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Evaluation of Probability Distribution Functions for Modeling Forest Tree Diameters on Agricultural Landscapes in Ogun State, Nigeria

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Abstract

The use of probability distribution functions for describing tree diameter at breast height provides useful information for forest resource evaluation and quantification. A series of probability distribution functions have been widely developed and applied for managing forest trees in conventional forest reserves without much consideration for trees outside forest reserves. The aim of this study is to evaluate and propose a suitable probability distribution function for trees in Agricultural landscapes. The study examined 3-parameter lognormal, Lognormal, 3-parameter Gamma, Gamma, 3-parameter Weibull and Weibull distribution functions, using the Maximum Likelihood method for fitting tree diameter at breast height. Three hundred and thirty-two temporary farmlands were randomly selected from which stem diameter of all living trees, with diameter ≥ 10.0 cm, were measured. Results of the statistical analysis showed that the 3-parameter lognormal distribution gave a superior description of the stem diameter with the least values of Anderson Darling (1.627) and Akaike Information Criterion (5962.0) statistics. Hence, the 3-parameter lognormal distribution function was found suitable for the stem diameter of trees in Agricultural landscapes in the study area.

Keywords

Probability Distribution, Tree Diameter, Agricultural Landscapes, Goodness of-Fit Statistics, Ogun State

1. Introduction

Tree stem diameter is the simplest, most common, and probably most important measurement in forestry (Zhang et al., 2003). Stem diameter has high relationships with other more difficult to quantify tree growth features such as wood volume, basal area and height and also represents a tree's competitive position in a forest stand (West & West, 2009). The probability distribution functions for univariate data, like diameter at breast height, are useful for facilitating tree population description and forecasting tree diameter and volume growth in a stand for effective forest management (Burkhart & Tomé, 2012). Tree diameter is assessed in forestry by measuring the diameter of the stem at breast height (dbh) and defining the frequency distribution with probability distributions, which are referred to as the model hereafter. Since the 1950s, foresters have been researching models that describe right-skewed diameter distributions (Meyer, 1952). Some classical models for diameter distribution are in the forms of Normal, beta, Exponential, Lognormal, Weibull and Gamma and Johnson's SB (Bliss & Reinker, 1964; Palahi et al., 2007). The choice of appropriate diameter distribution function depends on the result of statistical analysis of a given set of diameter measurements.

In a study carried out by Ezenwenyi et al. (2018), it was revealed that 3-parameter lognormal and Weibull (3-parameter) distributions were suitable for diameter distribution modeling of *Nauclea diderrichii* in Omo Forest Reserve, Ogun State, Nigeria. Also, in another work, Nanang (1998) fitted three different diameter distribution functions to data from neem plantations in Ghana. The outcome of goodness-of-fit statistics confirmed the use of lognormal distribution for the plantations in the study area. Moreso, Alo et al. (2017), assessed the performance and suitability of beta, 3- parameter gamma, 3-parameter lognormal and 3-parameter Weibull distribution functions for fitting diameter at breast height (Dbh) of the second rotation of *Tectona grandis* Linn. F plantation in Eda Forest Reserve, Nigeria. The test results of the study showed that 3-Parameter Weibull distribution was the most suitable for characterizing Dbh and it was recommended for effective management of the plantation. In a study for fitting diameter distribution models to a mixed tropical rainforest in Oban Forest Reserve, Nigeria, Aigbe & Omokhua (2014) discovered that Weibull 3P and Johnson SB distributions were the most suitable models for the management of natural forests in the study area.

The development and application of diameter distribution models have often been focused on protected forest reserves by forest researchers. Agricultural landscape (AL) is a type of terrain where farming activities such as planting, tilling, irrigation and fertilizing have a substantial impact on the natural land ecosystems (Etter, 1991). Forest trees in AL are an integral part of many farming landscapes (Sinclair, 1999). These trees are also categorized as a form Trees Outside Forests (TOF), which according to Food and Agriculture Organization (FAO), (FAO, 2001), comprise all trees excluded from the definition of forest and other wooded lands. Trees planted outside and inside forest reserves share similar attributes (Arnold, 1997), and are an important natural resource that contributes to the livelihood of people in many regions. Despite the widespread use of probability distribution models in forest management, there is little or no literature on the development and implementation of diameter distribution functions for trees on agricultural landscapes in Ogun State.

As a result, of this oversight, the purpose of this study was to investigate and select the most appropriate and adaptable probability distribution function for modeling stem diameter at the breast height of trees on the selected Agricultural Landscapes in the study area.

2. Materials and Methods

2.1. Study Area

Ogun State is located in South western part of Nigeria within latitudes 6˚N and 8˚N and longitudes 3˚E and 5˚E (Akanni, 2000). The state is made up of twenty Local Government Areas (LGAs) and bounded on the north by Oyo state and on the east by Ondo State. To the west is the Republic of Benin while Lagos State and the Atlantic Ocean are to the south. The state covers a land area of about 16,762 square kilometers which is approximately 1.81 percent of Nigeria's land mass of about 923,768 square kilometers and has an estimated population of 3,728,098 people (NBS, 2009) of which 67% were farmers (OGADEP, 1998). The state is within the tropical humid climatic zone of Nigeria, which is generally characterized by high rainfall and high relative humidity. This is attributable to the prevalence of moisture laden tropical Maritime air mass over the state for about nine months in a year. Agriculture, the mainstay of the economy, provides the major single occupation for the people of the state especially those in the rural areas. Arable crops like maize, yam, cassava, rice, cocoyam, groundnut, melon, banana plantain, oranges, pineapple, sugar-cane and kola nuts are produced in the state. The major export crops produced are cocoa, coffee, rubber and palm kernel. Logging activities also thrive because the state is endowed with appreciable forest resources.

2.2. Sampling Procedure and Data Collection

Data were collected through the adoption of 3-stage random sampling technique in the selection of sampled farmlands. The first stage involved the random selection of eight LGAs from the study area after which four villages were chosen at random from each of the previously selected LGAs. Ten farmlands were further randomly selected from each of the four villages which gave a total of 320 Agricultural farmlands for the study.

A complete enumeration of all live trees having dbh of ≥10 cm was conducted in all sample farmlands from which each tree diameter was measured at 1.3 m. Tree diameter tape was used for measuring dbh and all readings were recorded on the field at the time of enumeration.

3. Data Analysis

3.1. Diameter Distribution Fitting Procedure and Assessment

All data collected were first subjected to descriptive statistics. Thereafter, six different continuous probability distribution functions; Lognormal (3P), Gamma (3P), Lognormal, Gamma, Weibull (3P) and Weibull were tested on the basis of histogram plot, skewness and kurtosis values. The statistical distribution models considered in this study are:

1) 2-Parameter Lognormal Distribution:

This is a continuous probability distribution with two parameters and its natural logarithm has a normal distribution.

$$f(x) = \frac{1}{\sqrt{2\pi}\delta x} \exp\left[-\frac{1}{2\delta^2} (\ln x - \mu)^2 \right] \quad (1)$$

x: tree diameter, μ and δ: continuous parameters ($\delta > 0$).

2) 3-Parameter Lognormal Distribution:

The probability density function of the 3-parameter lognormal distribution as described by Aristizabal (2012) is given as:

$$f(x, \mu, \delta, \gamma) = \frac{1}{(x-\gamma)\delta\sqrt{2\pi}} \exp\left\{ \frac{\left[\ln(x-\gamma) - \mu \right]^2}{2\delta^2} \right\} \quad (2)$$

x: tree diameter, μ: mean, γ: location parameter, δ: continuous parameter ($\delta > 0$).

3) The 2-Parameter and 3-P Weibull Distributions (Weibull, 1951) are expressed below as:

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta} \right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta} \right)^{\alpha} \right] \quad (3)$$

x: tree diameter, α: continuous shape parameter ($\alpha > 0$) and β: continuous scale parameter ($\beta > 0$).

4) 3-Parameter Weibull Distribution:

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x-\gamma}{\beta} \right)^{\alpha} \exp\left[-\left(\frac{x-\gamma}{\beta} \right)^{\alpha} \right] \quad (4)$$

x: tree diameter, α: continuous shape parameter ($\alpha > 0$), β: continuous scale parameter ($\beta > 0$), γ: continuous location parameter.

5) 2-Parameter and 3-Parameter Gamma Distributions (Krishnamoorthy, 2006) are mathematically presented below as:

$$f(x) = \frac{x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} \exp\left(-\frac{x}{\beta} \right) \quad (5)$$

x: random variable, α: continuous shape parameter ($\alpha > 0$), β: continuous scale parameter ($\beta > 0$), $\Gamma(\alpha)$ gamma function.

6) 3-Parameter Gamma Distribution:

$$f(x) = \frac{(x-\gamma)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} \exp\left(-\frac{x-\gamma}{\beta} \right) \quad (6)$$

x: random variable, α: continuous shape parameter ($\alpha > 0$), β: continuous scale parameter ($\beta > 0$), γ: location parameter.

3.2. Method of Parameter Estimations

For this study, the Maximum Likelihood Estimation (MLE) method was adopted for the estimations of all unknown two and three parameters of the selected probability distributions. The MLE, according to Wackerly et al. (2014), is a method used for estimating the unknown value of parameters (θ) of a probability distribution function using observed data and it is mathematically expressed as; Let $(X_1,\cdots,X_n)$ be continuous random variable which are independent and identically distributed. The goal is to find a good value of the parameter θ to obtain the observed values using the likelihood function of the form;

$$\begin{aligned} L(\theta \mid X_1,\cdots,X_n) &= \text{joint probability density function pdf of } (X_1,\cdots,X_n) \\ &= f(X_1 \mid \theta)\cdots f(X_n \mid \theta) \\ &= \prod_{i=1}^{n} f(X_i \mid \theta) \end{aligned} \tag{7}$$

hence, the maximum likelihood estimator is the value of θ that maximizes the $L(\theta \mid X_1,\cdots,X_n)$, and this is achieved by applying the log-likelihood approach; $\ell(\theta \mid X_1,\cdots,X_n) = \log L(\theta \mid X_1,\cdots,X_n)$. When applied to a data set on tree diameter at breast height and given a statistical distribution function, maximum likelihood estimation provides estimates for the model's parameters. In this study, all the parameters were assumed to be unknown and were estimated by numerical iteration using SPC for Excel Version 5 and STATGRAPHICS Centurion 18 software.

3.3. Evaluation and Comparison Criteria

The goodness of fit tests and graphical representation for all selected distributions were assessed by the Log-Likelihood (LL) value which is used for comparing two or more distribution models and the value ranges between negative values to positive values. The higher the Log-Likelihood value, the better the model. Also, Anderson Darling Statistics and Akaike Information Criteria (AIC) are tests which must have minimum values for a given probability distribution function to be accepted for modeling (Hilbe & Robinson, 2013).

4. Results

4.1. Descriptive Statistics

The summary statistics for tree stem diameters on Agricultural landscapes are presented in **Table 1**. A total of 717 trees were measured for diameter at breast height. The Table further shows that the distribution of stem (Dbh) ranged from 10 cm to 150 cm, with a mean value of 40.05 and a standard deviation of 17.03. The values of skewness and kurtosis are 13.49 and 17.51 respectively and the values indicate that the distribution of diameter is positively skewed. The histogram in **Figure 1** further confirms the right skewness of the data. The majority of stem diameters at breast height were concentrated at the lower diameter class.

4.2. Parameter Estimation

The results of the parameter estimation for the fitted six probability functions, Lognormal (3P), Gamma (3P), Lognormal, Gamma, Weibull (3P) and Weibull are presented in Table 2. All distributions fitted came from continuous probability functions.

The graphical representations of empirical and theoretical densities of the six fitted probability functions are presented in Figure 2.

Table 1. Summary Statistics for Diameters (cm).

Number of Tree Diameters measured	717
Mean Stem Diameter	40.05
Standard Deviation	17.03
Coefficient of Variation	42.51
Minimum Stem Diameter	10.00
Maximum Stem Diameter	150.00
Standard Skewness	13.49
Standard Kurtosis	17.51

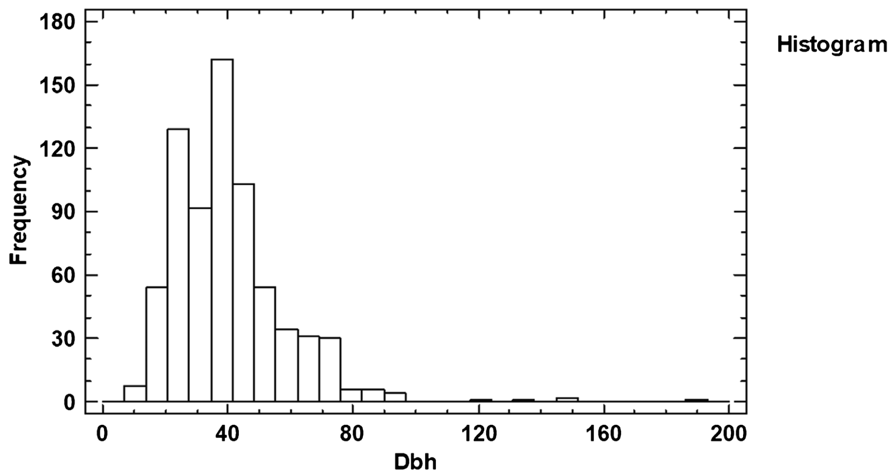

Figure 1. Histogram of the diameter at the breast height in centimeter.

Table 2. Parameter estimates from the distribution fitting.

Distribution	Location	Shape	Scale	Threshold
Lognormal (3P)	3.761		0.358	−5.797
Gamma (3P)		4.288	8.089	5.363
Lognormal	3.604		0.421	
Gamma		5.956	6.723	
Weibull (3P)		1.889	34.64	9.320
Weibull		2.461	45.17	

N.B.: 3P = Three-Parameter Distribution.

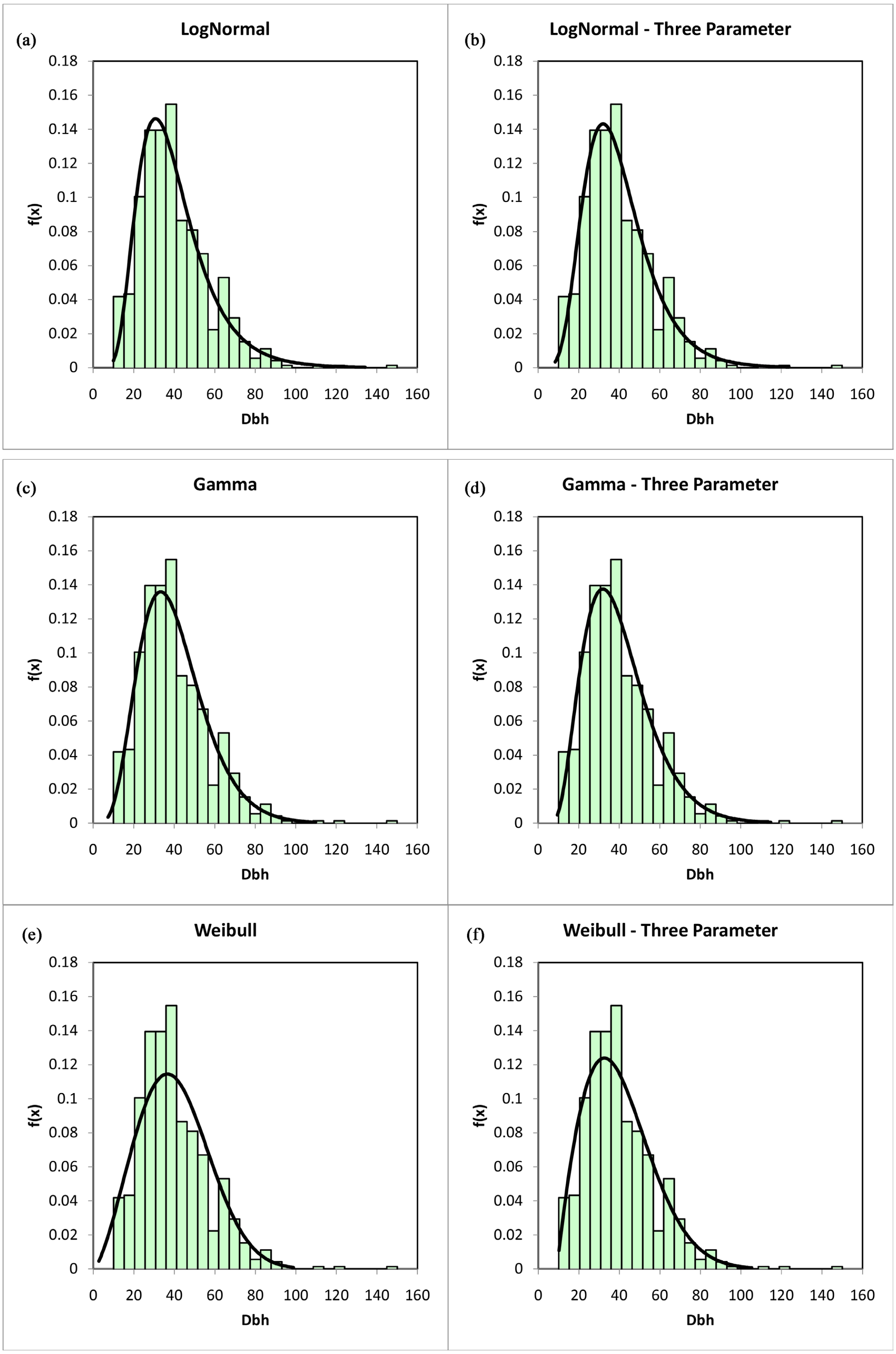

Figure 2. Histograms of the Observed and evaluated probability functions for (a) Lognormal (3P) (b) Gamma (3P) (c) Lognormal (d) Gamma (e) Weibull (3P) (f) Weibull.

The 3-Parameter lognormal distribution (Figure 2(a)) shows that the theoretical density captures most of the observed values while the 3-Parameter Gamma distribution (Figure 2(b)) has a very close and identical representation of empirical and theoretical densities to 3-Parameter Lognormal. Furthermore, Lognormal (Figure 2(c)) and Gamma (Figure 2(d)) distributions have a similar pattern with respect to Figure 2(a) and Figure 2(b). Other fitted distributions, the 3-Parameter Weibull (Figure 2(d)) and Weibull (Figure 2(e)) do not have a good overlay of both the empirical and theoretical densities.

4.3. Goodness of Fit

The results of the analysis on the goodness of fit are shown in Table 3. Three measures of goodness of fit, Log-Likelihood (LL), Anderson Darling (AD) and Akaike Information Criterion (AIC), were used for the objective evaluation and comparison of all the six fitted probability distributions.

The 3-Parameter Lognormal distribution has the highest value of Loglikelihood (−2978), the smallest values AD (1.627) and the AIC (5962.0). Weibull distribution has the smallest Loglikelihood value (−3020.1) with the highest values of AD (7.937) and AIC (6044.1). Only the 3-Parameter Gamma distribution is a bit close in values to the 3-Parameter Lognormal distribution. All other fitted distributions viz; Lognormal, Gamma, 3-Parameter Weibull and Weibull have their values between the 3-Parameter Lognormal and Weibull distributions.

5. Discussion

From this study on the probability distribution of diameter at breast height of farmland forest trees, the mean Dbh of 40.05 cm shows that most of the trees in the study area have Dbh of less than 48 cm permitted for timber harvest in south-western Nigeria. The mean value obtained serves as baseline information for use in the study area.

The distribution fitting using the maximum likelihood approach showed that the fitness of 3-P Lognormal, 3-P Gamma, Lognormal, Gamma, 3-P Weibull and Weibull distributions after had different performances in describing Dbh. Using Loglikelihood, Anderson Darling and Akaike Information Criterion, it was observed that little difference existed between 3-Parameter Lognormal and 3-Parameter Gamma

Table 3. Goodness of fit information for distribution.

Distribution	Loglikelihood	AD	AIC
Lognormal (3P)	−2978	1.627	5962.0
Gamma (3P)	−2979.1	1.906	5964.2
Lognormal	−2980.2	1.916	5964.4
Gamma	−2981.6	2.241	5967.1
Weibull (3P)	−2988.5	3.607	5983.1
Weibull	−3020.1	7.937	6044.1

distributions. In addition, with respect to the highest values of LL and lowest values of AD and lowest values of AIC, the 3-Parameter Lognormal and 3-Parameter Gamma distribution functions performed better than the Lognormal, Gamma, 3-Parameter Weibull and Weibull distribution functions. Therefore, the 3-Parameter Lognormal distribution function was found suitable for Dbh prediction of farmland trees in the study area. This was similar to the findings of Ezenwenyi et al. (2018) on the diameter distribution of *Nauclea diderrichii* plantation in a restricted tropical rainforest of Nigeria, who recommended the 3-Parameter Lognormal distribution using the maximum likelihood method for describing the diameter of *Nauclea diderrichii.* However, from a study on fitting diameter distribution model to a mixed tropical rainforest in Oban Forest Reserve, Nigeria, Aigbe & Omokhua (2014) discovered that Weibull 3P and Johnson SB distributions were the most suitable models for the management of natural forests in the study area.

Comparing the results of this study with other studies on trees from farmlands in the study area becomes unachievable as the majority of studies on diameter distribution fitting have been focused on protected forest reserves.

6. Conclusion

This work investigated a suitable probability distribution function for describing the diameter at breast height of trees in Agricultural landscapes. The study examined 3-parameter lognormal, Lognormal, 3-parameter Gamma, Gamma, 3-parameter Weibull and Weibull distribution functions, using the Maximum Likelihood method for fitting tree diameter at breast height. After performing the investigation, the corresponding results show that 3-parameter lognormal distribution gave a superior description of the stem diameter with the least values of Anderson Darling (1.627) and Akaike Information Criterion (5962.0) statistics. The use of appropriate statistical distributions for stem diameter prediction is helpful for forestry decision-making. In this study, therefore, it can be concluded that both the 3-Parameter Lognormal and 3-Parameter Gamma distribution functions are good. However, the 3-Parameter Lognormal distribution function, using the maximum likelihood method, is more robust and appropriate for effective prediction of Dbh and is hereby recommended for forest tree diameters prediction and description of Agricultural landscapes in the study area.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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