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The Onset of Buoyancy and Surface Tension Driven Convection in a Ferrofluid Layer by Influence of General Boundary Conditions

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Abstract

This paper investigated the buoyancy and surface tension-driven ferro-thermal-convection (FTC) in a ferrofluid (FF) layer due to influence of general boundary conditions. The lower surface is rigid with insulating to temperature perturbations, while the upper surface is stress-free and subjected to general thermal boundary condition. The numerically Galerkin technique (GT) and analytically regular perturbation technique (RPT) are applied for solving the problem of eigenvalue. It is analyzed that increasing Biot number, decreases the magnetic and Marangoni number is to postponement the onset. Additionally, magnetization nonlinearity parameter has no effect on FTC in the non-existence of Biot number. The results under the limiting cases are found to be in good agreement with those available in the literature.

Keywords

Marangoni Number, Ferrothermal Convection, Insulating, Regular Perturbation Technique, Galerkin Technique

1. Introduction

Until recently, there were liquids which could be magnetized to be comparable with the magnetization of magnetic nanoparticles. They have developed colloidal suspensions containing magnetic nanoparticles with a carrier liquid like water, hydrocarbon such as mineral oil or kerosene, or fluorocarbon referred as ferrofluids (FFs). Hence, FFs subjects have obtained much attention among the scientific communities [1] [2] [3] [4]. The magnetization of FFs depends on its magnetic field, temperature and density. Whereas when a horizontal FF layer is present with a magnetic field, it is heated from below and convective motions might take place which is called as FTC [5].

Thereby, FTC can also be induced by providing surface-tension and later with the function of temperature. Qin and Kaloni [6] have investigated both linear and non-linear stability of combined effects of buoyancy and surface tension forces in a FF layer. Hennenberg *et al.* [7] have examined the coupling effects on Marangoni and Rosensweig instabilities by considering two semi-infinite immiscible and incompressible viscous fluids. The results of different basic temperature gradients on FTC which is driven by buoyancy and surface tension forces discussed by Shivakumara *et al.* [8] with an plan following indulgent control of FTC concept. Shivakumara and Nanjundappa [9] have also examined the initiation of Marangoni FTC with differing initial temperature gradients. A very less number of researches address the effects of Bouyancy and surface tension forces on FTC (see [10] [11]) with viscosity variations ([12] [13] [14] [15]), heat source strength ([16] [17]) and Coriolis force ([18] [19] in a FF layer. Later, Shivakumara *et al.* [20] studied the onset of FTC in a horizontal FF layer with temperature dependent viscosity in exponentially. In many natural phenomena, the study of penetrative FTC in a saturated porous layer is studied by Nanjundappa *et al.* [21] with the internal heating source and applied Brinkman extended Darcy model in the momentum equation. Nanjundappa and co-workers ([22] [23] [24]) analyzed the internal heat generation effect on the onset of FTC in a FF saturated porous layer. Recently, Savitha *et al.* [25] investigated the penetrative FTC in a FF-saturated high porosity anisotropic porous layer via uniform internal heating.

The intent of the present work is to investigate Bénard-Marangoni FTC in a FF layer due to influence of general boundary conditions. The numerically Galerkin technique (GT) and analytically regular perturbation technique (RPT) are applied for solving the problem of eigenvalue when both the surfaces insulated to temperature perturbations.

2. Formulation of the Problem

Consider an incompressible FF horizontal layer of thickness d with temperatures $-k_1 \partial T/\partial z = q_0$ ($z = 0$) and $k_1 \partial T/\partial z = h_1 (T - T_\infty)$ ($z = d$). Where T is the temperature, q_0 is the conductive thermal flux, k_1 the overall thermal conductivity, h_t the heat transfer coefficient and T_∞ the temperature in the bulk of the environment.

Cartesian coordinates (x, y, z) system are chosen (see **Figure 1**). Gravity acts vertically downwards and is given by $\vec{g} = -g\hat{k}$, where $\hat{k}$ is the unit vector in the z-direction. The layer is bounded below by a rigid surface and above by a non-deformable free surface. At the upper free surface, the surface tension σ is assumed to vary linearly with temperature in the form

$$\sigma = \sigma_0 - \sigma_T (T - T_0) \tag{1}$$

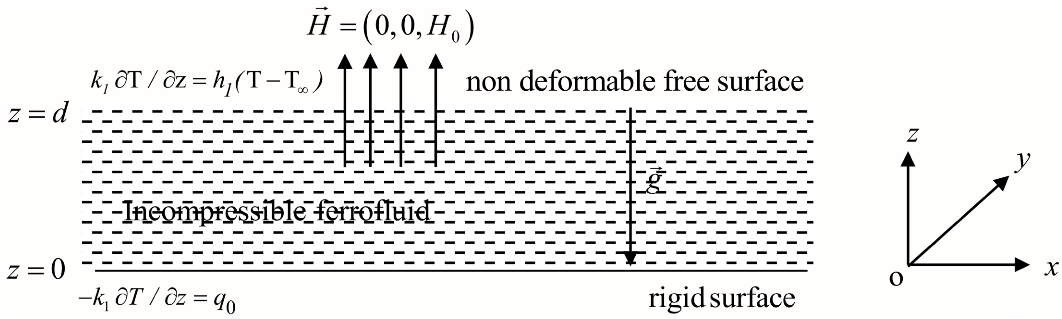


Figure 1. Physical configuration.

where σ_0 is the unperturbed value and $-\sigma_T$ is the rate of change of surface tension with temperature *T*. The fluid density ρ is assume to vary linearly with temperature in the form

$$\rho = \rho_0 \left[1 - \alpha_t \left(T - T_0\right)\right] \tag{2}$$

where α_t is the thermal expansion coefficient and ρ_0 is the density at $T = T_0$. The governing equations for the flow of an incompressible fluid are

$$\nabla \cdot \vec{q} = 0 \tag{3}$$

where $\vec{q} = (u, v, w)$ is the velocity vector.

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right] = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{q} + \mu_0 \left(\vec{M} \cdot \nabla\right)\vec{H} \tag{4}$$

where p is the pressure, t is the time and μ_0 the magnetic permeability of vacuum.

$$\left[\rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right] \frac{DT}{Dt} + \mu_0 T \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H} \cdot \frac{D\vec{H}}{Dt} = k_t \nabla^2 T \tag{5}$$

where C is the specific heat, $C_{V,H}$ is the specific heat at constant volume and magnetic field, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator.

The magnetic field ($\vec{H}$) in magnetic fluid obeys the Maxwell equations in the absence electric field and current are

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{H} = 0 \quad \text{or} \quad \vec{H} = \nabla \varphi \tag{6a,b}$$

where $\vec{B}$ is the magnetic induction and φ is the magnetic potential.

$$\vec{B} = \mu_0 \left(\vec{M} + \vec{H}\right) \tag{7}$$

Since the magnetization ($\vec{M}$) depends on the magnitude of magnetic field and temperature, we have

$$\vec{M} = \frac{\vec{H}}{H} M(H, T). \tag{8}$$

The linearized equation of magnetic state about H_0 and T_0 is

$$M = M_0 + \chi (H - H_0) - K (T - T_0) \tag{9}$$

where $\chi = (\partial M / \partial H)_{H_0, T_0}$ is the magnetic susceptibility, $K = -(\partial M / \partial T)_{H_0, T_0}$ is the pyromagnetic co-efficient and $M_0 = M(H_0, T_0)$.

It is clear that there exists the following solution for the basic state:

$$\vec{q}_b = 0, \quad p_b(z) = p_0 - \rho_0 g z - \frac{1}{2}\rho_0 \alpha_t g \beta z^2 - \frac{\mu_0 M_0 \kappa \beta}{1+\chi} z - \frac{\mu_0 \kappa^2 \beta^2}{2(1+\chi)^2} z^2$$

$$T_b(z) = T_0 - \beta z, \quad \vec{H}_b(z) = \left[H_0 - \frac{K\beta z}{1+\chi}\right]\hat{k}, \quad \vec{M}_b(z) = \left[M_0 + \frac{K\beta z}{1+\chi}\right]\hat{k} \tag{10}$$

where $\beta = \Delta T/d$ is the temperature gradient and the subscript b denotes the basic state.

To study the stability of the system, we perturb all the variables in the form

$$\vec{q} = \vec{q}\,', \ p = p_b(z) + p', \ T = T_b(z) + T', \ \vec{H} = \vec{H}_b(z) + \vec{H}', \ \vec{M} = \vec{M}_b(z) + \vec{M}' \tag{11}$$

where $\vec{q}\,'$, p', T', $\vec{H}'$ and $\vec{M}'$ are perturbed variables and are assumed to be small.

Substituting Equation (11) into Equations (8) and (9), and using Equation (7), we obtain (after dropping the primes)

$$\begin{aligned} H_x + M_x &= (1 + M_0/H_0) H_x, \\ H_y + M_y &= (1 + M_0/H_0) H_y, \\ H_z + M_z &= (1+\chi) H_z - KT. \end{aligned} \tag{12}$$

Again substituting Equation (11) into momentum Equation (4), linearizing, eliminating the pressure term by operating curl twice and using Equation (12) the z-component of the resulting equation can be obtained as (after dropping the primes):

$$\left(\rho_0 \frac{\partial}{\partial t} - \mu \nabla^2\right)\nabla^2 w = -\mu_0 K \beta \frac{\partial}{\partial z}\left(\nabla_h^2 \varphi\right) + \frac{\mu_0 K^2 \beta}{1+\chi}\nabla_h^2 T + \rho_0 \alpha_t g \nabla_h^2 T \tag{13}$$

where $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian operator. The temperature Equation (5), after using Equation (11) and linearizing, takes the form (after dropping the primes):

$$\frac{\partial T}{\partial t} - \mu_0 T_0 K \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z}\right) = k_1 \nabla^2 T + \left[\rho_0 C_0 - \frac{\mu_0 T_0 K^2}{1+\chi}\right] w\beta \tag{14}$$

where $\rho_0 C_0 = \rho_0 C_{V,H} + \mu_0 H_0 K$. Equations 6(a, b), after substituting Equation (11) and using Equation (12), may be written as (after dropping the primes)

$$\left(1 + \frac{M_0}{H_0}\right)\nabla_h^2 \varphi + (1+\chi)\frac{\partial^2 \varphi}{\partial z^2} - K\frac{\partial T}{\partial z} = 0. \tag{15}$$

The normal mode expansion of the dependent variables is assumed in the form

$$\{w, T, \varphi\} = \{W(z), \Theta(z), \Phi(z)\}\exp\left[i(\omega t + \ell x + m y)\right] \tag{16}$$

where ℓ and m are wave numbers in the x and y directions, respectively, and ω is the growth rate with is complex. On substituting Equation (16) into Equations (13)-(15) and non-dimesionalizing the variables by setting

$$z^* = \frac{z}{d},\ w^* = \frac{d}{\nu}w,\ t^* = \frac{\nu}{d^2}t,\ \Theta^* = \frac{\kappa}{\beta \nu d}\Theta,\ \Phi^* = \frac{(1+\chi)\kappa}{K\beta \nu d^2}\Phi \tag{17}$$

where $\nu = \mu/\rho_0$ is the kinematic viscosity and $\kappa = k_1/\rho_0 C_0$ is the effective thermal diffusivity, we obtain (after dropping the asterisks for simplicity)

$$\left[D^2 - a^2 - \omega\right]\left(D^2 - a^2\right)W + a^2\left[Ra_m D\Phi - \left(Ra + Ra_m\right)\Theta\right] = 0. \tag{18}$$

$$\left(D^2 - a^2 - \omega Pr\right)\Theta + W = 0. \tag{19}$$

$$\left(D^2 - M_3 a^2\right)\Phi - D\Theta = 0. \tag{20}$$

Here, W, Θ, Φ are respectively the z-component perturbed amplitudes of velocity, temperature and magnetization term. In addition $D \equiv \mathrm{d}/\mathrm{dz}$ differential operator, $a = \sqrt{l^2 + m^2}$ wave number, $Ra = \alpha_t g \beta d^4$ thermal Rayleigh number, $M_1 = \mu_0 K^2 \beta/(1+\chi)\alpha_t \rho_0 C$ magnetic number, $Ra_m = RaM_1 = \mu_0 K^2 \beta^2 d^4/(1+\chi)\mu\kappa$ magnetic thermal Rayleigh number, $M_2 = \mu_0 T_0 K^2/(1+\chi)\rho_0 C$ magnetic parameter, $M_3 = (1 + M_0/H_0)/(1+\chi)$ magnetization nonlinearity parameter and $Pr = \nu/\kappa$ Prandtl number.

We impose the boundary conditions (see Ref. [5] [12] [26]):

$$W = DW = 0,\ D\Theta = 0,\ \Phi = 0 \text{ at } z = 0 \tag{21}$$

$$W = D^2 W + a^2 Ma\,\Theta = 0,\ D\Theta + Bi\,\Theta = 0,\ D\Phi = 0 \text{ at } z = 1 \tag{22}$$

or

$$W = D^2 W + Ma\, a^2 \Theta = 0,\ D\Theta + Bi\,\Theta = 0,\ D\Phi - \Theta = 0 \text{ at } z = 1 \tag{23}$$

where, $Ma = \sigma_T \Delta T d/\mu\kappa$ the Marangoni number and $Bi = h_t d/k_1$ the Biot number.

3. Numerical Solution

The Galerkin method is applied to solve the problem of eigenvalue constituted by Equations (18)-(20) subject to Equations (21)-(23) and accordingly the expanded unknown variables are

$$\{W, \Theta, \Phi\}(z) = \sum_{m=1}^{n} \{A_m W_m, B_m \Theta_m, C_m \Phi_m\}(z) \tag{24}$$

where A_m, B_m, C_m are constants and basis functions W_m, Θ_m, Φ_m are trial chosen usually satisfying the considered boundary conditions as follows

$$W_m = \left(z^3 - 5z^2/2 + 3z/2\right)z^m,\ \Theta_m = z\left(1 - z/2\right)z^m,\ \Phi_m = z^2\left(1 - z/3\right)z^m. \tag{25}$$

By introducing Equation (25) into Equations (18)-(20), multiplying the resulting equations respectively by W_m, Θ_m and Φ_m, integrating between $z = 0$ and $z = 1$ and using Equations (21)-(23) yields

$$C_{nm} A_m + M_{nm} B_m + F_{nm} C_m = 0. \tag{26}$$

$$G_{nm} A_m + H_{nm} C_m = 0. \tag{27}$$

$$I_{nm} C_m + J_{nm} D_m = 0. \tag{28}$$

where

$$C_{mn} = \left\langle D^2W_mD^2W_n \right\rangle + 2a^2 \left\langle DW_mDW_n \right\rangle + a^4 \left\langle W_mW_n \right\rangle,$$
$$M_{mn} = -a^2Ra\left(1+M_1\right)\left\langle W_m\Theta_n \right\rangle + a^2MaDW_m\left(1\right)\Theta_n\left(1\right)$$
$$F_{mn} = a^2RaM_1\left\langle W_mD\Phi_n \right\rangle$$
$$G_{mn} = -\left\langle \Theta_mW_n \right\rangle,$$
$$H_{mn} = \left\langle a^2\Theta_m\Theta_n + D\Theta_mD\Theta_n \right\rangle + Bi\,D\Phi_m\left(1\right)\Theta_n\left(1\right)$$
$$I_{mn} = -\left\langle D\Phi_m\Theta_n \right\rangle$$
$$J_{mn} = \left\langle a^2M_3\Phi_m\Phi_n + D\Phi_mD\Phi_n \right\rangle$$

with $\left\langle \cdots \right\rangle = \int_0^1 \left(\cdots\right)\mathrm{d}z$

Equations (26)-(28) may have a solution of non-trivial solution if

$$\begin{vmatrix} C_{nm} & D_{nm} & E_{nm} \\ F_{nm} & G_{nm} & 0 \\ 0 & H_{nm} & I_{nm} \end{vmatrix} = 0. \tag{29}$$

It would be informative to seem at the results for $m = n = 1$ as it gives adequate physical insight into the problem with minimum mathematical computations. For this order, Equation (29) in terms of Ma gives the following characteristic equation (after omitting the subscript 1)

$$Ma = \frac{\eta_1 + \Omega\eta_2}{1260a^2\left\langle W\Theta \right\rangle}\left[\frac{70Bi}{\eta_3} + \left(a^2 + \Omega Pr\right)\right] + \frac{140R_m\left\langle WD\varphi \right\rangle}{\eta_3} - 2\left(R_a + R_m\right)\left\langle W\Theta \right\rangle \tag{30}$$

where $\eta_1 = 4536 + 432a^2 + 541a^4$, $\eta_2 = 216 + 541a^2$ and $\eta_3 = 56 + 11M_3a^2$.

To stability of the system is examined by taking $\Omega = i\omega$ in Equation (30) and the complex quantities have to be clearly yields

$$\begin{aligned} Ma = {} & \frac{1}{1260a^2\left\langle W\Theta \right\rangle}\left[\eta_1\left(\frac{70Bi}{\eta_3} + a^2\right) - Pr\eta_2\omega^2\right] \\ & + \frac{140R_m\left\langle WD\varphi \right\rangle}{\eta_3} - 2\left(R_a + R_m\right)\left\langle W\Theta \right\rangle \end{aligned} \tag{31}$$

where

$$N = \frac{1}{1260a^2\left\langle W\Theta \right\rangle}\left[Pr\eta_1 + \eta_2\left(\frac{70Bi}{\eta_3} + a^2\right)\right].$$

The steady onset (*i.e.*, direct bifurcation) is governed by $\omega = 0$ and it occurs at $Ma = Ma^s$, where

$$Ma = \frac{1}{1260a^2\left\langle W\Theta \right\rangle}\left[\eta_1\left(\frac{70Bi}{\eta_3} + a^2\right)\right] + \frac{140R_m\left\langle WD\Phi \right\rangle}{\eta_3} - 2\left(R_m + R_a\right)\left\langle W\Theta \right\rangle \tag{32}$$

4. Numerical Results and Discussion

Equation (29) leads to characteristic equation

$$f\left(Ra, Ma, Bi, M_1, M_3, a\right) = 0. \tag{33}$$

Here we note that the minimum of Ra corresponding to a_c is to be found that for various physical parameters Ma, Bi, M_1 and M_3. Mathematica 12.0 symbolic algebraic package is applied to compute numerically by Galerkin method for fixing the other parameters with three sets of boundary combinations. The value of $\left(Ra_c, a_c\right)$ obtained here are compared with Sparrow *et al.* [27]. The results established are in admirable agreement and thus validate the exactness of the numerical technique utilized (see **Table 1**).

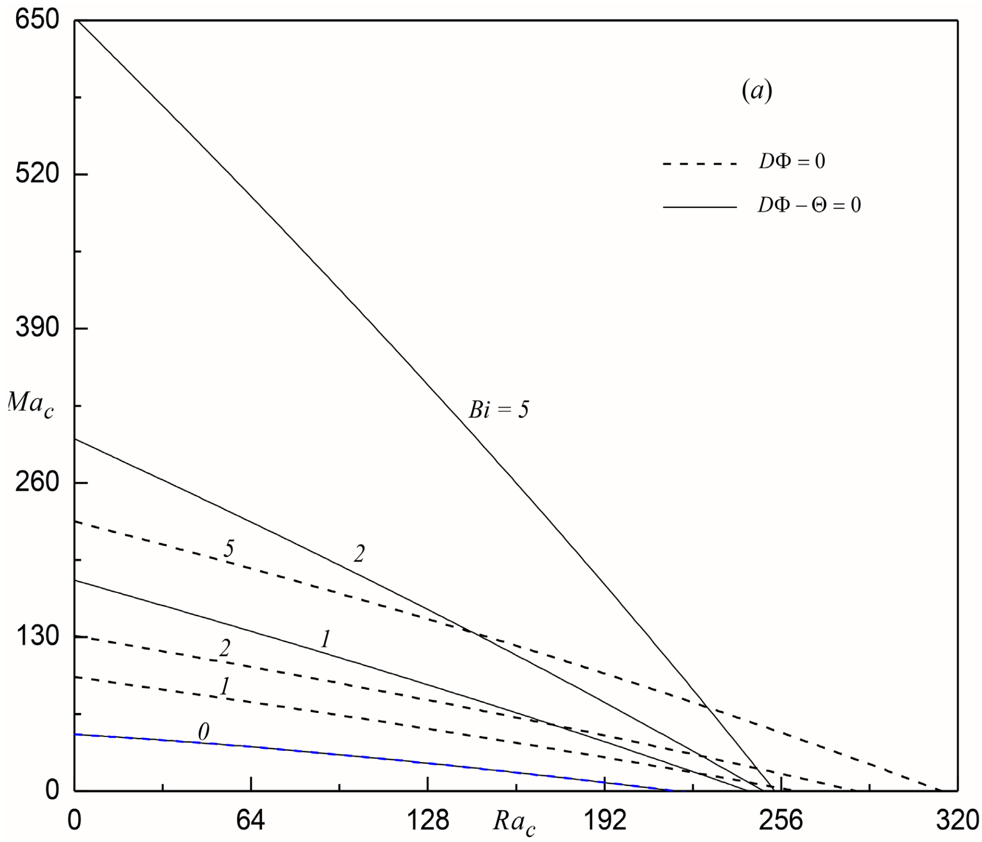
The loci of Ma_c and Ra_c with Bi, M_1 and M_3 are shown in **Figures 2(a)-4(a)** respectively as well as different magnetic boundaries at the upper surfaces like $D\Phi - \Theta = 0$ and $D\Phi = 0$ at $z = 1$. It is noticeable that, curves are slightly convex and there is a strong coupling between Ma_c and Ra_c. If the magnetic force is leading, then the surface tension becomes insignificant and vice-versa. A review of **Figure 2(a)**, further reveals that with increase in Bi it delays the FTC. This may be attributed to fact that with increasing Bi, the free surface gets deviated from good conductor of heat and there is an increase in Ma_c and Ra_c. Also $D\Phi - \Theta = 0$ surfaces offer more stabilizing effect compared to $D\Phi = 0$ against FTC. **Figure 2(b)** illustrates that increasing in a_c as Bi increases, hence its effect is to diminish the size of convection cells.

In **Figure 3(a)**, Ra_c and Ma_c presented with M_1 when $M_3 = 1$ and $Bi = 2$. This is expected that an increase in M_1 is to decrease Ma_c and Ra_c, thus leads to a more unstable system due to an increase in magnetic force. Moreover, it is remarkable Ma_c and Ra_c are diminishes as M_1 increases. From **Figure 3(b)**, increase M_1 is to increase a_c, thus leading to diminish the convection cell size..

The effect of increase in M_3 is shown in **Figure 4(a)** for $Bi = M_1 = 2$ and it is

Table 1. Comparison of $\left(Ra_c, a_c\right)$ with Bi for $Ma = M_1 = 0$.

Bi	Sparrow *et al.* [27]		Present study	
	Ra_c	a_c	Ra_c	a_c
0	320.000	0.00	320.000	-2.641×10^{-9}
0.01	338.905	0.58	338.904	0.5831
0.03	353.176	0.76	353.158	0.7624
0.1	381.665	1.015	381.665	1.0151
0.3	428.290	1.03	428.290	1.2992
1	513.792	1.64	513.790	1.6438
3	619.666	1.92	619.666	1.9211
30	780.240	2.18	780.237	2.1760
100	804.973	2.20	804.972	2.2029
∞	816.748	2.21	816.744	2.2147



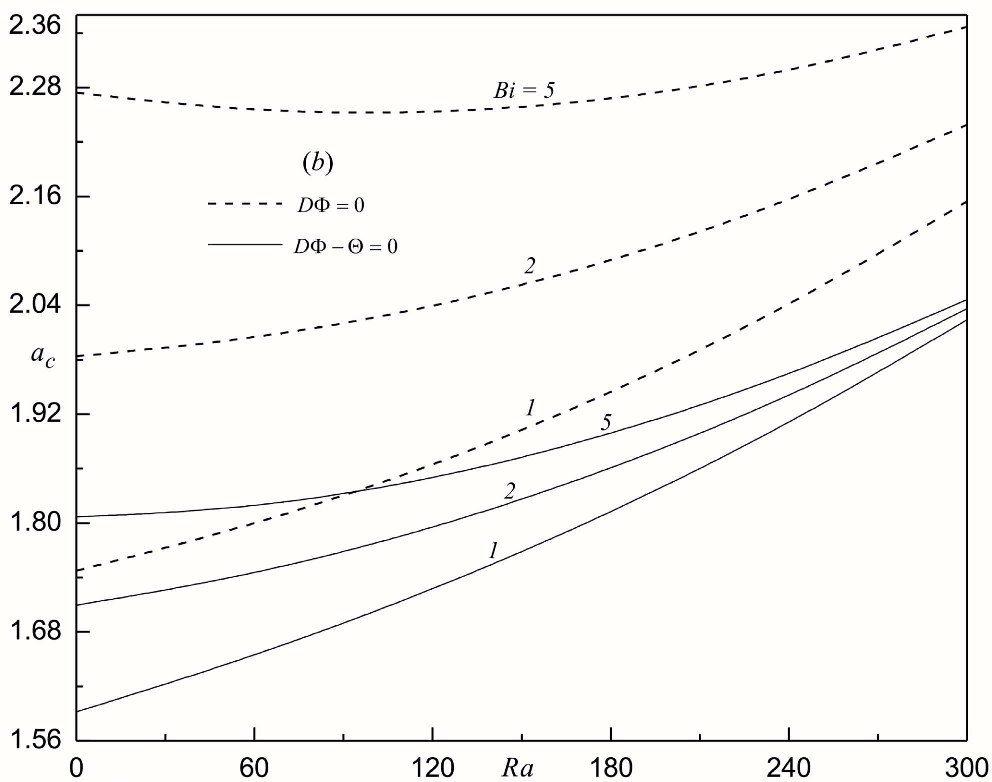
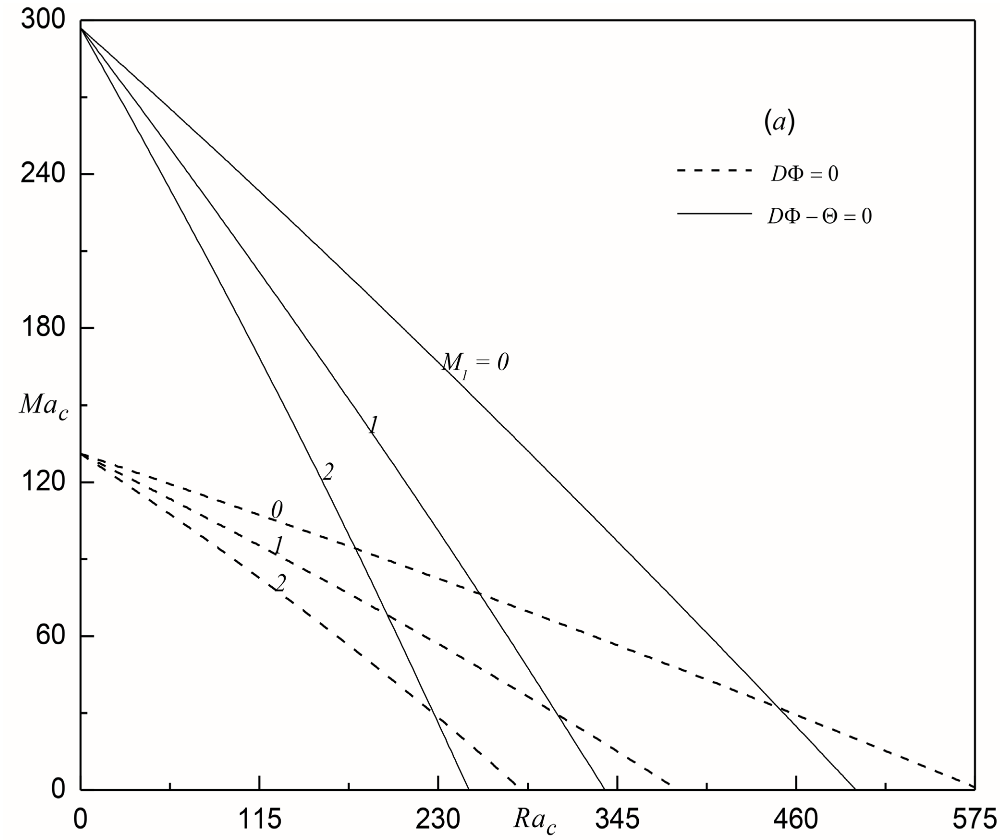


Figure 2. (a) Ma_c versus Ra_c with Bi when $M_3 = 1$ and $M_1 = 2$; (b) a_c versus Ra_c with Bi when $M_3 = 1$ and $M_1 = 2$.



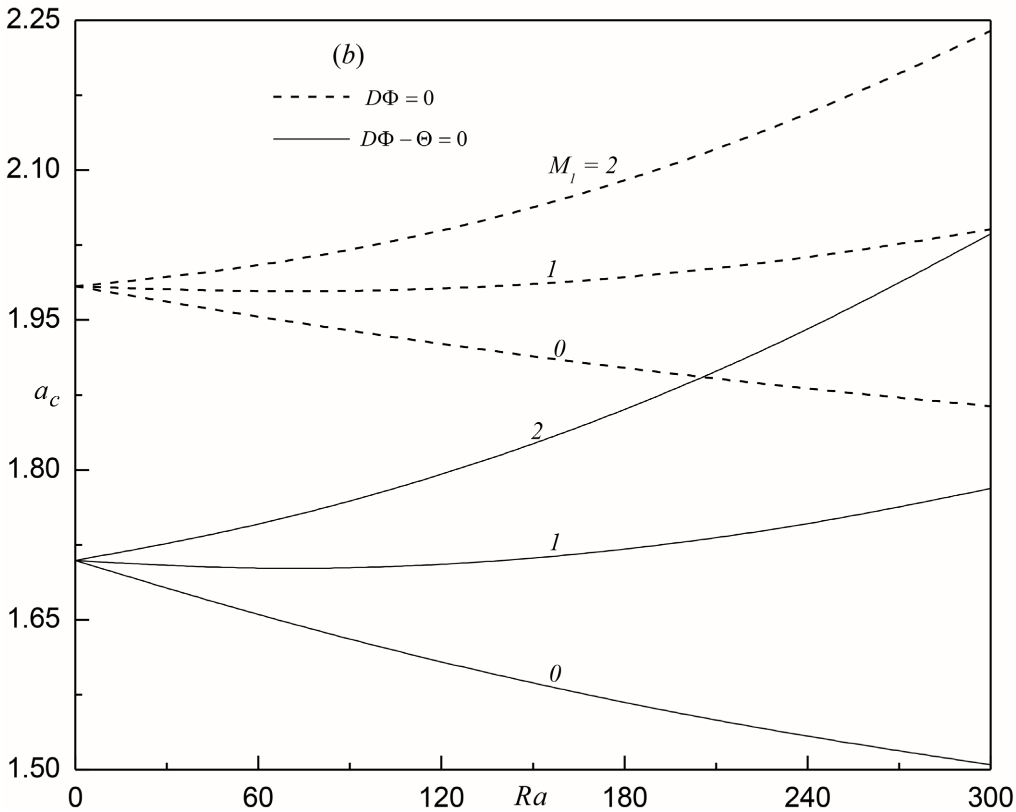


Figure 3. (a) Ma_c versus Ra_c with M_1 when $M_3 = 1$ and $Bi = 2$; (b) a_c versus Ra with M_1 when $M_3 = 1$ and $Bi = 2$.

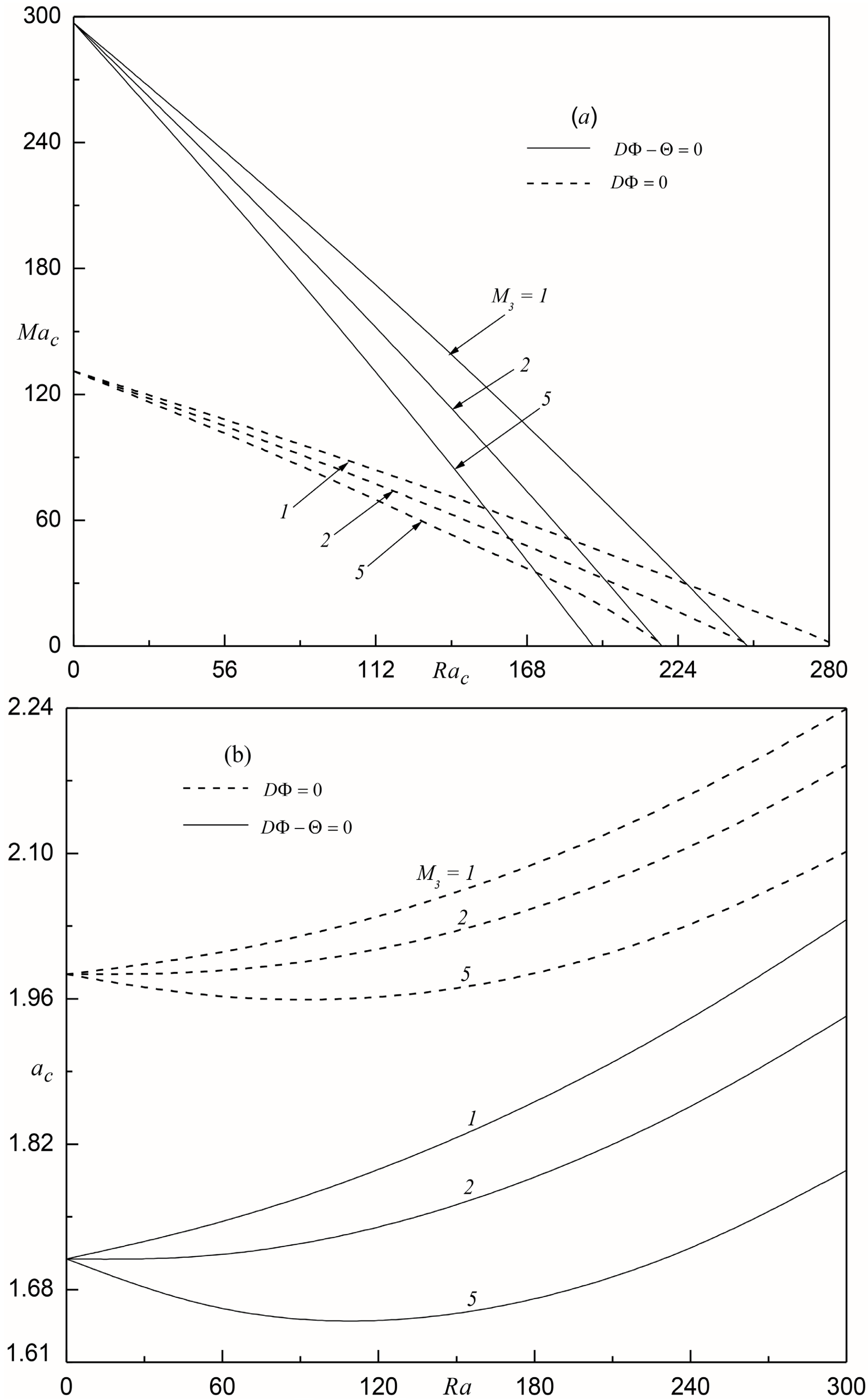


Figure 4. (a) Ma_c versus Ra_c with M_3 when $M_1 = 2$ and $Bi = 2$; (b) a_c versus Ra_c with M_3 when $M_1 = 2$ and $Bi = 2$.

observed the stability parameters Ra_c and Ma_c decreases as increasing M_3, thus the mechanism of magnetization non-linearity parameter has a destabilizing effect on the system. Nonetheless, Ra_c and Ma_c are found to be independent of M_3 for $Bi = 0$. While the value of a_c decreases as increasing in M_3 and thus the effect is to enlarge size of convection cells.

5. Conclusions

The influence of general boundary conditions on buoyancy and surface tension-driven FTC in a FF layer is investigated numerically Galrkin technique based on weighted residual technique. The following conclusions were resulting:

- The initiation of FTC is inhibited with increasing Biot number Bi.
- The magnetic parameter M_1 and fluid magnetization non-linearity parameter M_3 hasten the FTC.
- The magnetic bounding surfaces $D\Phi - \Theta$ offer more stabilizing while $D\Phi$ surfaces offer least stable effects against FTC. *i.e.* $\left(Ra_c \text{ or } Ma_c\right)_{D\Phi} < \left(Ra_c \text{ or } Ma_c\right)_{D\Phi-\Theta}$.
- The critical value (a_c) for $D\Phi$ is always higher than those of remaining boundaries. *i.e.* $\left(a_c\right)_{D\Phi-\Theta} < \left(a_c\right)_{D\Phi}$.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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