Scientific Research Publishing

Open Journal of Civil Engineering, 2022, 12, 559-571
https://www.scirp.org/journal/ojce
ISSN Online: 2164-3172
ISSN Print: 2164-3164

Detecting Damage in Reinforced Concrete Beams Using Vibrational Characteristics

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How to cite this paper: Hien, H.T., Thang, N.D. and Dang, N.N. (2022) Detecting Damage in Reinforced Concrete Beams Using Vibrational Characteristics. *Open Journal of Civil Engineering*, **12**, 559-571.
https://doi.org/10.4236/ojce.2022.124031

Received: September 27, 2022
Accepted: December 26, 2022
Published: December 29, 2022

Abstract

This paper evaluates two methods of diagnosing damage, Natural frequency and Stiffness-Frequency change-Based damage detection method in reinforced concrete beams under load using vibration characteristics such as natural frequency and mode shape. The research uses finite element method with crack damage instead of deleting or reducing the bearing capacity of the element like in previous studies. First, a theory of the damage diagnosis method based on the change of natural frequency and mode shape is presented. Next, the simulation results of reinforced concrete beams using ANSYS will be compared with the experiment. Particularly, the investigated damage cases are cracks in reinforced concrete beams under loads. Finally, we will evaluate the accuracy of the damage diagnosis methods and suggest the location of the vibration data and specify the failure threshold of the methods.

Keywords

Damage Detection, Vibration Method, Reinforced Concrete Beam, Natural Frequency, Mode Shape

1. Introduction

It is inevitable that construction works, after a long period of use under the influence of many subjective and objective factors, will occur unwanted damage. This affects safety factors, repair costs, and the longevity of the building. Therefore, monitoring, maintenance, and quality assessment of the building are essential. As a result, structural health monitoring (SHM) has been studied and applied more to architectural works. The diagnostic work is divided into 5 stages: damage detection, damage location, damage classification, damage assessment and damage prognosis [1]. This study will focus on the first two diagnosis tasks: damage detection and damage location.

Two common methods of diagnosing damage in structures today are destructive and non-destructive. Specifically, the advantages of the non-destructive method (*i.e.*, cost saving, time effectivity, and high applicability) give this method greater attention than the destructive method. Moreover, some preliminary studies have shown positive results in finding the relationship between damages and frequency characteristics [2] [3] [4] [5].

A non-destructive experimental technique proposed by Adams *et al.* showed an association between the damage and the reduction of natural frequency. The damage was assumed to be a reduction in the axial spring stiffness. The experiment analyzed the 1D model of a simple metal bar with and without a cut, allowing for the identification and location of the damage. The study also demonstrated how temperature affects vibration frequency [2].

Cawley and Adams have further developed the work of Adams *et al.* [2], which provides a method that can detect, locate the damage and estimate the degree of damage for aluminum and composite panels. The main basis of the research is based on the natural frequency shift (NFS) of two different mode shapes. The study shows a link between NFS and damage in the plate. The limitation of the study is single damage and does not apply to materials with multi-directional properties [3].

Gudmundson showed a method to model cracks or cuts in beams and how to calculate the changes in natural frequency for this local damage, which had not been done in earlier studies. He proposed two simulation techniques: the first technique is based on the theory of stress and energy, and the second technique simulates damage using a 2D finite element model. Both approaches demonstrate the relation between the resonant frequency change and damage size [4].

A study on the free vibrations of large beams and sudden changes in cross-section provides a stiffness transformation matrix at the position of the cross-sectional change and simulated by finite element. The finite element simulation results of the 1D beam show the similarity when compared with the experimental results. In addition, the correlation between the first natural frequency is influenced by the length and depth of the damage (the cross-sectional position changes) [5].

With the development of the finite element method in structural simulation, damage detection studies using this method are increasingly popular and have a high-reliability level. However, in previous studies, the damage was often assumed to be a result of the decrease in the element's stiffness or the removal of the element from the model. Consequently, the diagnostic results are not close to the actual damage in the structure.

This study will overcome the above factors by simulating the damage in the form of cracks under the effect of loads. From there, the damage detection methods using vibration will be applied to diagnose the damage.

2. Methodology

2.1. Natural Frequency

The damage diagnosis method based on the change of natural frequency is a simple, fast, and effective method in the SHM field. Theoretically, when damages appear in the structures, the structure's natural frequency will decrease simultaneously with the stiffness decrease in the structure [3] [4] [5] [6]. Based on this feature, the degree of variation of the natural frequency is calculated by the below equation:

$$\text{NFS} = \Delta f\,(\%) = \left| f - f^{*} \right| / f * 100\,(\%) \tag{1}$$

- NFS and $\Delta f\,(\%)$: natural frequency shift (%)
- *f*: the natural frequency of the undamaged state
- f^{*}: the natural frequency of the damaged state

2.2. Stiffness-Frequency Change-Based Damage Detection Method

A diagnostic method based on the changes in stiffness and frequency has been researched and developed [7] [8]. The method's foundation is based on the relationship between the change of the weightless spring stiffness between the elements and the strain energy in the structure when damage occurs. Following previous research, [9] has improved the relationship between elements in the link model by weightless spring, providing a calculation method for the damage vector by an inverse matrix.

Based on the proposed approximate calculation method and some calculation methods for the oscillation frequency, we have the following:

$$\omega^2 = \nu = \left(\frac{1}{2} \int_0^L EI(x) \left(\mathrm{d}^2 Z / \mathrm{d}x^2 \right)^2 \mathrm{d}x \right) \Big/ \left(\frac{1}{2} \int_0^L \rho A Z^2 \mathrm{d}x \right) = K/P \tag{2}$$

In which:

- *K*: strain energy

$$\frac{1}{2} \int_0^L EI(x) \left(\mathrm{d}^2 Z / \mathrm{d}x^2 \right)^2 \mathrm{d}x = \frac{1}{2} \int_0^L \psi \mathrm{d}x, \; \psi = EI(x) \left(\mathrm{d}^2 Z / \mathrm{d}x^2 \right)^2 \tag{3}$$

- *P*: kinematic energy

$$P = \frac{1}{2} \int_0^L \rho A Z^2 \mathrm{d}x \tag{4}$$

With a small change of frequency:

$$\frac{\Delta \nu}{\nu} = \frac{\Delta K}{K} - \frac{\Delta P}{P} \tag{5}$$

For beams with cracks are perpendicular to the beam axis and vibrating with $\Delta P = 0$. Set ΔK:

$$\Delta K = \frac{1}{2} \int_0^L S \psi \mathrm{d}x \tag{6}$$

With S being the parameter showing the change of K, in the case that K does not change or has no damage, we have $S = 0$ and $S = 1$ when the beam is completely cracked. If the beam is divided into n elements, the failure parameter S_i corresponds to the i^{th} element, L_i is the length of the i^{th} element, m measured frequencies, we adjust the Equation (5) as:

$$\frac{\Delta\omega_m}{\omega_m} = 2\sum_{i=1}^{n}\int_L g_m(\xi) L \mathrm{d}\xi S_i, \; g_m(\xi) = \frac{\psi_m}{4\int_L \psi_m \mathrm{d}x} \tag{7}$$

With the beams have n elements and m measured frequencies, the Equation (7) can be written in matrix form:

$$\left\{\frac{\Delta\omega}{\omega}\right\}_{m\times 1} = 2[H]_{m\times n}\{S_i\}_{n\times 1} \text{ in which } h_{ij} = \int_{L_i} g_i(\xi) L \mathrm{d}\xi \tag{8}$$

Ignoring the change of elastic modulus and moment of inertia of the structure, the value $\left(\mathrm{d}^2\Phi/\mathrm{d}x^2\right)^2$ in Equation (3) at each node is computed by central difference approximation. Rewrite the Equation (3):

$$\begin{aligned}\int_{L_i}\psi_i \mathrm{d}x &= EI(x)\int_{L_i}\left(\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2}\right)^2 \mathrm{d}x = EI(x)\int_{L_i}(\Phi_i'')^2 \mathrm{d}x \\ &= EI(x)\frac{(\Phi_{j+1}'')^2 + (\Phi_j'')^2}{2}(x_{j+1} - x_j) = EI(x) a_i\end{aligned} \tag{9}$$

In which Φ_i'' is the second derivative of the mode shape at the j_{th} node, x_j is the j_{th} node coordinate, L_i is the i_{th} element length. Considering the structure is homogeneous, there is no change in cross-section. The values of the matrix $[H]$ can be calculated according to the formula:

$$h_{i,j} = a_{i,j}\Big/\sum_{i=1}^{n} a_{i,j} \tag{10}$$

After calculating the matrix $[H]$ and the natural frequency change vector $\{\Delta\omega/\omega\}_{m\times 1}$, for $m = n$, it is now easy to calculate the element damage vector using the matrix inverse. However, in reality, the vibration data obtained is limited. In most cases, it is smaller than the number of measured points. Therefore, we have $m < n$, and the calculation of the damage vector now needs to use another method—the pseudo-inverse matrix. In the thesis, MATLAB software is used to calculate the matrix.

$$\{S_i\}_{n\times 1} = \frac{1}{2}[H]^{+}_{m\times n}\{\Delta\omega/\omega\}_{m\times 1} \tag{11}$$

With $[H]^+$ is the matrix's pseudo-inverse matrix $[H]$. S_i carries the meaning of the element's participation in the change of natural frequency, so the solution condition of Equation (11) is $S_i > 0$. For $S_i < 0$ values, set these values to 0 and recalculate until the condition is satisfied. To calculate the decreasing natural frequency of each element, we replace the vector $\{S_i\}_{n\times 1}$ in Equation (10) with the vector $\{0, 0, \cdots, S_i, \cdots, 0, 0\}^{T}_{1\times n}$. Now we have the formula:

$$\{\Delta\omega/\omega\}_{m\times 1,i} = 2[H]_{m\times n}\{0,0,\cdots,S_i,\cdots,0,0\}^{\mathrm{T}}_{1\times n} \tag{12}$$

After calculating all elements, the vectors are combined and separated into vectors that change the frequency of each element according to mode shape. For this type of data, this paper proposes a damage threshold using the normalizing data method to determine the damaged area:

$$Z = \left(\frac{\Delta\omega}{\omega} - \mu_{\frac{\Delta\omega}{\omega}}\right) \Big/ \delta_{\frac{\Delta\omega}{\omega}} \tag{13}$$

2.3. Evaluation Indicators

2.3.1. Accuracy of the Crack Zone: *A*

The crack zone accuracy A is the ratio of the diagnostic crack length to the actual crack length. This index is intended to evaluate the method's accuracy in determining the crack zone in the structure

$$A = \left(L_p/L_c\right)\cdot 100\% \tag{14}$$

In which

- L_P is the length of the diagnostic crack length,
- L_C is the actual crack length.

2.3.2. Accuracy of the Non-Cracked Zone: *B*

In addition to assessing the cracked region's accuracy, it is also necessary to evaluate the non-cracked region. The non-cracked zone accuracy is the ratio of the uncracked diagnostic length to the actual uncracked length:

$$B = \left(L_{pnc}/L_{nc}\right)\cdot 100\% \tag{15}$$

In which

- L_{pnc} is the length of the diagnostic non-cracked area, $L_{pnc} = L - L_C - L_{p,out}$;
- $L_{p,out}$ is the length of the diagnostic crack outside the actual crack;
- L_{nc} is the actual crack length.

Overall accuracy: C

An index is required to assess the accuracy of the full length of the beam. Therefore, the proposed overall accuracy index C is significant as the sum of the crack zone accuracy A and the uncracked zone accuracy B, then multiplied by the proportion of each zone. The proportion of each zone corresponds to the ratio of the crack zone's length or the length of the uncracked zone divided by the total length of the beam. The selection of the damage threshold of beams requires consideration of three indexes A, B, and C, to give an appropriate damage threshold.

$$C = A\left(L_c/L\right) + B\left(L_{nc}/L\right) \tag{16}$$

3. Model

A previous study tested reinforced concrete beams at three different load levels. Natural frequencies are obtained through sensors for each load level and different cracking states of the beam [10]. In this study, we have used the reinforced concrete beam parameters of the above study to simulate using ANSYS software and compare the results with the experiment. Beams have dimensions 120 × 240 × 3800. Dimensions and reinforcement details are shown in **Figure 1**.

Simple beam with a span length of 3800 mm and beam cross-section of 120 × 240 mm. The beam is arranged with 2ϕ16 reinforcements at the bottom and 2ϕ8 reinforcements above, ϕ8d100 reinforcement running along the beam. The thickness of the protective concrete layer is 25 mm. The beam is divided into 18 parts with the following dimensions: from E2 to E17 with dimensions of 200 mm, E1 and E18 with dimensions of 300 m, as shown in the figure. Load position between E8 and E9, 1.7 m from the end of beam, displacement data collection at the underside of the beam.

The parameters of reinforced concrete are based on standard ACI 318-14. Specifically, concrete has a compressive strength of 40 MPa, tensile strength of 3921 MPa, and elastic modulus of 29,726 MPa. The reinforcement has tensile strength 335 MPa, elastic modulus 210,000 MPa. The element types used in the concrete and reinforcement materials simulation are SOLID65 and BEAM188. The finite element model of beam is shown in **Figure 2**.

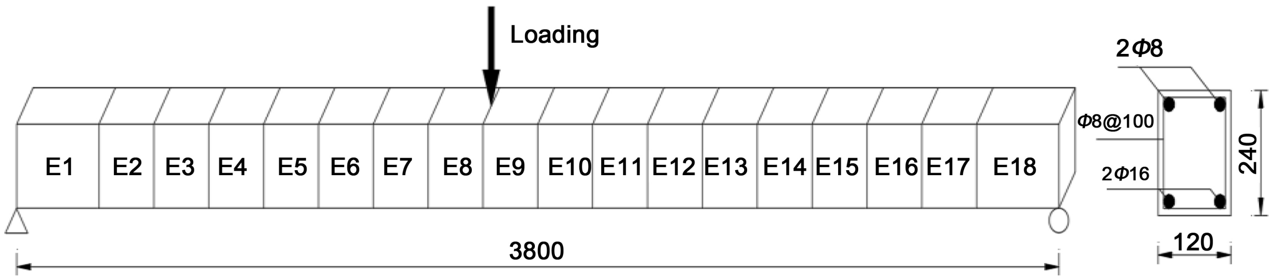

Figure 1. Parameters of beams [10].

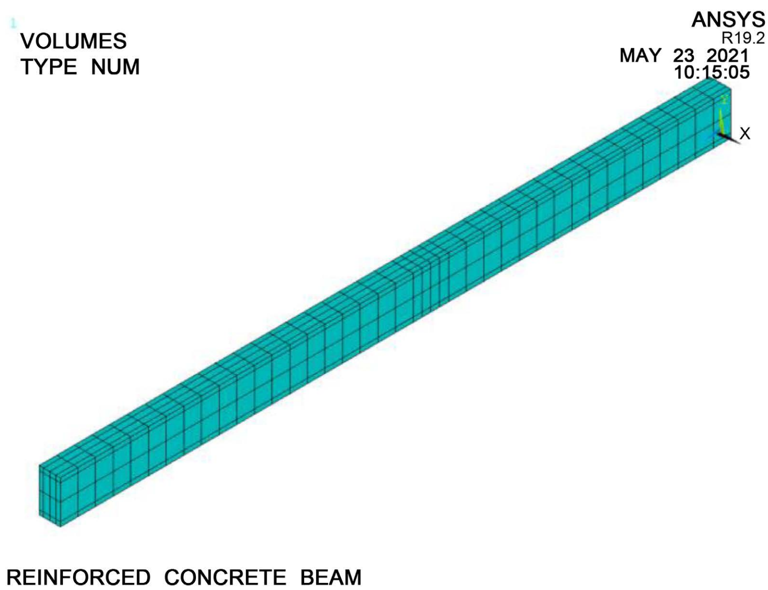

Figure 2. Simulation of beams by ANSYS.

The load is placed at a position 1.7 m from the beam end, with a transmission area of 0.1 × 1.2 m^2. In the paper, the simulation was carried out with 8 load levels: 0 kN, 3 kN, 6 kN are undamaged loads; 6.156 kN is the load causing cracking; 6.6 kN, 7.3 kN, 9.5 kN are the loads in the experiment [10]. The load level of 8 kN is used to investigate the transition between 7.3 kN and 9.5 kN loads.

Simulation results are compared with experimental and theoretical results. In **Table 1**, the cracking force is compared with the calculation according to the standard ACI 318-14 and the experiment [10]. The results show that the error of the cracking force is less than 6.8%.

In **Table 2**, the natural frequency of undamage beam is compared with the theoretical formula [11] and the experiment [10]. The error of the natural frequencie is less than 5.5%.

Previous studies mostly simulated homogeneous beams subjected to single or regional failure by deleting or reducing element stiffness [2]-[9]. In this study, the failure simulations of beams are cracks, according to the real damage of the beam. The finite element method uses ANSYS software to simulate reinforced concrete beams with high accuracy and reliability. The load-causing cracking and the initial natural frequency results of the model are similar to theoretical and experimental. Therefore, the finite element method can be used for research when it brings advantages such as cost and time effectiveness compared to experiments and can easily expand the problem with different load levels and states.

4. Results and Discussion

4.1. Analyze and Evaluate the Change in Natural Frequency

Vibration data of the beam at each load is collected from the finite element method and shown in **Table 3**.

The natural frequency changes are shown in **Figure 3**. In this figure, the analysis results show no difference in natural frequency at 3 kN, 6 kN—non-cracking

Table 1. Comparison table of cracking force results.

	Crack force	Error
Simulation	6.156 kN	0%
ACI 318-14	5.997 kN	−2.64%
Hong 2012	6.6 kN	6.73%

Table 2. Comparison table of natural frequency of undamaged beam results.

Mode shape	ANSYS	Chopra	Error (%)	Hong	Error (%)
Mode 1	27.516	26.268	4.53	26.375	4.15
Mode 2	102.405	105.073	−2.61	-	-
Mode 3	249.976	236.415	5.43	-	-
Mode 4	399.257	420.293	−5.27	-	-

Table 3. Natural frequency for each mode and load.

Natural frequency	Load (kN)							
	0	**3**	**6**	**6.156**	**6.6**	**7.3**	**8**	**9.5**
Mode 1	27.52	27.52	27.51	26.54	24.97	23.16	21.82	20.38
Mode 2	102.40	102.40	102.40	101.99	100.92	98.94	95.91	89.20
Mode 3	249.98	249.98	249.97	241.15	227.71	215.40	208.60	203.99
Mode 4	399.26	399.26	399.25	389.62	374.72	358.95	343.77	328.99

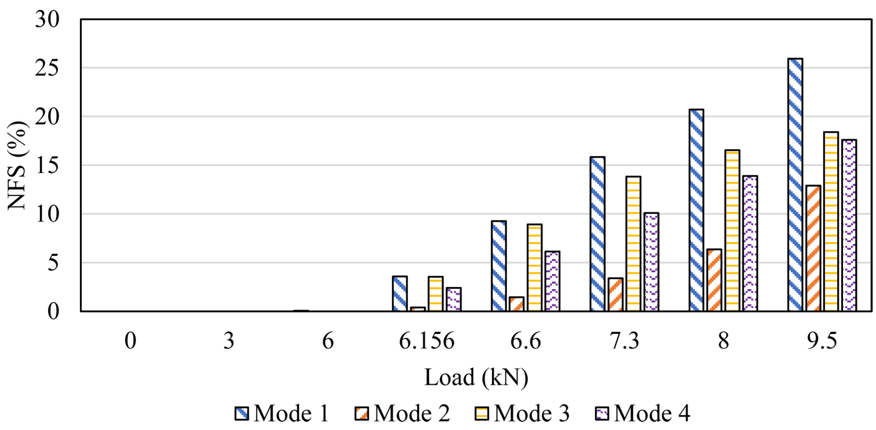

Figure 3. Natural frequency shift of the beam.

loads of beams. The natural frequency of the beam decreases when the beam has the first crack (load 6.156 kN) and decreases gradually when the load increases. Corresponding to the damage is an increasingly widening crack area. There is an unequal frequency change between mode shape; mode 1 and mode 3 show a clear change in natural frequency compared with modes 2 and 4. The crack appears and spreads from the position of loading to the beam end. This position is close to the peak of mode shape 1 and 3. From that, we can observe that the natural frequency sensitive to damage is close to the mode shape's peak position.

4.2. Stiffness-Frequency Change-Based Damage Detection Method

The method requires diagnostic data, including the natural frequency of the undamaged and damaged states and the mode shape of the undamaged states. Vibration data were collected from the ANSYS model at the beam's neutral axis position. This paper shows diagnostic data at load levels of 6.156 kN and 9.5 kN (**Figure 4**) and damage threshold Z = −0.2.

The degree of each beam element's participation in the frequency shift is calculated using this data, which is then utilized to apply the damage threshold using the normalized technique. Then, we will evaluate the model's accuracy using indicators A, B, and C. The S-FC-BDD technique is based on the identification of damaged parts. Therefore, the real failures are also presented as damaged elements to evaluate the accuracy. Diagnostic results are checked according to each load level. Particularly, there are 4 test modes for each load level and 1 composite mode (1, 2, 3, 4). The test failure threshold ranges from [−0.2, 0.2]. Z = −0.2 is the suggested damage threshold value, and the resulting graph is presented in **Figure 5** and **Figure 6**.

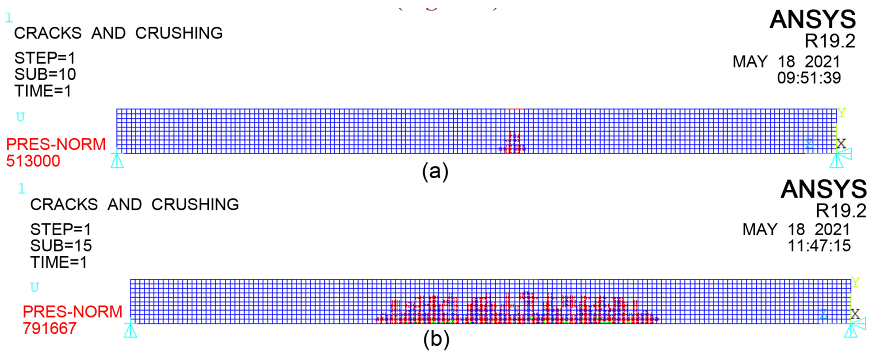

Figure 4. The damaged beam under load. (a) 6.156 kN. (b) 9.5 kN.

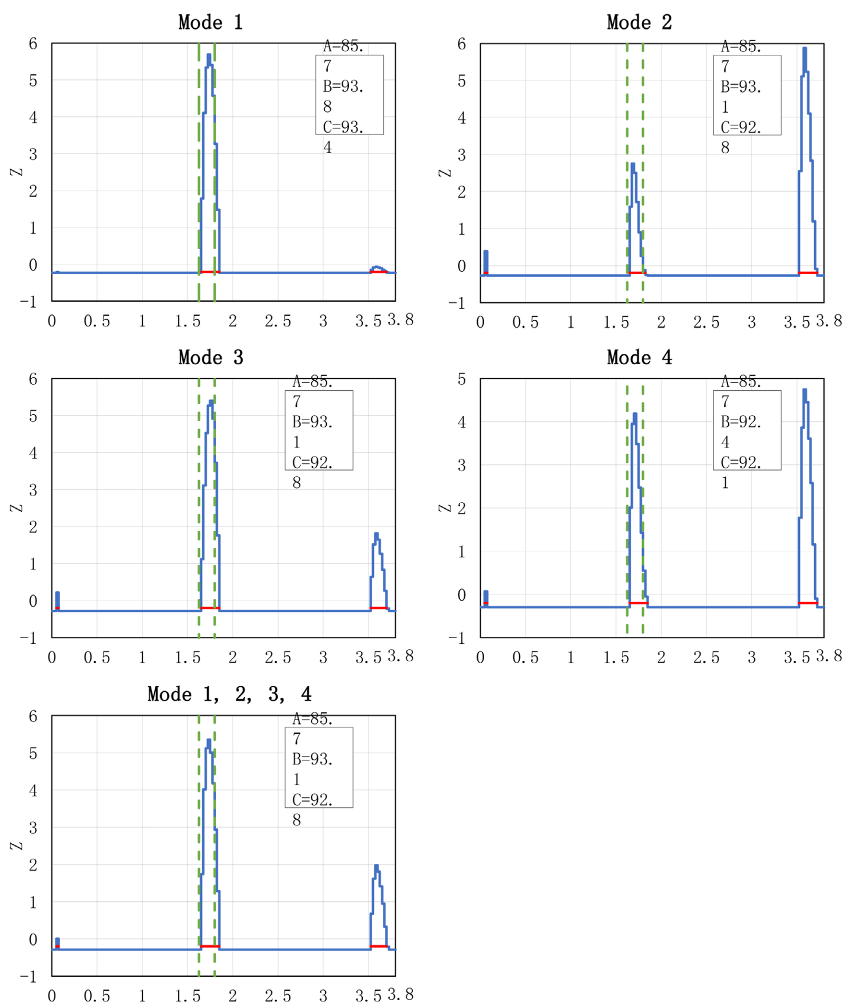

Figure 5. Diagnostic data at load level 6.156 kN with $Z = -0.2$.

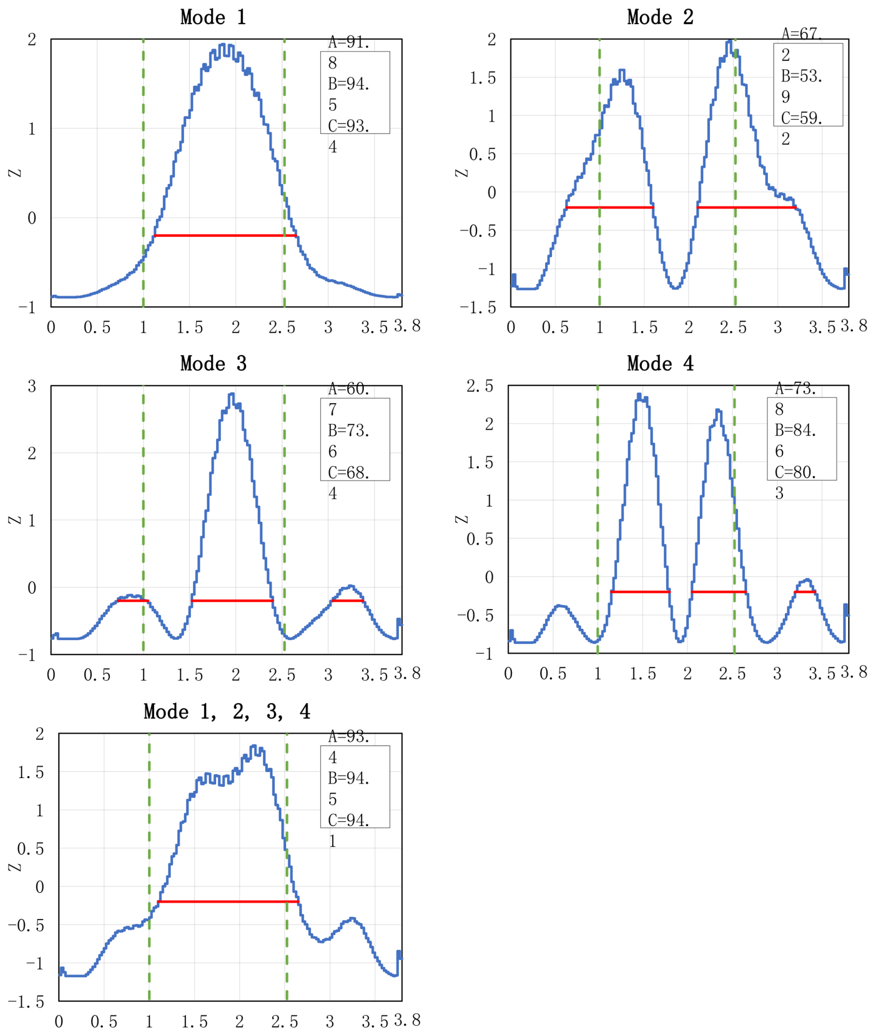

Figure 6. Diagnostic data at load level 9.5 kN with $Z = -0.2$.

The blue line is the normalized diagnostic result, the red line is the failure threshold or predicted damage area, and the area between two green line is real damaged area. From that, the diagnostic results of mode 1 and the combined mode give the closest results to the damaged area. Modes 2 and 4 have errors, but the diagnostic trend is still close to the actual damaged area. Besides, in the beam head area, there is a problem with diagnostic results, the cause comes from the boundary conditions of the model.

In Figure 7, with the remaining load levels corresponding to the damage threshold $Z = -0.2$, we have the results that the accuracy of A, B, and C are not uniform for each mode of oscillation. Accuracy A decreases with increasing load or increasing failure. Mode 1 and combined mode give results A, B, C, all higher than 75%, while mode 2, 3, 4 give negative results at high loads. The cause comes from the location of the beam's failure area near the top of mode 1, and mode 3. Therefore, these modes are more sensitive to damage, with the composite mode being the combination of all four modes. So, there will be more balanced and

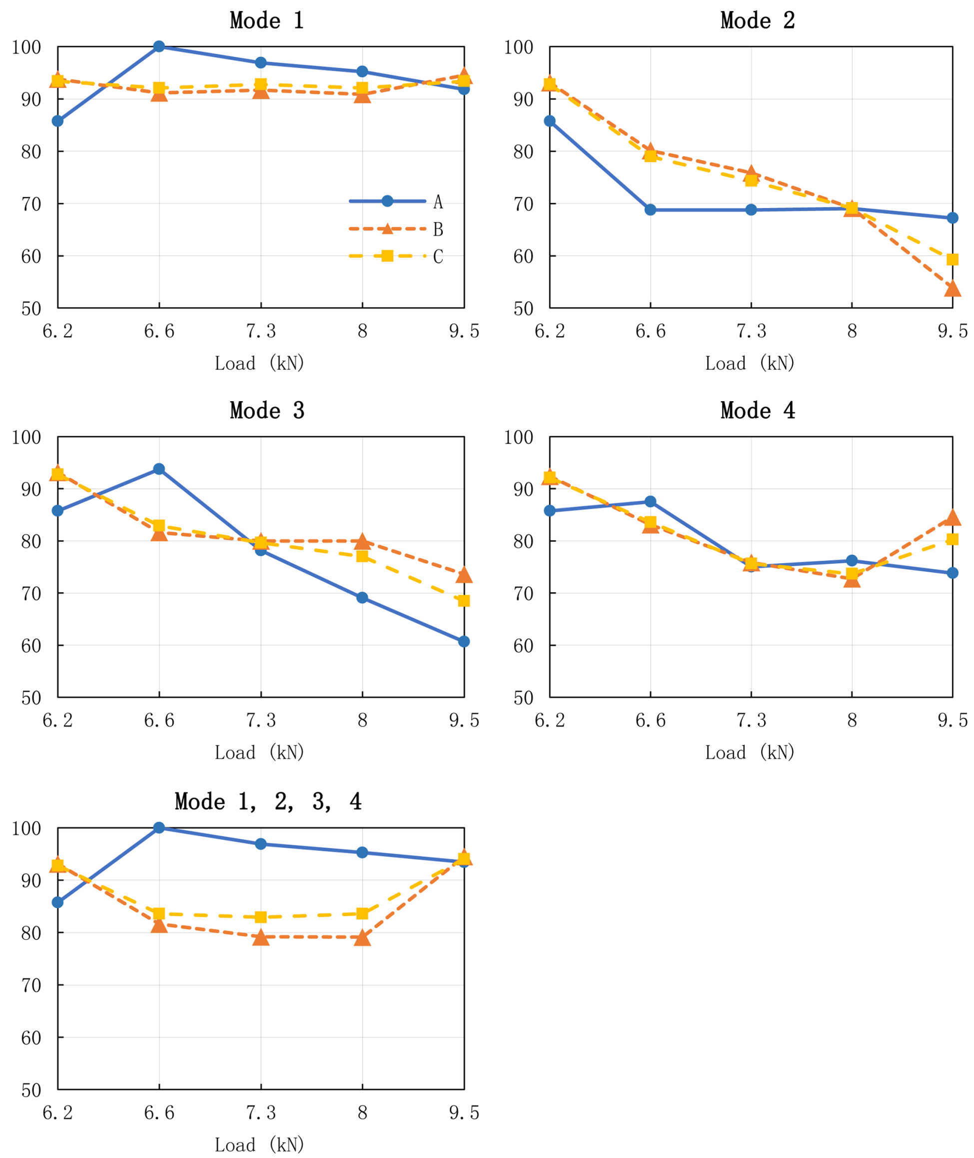

Figure 7. The average accuracy of the S-FC-BDD method for each load level with $Z = -0.2$.

objective than using an oscillating mode.

In **Figure 8**, the average A indicator decreases as Z increases, while the average B and C indicators increase as Z increases in the survey range $Z = [-0.2; 0.2]$. It can be shown that all three indicators A, B, and C average above 80% at the damage threshold $Z = -0.2$ and that there is not a difference between the three indicators. As a result, $Z = -0.2$ was selected as the suggested damage threshold.

Stiffness-Frequency change-Based damage detection method is a simple method that quickly determines whether a structure has been damaged. The method's limitation is that a large variation in frequency must exist for the best result. Additionally, choosing the right mode shape is an issue worth concerning. Moreover, the method only detects the damage location but does not evaluate the damage level.

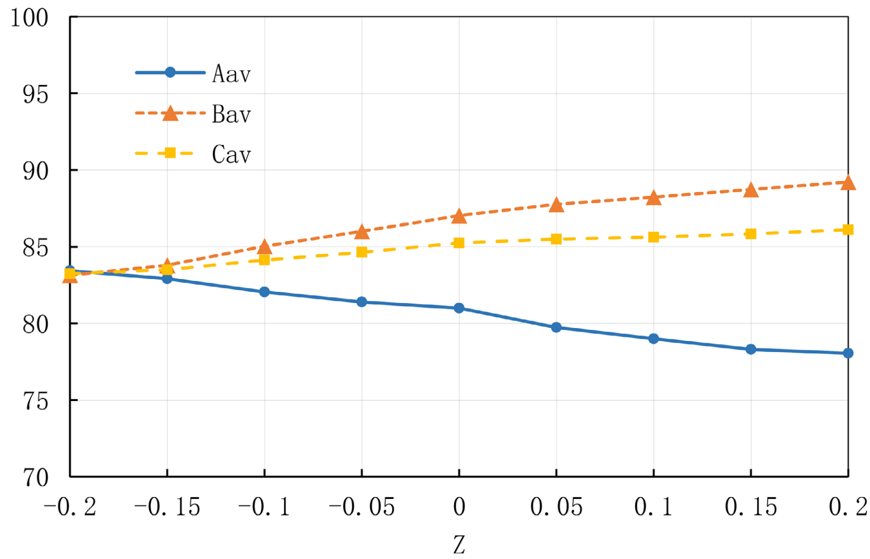

Figure 8. Accuracy indicators with $Z = [-0.2; 0.2]$.

5. Conclusions

The method based on the frequency change can determine the damage but not the damage location. Stiffness-Frequency change-Based damage detection method can detect and locate the damage. This method requires 3 inputs: the natural frequency of the damaged and undamaged states; mode shape of the undamaged state. In which, the natural frequency and mode shape of the undamaged states can be obtained from the finite element method, and only the value of the natural frequency of the damaged state needs to be measured. This shows its applicability in practice in diagnosing damage in reinforced concrete beams and, by extension, other complex structures.

After normalizing the diagnostic results, the suitable failure threshold is chosen. 3 indexes are used to measure the method's accuracy: *A*—crack zone accuracy, *B*—non-cracked zone accuracy, and *C*—overall accuracy. The study shows that the reinforced concrete beam model has a failure threshold of $Z = -0.2$ for the average diagnostic accuracy of A, B, and C above 80%. For each individual natural frequency, modes 1, 3, and combined modes give better results than modes 2 and 4, corresponding to modes with large changes in vibration frequency.

Some points that can be improved in further studies such as:

1) To understand the damage to the beam when the crack is wide open, a more thorough examination of the load level is required.

2) To change the damaged position of the beam and the number of damaged positions on the beam.

3) An extension of the study with a more complex structure is needed to evaluate the effectiveness of the diagnostic method.

In summary, the method provides an average accuracy of damage location of over 80% for simple reinforced concrete beams when using 4 types of vibrations and the suitable damage threshold. With the above advantages and disadvantages, the method can be used as a method of damage diagnosis in practice.

Acknowledgements

This research is funded by Vietnam National University HoChiMinh City (VNU-HCM) under grant number C2020-54-01.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Farrar, C.R., *et al.* (2009) Principles of Structural Degradation Monitoring. *Encyclopedia of Structural Health Monitoring*, **82**, 1350-1370. https://doi.org/10.1002/9780470061626.shm105

[2] Adams, R.D. *et al.* (1978) A Vibration Technique for Non-Destructively Assessing the Integrity of Structures. *Journal of Mechanical Engineering Science*, **20**, 93-100. https://doi.org/10.1243/JMES_JOUR_1978_020_016_02

[3] Cawley, P. and Adams, R.D. (1979) The Location of Defects in Structures from Measurements of Natural Frequencies. *Journal of Strain Analysis*, **14**, 49-57. https://doi.org/10.1243/03093247V142049

[4] Gudmundson, P. (1982) Eigenfrequency Changes of Structures due to Cracks, Notches or Other Geometrical Changes. *Journal of the Mechanics and Physics of Solids*, **30**, 339-353. https://doi.org/10.1016/0022-5096(82)90004-7

[5] Sato, H. (1983) Free Vibration of Beams with Abrupt Changes of Cross-Section. *Journal of Sound and Vibration*, **89**, 59-64. https://doi.org/10.1016/0022-460X(83)90910-0

[6] Salawu, O.S. (1997) Detection of Structural Damage through Changes in Frequency: A Review. *Engineering Structures*, **19**, 718-723. https://doi.org/10.1016/S0141-0296(96)00149-6

[7] Liang, R.Y., *et al.* (1992) Quantitative NDE Technique for Assessing Damages in Beam Structures. *Journal of Engineering Mechanics*, **118**, 1468-1487. https://doi.org/10.1061/(ASCE)0733-9399(1992)118:7(1468)

[8] Hu, J. and Liang, O.Y. (1993) An Integrated Approach Cracks Using Vibration Characteristics. *Journal of the Franklin Institute*, **330**, 841-853. https://doi.org/10.1016/0016-0032(93)90080-E

[9] Patil, D.P. and Maiti, S.K. (2003) Detection of Multiple Cracks using Frequency Measurements. *Engineering Fracture Mechanics*, **70**, 1553-1572. https://doi.org/10.1016/S0013-7944(02)00121-2

[10] Hong, W., *et al.* (2012) Condition Assessment of Reinforced Concrete Beams Using Dynamic Data Measured with Distributed Long-Gage Macro-Strain Sensors. *Journal of Sound and Vibration*, **331**, 2764-2782. https://doi.org/10.1016/j.jsv.2012.02.007

[11] Chopra, A.K. (2016) Dynamics of Structures, 5th Edition. Pearson, London.