Open Journal of Discrete Mathematics, 2021, 11, 31-42
https://www.scirp.org/journal/ojdm
ISSN Online: 2161-7643
ISSN Print: 2161-7635

Cordial Labeling of Corona Product of Path Graph and Second Power of Fan Graph

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How to cite this paper: Elrokh, A.I.H., Nada, S.I.M. and El-Shafey, E.M.E.-S. (2021) Cordial Labeling of Corona Product of Path Graph and Second Power of Fan Graph. *Open Journal of Discrete Mathematics*, **11**, 31-42.
https://doi.org/10.4236/ojdm.2021.112003

Received: January 6, 2021
Accepted: April 12, 2021
Published: April 15, 2021

Abstract

A graph is said to be cordial if it has 0 - 1 labeling which satisfies particular conditions. In this paper, we construct the corona between paths and second power of fan graphs and explain the necessary and sufficient conditions for this construction to be cordial.

Keywords

Corona, Second Power of Fan, Cordial Graph

1. Introduction

Labeling problem is important in graph theory. It is known that graph theory and its branches have become interesting topics for almost all fields of mathematics and also other areas of science such as chemistry, biology, physics, communication, economics, engineering, and especially computer science. A graph labeling is an assignment of integers to the vertices or edges or both. There are many contributions and different types of labeling. [1] [2] [3] [4] suppose that $G=(V,E)$ is a graph, where V is the set of its vertices and E is the set of its edges. Throughout, it is assumed G is connected, finite, simple and undirected. A binary vertex labeling of G is a mapping $f:V\rightarrow\{0,1\}$ in which $f(u)$ is said to be the labeling of $u\in V$. For an edge $e=uv\in E$, where $u,v\in V$, the induced edge label $f^*:E\rightarrow\{0,1\}$ is defined by the formula $f^*(vw)=(f(v)+f(w))(\text{mod}\,2)$. Thus, for any edge e, $f^*(e)=0$ if its two vertices have the same label and $f^*(e)=1$ if they have different labels. Let us denote v_0 and v_1 be the numbers of vertices labeled by 0 and 1 in V respectively, and let e_0 and e_1 be the corresponding numbers of edge in E labeled by 0 and 1 respectively. A binary vertex labeling f of G is said to be cordial if $|v_0 - v_1| \le 1$ and $|e_0 - e_1| \le 1$ hold. A graph G is cordial if it has cordial labeling. Cordial graphs were introduced by Cahit [5] [6] as a weaker version of both graceful graphs and harmonious graphs [2] [3] [4]. A recommended reference on this subject is the survey by Gallian [1]. A path with n vertices and $n-1$ edges is denoted by P_n, and second power of fan graph has $n+1$ vertices and $3n-3$ edges is denoted by F_n^2. Let G (with n_1 vertices and m_1 edges) and H (with n_2 vertices and m_2 edges) are two graphs. The corona between G and H is the graph denoted by $G \odot H$ and is obtained by taking one copy of G and n_i copies of H, and then joining the *i-th* vertex of G with an edge to every vertex in the *i-th* copy of H [9]. It follows from the definition of the corona that $G \odot H$ has $n_1 + n_1 \cdot n_2$ vertices and $m_1 + n_1 \cdot m_2 + n_1 \cdot n_2$ edges. It is easy to see that $G \odot H$ is not in general isomorphic to $H \odot G$. A second power of a fan F_m^2 is the graph obtained from the join of the second power of a path P_m^2 and a null graph N_1, *i.e.* $F_m^2 = P_m^2 + N_1$. So the order of F_m^2 is $m+1$ and its size is $3m-3$, in particular $F_1^2 \equiv P_2$, $F_2^2 \equiv C_3$ and $F_3^2 \equiv K_4$. In this paper we study the corona $P_K \odot F_m^2$ and show that is cordial for all $K \ge 1$ and $m \ge 4$.

2. Terminology and Notation

We introduce some notation and terminology for a graph with $4r$ vertices [7] [8] [9]. Let M_r denote the labeling $0101\cdots01$, zero-one repeated r-times if r is even and $0101\cdots010$ if r is odd; for example, $M_6 = 010101$ and $M_5 = 01010$. Welet M'_{2r} denote the labeling $1010\cdots10$. Sometimes, we modify the labeling M_r or M'_r by adding symbols at one end or the other (or both). We let L_{4r} denote the labeling $0011\,0011\cdots0011$ (repeated r-times) where $r \ge 1$ and, L'_{4r} denote the labeling $1100\,1100\cdots1100$ (repeated r-times) where $r \ge 1$ and S_{4r} denotes the labeling $1001\,1001\cdots1001$ (repeated r-times) and S'_{4r} denotes the labeling $0110\,0110\cdots0110$ (repeated r-times). In most cases, we then modify this by adding symbols at one end or the other (or both), thus $L_{4r}101$ denotes the labeling $0011\,0011\cdots0011\,101$ (repeated r-times) when $r \ge 1$ and 101 when $r = 0$. Similarly, $1L'_{4r}$ is the labeling $1\,1100\,1100\cdots1100$ (repeated r-times) when $r \ge 1$ and 1 when $r = 0$. Similarly, $0L'_{4r}1$ is the labeling $0\,1100\,1100\cdots1100\,1$ when $r \ge 1$ and 01 when $r = 0$. Also, we write 0_r for the labeling $0\cdots0$ (repeated r-times) and 1_r for the labeling $1\cdots1$ (repeated r-times) [7] [8] [9] [10]. For specific labeling L and M of $G \odot H$ where G is path and H is a second fans, we let $[L;M]$ denote the corona labeling. Additional notation that we use is the following. For a given labeling of the corona $G \odot H$, we let v_i and e_i (for $i = 0,1$) be the numbers of labels that are i as before, we let x_i and a_i be the corresponding quantities for G, and we let y_i and b_i be those for H, which are connected to the vertices labeled 0 of G. Likewise, let y'_i and b'_i be those for H, which are connected to the vertices labeled 1 of G. In case it increases by one more vertexes, so y''_i and b''_i will be those for H, which are connected to the vertex labeled 1 or 0 of G. It is easy to verify that,

$$v_0 = x_0 + y_0 x_0 + y_0' (x_1 - 1) + y_0'', \quad v_1 = x_1 + y_1 x_0 + y_1' (x_1 - 1) + y_1''$$

and

$$e_0 = a_0 + b_0 x_0 + b_0' (x_1 - 1) + b_0'' + y_0 x_0 + y_1' (x_1 - 1) + y_0'',$$
$$e_1 = a_1 + b_1 x_0 + b_1' (x_1 - 1) + b_1'' + y_1 x_0 + y_0' (x_1 - 1) + y_1''.$$

Thus,

$$v_0 - v_1 = (x_0 - x_1) + x_0 (y_0 - y_1) + (x_1 - 1)(y_0' - y_1') + (y_0'' - y_1'')$$

and

$$e_0 - e_1 = (a_0 - a_1) + x_0 (b_0 - b_1) + (x_1 - 1)(b_0' - b_1') + (b_0'' - b_1'') + x_0 (y_0 - y_1) - (x_1 - 1)(y_0' - y_1') - (y_1'' - y_0'')$$

when it comes to the proof, we only need to show that, for each specified combination of labeling, $|v_0 - v_1| \le 1$ and $|e_0 - e_1| \le 1$.

3. The Corona between Paths and Second Fans

In this section, we show that the corona between paths and second power of Fan graphs $P_K \odot F_m^2$ is cordial for all $k \ge 1$, and $m \ge 4$. This target will be achieved after the following series of lemmas.

Lemma 3.1 $P_K \odot F_m^2$ is cordial for all $k \ge 1$ and $m \equiv 0 \pmod 4$.

Proof. We need to examine the following cases:

Case (1). $k \equiv 0 \pmod 4$.

Let $k = 4r$, $r \ge 1$. Then, one can choose the labeling $[L_{4r} : 0M_{4s}', 0M_{4s}', 1M_{4s}, 1M_{4s}, \cdots (r\text{-times})]$ for $P_{4r} \odot F_{4s}^2$. Therefore, $x_0 = x_1 = 2ra_0 = 2r$, $a_1 = 2r - 1$, $y_0 = 2s + 1$, $y_1 = 2s$, $b_0 = 6s - 2$, $b_1 = 6s - 1$, $y_0' = 2s$, $y_1' = 2s + 1$, $b_0' = 6s - 2$ and $b_1' = 6s - 1$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$ Thus $P_{4r} \odot F_{4s}^2$, $s \ge 1$ is cordial.

As an example, Figure 1 illustrates $P_4 \odot F_4^2$.

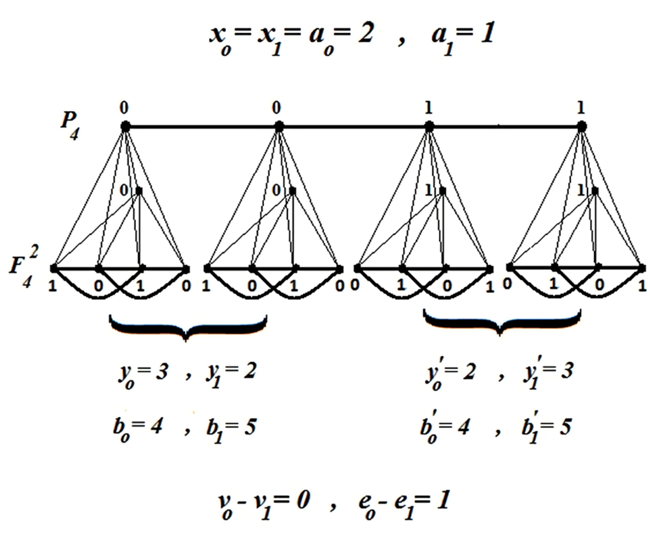

Figure 1. The corona between paths and second power of Fan graphs $P_4 \odot F_4^2$.

Case (2). $k \equiv 1(\bmod 4)$.

Let $k = 4r+1$, $r > 0$. Then, one can choose the labelling $[L_{4r}\ 1:0M_{4s}, 0M_{4s}, 1M_{4s}, 1M_{4s}, \cdots (r\text{-times}), 0M_{4s}]$ for $P_{4r+1} \odot F_{4s}^2$. Therefore $x_0 = 2r$, $x_1 = a_0 = 2r+1$, $a_1 = 2r-1$, $y_0 = 2s+1$, $y_1 = 2s$, $b_0 = 6s-2$, $b_1 = 6s-1$, $y_0' = 2s$, $y_1' = 2s+1$, $b_0' = 6s-2$, $b_1' = 6s-1$, $y_0'' = 2s+1$, $y_1'' = 2s$, $b_0'' = 6s-2$ and $b_1'' = 6s-1$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Thus $P_{4r+1} \odot F_{4s}^2, s \geq 1$ is cordial.

As an example, Figure 2 illustrates $P_5 \odot F_4^2$.

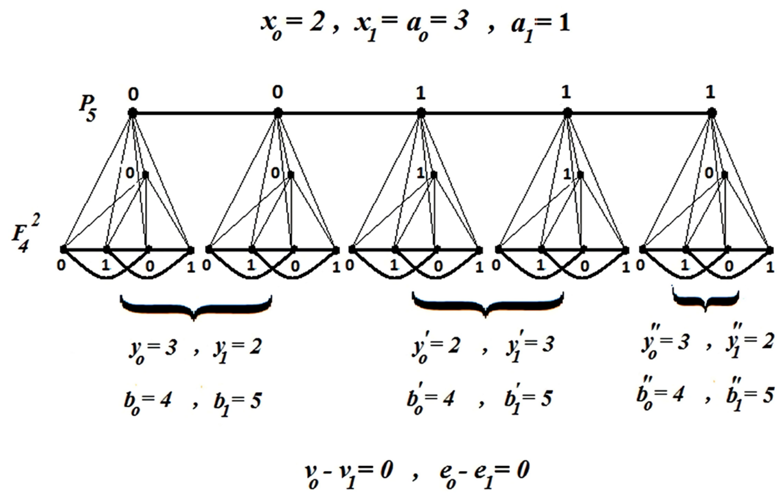

Figure 2. The corona between paths and second power of Fan graphs $P_5 \odot F_4^2$.

Case (3). $k \equiv 2(\bmod 4)$.

Let $k = 4r+2$, $r > 0$. Then, one can choose the labelling $[L_{4r}\ 10:0M_{4s}', 0M_{4s}', 1M_{4s}, 1M_{4s}, \cdots (r\text{-times}), 1M_{4s}, 0M_{4s}']$ for $P_{4r+2} \odot F_{4s}$. Therefore $x_0 = x_1 = 2r+1$, $a_0 = 2r+1$, $a_1 = 2r$, $y_0 = 2s+1$, $y_1 = 2s$, $b_0 = 6s-2$, $b_1 = 6s-1$, $y_0' = 2s$, $y_1' = 2s+1$, $b_0' = 6s-2$ and $b_1' = 6s-1$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Thus $P_{4r+2} \odot F_{4s}^2$, $s \geq 1$ is cordial.

As an example, Figure 3 illustrates $P_6 \odot F_4^2$.

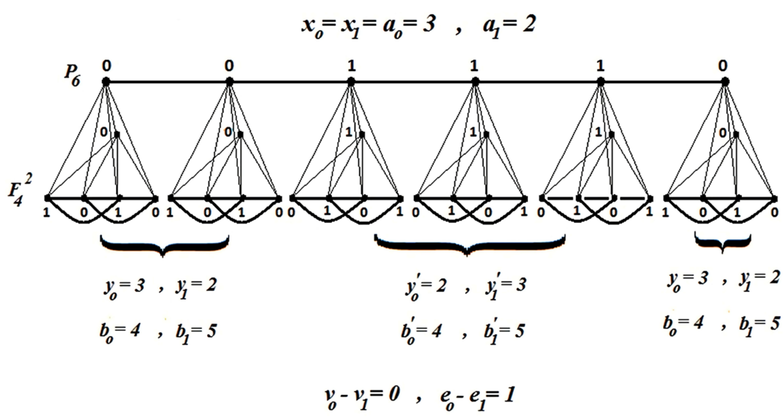

Figure 3. The corona between paths and second power of Fan graphs $P_6 \odot F_4^2$.

Case (4). $k \equiv 3 \pmod 4$.

Let $k = 4r+3$, $r > 0$. Then, one can choose the labelling $\left[L_{4r}\ 110 : 0M'_{4s}, 0M'_{4s}, 1M'_{4s}, 1M'_{4s}, \cdots (r\text{-times}), 1M'_{4s}, 1M_{4s}, 0M'_{4s}\right]$ for $P_{4r+3} \odot F_{4s}^2$. Therefore, $x_0 = 2r+1$, $x_1 = a_0 = 2r+2$, $a_1 = 2r$, $y_0 = 2s+1$, $y_1 = 2s$, $b_0 = 6s-2$, $b_1 = 6s-1$, $y'_0 = 2s$, $y'_1 = 2s+1$, $b'_0 = 6s-2$, $b'_1 = 6s-1$, $y''_0 = 2s+1$, $y''_1 = 2s$, $b''_0 = 6s-2$ and $b''_1 = 6s-1$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Thus $P_{4r+3} \odot F_{4s}^2, s \ge 1$ is cordial.

As example, Figure 4 illustrates $P_7 \odot F_4^2$.

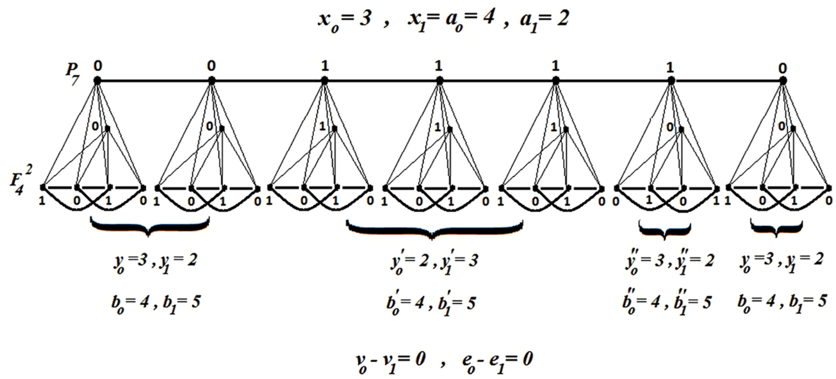

Figure 4. The corona between paths and second power of Fan graphs $P_7 \odot F_4^2$.

Lemma 3.2 $P_K \odot F_m^2$ is cordial for all $k \ge 1$ and $m \equiv 1 \pmod 4$.

Proof. We need to examine the following cases:

Case (1). $k \equiv 0 \pmod 4$.

Let $k = 4r$, $r \ge 1$. Then, one can choose the labeling $\left[L_{4r} : 11_3 0_2 M_{4s-4}, 11_3 0_2 M_{4s-4}, 00_3 1_2 M'_{4s-4}, 00_3 1_2 M'_{4s-4}, \cdots (r\text{-times})\right]$ for $P_{4r} \odot F_{4s+1}^2$. Therefore $x_0 = x_1 = a_0 = 2r$, $a_1 = 2r-1$, $y_0 = 2s$, $y_1 = 2s+2$, $b_0 = 6s+1$, $b_1 = 6s-1$, $y'_0 = 2s+2$, $y'_1 = 2s$ and $b'_0 = 6s+1$, $b'_1 = 6s-1$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Thus $P_{4r} \odot F_{4s+1}^2$, $s \ge 1$ is cordial.

As an example, Figure 5 illustrates $P_4 \odot F_5^2$.

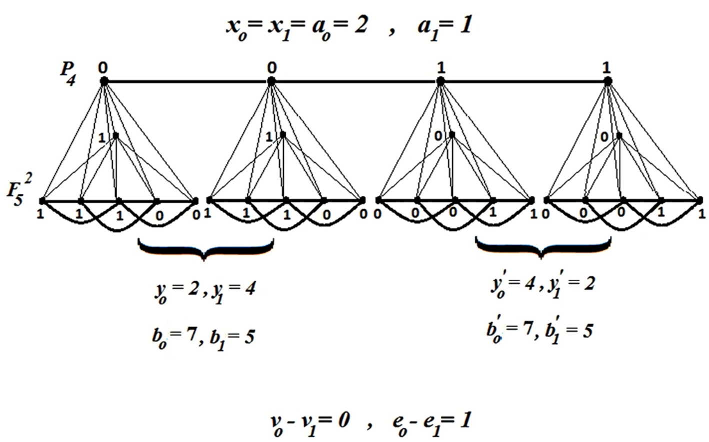

Figure 5. The corona between paths and second power of Fan graphs $P_4 \odot F_5^2$.

Case (2). $k \equiv 1(\bmod 4)$.

Let $k = 4r+1$, $r > 0$. Then, one can choose the labeling $[L_{4r}\ 0:11_3 0_2 M_{4s-4}, 11_3 0_2 M_{4s-4}, 00_3 1_2 M'_{4s-4}, 00_3 1_2 M'_{4s-4}, \cdots(r\text{-times}), 10_3 1_2 M_{4s-4}]$ for $P_{4r+1} \odot F^2_{4s+1}$. Therefore $x_0 = 2r+1$, $x_1 = a_0 = a_1 = 2r$, $y_0 = 2s+2$, $y_1 = 2s$, $b_0 = 6s-1$, $b_1 = 6s+1$, $y'_0 = 2s$, $y'_1 = 2s+2$, $b'_0 = 6s-1$, $b'_1 = 6s+1$, $y''_0 = y''_1 = 2s+1$, and $b''_0 = b''_1 = 6s$. Hence, one can easily show that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Thus $P_{4r+1} \odot F^2_{4s+1}$, $s \ge 1$, is cordial.

As an example, **Figure 6** illustrates $P_5 \odot F^2_5$.

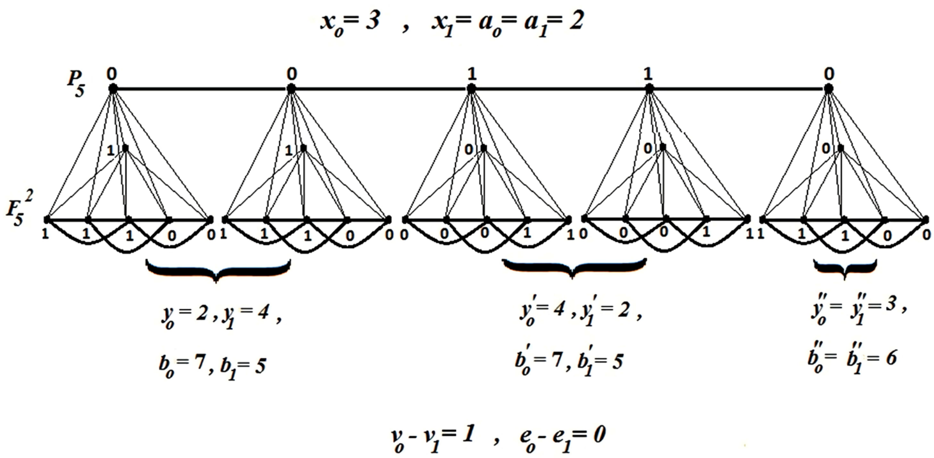

Figure 6. The corona between paths and second power of Fan graphs $P_5 \odot F^2_5$.

Case (3). $k \equiv 2(\bmod 4)$.

Let $k = 4r+2$, $r \ge 0$. Then, one can choose the labeling $[L_{4r}\ 10:11_3 0_2 M_{4s-4}, 11_3 0_2 M_{4s-4}, 00_3 1_2 M'_{4s-4}, 00_3 1_2 M'_{4s-4}, \cdots(r\text{-times}), 00_3 1_2 M'_{4s-4}, 11_3 0_2 M_{4s-4}]$ for $P_{4r+2} \odot F^2_{4s+1}$. Therefore $x_0 = x_1 = a_0 = 2r+1$, $a_1 = 2r$, $y_0 = 2s+2$, $y_1 = 2s$, $b_0 = 6s-1$, $b_1 = 6s+1$, $y'_0 = 2s$, $y'_1 = 2s+2$ and $b'_0 = 6s-1$, $b'_1 = 6s+1$. Hence one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Thus $P_{4r+2} \odot F^2_{4s+1}$, $s \ge 1$, is cordial.

As an example, **Figure 7** illustrates $P_6 \odot F^2_5$.

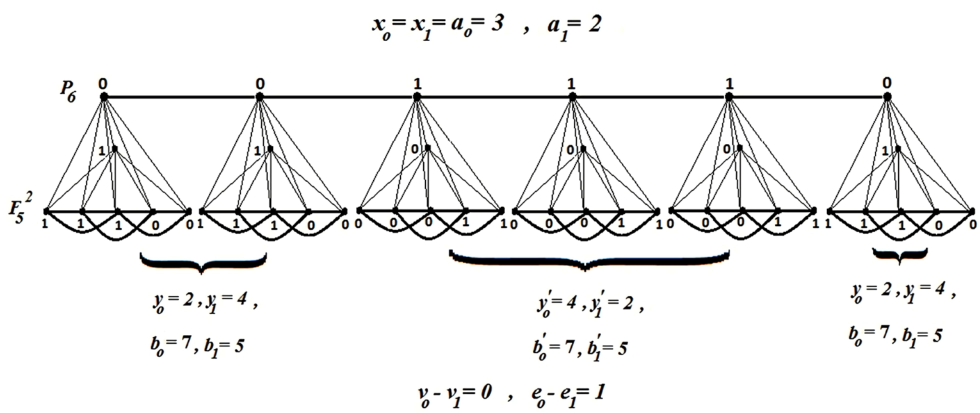

Figure 7. The corona between paths and second power of Fan graphs $P_6 \odot F^2_5$.

Case (4). $k \equiv 3 \pmod 4$.

Let $k = 4r+3$, $r \geq 0$. Then, one can choose the labeling [L_{4r} $101:11_3 0_2 M_{4s-4}$, $11_3 0_2 M_{4s-4}$, $00_3 1_2 M'_{4s-4}$, $00_3 1_2 M'_{4s-4}$, $\cdots(r\text{-times})$, $00_3 1_2 M'_{4s-4}$, $11_3 0_2 M_{4s-4}$, $00_2 1_3 M'_{4s-4}$] for $P_{4r+3} \odot F^2_{4s+1}$. Therefore $x_0 = a_0 = a_1 = 2r+1$, $x_1 = 2r+2$, $y_0 = 2s$, $y_1 = 2s+2$, $b_0 = 6s+1$, $b_1 = 6s-1$, $y'_0 = 2s+2$, $y'_1 = 2s$, $b'_0 = 6s+1$, $b'_1 = 6s-1$, $y''_0 = y''_1 = 2s+1$, and $b''_0 = b''_1 = 6s$. Hence, one can easily show that $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$. Thus $P_{4r+3} \odot F^2_{4s+1}$, $s \geq 1$, is cordial.

As an example, Figure 8 illustrates $P_7 \odot F^2_5$.

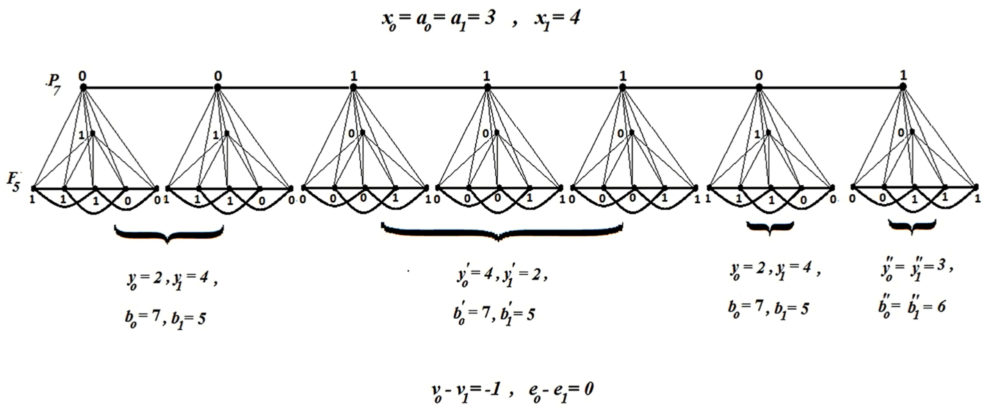

Figure 8. The corona between paths and second power of Fan graphs $P_7 \odot F^2_5$.

Lemma 3.3 $P_k \odot F^2_m$ is cordial for all $k \geq 1$ and $m \equiv 2 \pmod 4$.

Proof. We need to study the following cases:

Case (1). $k \equiv 0 \pmod 4$.

Let $k = 4r$, $r \geq 1$. Then, one can choose the labeling $\left[L_{4r} : 0M'_{4s+2}, 0M'_{4s+2}, 1M_{4s+2}, 1M_{4s+2}, \cdots(r\text{-times})\right]$ for $P_{4r} \odot F^2_{4s+1}$. Therefore $x_0 = x_1 = a_0 = 2r$, $a_1 = 2r-1$, $y_0 = 2s+2$, $y_1 = 2s+1$, $b_0 = 6s+1$, $b_1 = 6s+2$, $y'_0 = 2s+1$, $y'_1 = 2s+2$, $b'_0 = 6s+1$ and $b'_1 = 6s+2$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Thus $P_{4r} \odot F^2_{4s+1}, s \geq 1$ is cordial.

As an example, Figure 9 illustrates $P_4 \odot F^2_6$.

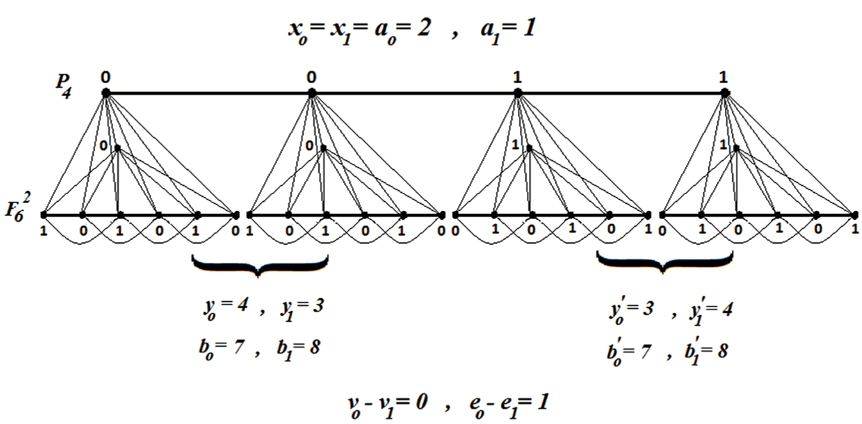

Figure 9. The corona between paths and second power of Fan graphs $P_4 \odot F^2_6$.

<u>**Case (2).**</u> $k \equiv 1(\bmod 4)$.

Let $k = 4r+1$, $r \geq 0$. Then, one can choose the labeling $\left[L_{4r}\ 1: 0M'_{4s+2}, 0M'_{4s+2}, 1M'_{4s+2}, 1M'_{4s+2}, \cdots (r\text{-times}), 0M_{4s+2}\right]$ for $P_{4r+1} \odot F^2_{4s+2}$. Therefore $x_0 = 2r$, $x_1 = a_0 = 2r+1$, $a_1 = 2r-1$, $y_0 = 2s+2$, $y_1 = 2s+1$, $b_0 = 6s+1$, $b_1 = 6s+2$, $y'_0 = 2s+1$, $y'_1 = 2s+2$, $b'_0 = 6s+1$, $b'_1 = 6s+2$, $y''_0 = 2s+2$, $y''_1 = 2s+1$, $b''_0 = 6s+1$ and $b''_1 = 6s+2$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Thus $P_{4r+1} \odot F^2_{4s+2}$, $s \geq 1$ is cordial.

As an example, **Figure 10** illustrates $P_5 \odot F_6^2$.

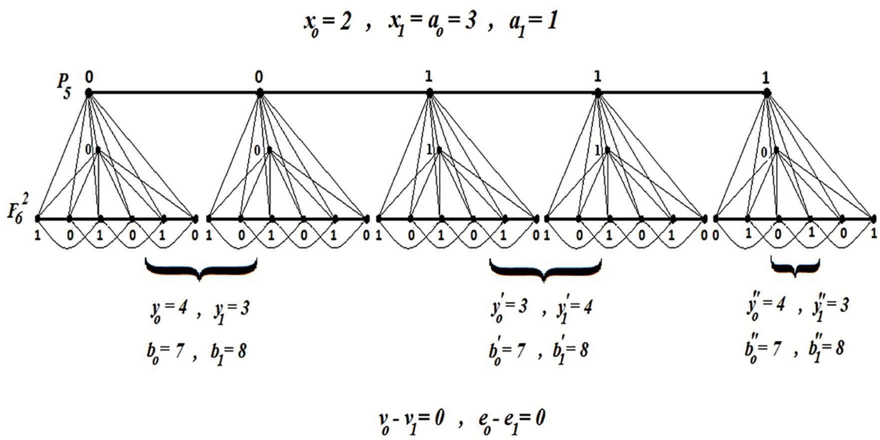

Figure 10. The corona between paths and second power of Fan graphs $P_5 \odot F_6^2$.

<u>**Case (3).**</u> $k \equiv 2(\bmod 4)$.

Let $k = 4r+2$, $r \geq 0$. Then, one can choose the labeling $\left[L_{4r}\ 10: 0M'_{4s+2}, 0M'_{4s+2}, 1M_{4s+2}, 1M_{4s+2}, \cdots (r\text{-times}), 1M_{4s+2}, 0M'_{4s+2}\right]$ for $P_{4r+2} \odot F^2_{4s+2}$. Therefore $x_0 = x_1 = a_0 = 2r+1$, $a_1 = 2r$, $y_0 = 2s+2$, $y_1 = 2s+1$, $b_0 = 6s+1$, $b_1 = 6s+2$, $y'_0 = 2s+1$, $y'_1 = 2s+2$, $b'_0 = 6s+1$ and $b'_1 = 6s+2$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Thus $P_{4r+2} \odot F^2_{4s+2}$, $s \geq 1$ is cordial.

As an example, **Figure 11** illustrates $P_6 \odot F_6^2$.

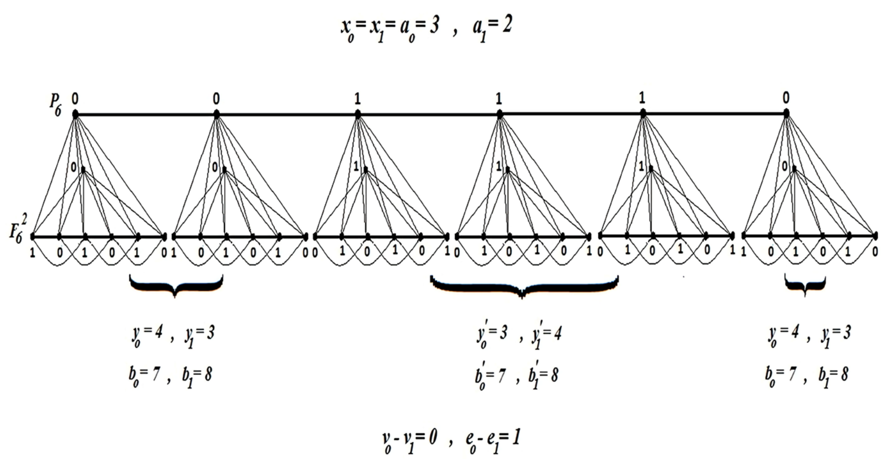

Figure 11. The corona between paths and second power of Fan graphs $P_6 \odot F_6^2$.

Case (4). $k \equiv 3(\text{mod}\,4)$.

Let $k = 4r+3$, $r \geq 0$. Then, one can choose the labeling $\left[L_{4r}\,110 : 0M'_{4s+2}, 0M'_{4s+2}, 1M_{4s+2}, 1M_{4s+2}, \cdots(r\text{-times}), 1M_{4s+2}, 00_2 1_2 M'_{4s-2}, 0M'_{4s+2}\right]$ for $P_{4r+3} \odot F^2_{4s+2}$. Therefore $x_0 = 2r+1$, $x_1 = a_0 = 2r+2$, $y_0 = 2s+2$, $a_1 = 2r$, $y_0 = 2s+2$, $y_1 = 2s+1$, $b_0 = 6s+1$, $b_1 = 6s+2$, $y'_0 = 2s+1$, $y'_1 = 2s+2$, $b'_0 = 6s+1$, $b'_1 = 6s+2$, $y''_0 = 2s+2$, $y''_1 = 2s+1$, $b''_0 = 6s+1$ and $b''_1 = 6s+2$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 0$. Thus $P_{4r+3} \odot F^2_{4s+2}$, $s \geq 1$ is cordial.

As an example, Figure 12 illustrates $P_7 \odot F^2_6$.

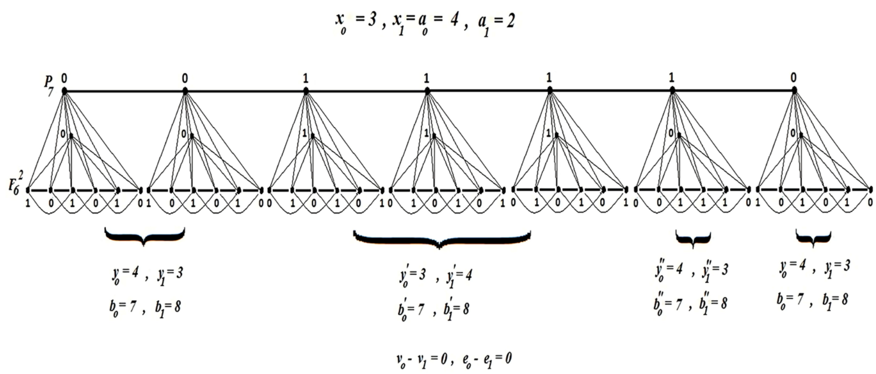

Figure 12. The corona between paths and second power of Fan graphs $P_7 \odot F^2_6$.

Lemma 3.4 $P_k \odot F^2_m$ is cordial for all $k \geq 1$ and $m \equiv 3(\text{mod}\,4)$.

Proof: Will be examined following cases:

Case (1). $k \equiv 0(\text{mod}\,4)$.

Let $k = 4r$, $r \geq 1$. Then, one can choose the labeling $\left[L_{4r} : 10_3 M'_{4s}, 10_3 M'_{4s}, 101_2 M_{4s}, 101_2 M_{4s}, \cdots(r\text{-times})\right]$ for $P_{4r} \odot F^2_{4s+3}$. Therefore $x_0 = x_1 = a_0 = 2r$, $a_1 = 2r-1$, $y_0 = 2s+3$, $y_1 = 2s+1$, $b_0 = 6s+2$, $b_1 = 6s+4$, $y'_0 = 2s+1$, $y'_1 = 2s+3$ and $b'_0 = 6s+2$, $b'_1 = 6s+4$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Thus $P_{4r} \odot F^2_{4s+3}$, $s \geq 1$ is cordial.

As an example, Figure 13 illustrates $P_4 \odot F^2_7$.

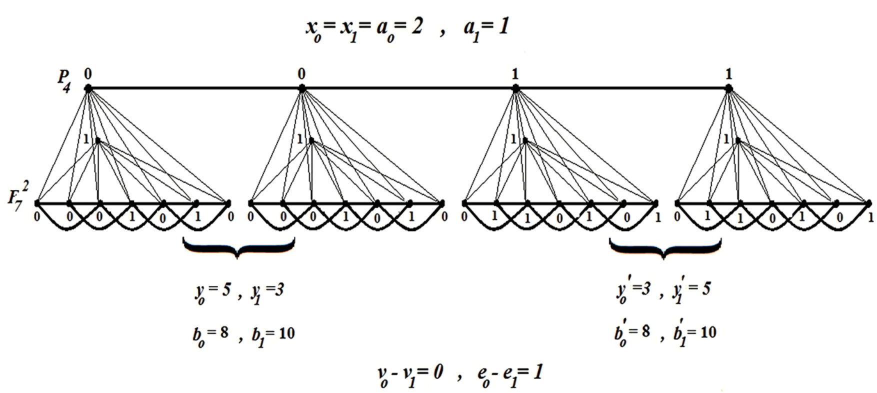

Figure 13. The corona between paths and second power of Fan graphs $P_4 \odot F^2_7$.

Case (2). $k \equiv 1(\bmod 4)$.

Let $k = 4r+1$, $r \geq 0$. Then, one can choose the labeling $\left[L_{4r}\ 1:10_3 M'_{4s}, 10_3 M'_{4s}, 101_2 M_{4s}, 101_2 M_{4s}, \cdots (r\text{-times}), 01_2 0 M'_{4s}\right]$ for $P_{4r+1} \odot F^2_{4s+3}$. Therefore $x_0 = 2r$, $x_1 = a_0 = 2r+1$, $a_1 = 2r-1$, $y_0 = 2s+3$, $y_1 = 2s+1$, $b_0 = 6s+2$, $b_1 = 6s+4$, $y'_0 = 2s+1$, $y'_1 = 2s+3$, $b'_0 = 6s+2$, $b'_1 = 6s+4$, $y''_0 = y''_1 = 2s+2$, $b''_0 = 6s+2$ and $b''_1 = 6s+4$. Hence one can easily show that $v_0 - v_1 = -1$ and $e_0 - e_1 = 0$. Thus $P_{4r+1} \odot F^2_{4s+3}$, $s \geq 1$ is cordial.

As an example, Figure 14 illustrates $P_5 \odot F^2_7$.

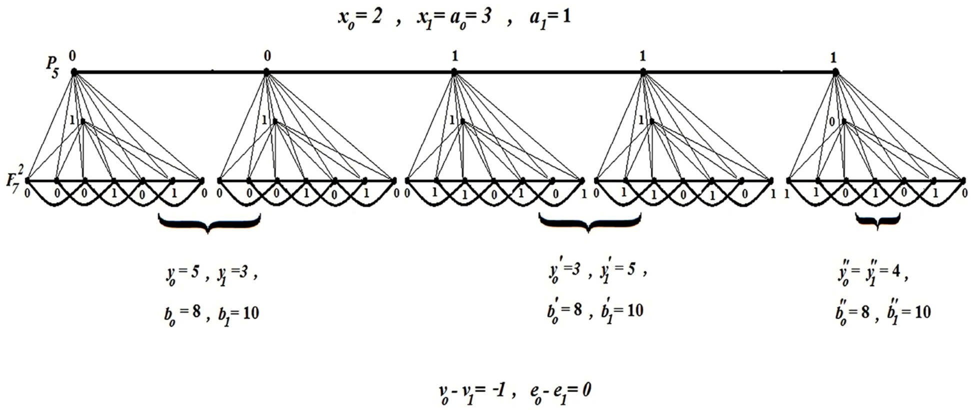

Figure 14. The corona between paths and second power of Fan graphs $P_5 \odot F^2_7$.

Case (3). $k \equiv 2(\bmod 4)$.

Let $k = 4r+2$, $r \geq 0$. Then, one can choose the labeling $\left[L_{4r}\ 10:10_3 M'_{4s}, 10_3 M'_{4s}, 101_2 M_{4s}, 101_2 M_{4s}, \cdots (r\text{-times}), 101_2 M_{4s}, 10_3 M'_{4s}\right]$ for $P_{4r+2} \odot F^2_{4s+3}$. Therefore $x_0 = x_1 = a_0 = 2r+1$, $a_1 = 2r$, $y_0 = 2s+3$, $y_1 = 2s+1$, $b_0 = 6s+2$, $b_1 = 6s+4$, $y'_0 = 2s+1$, $y'_1 = 2s+3$, $b'_0 = 6s+2$ and $b'_1 = 6s+4$. Hence, one can easily show that $v_0 - v_1 = 0$ and $e_0 - e_1 = 1$. Thus $P_{4r+2} \odot F^2_{4s+3}$, $s \geq 1$ is cordial.

As an example, Figure 15 illustrates $P_6 \odot F^2_7$.

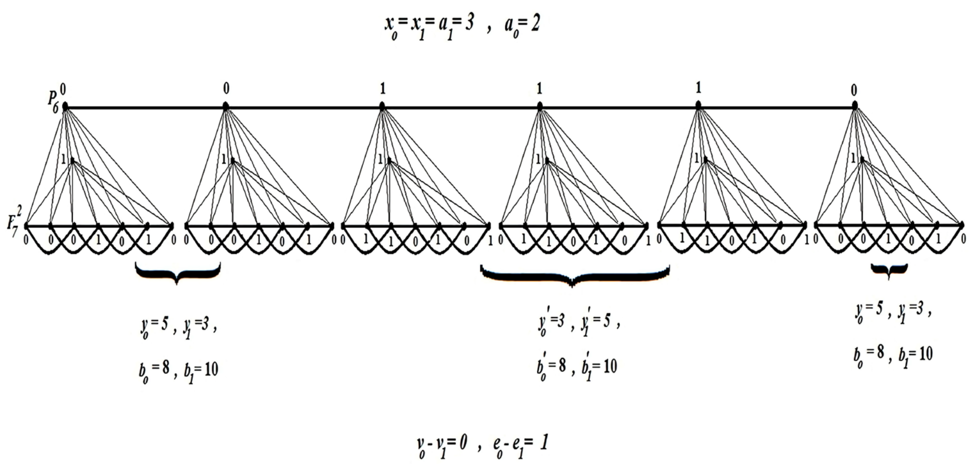

Figure 15. The corona between paths and second power of Fan graphs $P_6 \odot F^2_7$.

Case (4). $k \equiv 3(\bmod 4)$.

Let $k = 4r + 3$, $r \geq 0$. Then, one can choose the labeling [L_{4r} $100:10_3 M'_{4s}$, $10_3 M'_{4s}$, $101_2 M_{4s}$, $101_2 M_{4s}$, $\cdots(r\text{-times})$, $101_2 M_{4s}$, $10_3 M'_{4s}$, $01_2 0 M'_{4s}$] for $P_{4r+3} \odot F^2_{4s+3}$. Therefore $x_0 = a_0 = 2r + 2$, $x_1 = 2r + 1$, $a_1 = 2r$, $y_0 = 2s + 3$, $y_1 = 2s + 1$, $b_0 = 6s + 2$, $b_1 = 6s + 4$, $y'_0 = 2s + 1$, $y'_1 = 2s + 3$, $b'_0 = 6s + 2$, $b'_1 = 6s + 4$, $y''_0 = y''_1 = 2s + 2$, $b''_0 = 6s + 2$ and $b''_1 = 6s + 4$. Hence one can easily show that $v_0 - v_1 = 1$ and $e_0 - e_1 = 0$. Thus $P_{4r+3} \odot F^2_{4s+3}$, $s \geq 1$ is cordial.

As an example, Figure 16 illustrates $P_7 \odot F_7^2$.

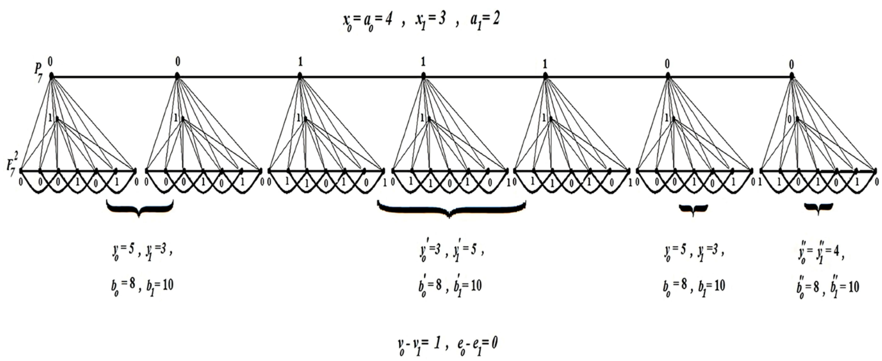

Figure 16. The corona between paths and second power of Fan graphs $P_7 \odot F_7^2$.

As a consequence of all previous lemmas one can establish the following theorem.

Theorem. The corona between path P_k & F_m^2 is cordial for all k and m.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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