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Study on Vibration Characteristics of Stay Cable-Nonlinear Viscous Damper System

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Abstract

Tension cables are easily prone to generating varied vibrations under the action of external loads, which adversely affects the safety of bridges. Therefore, it is necessary to take effective measures to suppress the vibrations of tension cables. Cable end dampers are widely used in vibration reduction for cable-stayed bridges due to their convenient installation and low costs. However, the previous studies on the tension cable-viscous damper systems mostly adopt the linear method, and the weakening effect of the flexibility of mounting brackets on the damper vibration reduction is not sufficiently taken into account. Therefore, this paper adopts the improved Kelvin model to conduct the derivation, solution, and parametric analysis of vibration equations for the stay cable-nonlinear viscous damper systems. The results of parametric analysis show that the maximum modal damping ratio that can be obtained by cables and the corresponding optimal damping coefficient of dampers are correlated with the damping nonlinear coefficient α, stiffness nonlinear coefficient β, vibration order n, installation position a/L, and stiffness coefficient μ, etc.; among them, n damping nonlinear coefficient α and stiffness nonlinear coefficient β are the key parameters that affect the parameter design of dampers, where damping nonlinear coefficient α mainly controls the optimal damping coefficient and stiffness nonlinear coefficient β mainly controls the maximum damping ratio. Based on the parametric analysis, the design principles of dampers and value requirements of key parameters under different vibration suppression objectives are presented.

Keywords

Stay Cable, Vibration, Nonlinear Viscous Damper, Damper Design Method, Parametric Analysis

1. Introduction

Stay cables are widely used as load-bearing structures for cable-stayed bridges.

Because of their considerable flexibility, low damping, lightweight and other characteristics, tension cable components of cable-bearing large-span bridges are prone to vibration under the excitation of various external environmental factors. Continuous cable vibration may be the cause that leads to fatigue and corrosion of cable strands as well as the concerns of pedestrians for the safety of cable-stayed bridges.

The vibration mechanism of tension cables of cable-stayed bridges is complicated, and can be classified into vortex-induced vibration, wake galloping vibration, icing cable galloping vibration, rain-wind induced vibration, parametric vibration, etc. It is generally believed that as long as the logarithmic decrement of the low-order vibration mode of tension cables is increased to 0.02 - 0.03, the occurrence of various abnormal vibrations of tension cables can be effectively prevented.

At present, there are actually three methods to preventing or suppressing the wind-induced vibration of tension cables on bridges, as follows [1]: 1) Tension cable dampers, 2) Auxiliary cables, 3) Pneumatic measures. Compared with the current technical and theoretical conditions, the method of using tension cables with vibration dampers is more economical, simpler and more effective, thus it has been widely adopted. The design method of linear viscous dampers has been well developed over nearly 30 years of evolution.

Carne [2] (1981), and Kovacs [3] (1982) studied the modal damping characteristics of the tensioned string-cable end damper system for the first time. Carne obtained an approximate analytical solution and derived the transcendental equation of complex eigenvalues; Kovacs used a similar method and approximately obtained the maximum modal damping ratio and the corresponding optimal damping coefficient of the system; Kovacs believes that if the damping coefficient is too small, the damping effect of the damper will be small too, and If the damping coefficient is too big, the damping effect will also be very limited for the equivalence that the restraint point is fixed, thus there is an optimal damping coefficient.

Pacheco *et al.* [4] used the modal shape of the undamped and horizontally tensioned string as the primary function, adopted the Galerkin method to discretize the differential equation of the horizontally tensioned string-cable end damper system, and took the dimensionless damping coefficient $\frac{c}{mL\omega_0} i \frac{x_c}{L}$ as the abscissa axis and the dimensionless system modal damping ratio $\frac{c}{mL\omega_0} i \frac{x_c}{L}$ as the ordinate axis respectively, which obtained a design curve of general oil dampers. These scholars have not taken into account many practical factors such as the bending stiffness and sag of tension cables, the stiffness of dampers, etc.

Krenk [5], Main and Jones [6] [7] [8] [9], *et al.* analyzed the influence of installation positions of dampers on the damping characteristics of the system; Linear and nonlinear conditions of dampers are taken into account respectively. Sun Limin, Zhou Haijun and Shi Chen [1] [10] [11] [12] [13] [14] discovered that the support stiffness of dampers has a significant influence on the damping performance of the tension cable-cable end damper system through the real cable test, and put forward the support stiffness requirements for damper installation; Various types of dampers are tested and studied in details through the single-damper test and real cable test, and the influence of various factors such as damper nonlinearity on the damping performance of tension cables is analyzed.

Jones, Hoang [15] [16] Studied the vibration reduction effect of two dampers attached to the cable; Hoang [17] studied the influence of cable bending stiffness.

In the above theoretical research, only Zhou Haijun [13] took into account the influence of damper stiffness; However, the nonlinear factor of stiffness was not considered. According to practices, the damping characteristics of viscous dampers and the stiffness characteristics of the supports and dampers themselves all have a certain degree of nonlinearity, and the current researches universally adopt the linear assumption to simplify the analysis. In order to analyze the influence of those nonlinear factors, this paper takes into account of both the damping nonlinearity and stiffness nonlinearity and adopts the improved Kelvin model to carry out derivation and solution of the vibration equation and analyze the parameters for the theoretical study of the cable-viscous damper system.

2. Symbols

The definitions of symbols in this paper are shown in **Table 1**.

3. Derivation and Solution of the Vibration Equation of Cable-Nonlinear Viscous Damper

3.1. Analytical Model of Cable-Nonlinear Viscous Damper

The model considered in this paper for improvement is the cable-nonlinear Kelvin damper system, and a derived model of the Kelvin model is adopted for the damper: A power function nonlinear spring and a power function nonlinear damper are connected in parallel. Both the nonlinearity of damping and nonlinearity of stiffness are considered for this type of dampers, and the system calculation model is shown in **Figure 1**. L is the length of the cable, C is the damping coefficient of the damper, K is the stiffness coefficient of the damper, m is the mass of the cable per unit length, T is the cable tension, a and a' are the distances between the installation point of the damper and the left and right anchor ends, and x and x' are the axial coordinate of the cable.

When $\alpha = 1$ and $\beta = 0$, it is an ideal linear viscous damper model, and for $\alpha \neq 1$, it is a rate-dependent nonlinear model; $\beta \neq 0$ means that the damper has stiffness, and when $\beta = 1$, it is linear stiffness, which is represented as a linear spring in the model. The relationship between damping force and velocity under different α is shown in **Figure 2**.

Table 1. Definitions of symbols.

symbols	definitions
L	length of the cable
C	damping coefficient of the damper
K	stiffness coefficient of the damper
m	mass of the cable per unit length
T	cable tension
a, a'	distances between the installation point of the damper and the left and right anchor ends
x, x'	axial coordinate of the cable
α	nonlinear coefficient of damping
β	nonlinear coefficient of stiffness
$v(x,t)$	cable vibration amplitude
ω_n	natural frequency of the n order
F_k	force caused by the spring
F_c	force caused by the dashpot
η	dimensionless damping coefficient
μ	dimensionless stiffness coefficient
$\xi_{n,\max}$	Maximum damping ratio
η_{opt}	Optimal damping coefficient

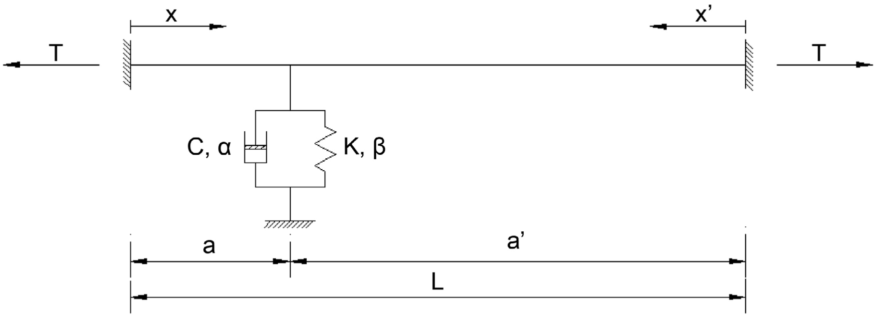

Figure 1. Nonlinear Kelvin model.

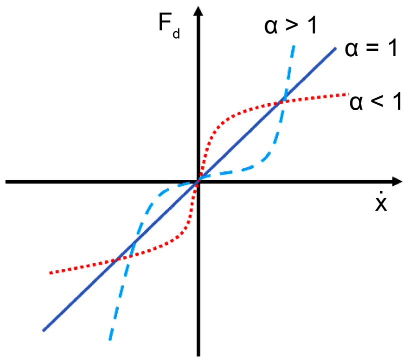

Figure 2. Relationship between damping force and velocity under the nonlinear coefficient α.

The following assumptions are adopted in derivation of the vibration equation:

1) Cable sag, bending stiffness and self-damping are ignored;

2) The tension of the cable remains unchanged during the vibration process;

3) The installation position of the damper is close to the anchor ends;

4) Damping is small, that is, the modal damping ratio ξ_n is small.

3.2. Derivation of the Vibration Equation of the Cable-Nonlinear Viscous Damper

Based on the parallel tensioned string theory, the free vibration equation of a cable without dampers is [2]:

$$T\frac{\partial^2 v(x,t)}{\partial x^2} - m\frac{\partial^2 v(x,t)}{\partial t^2} = 0,\ \ 0 \le x \le l \tag{1}$$

where, T is the cable tension, m is the unit mass of the cable, l is the cable length, and $v(x, t)$ is the cable amplitude, which is a function of the time t and the distance x along the length of the cable.

The vibration equation is solved by the separation variable method, and the natural frequency of each order of the cable is:

$$\omega_n = \frac{n\pi}{l}\sqrt{\frac{T}{m}} \tag{2}$$

where, n is the vibration order.

The vibration equation of the cable-nonlinear Kelvin damper system is as follows:

$$T\frac{\partial^2 v(x,t)}{\partial x^2} - m\frac{\partial^2 v(x,t)}{\partial t^2} = F_d\delta(x-a) \tag{3}$$

where:

$$\begin{aligned} F_d &= F_k + F_c = K|x|^{\beta}\,sign(x) + C|\dot{x}|^{\alpha}\,sign(\dot{x}) \\ &= K|v(x,t)|^{\beta-1}v(x,t) + C\left|\frac{\partial v(x,t)}{\partial x}\right|^{\alpha-1}\frac{\partial v(x,t)}{\partial x} \end{aligned} \tag{4}$$

F_k is the force caused by the spring in the nonlinear damper, F_c is the force caused by dashpot (damping) of the nonlinear damper, K and C are the stiffness coefficient and the damping coefficient respectively. The separation variable method is used to solve the problem, which assumes that:

$$v(x,t) = V(x)\mathrm{e}^{i\omega t} \tag{5}$$

$V(x)$ is the lateral displacement of in-plane vibration of the cable.

The Formula (5) is substituted into Formula (3), and the derivation method [10] similar to that of the linear viscous damper is adopted to obtain:

$$\begin{aligned} \cot(\lambda a) + \cot(\lambda a') &= -\frac{i\omega C|V_a\omega\sin(\omega t)|^{\alpha-1} + K|V_a\cos(\omega t)|^{\beta-1}}{T\lambda} \\ &= -i\eta|V_a\omega\sin(\omega t)|^{\alpha-1} + \frac{\mu}{\lambda a}|V_a\cos(\omega t)|^{\beta-1} \end{aligned} \tag{6}$$

The dimensionless damping coefficient η and the dimensionless stiffness coefficient μ are respectively:

$$\eta = \frac{C}{\sqrt{Tm}}, \mu = \frac{Ka}{T} \tag{7}$$

Make:

$$\begin{aligned} A_1 &= \eta \left| V_a \omega \sin(\omega t) \right|^{\alpha-1} \\ A_2 &= \mu \left| V_a \cos(\omega t) \right|^{\beta-1} \end{aligned} \tag{8}$$

Obtain:

$$\tan(\lambda L) = \frac{\left(iA_1 + \dfrac{A_2}{\lambda a} \right) \sin^2(\lambda a)}{1 + \left(iA_1 + \dfrac{A_2}{\lambda a} \right) \sin(\lambda a) \cos(\lambda a)} \tag{9}$$

3.3. Approximate Solution and Iterative Numerical Solution of the Vibration Equation

The approximate solution of the vibration Equation (9) can be obtained by using the method [10] similar to that of the linear viscous damper, according to the near anchor end assumption $a/L \ll 1$

$$\lambda L \approx n\pi + \frac{iA_1 \left(n\pi \dfrac{a}{L} \right)^2 + A_2 \left(n\pi \dfrac{a}{L} \right)}{(1 + A_2) + iA_1 \left(n\pi \dfrac{a}{L} \right)} \tag{10}$$

$$\operatorname{Im}(\lambda L) \approx \frac{A_1 \left(n\pi \dfrac{a}{L} \right)^2}{(1 + A_2)^2 + \left[A_1 \left(n\pi \dfrac{a}{L} \right) \right]^2} \tag{11}$$

From [10], when ξ_n is very small, the approximate calculation can be made by the following formula:

$$\xi_n = \frac{\operatorname{Im}(\lambda L)}{|\lambda L|} \approx \frac{\operatorname{Im}(\lambda L)}{\operatorname{Re}(\lambda L)} = \frac{\operatorname{Im}(\lambda L)}{n\pi} \tag{12}$$

Obtain:

$$\begin{aligned} \frac{\xi_n}{a/L} &\approx \frac{A_1}{(1 + A_2)^2 + \left[A_1 \left(n\pi \dfrac{a}{L} \right) \right]^2} \\ &= \frac{n\eta \dfrac{\pi a}{L} \left| V_a \omega \sin(\omega t) \right|^{\alpha-1}}{\left(1 + \mu \left| V_a \cos(\omega t) \right|^{\beta-1} \right)^2 + \left[n\eta \dfrac{\pi a}{L} \left| V_a \omega \sin(\omega t) \right|^{\alpha-1} \right]^2} \end{aligned} \tag{13}$$

In order to obtain the maximum damping ratio and the optimal damping coefficient [4], make:

$$\frac{\partial \xi_n}{\partial \eta} = 0 \tag{14}$$

Obtain:

Optimal damping coefficient:

$$\eta_{opt} = \frac{1 + \mu \left| V_a \cos(\omega t) \right|^{\beta - 1}}{n \frac{\pi a}{L} \left| V_a \omega \sin(\omega t) \right|^{\alpha - 1}} \tag{15}$$

Maximum damping ratio:

$$\xi_{n,\max} = \frac{1}{2} \frac{1}{1 + \mu \left| V_a \cos(\omega t) \right|^{\beta - 1}} \frac{a}{L} \tag{16}$$

These are the approximate solutions of the vibration equation. As a matter of convenience, make:

$$\eta' = \eta \frac{\pi a}{L} = \frac{C}{\sqrt{Tm}} \frac{\pi a}{L} \tag{17}$$

Equation (9) is solved iteratively by the numerical method. The fixed point iteration is carried out, and:

$$\lambda L = n\pi + \arctan \frac{\left(iA_1 + \frac{A_2}{\lambda a} \right) \sin^2(\lambda a)}{1 + \left(iA_1 + \frac{A_2}{\lambda a} \right) \sin(\lambda a) \cos(\lambda a)} \tag{18}$$

By using this equation, the exact numerical solution can be obtained iteratively.

4. Parametric Analysis

Based on the vibration equation of the cable-nonlinear damper system derived above, the approximate solution and numerical exact solution are carried out, and parametric analysis of the various parameters in the equation is conducted by the control variable method, to determine the influence of various parameters on the damper model. In the following analysis, except for the changed parameters, the default values of various parameters are the values in Table 2:

Table 2. Default values of parameters.

V_a	0.01 m
f	0.658 Hz
$\sin(\omega t)$	0.5
μ	0.2
α	0.5
β	1.5
n	1
a/L	0.02

4.1. Influence of Stiffness Coefficient μ

In order to analyze the influence of stiffness coefficient μ (see Equation (6)) on the general design curve of the damper, other parameters remain unchanged, and different μ values are taken for calculation, and the results are as shown in Figure 3 (the abscissa indicates the damping coefficient, and the ordinate indicates the damping ratio index. When the damping ratio index is maximum, the corresponding damping coefficient is the optimal damping ratio).

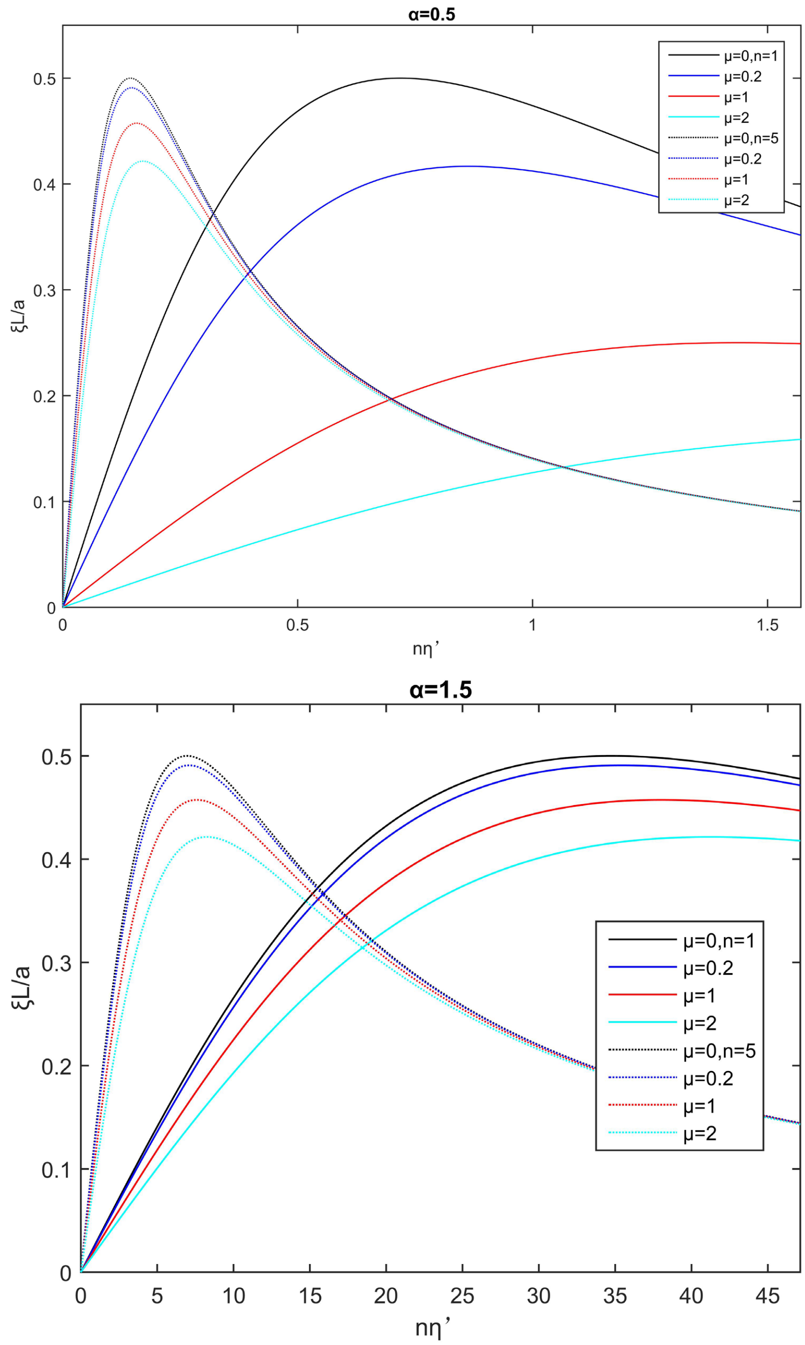

Figure 3. Influence of stiffness coefficient μ of dampers.

It can be seen that the larger the stiffness coefficient μ, the smaller the attainable damping ratio, that is, the damping effect of the damper that can be achieved under the same conditions, so the damping stiffness coefficient has an adverse effect on the damping effect of the damper.

4.2. Influence of Nonlinear Coefficient *α* and Vibration Order *n* of Dampers

The influence of a is discussed in the following two cases.

1) Variation of value α, $n = 1$

At this point, only the first-order vibration of cables is controlled. The corresponding results or different α values are shown in **Figure 4**.

It can be seen that the larger the α, the larger the corresponding optimal damping coefficient, that is, the larger the damping force the damper is required to provide; *the smaller the α is, the smaller the damping force the damper is required to provide*.

2) Combined variation of α and n

At this point, only the first-order to fifth-order vibrations of cables are controlled at the same time, and the corresponding results are shown in **Figure 5**.

Although the dispersion degree of the optimal damping coefficients for the first five orders is varied for different α values, however, since the curves of various orders are coincident with $n\eta'$ as abscissa, the order of vibration that can be suppressed with different α values is the same (when n is the same, the attainable maximum damping ratio is the same).

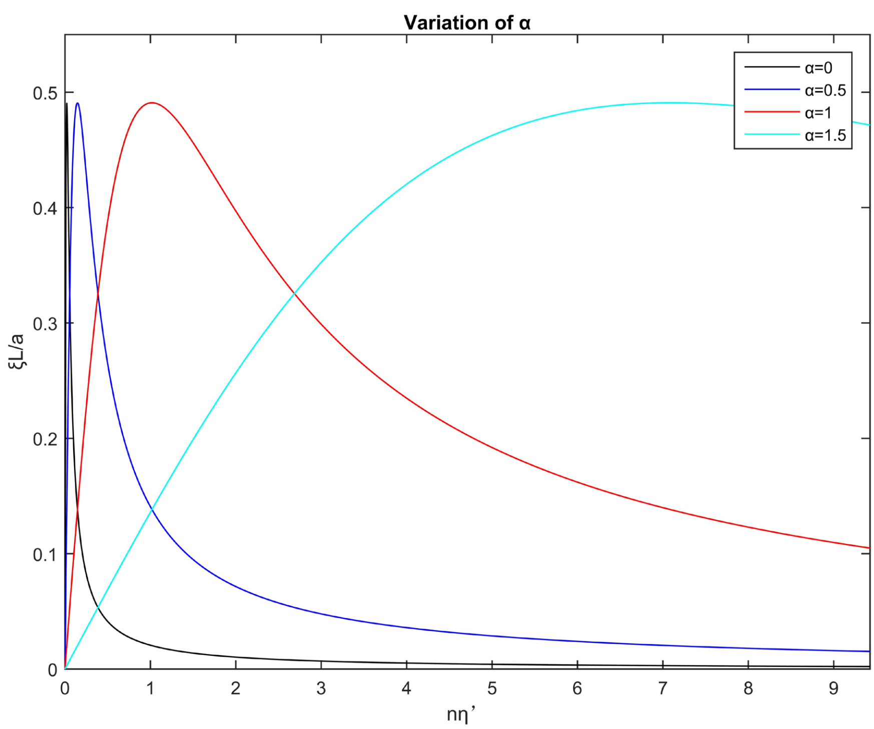

Figure 4. Influence of damping nonlinear coefficient.

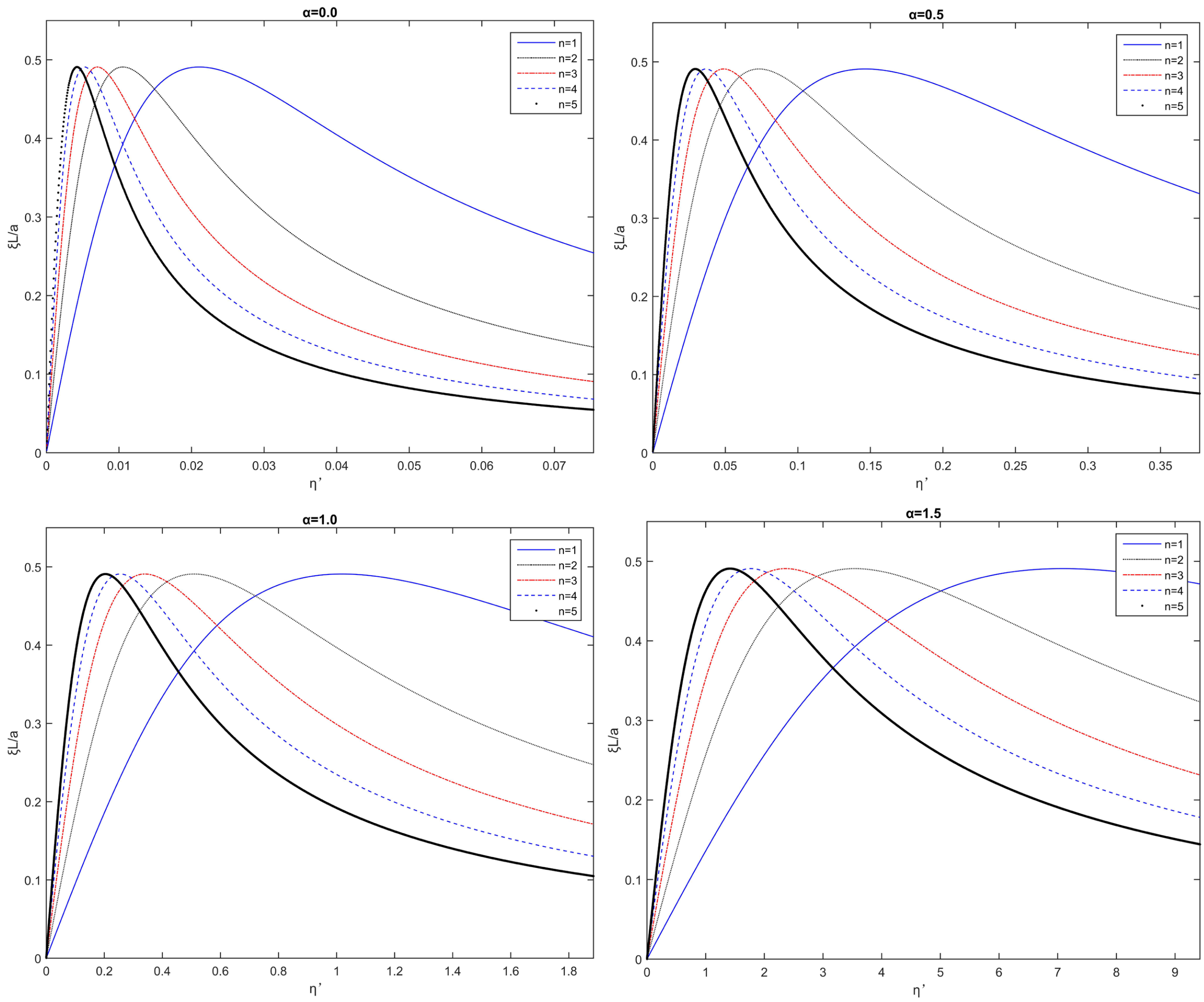

Figure 5. Influence of different α values and different order n.

It can be seen from the above figure that, to suppress the vibrations of the first five orders, the optimal damping coefficient selected should be taken as the value at the intersection of the first-order curve and the fifth-order curve marked in the figure, and the attainable damping ratios of the first and fifth orders are the same for different α values, which is about $0.37^* a/L$ and slightly smaller than the damping ratios of the second, third and fourth orders.

4.3. Influence of Nonlinear Coefficient β of Dampers

For different μ values ($\mu = 0.2$, $\mu = 1$, $\mu = 1.5$, $\mu = 2$), when β changes from 0 to 2, both the optimal damping coefficient and the corresponding maximum damping ratio change accordingly, as shown in Figure 6.

As can be seen from the above figure, when $\beta = 1$, it is equivalent that the nonlinear spring in model 1 is degraded into a linear spring. When $\beta < 1$, the smaller β is, the larger the optimal damping coefficient and the smaller the

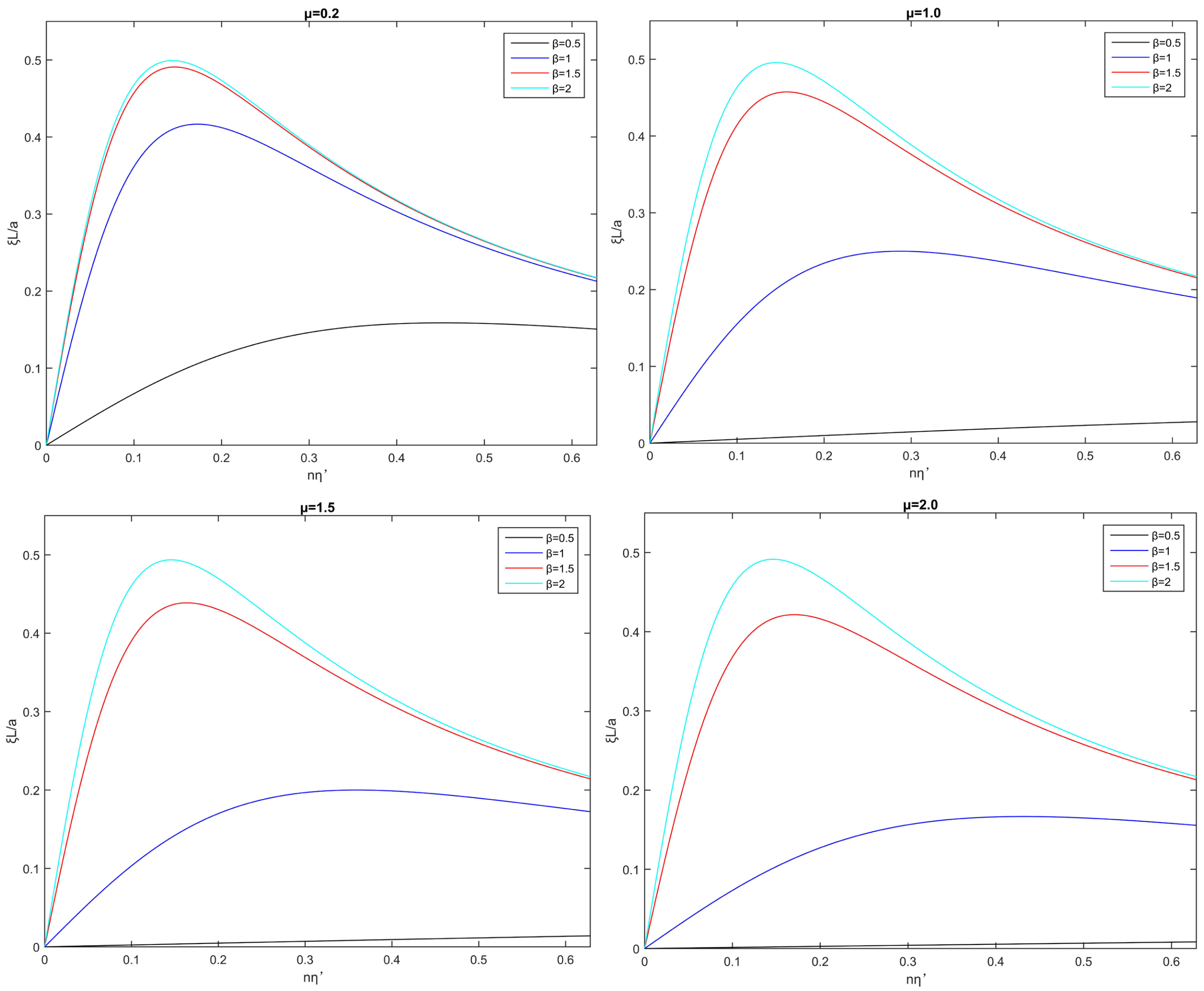

Figure 6. Influence of stiffness nonlinear coefficient β.

corresponding maximum damping ratio. When $\beta > 1$, the larger β is, the smaller the optimal damping coefficient and the larger the corresponding maximum damping ratio. When μ is relatively small ($\mu < 1$), if $\beta > 2$, the curve increases very little with the increase of β, and the longitudinal axis $\xi_{n,\max}\dfrac{L}{a}$ is basically close to the maximum value of 0.5 in the linear case. Therefore, the larger the value of β, the better, but it is not much more effective when the value of β is greater than 1.5. When μ is relatively large ($\mu > 1$), to make the longitudinal axis $\xi_{n,\max}\dfrac{L}{a}$ close to the maximum value of 0.5 in the linear case, the required β value is larger, but its variation trend is the same as that when μ is relatively small. Therefore, the larger the value of β, the better, but it is not much more effective when the value of β is greater than a certain value.

The analysis results show that, in the design and engineering application of tension cable dampers, stiffness characteristics should be emphasized and the stiffness nonlinear coefficient should be increased as much as possible to ensure that a higher maximum damping ratio can be obtained.

4.4. Comparison between Approximate Calculation and Iterative Calculation

1) The damper is an ideal linear damper

When the damper is an ideal linear damper (*i.e.* $\mu = 0$, $\alpha = 1$, $\beta = 1$), the approximate calculation results of the first five orders are compared with the iterative calculation results of the first five orders to evaluate whether the approximate calculation is accurate and reasonable. The comparison results are shown in Figure 7:

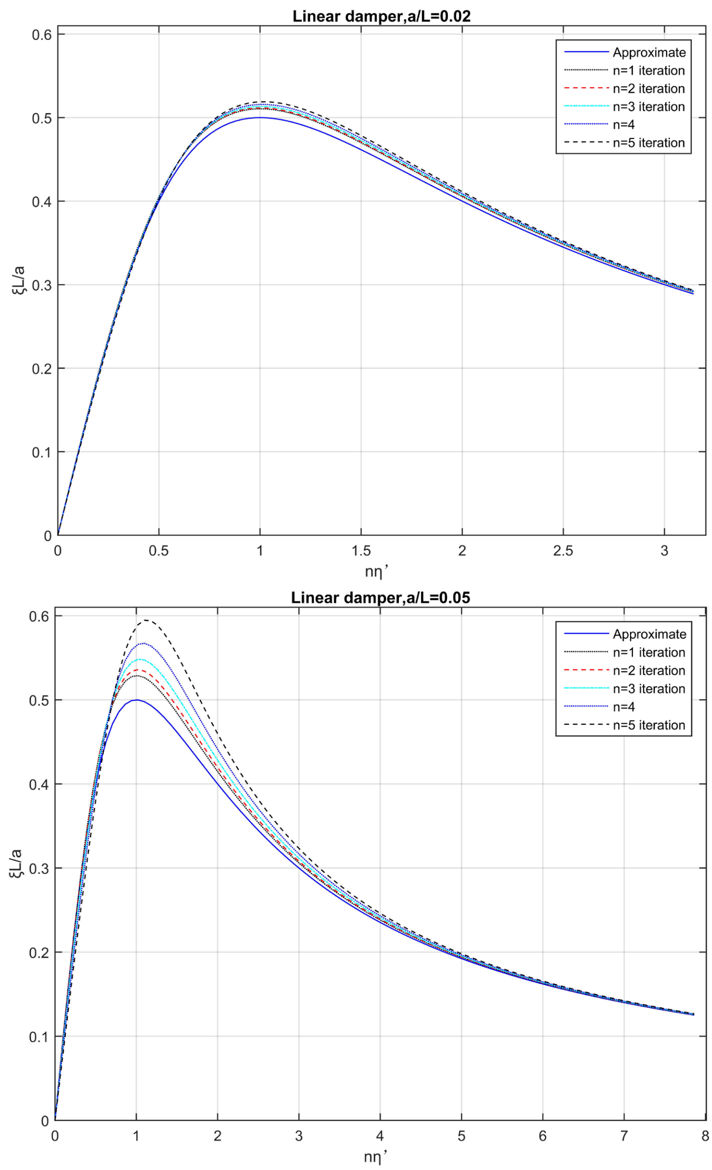

Figure 7. Comparison between approximate calculation results and iterative calculation results.

Table 3. Approximate calculation error.

$\xi_{n,\max}\dfrac{L}{a}$	Approximation	Iteration		Error	
		$n = 1$	$n = 5$	$n = 1$	$n = 5$
$a/L = 0.02$	0.5	0.5106	0.519	2.12%	3.80%
$a/L = 0.05$	0.5	0.5288	0.5945	5.76%	18.90%

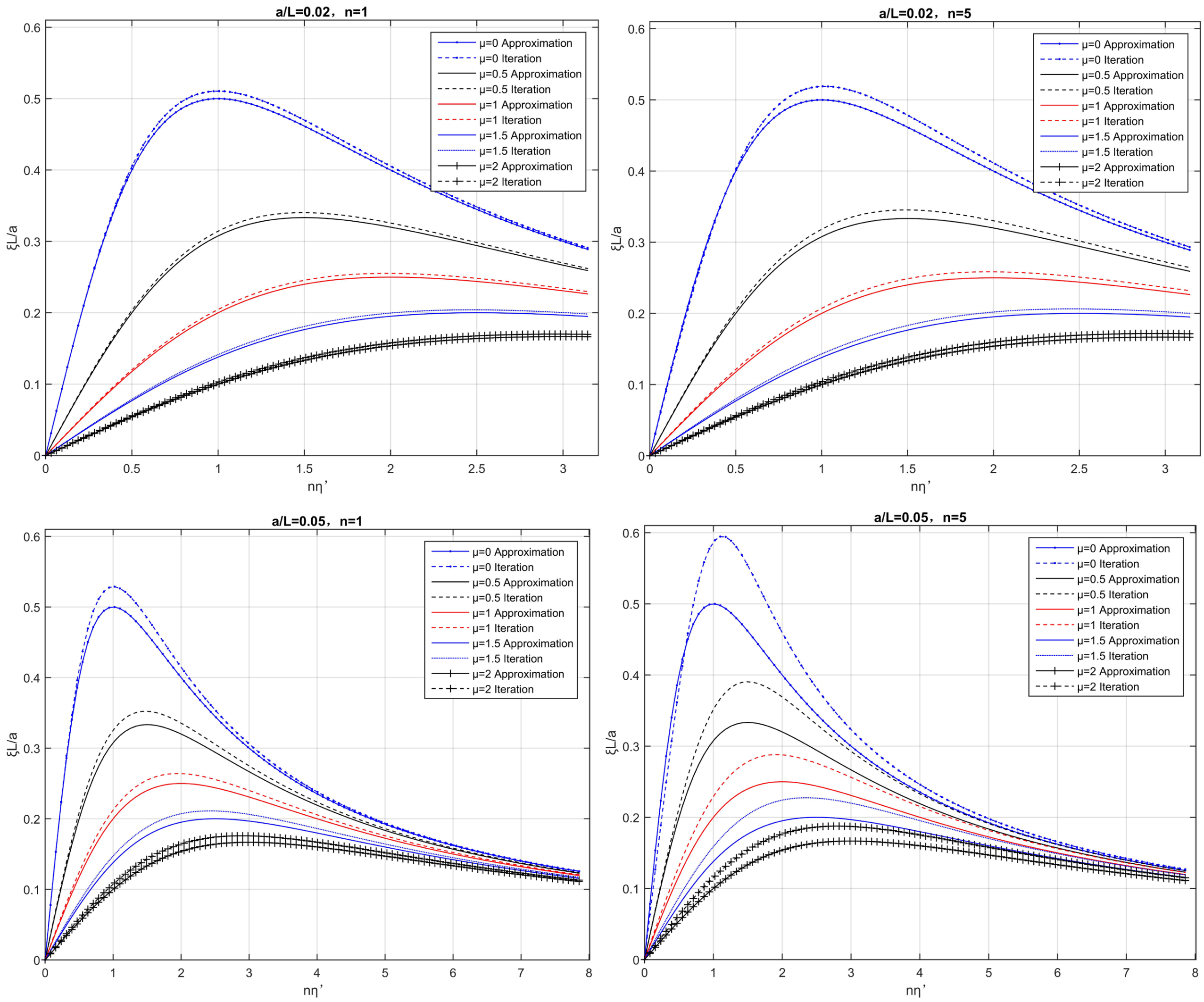

Figure 8. Comparison between approximate calculation and iterative calculation when $\mu \neq 0$.

As can be seen from Figure 7, the error of the approximate calculation is smaller for the first order, and the higher the order, the larger the error. When $a/L = 0.02$, the error of the fifth-order approximate calculation is less than 5%; However, when $a/L = 0.05$, the approximate calculation error is quite large, and the error calculation is shown in Table 3:

It can be seen that the larger the damper position parameter a/L, the larger the approximate calculation error; the low-order error is smaller, and the high-order error is larger. This is consistent with the research results of Krenk [5].

2) Consider the stiffness of the damper

When the damper has stiffness (that is, $\mu \neq 0$, $\alpha = 1$, $\beta = 1$), the approximate calculation results of the first five orders are compared with the iterative calculation results of the first five orders. The comparison results are shown in **Figure 8**:

As can be seen from **Figure 8**, when $\mu \neq 0$, the difference between the approximate calculation results and the iterative calculation results is basically close to that of $\mu = 0$. Still, the larger the damper position parameter a/L, the larger the approximate calculation error; the low-order error is smaller, and the high-order error is larger. The maximum error is close to 20% when $a/L = 0.05$ and $n = 5$.

When $\mu \neq 0$, the larger the μ value, the smaller the maximum damping ratio and the larger the optimal damping coefficient, which is the same as the results of approximate calculation and analysis.

5. Conclusions

This paper provides systematic theoretical research on the cable-damper system, adopts the linear model and the improved Kelvin model for analysis, derives the vibration equation and analyzes the parameters, and presents a comparative analysis between the approximate calculation and the numerically exact solution. The research results show that:

1) The maximum modal damping ratio that can be obtained by cables and the corresponding optimal damping coefficient of dampers are correlated with the damping nonlinear coefficient α, stiffness nonlinear coefficient β, vibration order n, installation position a/L, and stiffness coefficient μ, etc.

2) The damping nonlinear coefficient α mainly controls the optimal damping coefficient and the optimization design of dampers can be carried out with reference to the vibration orders of cables to be suppressed. The design principles for vibration suppression of the first five orders and the corresponding attainable maximum damping ratios are put forward respectively.

3) The stiffness nonlinear coefficient β mainly controls the maximum damping ratio, and the higher the β value, the greater the attainable maximum damping ratio, that is, the stronger the corresponding damping capacity. The damping stiffness coefficient μ has an adverse effect on the damping effect of the dampers, and the larger the μ value, the smaller the maximum damping ratio and the larger the optimal damping coefficient. Therefore, the additional stiffness should be reduced and the stiffness nonlinear coefficient should be increased as much as possible in the design of dampers.

In this paper, the proposed nonlinear damper model and related parameter analysis have not been tested and verified on the actual cable. In further research, the corresponding damper model will be made and the actual cable test for verification will be carried out to provide the basis for the design and manufacture of the corresponding damper.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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