V}+\beta\rho_c\left|\boldsymbol{V}\right|\boldsymbol{V} \tag{1}$$

The works [8] [9] [10] [11] present filtration in an isotropic porous medium, described by the Navier-Stokes-Brickman equation, which combines the hydrodynamic equation with the filtration equation

$$-\nabla\left(\eta_{eff}\nabla\boldsymbol{V}\right)+\left(\rho_c\boldsymbol{V}\nabla\right)\boldsymbol{V}+\eta_{rff}k_p^{-1}\boldsymbol{V}=f-\Delta P=\rho_c g-\Delta P \tag{2}$$

Here, f is the external gravitational force.

In this equation, the first and second terms determine the viscous and convective fluid flow, and the subsequent terms characterize the filtration through a porous medium, considering body forces. If we neglect the viscous and convective terms, then we obtain the Darcy filtration equation in the form

$$\eta_{rff}k_p^{-1}\boldsymbol{V}=-\Delta P \tag{3}$$

The transfer of some substance (mass, heat, and momentum) Ψ in the assumed continuous porous medium (inside the reservoir) is described by the following equation.

$$\frac{\partial\varepsilon\Psi}{\partial t}+\operatorname{div}J_\psi=w\left(\Psi\right) \tag{4}$$

Here, $J_{\psi}\left(\Psi\right)$ is the flow of substance in a unit volume of the reservoir, $w\left(\Psi\right)$ is the amount of transported substance formed within a unit volume of the reservoir per unit time. Substituting (4) into expression (3), after a series of simple transformations, for non-Newtonian fluids we obtain [1].

$$\eta_{eff}\left(\tau\right)\left(\beta_c+\varepsilon\beta_p\right)\frac{\partial P}{\partial t}-\frac{\partial}{\partial x_i}\left(\vec{k}_p\frac{\partial P}{\partial x_i}\right)=w \tag{5}$$

Here, $\frac{1}{\rho_c}\frac{\partial\rho}{\partial P}=\beta_c,\frac{\partial\varepsilon}{\partial P}=\beta_p$ —coefficients of isothermal compressibility of liquid and pores. Equations (3) and (5) serve as the basis for calculating the processes of filtration of non-Newtonian fluids in a porous medium.

2. Capillary Flow of a Non-Newtonian Fluid in Pores

Let us consider the flow of a viscous-plastic Bingham fluid, $\tau=\tau_0+\eta\dot{\gamma}$ in cylindrical rectilinear pores in the absence of external forces. The equation of hydrodynamics in this case (2) for the stationary case in cylindrical coordinates will be represented as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)=\frac{1}{\eta_{eff}}\frac{\partial P}{\partial x} \tag{6}$$

The boundary conditions for the flow of a viscous-plastic fluid in a pipe are shown in **Figure 1**.

$$r=R,\tau<\tau_0,\tau=\tau_0+\eta_{eff}\frac{\partial V}{\partial r}=0,V=0 \tag{7}$$

$$r=R,\tau>\tau_0,\tau=\tau_0+\eta_{eff}\frac{\partial V}{\partial r}\neq 0,V=0 \tag{8}$$

Integrating condition (7), we have

$$V=-\frac{\tau_0}{\eta_{eff}}R \tag{9}$$

Integrating condition (8), we obtain

$$V=\frac{\tau_0 R}{\eta_{eff}}\left(1-\frac{\tau}{\tau_0}\right) \tag{10}$$

Integrating expression (6) twice, we obtain

$$V=-\frac{1}{4\eta_{eff}}\frac{\partial P}{\partial x}r^2+A \tag{11}$$

Here A is the integration coefficient, equal to the first boundary condition (7)

$$A=\frac{1}{4\eta_{eff}}\frac{\partial P}{\partial x}R^2-\frac{\tau_0}{\eta_C}R \tag{12}$$

and for the second boundary condition (8)

$$A=\frac{1}{4\eta_{eff}}\frac{\partial P}{\partial x}R^2-\frac{\tau_0 R}{\eta}\left(1-\frac{\tau}{\tau_0}\right) \tag{13}$$

As a result of solving the hydrodynamics of Equation (6) for a viscous-plastic fluid described by the expression $\tau = \tau_0 + \eta \mathrm{d}V/\mathrm{d}r$ in a pore with boundary conditions (7) and (8) and assuming that, $\partial P/\partial x = \Delta P/l$, taking into account (11)-(13), in the simplest case, we obtain the following distribution for the velocity currents (**Figure 1**).

$$\begin{aligned} V(r) &= \frac{\Delta P R^2}{4\eta_{eff} l}\left(1 - \frac{2l}{R}\frac{\tau_0}{\Delta P}\right), r < r_0 \\ V(r) &= \frac{\Delta P R^2}{4\eta_{eff} l}\left[1 - \frac{r^2}{R^2} - \frac{4l}{R}\frac{\tau_0}{\Delta P}\left(1 - \frac{r}{r_0}\right)\right], r_0 \le r < R \end{aligned} \quad (14)$$

where l is the pore length, r_0 is the plug radius. It is important to note that the effective viscosity of a viscous-plastic fluid depends on its composition. For heavy oils, the effective viscosity depends on the content of the solid phase, water droplets and asphalt-tar substances in the oil. The deposition of these particles on the inner surface of the pores leads to the formation of a dense layer of particles on the inner surface of the pores, which leads to a significant change in the flow structure. For the case of the formation of a dense layer of particles on the inner surface of a pore with a thickness δ, which is typical for the flow of a viscous-plastic fluid and, assuming that $R = R_0\beta, \beta = 1 - \frac{\delta}{R_0}$, then these expressions (14) for the quasi-stationary case will be presented as

$$\begin{aligned} V(r) &= \frac{\Delta P R_0^2 \beta^2}{4\eta_{eff} l}\left(1 - \frac{2l}{R_0\beta}\frac{\tau_0}{\Delta P}\right), r < r_0 \\ V(r) &= \frac{\Delta P R_0^2 \beta^2}{4\eta_{eff} l}\left[1 - \frac{r^2}{R_0^2\beta^2} - \frac{4l}{R_0\beta}\frac{\tau_0}{\Delta P}\left(1 - \frac{r}{r_0}\right)\right], r_0 \le r < R_0\beta \end{aligned} \quad (15)$$

It should be noted that if $\beta = 1$, then the thickness of the deposits is absent, at $\beta = 0$, then $\delta = R_0$, there is a complete blockage of the pore and the flow velocity $V(r) \to 0$, *i.e.* throughput is reduced to almost zero.

This distribution of velocities in hydrodynamics is called the "structural regime of motion". Note that the presence of various dispersed particles in non-Newtonian oil leads to the formation of coagulation structures and aggregates that fill the pore space until condition $\tau = \tau_0$ is met, at which the flow

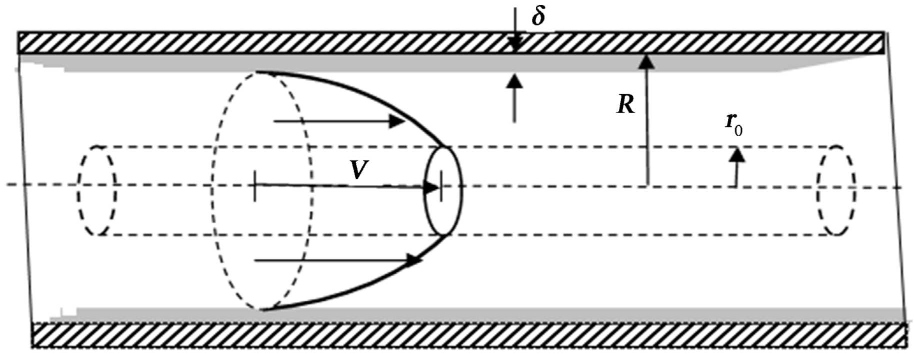

Figure 1. Velocity distribution of a viscous-plastic fluid over the pore cross section.

$V = 0$ completely stops. Obviously, for a system of pores forming a porous medium with a complex geometry, the flow of a viscous-plastic fluid will be described by the equations of hydraulic diffusion and filtration.

3. Equation of Hydraulic Diffusion in a Porous Cylindrical Medium

Hydraulic diffusion Equation (5) characterize the pressure distribution in a porous medium with allowance for diffusion and convective momentum transfer. The existing literature offers various options for solving the equation of cylindrical filtration, based on the Darcy equation [12] [13], for an infinite porous medium.

Having assumed the cylindrical shape of a porous medium (**Figure 2**), the general equation of hydraulic diffusion (5) for its final dimensions will be represented as

$$\varepsilon \frac{\partial P}{\partial t} + V \frac{\partial P}{\partial r} = \frac{\chi}{r} \frac{\partial}{\partial r}\left(r \frac{\partial P}{\partial r} \right) - b(t)\left(P - P_e\right)^m \quad (16)$$
$$r = R_i, P = P_i; r = R_e, P = P_e; t = 0, P = P_0(r)$$

Here, $\vec{\chi} = \dfrac{\vec{k}_p}{\eta_{eff}(\tau)\left(\beta_c + \varepsilon \beta_p\right)}$—coefficient of hydraulic diffusion. This linear differential equation is derived based on the synthesis of the continuity equation, the dynamic filtration equation—Darcy's law, the equation of state of a porous medium and a saturating liquid.

The equation makes it possible to calculate the distribution of pressure along the radius of the porous medium $P_0(r)$ depending on the contour pressure and porosity of the medium. To solve this boundary value problem, we introduce dimensionless variables $\upsilon = \dfrac{V}{V_0}$, $\rho = \dfrac{r}{R_e}$, $\tau = \dfrac{V_0 t}{R_e}$, $u = P - P_e$ considering which we transform Equation (16) to a dimensionless form

$$\varepsilon \frac{\partial u}{\partial \tau} + \upsilon \frac{\partial u}{\partial \rho} = \frac{1}{Ke} \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial u}{\partial \rho} \right) - \gamma(t) u^m \quad (17)$$

with boundary conditions

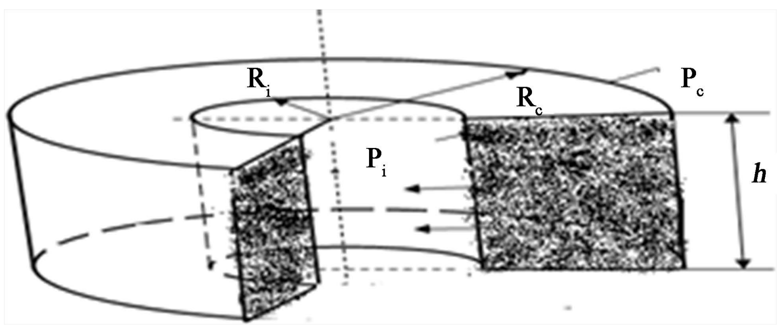

Figure 2. Schematic diagram of an oil reservoir: R_i, R_e—internal and external radii of the medium, respectively, P_i, P_e—internal and external pressure.

$$r = R_c, \rho = \frac{R_i}{R_e}, u = P_i - P_e, r = R_e, \rho = 1, u = 0, \tau = 0, u = P_0(r) - P_e \tag{18}$$

Here, $Ke = \frac{V_0 R_e}{\chi} = ReQ_K$—a criterion that characterizes the ratio of convective transfer to momentum transfer by hydraulic diffusion and represents an analogy of the number Pe in the hydrodynamics of fluid flow and the number Pe in the processes of mass and heat transfer, χ is the hydraulic diffusion coefficient, $Re = \frac{V_0 R_e}{\nu_c}$—the Reynolds number, $\gamma(t) = b(t)V_0/R$, $Q_K = \frac{\nu_s}{\chi}$ is a criterion characterizing the physical properties of the fluid and reservoir, similar to the Schmidt number for mass transfer and the Prandtl number for heat transfer. The value of the criterion determines the nature and area of transfer and filtration of fluid in the reservoir and depends on the flow rate and the coefficient of hydraulic diffusion. It should be noted that for a viscous-plastic fluid, we have $Ke = \frac{\dot{\gamma}R^2}{\chi}$, and for a power-law non-Newtonian fluid we have

$Ke = \frac{k_0 \beta}{k_p} V^n R^{2-n}$, $\beta = \beta_c + \varepsilon \beta_p$, $Re = \frac{\rho_C \dot{\gamma}^{2-n} R^2}{k_0} = \frac{\rho_C V^{2-n} R^n}{k_0}$.

Note that Equation (16) is questioned on the grounds that it comes from experiments with homogeneous isotropic porous media, although in a real situation the porous medium is inhomogeneous, with a characteristic distribution of pores in the volume and their geometric structure inherent in each layer. Obviously, such a simplification often makes it possible to use empirical expressions when modelling these systems.

The solution of Equation (17) with boundary conditions (18) with small convective transfer can be carried out by the method of separation of variables, setting $m = 1$ and introducing the following transformation [14] [15]

$$u(\rho, \tau) = \psi(\tau)\varphi(\rho) \tag{19}$$

Substituting expression (19) into (17) and dividing the variables into separate terms, we obtain the following two equations

$$\begin{aligned} &\frac{\partial \psi}{\partial t} = -\frac{1}{\varepsilon}\left(\gamma(t) + \frac{\mu^2}{Ke}\right) \\ &\frac{\partial^2 \varphi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \varphi}{\partial \rho} + \mu^2 \varphi = 0 \end{aligned} \tag{20}$$

where μ^2 are eigenvalues. The solution of the first Equation (20) will be represented as

$$\psi(\tau) = C_1 \exp\left[-\frac{\mu^2 \tau}{\varepsilon Ke} - \int \frac{1}{\varepsilon}\gamma(\tau)\mathrm{d}\tau\right] \tag{21}$$

The second expression (20) is the zero-order Bessel equation of the real argument. Therefore, the limited solution of the second Equation (20) can be represented as

$$\varphi(\rho)=C_2J_0(\mu\rho) \tag{22}$$

Finally, solution (17), taking into account (20) and (22), will be represented as

$$u(\rho,\tau)=\sum_{n=0}^{\infty}A_nJ_0(\mu_n\rho)\exp\left[-\frac{\mu_n^2\tau}{\varepsilon Ke}-\int\frac{1}{\varepsilon}\gamma(\tau)\mathrm{d}\tau\right] \tag{23}$$

where $J(r)$ is the Bessel function, μ_n are the roots of the equation $J_0(\mu_n)=0$, obtained according to the second condition (18) and are equal to: $\mu_1=2.4048;\mu_2=5.5201;\mu_3=8.6537;\cdots$.

The constant coefficients of the series are determined according to the third condition (9) and the conditions for the orthogonality of the Bessel functions at $R_i \ll R_e$ in the form

$$A_n=\frac{2\int_0^1\rho(P_0(r)-P_e)J_0(\mu_n\rho)\mathrm{d}\rho}{[J_1(\mu_n)]^2} \tag{24}$$

The final general solution (17) can be represented as

$$P(r,t)=P_e+\exp\left(-\int\frac{1}{\varepsilon}\gamma(t)\mathrm{d}t\right)\sum_{n=1}^{\infty}A_nJ_0\left(\mu_n\frac{r}{R_e}\right)\exp\left(-\frac{\mu_n^2\tau}{\varepsilon Ke}\right) \tag{25}$$

Due to the large values of μ_n^2, the series (25) converges rapidly and, therefore, in practical calculations, only the first terms of the series can be used. This allows using simpler expressions to estimate the main hydrodynamic characteristics of a porous medium. Let us consider special cases of solving the filtration equation.

When solving applied problems, Equation (16) under certain assumptions is simplified to a simpler form, and therefore, we will consider special cases of solving Equation (16):

1) Provided that $Ke \ll 1$, *i.e.*, for large values of the hydraulic diffusion coefficient, Equation (16) is transformed into the equation of unsteady cylindrical filtration

$$\varepsilon\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{\chi}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) \tag{26}$$

with boundary conditions

$$t=0,P=P_0(r);r=R_e,P=P_e;r=R_i,P=P_i$$

Solution (26) is obtained, similarly to the above, in the form

$$P(r,t)=P_e+\sum_{n=1}^{\infty}A_nJ_0\left(\mu_n\frac{r}{R_e}\right)\exp\left(-\mu_n^2\frac{\chi}{R_e^2\varepsilon}t\right) \tag{27}$$

Here the coefficients A_n are determined from Equation (24). Having defined the derivative in (27) in the form

$$\frac{\partial P}{\partial r}=-\sum_{n=1}^{\infty}\frac{A_n}{R_e}J_1\left(\mu_n\frac{r}{e}\right)\exp\left(-\mu_n^2\frac{\chi}{R_e^2\varepsilon}t\right) \tag{28}$$

We find the flow rate of the liquid depending on the time at $r=R_i$

$$q = \frac{2\pi h k_p(\varepsilon)}{\eta_{eff}(\tau)} \sum_{n=1}^{\infty} A_n J_1\left(\mu_n \frac{R_i}{R_e}\right) \exp\left(-\mu_n^2 \frac{\chi}{R_e^2 \varepsilon} t\right) \tag{29}$$

2) Consider the stationary solution of the filtration Equation (16), setting $\frac{\mathrm{d}P}{\mathrm{d}t} \approx 0$. Then we have the following equation

$$\frac{\chi}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = 0 \tag{30}$$

$$r = R_e, P = P_e; r = R_i, P = P_i$$

This equation can be represented as

$$\frac{\chi}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = 0 \tag{31}$$

Integrating expression (31) twice, taking into account the boundary conditions, we finally obtain

$$P(r) = P_e - \frac{P_e - P_i}{\ln\left(\frac{R_e}{R_i}\right)} \ln r \tag{32}$$

Let us determine the fluid filtration rate in a porous medium and determine the fluid flow rate in the form

$$q = 2\pi h R_e V = \frac{2\pi h k_p(\varepsilon)}{\eta_{eff}(\tau)} \frac{P_e - P_i}{\ln\left(\frac{R_e}{R_i}\right)} \tag{33}$$

Equation (33) in the relevant literature [16] [17] is called the Dupuis formula, although it is a particular case of solving the stationary filtration Equation (16) for $K_e \ll 1$.

3) For large values of the number of $K_e \gg 1$, *i.e.* for small values of the hydraulic diffusion coefficient, Equation (16) is transformed to the form

$$\varepsilon\frac{\mathrm{d}P}{\mathrm{d}t} = -b(t)(P - P_e)^m$$
$$t = 0, P = P_0 \tag{34}$$

In particular, if $m = 1$, which is confirmed experimentally for many oils, the solution to Equation (34) can be represented as

$$P(t) = P_e - (P_e - P_i)\exp\left[-\int_0^t \frac{1}{\varepsilon} b(t)\,\mathrm{d}t\right] \tag{35}$$

The choice of the structure and type of coefficient $b(t)$ is carried out on the basis of experimental studies of the pressure recovery curve, taking into account the experience and intuition of the researcher. It should be noted that in the existing literature, the experimental data are presented in the coordinates $P(t) \sim \ln t$, based on this, in the first approximation, we can take

$$b(t) = \alpha(\ln t)^m, \int_0^t \alpha(\ln t)^m \,\mathrm{d}(\ln t) = \frac{\alpha}{m+1}(\ln t)^{m+1} \tag{36}$$

Then expression (35) will be presented in the form

$$P(t) = P_e - (P_e - P_i)\exp\left[-m_0(\ln t)^{m+1}\right] = P_e - (P_e - P_i)\exp\left[-\left(\frac{\ln t}{\ln \tau_p}\right)^{m+1}\right] \quad (37)$$

Here, $m_0 = \frac{\alpha}{\varepsilon(m+1)} = \frac{1}{(\ln \tau_p)^{m+1}}$, τ_p —relaxation time. It is important to note that expression (37) is not the only formula for estimating the pressure recovery curve, since other options for determining the coefficient $b(t)$ are also possible. Figure 3 below shows a comparison of experimental data from the literature [3] [17] and our own studies of the pressure distribution curve in an oil well with their calculated values according to Formula (37):
$P(t) = 0.3 - 0.235\exp\left[-0.05(\ln t)^3\right]$, MPa.

The advantage of Equation (37) is that it describes the entire pressure recovery curve as a single equation, which makes it possible to estimate the hydrodynamic properties of the oil reservoir without additional tangents or approximations of the quasi-linear part of the curve. As shown in Figure 3, the time derivative of pressure (curve 2) practically does not become constant anywhere, which indicates the absence of a linear section of the pressure recovery curve. Until now, the estimation of the parameters of a porous medium (the coefficients of hydraulic diffusion and permeability has been carried out by approximating the experimental data of the steady-state region with linear functions, which leads to significant errors. In this case, all information about the pressure recovery curve

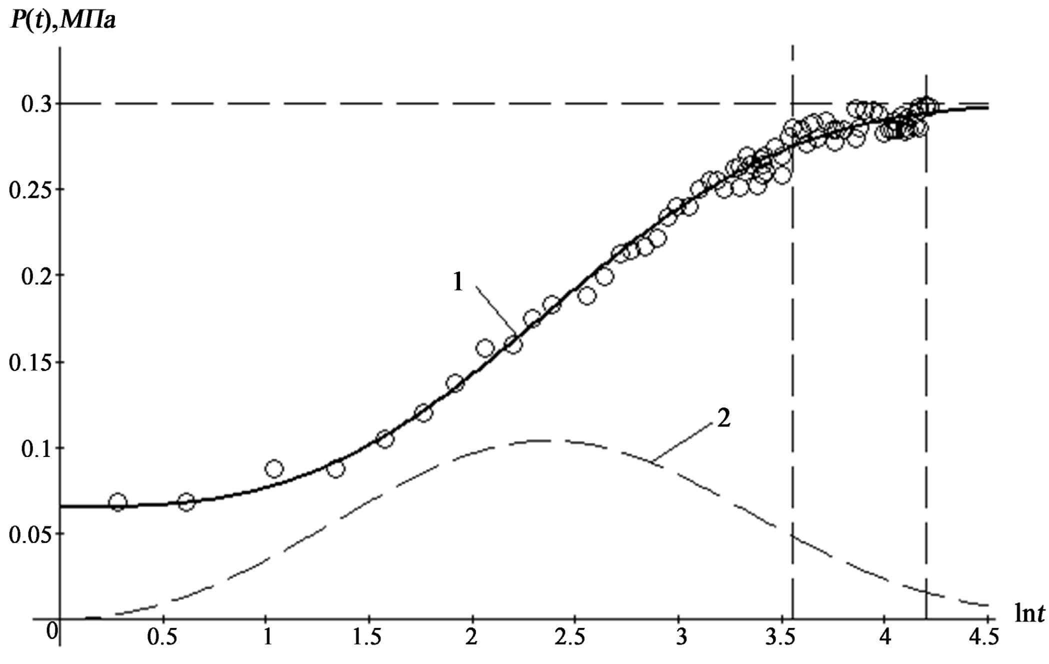

Figure 3. Comparison of the experimental pressure recovery curve [18] with the calculated values (37) ($m = 2, m_0 = 0.05$); 1: pressure recovery curve; 2: derivative of pressure with respect to time.

is not used, and in the steady region of this curve, information about its origin is lost, *i.e.*, the hereditary memory of the curve is lost. Consider the determination of hydrodynamic parameters for a shut-in well using the solution of the filtration equation for a limited reservoir with $Ke \ll 1$. To calculate the hydrodynamic characteristics of the reservoir, it is desirable to use the obtained analytical solutions (25) and (27). Since the series (25) and (27) converge rapidly for the given values of μ_n, it suffices to confine ourselves to the first term and determine that $J_1(2.408) \approx 0.52$, $A_1 = -\frac{2(P_e - P_i)}{0.52 \times 2.408} = -1.5974(P_e - P_i)$. Solution (27) can be written as

$$\frac{P_e - P}{P_e - P_i} = 1.5974 J_0\left(\mu_1 \frac{R_i}{R_e}\right)\exp\left(-5.783\frac{\chi}{\varepsilon R_e^2}t\right) \tag{38}$$

From this equation, assuming that $R_i \ll R_e$ or $\frac{R_i}{R_e} \approx 10^{-4}$ and $J_0(2.408 \times 10^{-4}) \approx 1.0$, by simple transformations, we find an estimate for the effective coefficient of hydraulic diffusion in the form

$$\chi = -\frac{0.1729\varepsilon R_e^2}{t}\ln\frac{0.626(P_e - P)}{P_e - P_i} \tag{39}$$

Using the dependence of pressure on time (**Figure 3**), with the following data: $\ln t = 3.5$, $t = 33.115$ hour = 119214 sec, $R_e = 300\text{m}$, $P_e = 0.3$ MPa, $P_i = 0.06$ MPa, $P = 0.27$ MPa, $\varepsilon \approx 0.2$, from Equation (39) we determine that the coefficient of hydraulic diffusion is. Below in **Figure 4** shows the interpretation of the surface of the change in the coefficient of hydraulic diffusion from time and pressure. Thus, for a sufficiently large area of the pressure recovery curve, the

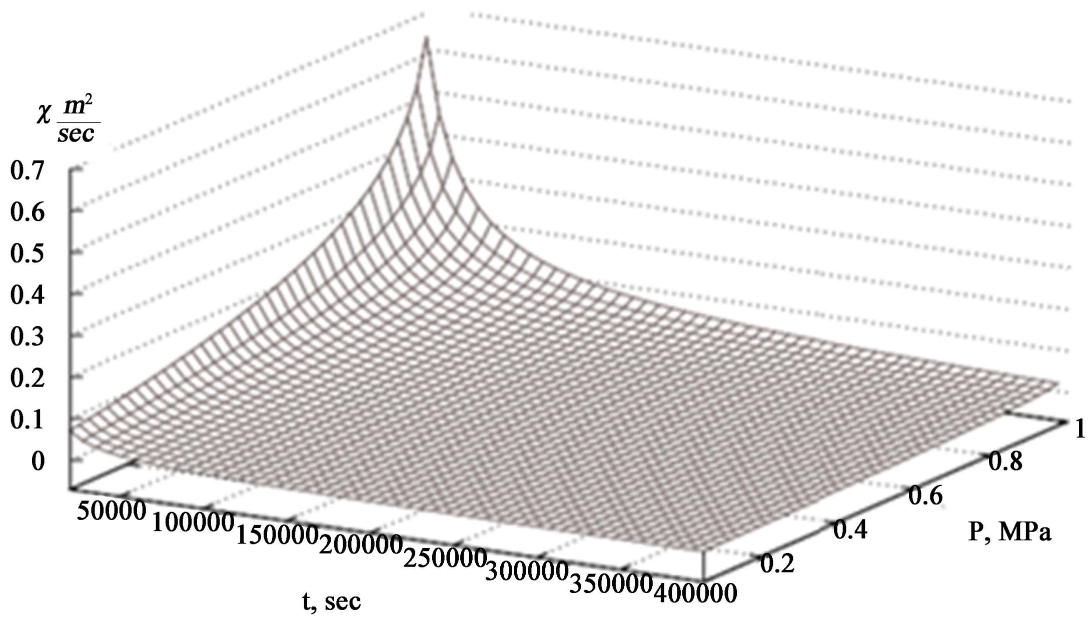

Figure 4. Graphical interpretation of the dependence of the hydraulic diffusion coefficient on time and pressure.

hydraulic diffusion coefficient tends to a constant value for all values of time and pressure, equal to $\chi = 7.181\times10^{-2}\ \mathrm{m}^2/\mathrm{sec}$, although for small values of these parameters it gives a significant deviation. The dimensions of this area are determined from the condition $\frac{\varepsilon R_e^2}{\chi t} > 2.3712$, which corresponds to the condition $\frac{\partial P}{\partial t} < 0$ (**Figure 3**). This means that Formula (39) can be suitable for calculating the hydraulic diffusion coefficient for various values of time and pressure of the pressure recovery curve in the indicated region.

The permeability coefficient, using the value of the hydraulic diffusion coefficient, is determined by the following formula

$$k_p = \chi \eta_{tff} \left(\beta_c + \varepsilon \beta_f\right)$$

4. Filtration Rate of a Viscous-Plastic Liquid in a Porous Medium

Anomalous viscous-plastic fluids differ in their properties from ordinary fluids and their rheological description obeys the laws of flow of non-Newtonian Bingham fluids.

$$\tau = \tau_0 + \eta \dot{\gamma} \tag{40}$$

Expressing the viscosity of a viscous-plastic fluid in terms of the effective viscosity in the form [1] [2]

$$\eta_{eff} = \eta \frac{\tau - \tau_0}{\tau -}, \tag{41}$$

we obtain a nonlinear filtration equation for a structured oil system

$$V_p = -\frac{k_p}{\eta_{eff}}\left(1 - \frac{\tau_0}{\tau}\right)\frac{\partial P}{\partial x} \tag{42}$$

At $\tau \gg \tau_0$, this expression transforms into the usual Darcy equation for unstructured oil. An analysis of experimental data on the filtration of non-Newtonian oils made it possible to approximate the ratio $\frac{\tau_0}{\tau}$ in the form

$$\ln\frac{\tau}{\tau_0} = \alpha\left(\frac{\mathrm{grad}\,P}{\left(\mathrm{grad}\,P\right)_0}\right)^n \tag{43}$$

Here, α is a coefficient determined based on experimental data, n is an exponent. The rheological equation can be written in the form (**Figure 5**).

$$\tau = \tau_0 \exp\left(\alpha\left(\dot{\gamma}/\dot{\gamma}_0\right)^n\right) \tag{44}$$

Expression (41) can be considered as a new rheological equation describing the viscous-plastic flow of non-Newtonian fluids. Obviously, the exponent n, depending on the temperature and properties of the porous layer, characterizes the complete destruction of the structure. To date, many concepts and models have been put forward to describe the shear flow of oil dispersed systems, resulting in a wide variety of rheological dependences of effective viscosity on shear stress τ and shear rate $\dot{\gamma}$.

Using the experimental data of the work [3] and Equations (42) and (43), we represent the filtration rate in the following form

$$V_p = K_2(T)\left(1-\exp\left(-\alpha_2(T)(z/z_0)^6\right)\right)z \tag{45}$$

here $\alpha_2 = 0.1422\exp(-0.0247T)$, $K_2(T) = 1.4\times10^{-5}\exp(0.0364T)$, $z = \text{grad}P$, $z_0 = (\text{grad}P)_0$, $K_2 = k_p/\eta_{eff}$ —fluid mobility. The high value of the exponent is explained by a sharp drop in viscosity during the destruction of the formed structure. Figure 6 below shows a comparison of the calculated (45) and experimental values of the filtration rate of abnormal oils at different

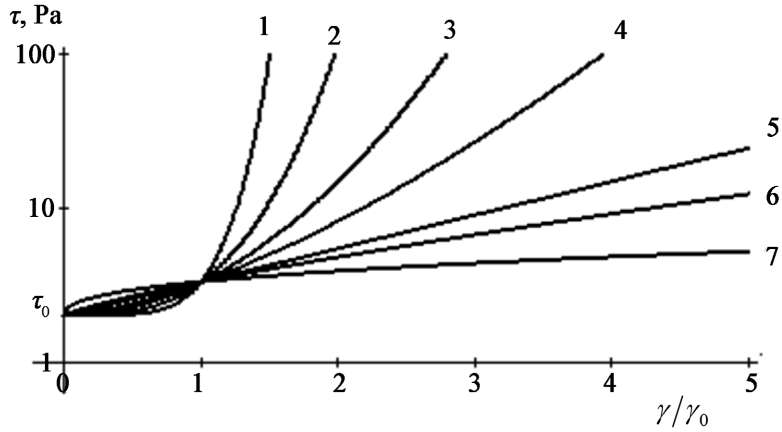

Figure 5. Dependence of shear stress on shear rate at n, equal to: 1—5.0; 2—3.0; 3—2.0; 4—1.5; 5—1.0; 6—0.8; 7—0.4.

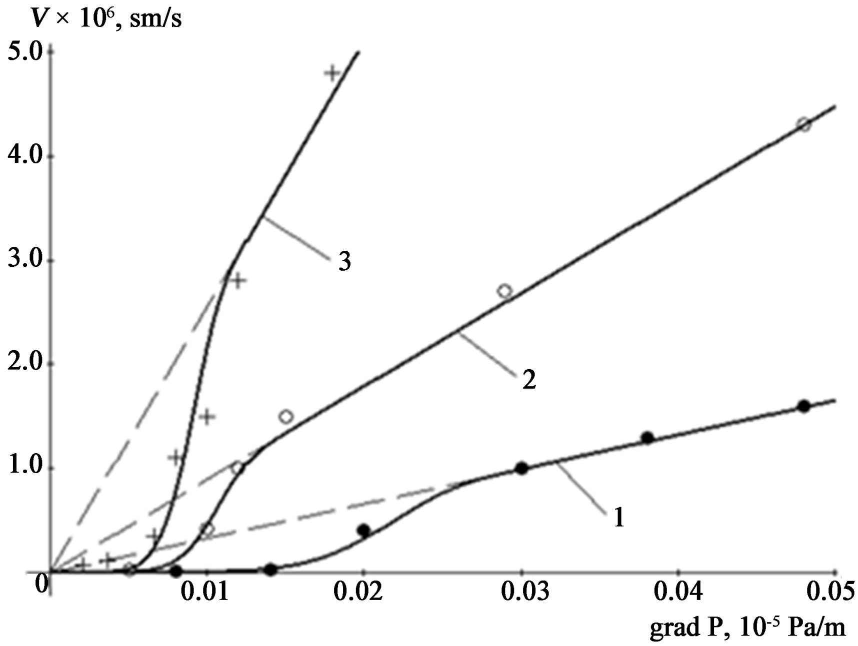

Figure 6. Change in filtration rate of abnormal oils for different temperatures: 1—T = 24˚C; 2—T = 50˚C; 3—T = 80˚C.

temperatures for different fields [3].

The change in the effective viscosity of anomalous oil from the pressure gradient based on experimental data and expression (45) is determined by the empirical formula

$$\eta_{eff} = (\eta_0 - \eta_\infty)\exp(-30z^6) + 28\exp(-26.65z) + \eta_\infty \tag{46}$$

here, η_0, η_∞—initial ($\tau \le \tau_0$) and final oil viscosity ($\tau \gg \tau_0$).

In **Figure 7**, the solution of Equation (46) is compared with the experimental data at a temperature of $T = 24^\circ\text{C}$ [3].

As follows from Fig., at low flow rates, the effective viscosity of the abnormal oil depends on the shear rate or on the pressure gradient, and at $\tau_0 < \tau \le \tau_P$, the effective viscosity decreases from the maximum η_0 value η_∞ to the minimum and then stabilizes. A sharp decrease in the effective viscosity of non-Newtonian oil indicates an instantaneous destruction of the structure with increasing shear stress and a high nonlinear dependence on shear stress or pressure gradient.

As follows from **Figure 7**, the ratio $\dfrac{k}{\eta_{eff}}$ or the mobility of oil at shear stress values $\tau \le \tau_0$ increases very slowly and practically remains constant; at $\tau_0 < \tau \le \tau_P$. The mobility of oil increases intensively to a maximum value, and the transition from the minimum to the maximum value occurs in a narrow range of pressure gradient changes and stabilizes only at pressure gradient values corresponding to the limiting destruction of the structure $\tau > \tau_P$.

5. Results and Discussions

Problems of the rheology of viscous-plastic fluids associated with the solution of various problems of hydrodynamics and filtration are considered. A solution to

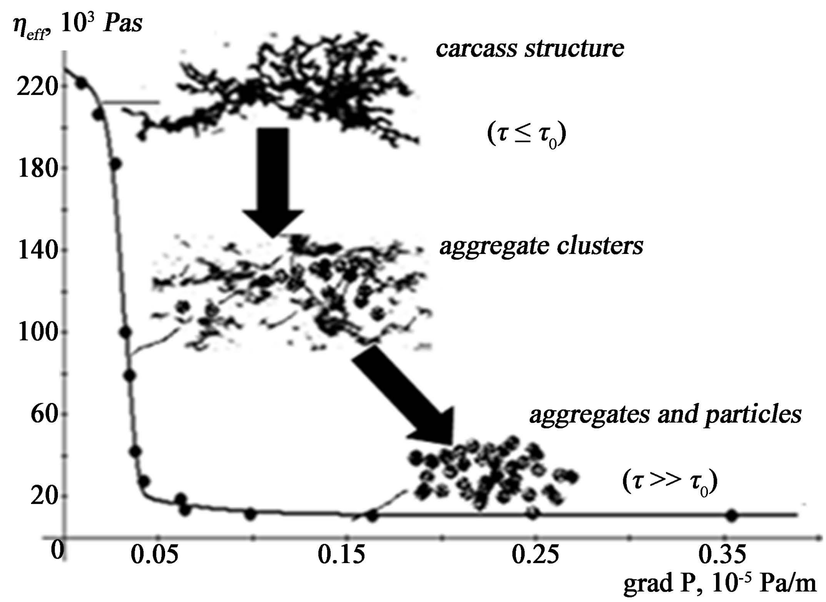

Figure 7. Dependence of viscosity on the pressure gradient during the destruction of disordered structures in oil.

the problems of the flow of viscous-plastic liquids in capillary pores is proposed, as a result of which the structure of the flow and the distribution of the flow velocity in various regions are determined (14). These equations consider the formation of a dense layer of particles on the inner surface of the pores and their influence on the distribution of the flow velocity over the cross section (15). A solution to the equation of hydraulic diffusion of viscous-plastic fluids for a cylindrical porous medium is proposed, taking into account the rheological properties (25) that are important for estimating the coefficients of hydraulic diffusion (39) and the permeability of the porous medium. A nonlinear equation for the filtration of viscous-plastic fluids (42) and a rheological model are proposed that consider the effective viscosity and shear stress (44). All proposed models are tested on experimental data for non-Newtonian oil. The proposed models allow for solving important applied problems related to the transport and production of viscous-plastic oil in porous media.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Symbols

k_p	permeability coefficient
R	radius
υ	immeasurable speed
P	pressure
T	temperature
V	fluid velocity
V_0	average speed
V_p	filtration rate
β	dimensionless deposit thickness
β_c, β_f	the elasticity coefficients of the liquid and the skeleton of the porous medium
ρ_C	the density of the liquid
r	current radius
χ	hydraulic diffusion coefficient
δ	particle layer thickness
ε	porosity; medium density
$\dot{\gamma}$	shear rate
η	viscosity
τ	shear stress
ε_{eff}	the effective viscosity of the liquid
τ_0	yield point
τ_p	relaxation time