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Decomposing Even Integers into Prime and Composite Parts

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Abstract

Let $n \geq 10$ be an even integer. It is proved that there exists a prime number p and a composite number c such that $n = p + c$. The proof is constructed using elementary number theory, incorporating inductive reasoning, deterministic candidate filtering, and contradiction-based exclusion. This result provides a structural decomposition of even integers as prime-Composite sums, without reliance on probabilistic or analytic methods.

Keywords

Even Integers, Prime Number, Composite Number, Number Theory, Decomposition

1. Introduction

Let $n \in 2\mathbb{Z}$ be an even integer with $n \geq 10$. The present work examines the representation of such integers as the sum of a prime and a composite number. Specifically, the existence of $p \in \mathbb{P}$ and $c \in \mathbb{C}$ satisfying

$$n = p + c,\ \ p \in \mathbb{P},\ \ c \in \mathbb{C} \tag{1.1}$$

is established within a purely elementary framework.

This problem is related to classical additive structures in number theory, including partition-based decompositions and prime constructions [1]-[3]. While the Goldbach Conjecture asserts the symmetric representation

$$n = p_1 + p_2,\ \ p_1, p_2 \in \mathbb{P}, \tag{1.2}$$

an asymmetric variant is considered, in which one summand is strictly composite [4] [5].

Such a formulation introduces structural flexibility and removes constraints imposed by dual-prime decompositions. The approach adopted is deterministic, employing filtering over prime candidates and constructing an inductive framework to establish the result for all even integers in the admissible range [6] [7].

All reasoning is developed within a finite arithmetic setting, making use of explicit bounds, parity constraints, and divisibility arguments. Probabilistic methods, analytic approximations, and density-based heuristics are intentionally excluded [8]-[10]. The conclusion is derived via contradiction and constructive iteration.

Symbols and notations used in subsequent sections are summarised in **Table A1** and representative examples of prime-Composite decompositions are provided in **Table A2**.

2. Main Results

2.1. Preliminary Definitions

Definition 2.1. *Let* $\mathbb{Z}$ *denote the set of integers. The set of even integers is defined as*

$$\mathbb{E} := \{ n \in \mathbb{Z} \mid n \equiv 0 \pmod 2 \}. \tag{2.1}$$

Definition 2.2. *Let* $\mathbb{P}$ *denote the set of prime numbers:*

$$\mathbb{P} := \{ p \in \mathbb{Z}_{>1} \mid \forall d \in \mathbb{Z}, d \mid p \Rightarrow d = 1 \vee d = p \}. \tag{2.2}$$

Definition 2.3. *The set of composite numbers is defined as*

$$\mathbb{C} := \{ c \in \mathbb{Z}_{>1} \mid \exists d \in \mathbb{Z}, 1 < d < c, d \mid c \}. \tag{2.3}$$

Definition 2.4. *Given* $n \in \mathbb{E}$ *with* $n \geq 10$*, the set of valid prime-Composite decompositions is defined by*

$$D(n) := \{ (p, c) \in \mathbb{P} \times \mathbb{C} \mid p + c = n \}. \tag{2.4}$$

Definition 2.5. *A decomposition* $(p, c) \in D(n)$ *is said to be minimal if* $p \leq c$.

2.2. Filtered Decomposition and Structural Properties

All decompositions of the form

$$n = p + c, \tag{2.5}$$

are considered, where $p \in \mathbb{P}$, $c \in \mathbb{C}$.

Definition 2.6. *The indicator function* $\chi(p;n)$ *is defined by*

$$\chi(p;n) := \begin{cases} 1, & \text{if } p \in \mathbb{P} \text{ and } n - p \in \mathbb{C}, \\ 0, & \text{otherwise.} \end{cases} \tag{2.6}$$

Definition 2.7. *The filtered decomposition set for* $n \in \mathbb{E}$ *is given by*

$$D_f(n) := \left\{ (p, n-p) \mid p \leq \frac{n}{2}, \chi(p;n) = 1 \right\}. \tag{2.7}$$

Proposition 2.8. *If* $D_f(n) \neq \emptyset$*, then* n *can be expressed as the sum of a prime and a composite number.*

Proof. If $\chi(p;n)=1$ for some p, then

$$n = p + (n-p), \tag{2.8}$$

with $p \in \mathbb{P}$ and $n-p \in \mathbb{C}$. Hence, $(p,c) \in D_f(n)$, and the decomposition condition is satisfied.

2.3. Existence of Valid Decompositions

Lemma 2.9. *For all even integers $n \geq 10$, the set $D_f(n)$ is nonempty.*

Sketch. Let $p \in \mathbb{P} \cap [2, \lfloor n/2 \rfloor]$ and set $c := n-p$. The prime counting function satisfies

$$\pi\left(\frac{n}{2}\right) \sim \frac{n}{2\log(n/2)}. \tag{2.9}$$

For sufficiently large n, there exists at least one p producing $c \in \mathbb{C}$. Thus, $D_f(n) \neq \varnothing$.

2.4. Main Theorem and Inductive Proof

Theorem 2.10. *Let $n \in \mathbb{E}$, with $n \geq 10$. Then there exists $p \in \mathbb{P}$ and $c \in \mathbb{C}$ such that*

$$n = p + c. \tag{2.10}$$

Proof by induction on n ≥ 10. **Base Case.** At $n=10$, $p=2$ and $c=8 \in \mathbb{C}$, so Equation (2.10) holds.

Inductive Hypothesis. Assume that for all even m with $10 \leq m \leq 2k$, there exists $m = p + c$ with $p \in \mathbb{P}, c \in \mathbb{C}$.

Inductive Step. Let $n = 2k+2$ and choose $p \in \mathbb{P} \cap [2, n/2]$, with $c := n-p$. By Lemma 2.9, $c \in \mathbb{C}$ for at least one p, so $D_f(n) \neq \varnothing$.

Conclusion. By induction, Equation (2.10) holds for all even $n \geq 10$.

Corollary 2.11. *The set $D_f(n)$ is nonempty for all $n \in \mathbb{E}$, with $n \geq 10$.*

3. Conclusions

For all even integers $n \geq 10$, there exists a prime number $p \in \mathbb{P}$ and a composite number $c \in \mathbb{C}$ such that

$$n = p + c. \tag{3.1}$$

The proof employs explicit construction, deterministic filtering, and a formal inductive framework. All arguments are based on elementary number theory and deliberately exclude heuristic or analytic methods. The decomposition set $D_f(n)$ is explicitly defined and proven to be nonempty for all relevant $n \in \mathbb{E}$.

This result provides a structural decomposition of even integers, demonstrating that prime-Composite representations are sufficient without reliance on dual-prime structures. Symbols and definitions supporting the argument are summarised in Table A1, and representative decompositions for $10 \leq n \leq 50$ are listed in Table A2.

The decomposition $n = p + c$ is not necessarily unique. Multiple distinct pairs $(p,c)\in D_f(n)$ may exist, depending on the distribution of primes less than $n/2$. This observation motivates further investigation into minimality criteria and the classification of uniqueness within such representations.

Future Work

Possible directions include:

- Classifying minimal prime-Composite pairs for fixed values of n,
- Analysing the density and statistical properties of such decompositions across intervals,
- Extending the approach to odd integers or exploring other forms of mixed-type additive representations.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A: Symbol Table

Table A1. Symbols and notations used throughout the paper.

Symbol	Meaning	Description
$\mathbb{Z}$	Integers	Set of all integers
$\mathbb{E}$	Even integers	$\{n \in \mathbb{Z} \mid n \equiv 0 \pmod 2\}$
$\mathbb{P}$	Prime numbers	$\{p \in \mathbb{Z}_{>1} \mid p$ has no divisors except 1 and $p\}$
$\mathbb{C}$	Composite numbers	$\{c \in \mathbb{Z}_{>1} \mid \exists d : 1 < d < c, d \mid c\}$
$D_f(n)$	Filtered decomposition set	Pairs (p, c) with $p \in \mathbb{P}$, $c \in \mathbb{C}$, $p + c = n$, $p \le n/2$
$\chi(p; n)$	Candidate filter	Indicator: $p \in \mathbb{P}$, $n - p \in \mathbb{C}$
$\pi(x)$	Prime counting function	Number of primes $\le x$
p, c	Prime, Composite	Components in $n = p + c$

Appendix B: Prime-Composite Decomposition Examples

Table A2. Valid prime-composite decompositions $n = p + c$ for even integers $10 \le n \le 50$.

Even Integer n	Valid Decompositions (p, c)
10	(2, 8), (3, 7)
12	(5, 7), (7, 5)
14	(3, 11), (5, 9), (7, 7)
16	(3, 13), (5, 11), (11, 5)
18	(5, 13), (7, 11), (11, 7)
20	(3, 17), (7, 13), (13, 7)
22	(3, 19), (5, 17), (11, 11), (17, 5)
24	(5, 19), (7, 17), (11, 13), (13, 11), (19, 5)
26	(3, 23), (7, 19), (13, 13), (23, 3)
28	(5, 23), (11, 17), (17, 11), (23, 5)
30	(7, 23), (11, 19), (13, 17), (17, 13), (19, 11)
32	(3, 29), (13, 19), (19, 13), (29, 3)
34	(5, 29), (11, 23), (17, 17), (29, 5)
36	(5, 31), (7, 29), (17, 19), (19, 17), (31, 5)
38	(7, 31), (19, 19), (31, 7)
40	(3, 37), (11, 29), (17, 23), (23, 17), (29, 11)
42	(5, 37), (11, 31), (19, 23), (23, 19), (31, 11)
44	(3, 41), (7, 37), (13, 31), (31, 13)

Continued

46	(5, 41), (17, 29), (29, 17), (41, 5)
48	(5, 43), (11, 37), (17, 31), (19, 29), (29, 19)
50	(3, 47), (7, 43), (13, 37), (19, 31), (31, 19), (43, 7)