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A Review on Methods for Determining the Vibratory Damping Ratio

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Abstract

This article aims to popularize the methods for determining the vibratory damping ratio, to explain the various mathematical and physical theorems related to the establishment of literal expressions. Vibration damping is an essential parameter to reduce the dynamic responses of structures. The study aimed at its determination is necessary and essential for the safeguard of buildings and human lives during the earthquake. Among the main methods studied in this article, the free vibration attenuation method seems to be easy to implement but requires a state-of-the-art device to capture the responses. In addition to this device, the other methods require other equipment for the vibration of the system and the transformation of the responses in the frequency domain.

Keywords

Damping, Oscillator, Attenuation of Free Vibrations, Amplification at Resonance, Resonance Peak Width, Energy Dissipated by Damping, Stored Elastic Energy

1. Introduction

The problem of structural vibrations remains topical given the damage that occurred after the recent deadly earthquakes in Turkey and Haiti. Predicting the dynamic behavior of structures subjected to earthquakes is an important issue for the prevention and reduction of seismic risk. This behavior is not only dependent on the physical parameters of the structure, but also on the capacity of the constituent material to dissipate vibration energy. The latter, called intrinsic damping of the material, is an essential factor in reducing structural vibrations [1]. During the last decades, several states of the world have developed paraseismic calculation standards and optimized the use of materials combining mechanical performance with good dissipative properties. However, despite this popularization of standards, the techniques for evaluating the vibration damping ratio have not evolved [2] and do not take into account the degradation of the material during previous vibrations [3].

The standards [4] [5] [6] fix the value of this ratio for the regulatory response spectrum and propose a relation for its correction, without however exposing the physical and mathematical approaches allowing its evaluation. The experiment carried out by Ali Mikael *et al.* (2011) in [2] shows that the measured values are different from those fixed by the standards. Although the physical origin of damping remains imprecise and less well known, it is roughly modeled by viscous damping. This study presents some methods (vibratory and energetic) allowing its estimation. For this, after the modeling of the system and the setting in vibration of the system under free or forced solicitations, the free responses obtained are exploited directly while the force responses are first transformed in the frequency domain before the exploitation. The final literal expressions of the vibration damping ratio are obtained as follows:

- The ratio of two free responses taken one cycle apart;
- The ratio of the response to resonance and the static response;
- Measurement of the bandwidth of the frequency spectrum of harmonic excitations;
- The ratio of the energy dissipated by damping and the maximum elastic energy during a cycle of vibration.

The importance of this work is to popularize and explain the techniques for evaluating the vibration damping ratio of a material.

2. Description of the Methods for Determining the Vibration Damping Ratio

Before the description of the different methods, the system is modeled by a one-dimensional discrete model (simple oscillator). It consists of a concentrated mass, where the inertia forces are applied, and linked to the ground using a vertical rod of negligible mass. The rod represents the bending stiffness of the system. From this model will be submitted the support conditions and a type of solicitation depending on the procedure. The demonstrations will be made on this model, considering that it oscillates with damping (**Figure 1**).

The equation of motion generated for such a system is written:

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=0 \tag{1.1}$$

where m, c, k are the mass, the damping constant and the rod stiffness, respectively.

$\ddot{u}(t),\dot{u}(t),u(t)$ denote respectively, the acceleration, the velocity and the displacement of the mass of the oscillator.

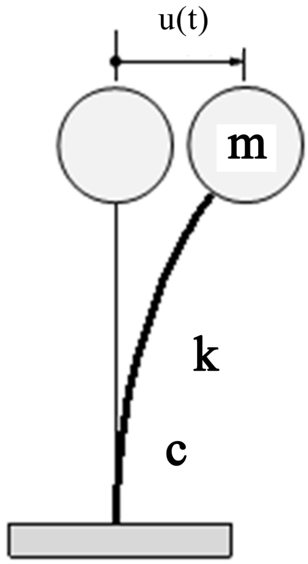

Figure 1. Single degree of freedom oscillator.

The vibration of a mechanical system results from the non-dissipation of the energy received and which persists. When it dissipates, the amplitudes of the movement attenuate until static equilibrium. The methods (vibratory and energetic) related to its determination are briefly explained in the following section.

2.1. Description of Vibration Methods

The process consists in subjecting the simple oscillator to a free excitation on the one hand and harmonic on the other hand. The harvested temporal process is processed directly by free vibration attenuation, so long as the other two methods (resonance amplification and resonance peak width) are first transformed into the frequency domain before processing. The description of the vibration methods is described in the following paragraphs.

2.1.1. System Free Vibration Mitigation Method

Free vibrations are induced in the simple oscillator by the application of an impact force or by a drop with no initial velocity from its displaced position or by an impulse-like movement. During the free vibrations, the responses are recorded and plotted on a graph as a function of time. The amplitudes of movement attenuate until static equilibrium. This attenuation describes the vibration damping of the system. The determination of the damping ratio of the system consists in measuring the rate of decay of the free oscillations. This calculation procedure was implemented by Clough & Penzien in 1993, and is called the logarithmic (δ) decrement of free vibrations [7] [8]. The analytical expression of the response of such a system is obtained by the relation below.

$$u(t) = U\mathrm{e}^{-\xi\omega t}\sin(\omega_d t + \theta) \text{ or } u(t) = U\mathrm{e}^{-\xi\omega t}\cos(\omega_d t - \theta) \tag{1.2}$$

where $\xi = c/2m\omega$ is the damping ratio, $\omega = \sqrt{k/m}$ is the oscillator's pulsation and $\omega_d = \omega\sqrt{1-\xi^2}$ the pulsation of damped oscillations.

Taking into account the initial conditions($u(0) = u_o = 0, \dot{u}(0) = \dot{u}_o = 0$), we obtain:

$$\begin{cases} U = \dfrac{1}{\omega_d}\sqrt{(u_0\omega_d)^2 + (\dot{u}_0 + \xi\omega u_o)^2} \\ \theta = \tan^{-1}\left(\dfrac{u_0\omega_d}{\dot{u}_0 + \xi\omega u_o}\right) \end{cases} \tag{1.3}$$

Consider the displacements measured at two consecutive peaks, the associated equations are:

At time *t*:

$$u(t) = U\mathrm{e}^{-\xi\omega t}\sin(\omega_d t + \theta) \tag{1.4}$$

At time $t + T_d$:

$$u(t+T_d) = U\mathrm{e}^{-\xi\omega(t+T_d)}\sin(\omega_d(t+T_d)+\theta) \tag{1.5}$$

The two displacements are illustrated graphically in **Figure 2**.

Let us make the report of these two displacements,

$$\frac{u(t)}{u(t+T_d)} = \frac{U\mathrm{e}^{-\xi\omega t}\sin(\omega_d t+\theta)}{U\mathrm{e}^{-\xi\omega(t+T_d)}\sin(\omega_d(t+T_d)+\theta)} \tag{1.6}$$

With

$$\sin(\omega_d(t+T_d)+\theta) = \sin((\omega_d t+\theta)+\omega_d T_d) \tag{1.7}$$

$$\sin((\omega_d t+\theta)+2\pi) = \sin(\omega_d t+\theta) \tag{1.8}$$

By introducing Equations (1.7) and (1.8) into Equation (1.6), it becomes:

$$\frac{u(t)}{u(t+T_d)} = \frac{\mathrm{e}^{-\xi\omega t}}{\mathrm{e}^{-\xi\omega(t+T_d)}} = \mathrm{e}^{\xi\omega T_d} \tag{1.9}$$

By taking the logarithm of both sides of Equation (1.9), we get the logarithmic decrement:

$$\delta = \ln\left(\frac{u(t)}{u(t+T_d)}\right) = \xi\omega T_d = \xi\omega\frac{2\pi}{\omega_d} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \tag{1.10}$$

On the other hand, if the attenuation of the oscillations is provided slowly, the logarithmic decrement can be calculated as an average of several peaks.

$$\delta = \frac{1}{n}\ln\left(\frac{u(t)}{u(t+nT_d)}\right) \tag{1.11}$$

he free vibration damping ratio of the system is obtained from the logarithmic decrement, by doing the inverse of Equation (1.10).

$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \tag{1.12}$$

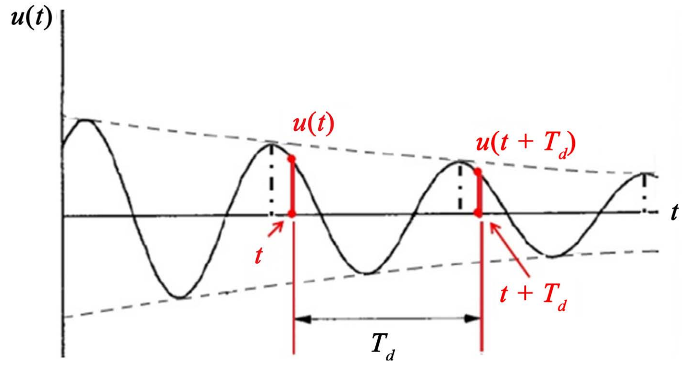

Figure 2. Two free answers taken at an interval cycle.

2.1.2. Resonance Amplification Method

For this method, the damping ratio is obtained by the report of the amplitude of the resonance response to the displacement if the force is applied statically [3]. For this, the forced vibrations are induced to the system under study by the application of a forced harmonic movement, with a constant amplitude. This type of vibration is expressed by the following relationship.

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=F_0\sin\bar{\omega}t \tag{1.13}$$

During the forced vibrations, let us also draw as before the graph of the movement which follows. This graph obtained is analytically described by the expression (1.14) and represents the solution of the equation of motion (1.13) for an underdamped system ($\xi < 1$) forced by a harmonic force [7] [8] [9].

$$u(t)=U\mathrm{e}^{-\xi\omega t}\sin(\omega_d t+\theta)+\frac{F_0}{k}\frac{1}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}}\sin(\bar{\omega}t-\theta) \tag{1.14}$$

The first term of Equation (1.14) constitutes the free response of the system which was studied in the previous paragraph and the second the forced response. The free response is damped over time, all the more rapidly as the critical damping percentage is high and the response tends towards the forced solution. The contribution of the transient response can be neglected. The forced response can then be written, similar to Equation (1.4):

$$u(t)=\rho\sin(\bar{\omega}t-\theta) \tag{1.15}$$

where ρ represents the amplitude of the response and θ the phase which characterizes the phase shift between the force applied and the resulting displacement. The amplitude of the response and the phase are equal to:

$$\begin{cases}\rho=\dfrac{F_0}{k}\dfrac{1}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}}=\dfrac{F_0}{k}D & (1.16)\\[2ex] \theta=\operatorname{artan}\dfrac{2\xi\beta}{1-\beta^2}\ \text{ et }\ \beta=\dfrac{\bar{\omega}}{\omega} & (1.17)\end{cases}$$

F_0/k represents the static displacement (u_{stat}) of the mass when the stress is worth F_0 and D the dynamic amplification factor.

To obtain the vibration damping ratio by resonance amplification, we will construct the frequency spectrum made up of amplitudes of the series of discrete pulsations of the harmonic excitation force. By plotting these couples (amplitude-pulsation), we obtain the graph below (**Figure 3**) [10] [11] [12].

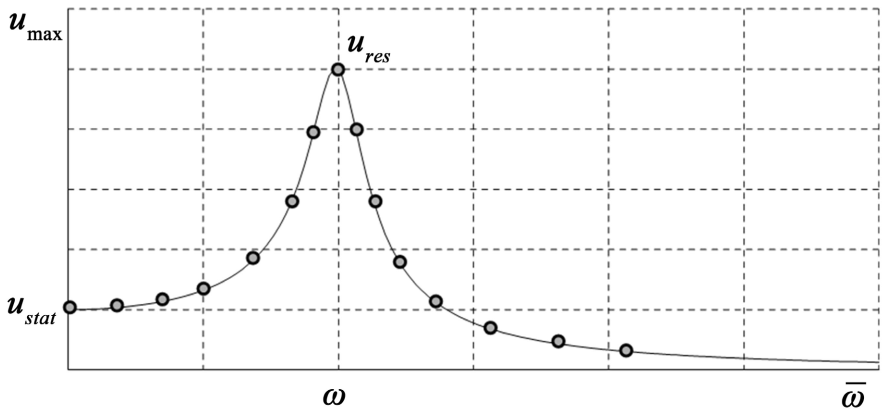

Figure 3. Frequency spectrum of harmonic excitations of the system.

From the Equation (1.16), we can say that when the pulsation $\overline{\omega}$ of the solicitation coincides with the own pulsation ω of the oscillator, we reach the resonance ($\beta = 1$) and the amplification factor D passes by a maximum.

$$D_{\max} = \frac{1}{2\xi} \tag{1.18}$$

From the previous expression (1.18), we deduce that the maximum displacement or the amplitude at resonance is directly related to the damping and is equal to:

$$u_{\max} = u_{res} = \frac{F_0}{k}\frac{1}{2\xi} \tag{1.19}$$

The vibration damping ratio obtained by resonance amplification is determined from Equation (1.19) and taking into account the previous considerations by the following expression:

$$\xi = \frac{u_{stat}}{2u_{\max}} \tag{1.20}$$

2.1.3. Resonance Peak Width Method

It makes it possible to estimate the resonance frequency and the associated damping ratio [8]. Proceeding in the same way as before, to obtain the bandwidth, the forced responses of the system are also represented on a graph as a function of the series of discrete pulsations of the harmonic excitation force (**Figure 4**). By substituting the frequency of the maximum amplitude($\beta_{\max}$), in the solution of the forced response (1.16), the amplitude at the resonance becomes:

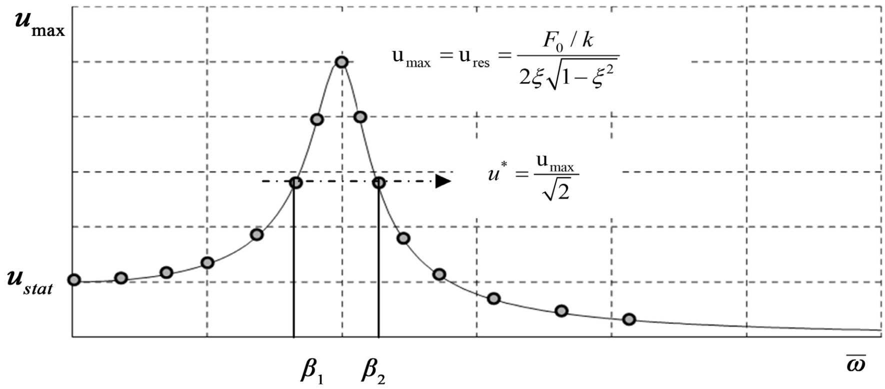

Figure 4. Measurement of the damping ratio by the half-power bandwidth method.

$$\beta_{max} = \sqrt{1-2\xi^2} \text{ et } D(\beta = \beta_{max}) = \frac{1}{2\xi\sqrt{1-\xi^2}} \tag{1.20}$$

$$u_{max} = u_{res} = \frac{F_0/k}{2\xi\sqrt{1-\xi^2}} \tag{1.21}$$

On the system response curve to harmonic excitation, identify two points corresponding to the amplitude at resonance multiplied by $1/\sqrt{2}$ and their respective pulsations. By equating the answer to Equation (1.16) by $1/\sqrt{2}$ multiplying the answer to Equation (1.21), we get the vibration damping ratio, using the pulsation values corresponding to the reduced amplitude.

$$\frac{F_0/k}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}} = \frac{1}{\sqrt{2}} \frac{F_0/k}{2\xi\sqrt{1-\xi^2}} \tag{1.22}$$

By expanding both sides, Equation (1.22) becomes:

$$\beta^4 - 2(1-2\xi^2)\beta^2 + 1 - 8\xi^2(1-\xi^2) = 0 \tag{1.23}$$

Its roots are:

$$\beta^2 = (1-2\xi^2) \pm 2\xi\sqrt{1-\xi^2} \tag{1.24}$$

Taking into account that the system is under damped, with the small values of damping [1], we have:

$$\beta = (1 \pm 2\xi)^{\frac{1}{2}} \tag{1.25}$$

According to the Taylor series expansion, we can write that:

$$\beta = 1 \pm \xi \tag{1.26}$$

The two roots $\bar{\omega}_1$ and $\bar{\omega}_2$ are written:

$$\frac{\bar{\omega}_1}{\omega} = 1 - \xi \text{ et } \frac{\bar{\omega}_2}{\omega} = 1 + \xi \tag{1.27}$$

By taking the difference of the two distinct roots, we obtain the damping ratio by:

$$\xi = \frac{\bar{\omega}_2 - \bar{\omega}_1}{2\omega} \tag{1.28}$$

2.2. Description of the Energy Method

The process also consists in subjecting the simple oscillator to a harmonic excitation which puts it in forced vibration. During a vibration cycle, the energy dissipated by damping and the maximum elastic energy are calculated. By calculating the ratio of the two energies, the value of the vibration damping ratio is deduced.

2.2.1. Determination of Energy Dissipated by Damping

The description of the phenomenon of energy dissipation by damping is obtained using an equivalent damper, assuming that the energy dissipated in one cycle of vibration of the system is equal to the energy dissipated in a linear damper for a cycle of the same displacement amplitude. For that, let us consider a system subjected to a cycle characterized by a maximum amplitude of the displacement equal to $u_{\max}$. During this cycle, the force required to deform the structure is measured. The force-displacement diagram is represented by the curve limiting the hatched area in **Figure 5** [9]. The area of the loop represents the energy dissipated E_D by the structure during a stress cycle.

Now consider that the linear damper in **Figure 5** is subjected to a pulsating harmonic force ϖ.

$$f(t) = f_0 \sin \varpi t \tag{1.29}$$

For this system, the damper constant c is given by:

$$c = \frac{f_{D\max}}{\dot{u}_{\max}} = \frac{f_0}{\varpi u_{\max}} \tag{1.30}$$

The force-displacement curve described by this system is represented by an ellipse, shown as a dotted line in **Figure 5**. The energy dissipated during a cycle by the linear damper is given by:

$$E_D = \oint f_D \mathrm{d}u = \int_0^T c\dot{u}\mathrm{d}u = \int_0^T c\dot{u}^2 \mathrm{d}t \tag{1.31}$$

Assuming that the excitation and the displacement are cyclic, we have:

$$u(t) = u_{\max} \sin \varpi t \tag{1.32}$$

As a result:

$$\dot{u}(t) = \varpi u_{\max} \cos \varpi t \tag{1.33}$$

The energy dissipated during an excitation cycle is therefore:

$$E_D = \int_0^{T=\frac{2\pi}{\varpi}} c\left(\varpi u_{\max} \cos \varpi t\right)^2 \mathrm{d}t = c\varpi^2 u_{\max}^2 \int_0^{T=\frac{2\pi}{\varpi}} \cos^2 \varpi t \mathrm{d}t \tag{1.34}$$

With $\cos^2 \varpi t = \dfrac{1+\cos 2\varpi t}{2}$, therefore:

$$E_D = \frac{c\varpi^2 u_{\max}^2}{2} \int_0^{T=\frac{2\pi}{\varpi}} \left(1+\cos 2\varpi t\right) \mathrm{d}t \tag{1.35}$$

$$E_D = \frac{c\varpi^2 u_{\max}^2}{2} \left[t + \frac{1}{2\varpi} \sin 2\varpi t \right]_0^{T=\frac{2\pi}{\varpi}} = \pi c \varpi u_{\max}^2 \tag{1.36}$$

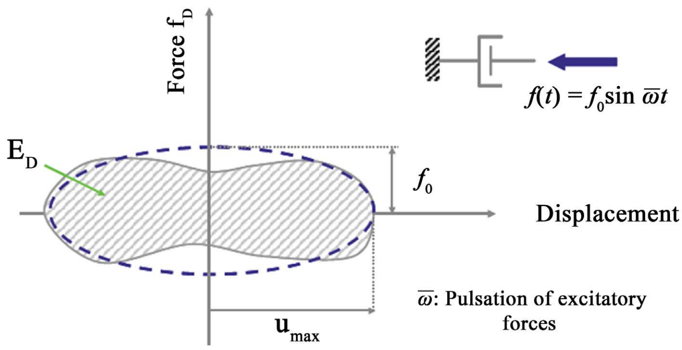

Figure 5. Force-displacement diagram of a vibration cycle of the linear damper.

2.2.2. Determination of the Stored Elastic Energy

The description of the phenomenon of stored elastic energy is obtained using a linear elastic spring, assuming that the elastic energy stored for an amplitude of displacement of the system is restored when the load is removed. For this, the restoring force is therefore proportional to the displacement and is equal to.

$$f_R = -ku \tag{1.37}$$

The work done by the restoring force of a linear spring when it is stretched from the initial configuration (unstretched) to the final configuration (stretched) is easily obtained as follows:

$$W = \int_0^u -ku\mathrm{d}u = -\frac{1}{2}ku^2 \tag{1.38}$$

More precisely, the variation of potential energy of a body when it moves between two points is the opposite of the work provided by the force to which it is subjected between these two points [10]. Thus the work of a conservative force verifies the relation:

$$\Delta W = -\Delta U \tag{1.39}$$

With

$$\Delta W = f_R \cdot \mathrm{d}u = -\Delta U \tag{1.40}$$

So

$$\Delta U = ku \cdot \mathrm{d}u \tag{1.41}$$

The corresponding potential energy of the deformed spring is then deduced from Equation (1.41), by integration.

$$U = \frac{1}{2}ku^2 \tag{1.42}$$

The maximum elastic potential energy is obtained when the displacement is maximum.

$$U_{max} = \frac{1}{2}ku_{max}^2 \tag{1.43}$$

During a complete cycle, the energy stored in potential form in the spring is entirely restored, on the other hand an energy is dissipated E_D in the shock absorber. The ratio of the two energies makes it possible to obtain the vibration damping ratio. Assuming that the damper constant $c = 2\xi m\bar{\omega}$ and the rod stiffness $k = \bar{\omega}^2 m$.

$$\xi = \frac{E_D}{4\pi U_{\max}} \tag{1.44}$$

3. Conclusion

The popularization of techniques for evaluating the vibration damping ratio can lead to a better estimation of the response of structures and a reduction in damage after the earthquake. The implementation of these methods requires high precision equipment. The logarithmic decrement method is less complex than the others and easy to perform, because all you need is a displacement sensor. The determination of the damping ratio by the vibration methods is obtained from the forced or free responses of the system, while for the energy method, it is obtained from the alternating transfer of the stored and dissipated energies causing the vibration of the system. It is desirable to take into account in the literal expressions, the degradations undergoes by the structures during the dynamic requests.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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