

# Effect of Water Content and Grains Size Distribution on the Characteristic Resilient Young's Modulus ( $E_c$ ) Obtained Using Anisotropic Boyce Model on Gravelly Lateritic Soils from Tropical Africa (Burkina Faso and Senegal)

Bibalo Ida Josiane Ki, Makhaly Ba, Rokhaya Gueye, Pierre Hornych, Ali Sana

Laboratoire de Mécanique et Modélisation (L2M), UFR Sciences de l'Ingénieur, Université de Thiès, Thiès, Sénégal  
Email: bijosiane.ki@univ-thies.sn

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## Abstract

This research was carried out to determine the rheological parameters of lateritic soils in order to contribute to the improvement of the technical documents used for pavement design in tropical Africa. The study is based on the loading repeated of cyclic triaxial tests (LRT) performed at University Gustave Eiffel (formerly Institut Français des Sciences et Technologies des Transports de l'Aménagement et des Réseaux (IFSTTAR)) in Nantes with the application of the European standard EN 13286-7: 2004 [1]. The tests were performed at constant confinement stress and using the stepwise method to determine the resilient axial ( $\varepsilon_1^r$ ) and radial ( $\varepsilon_3^r$ ) deformation as a function of the axial and radial stresses. Four gravel lateritic soils from different sites selected in Burkina Faso and Senegal were the subject of this research for the triaxial tests. These materials have a maximum diameter of 20 mm and a percentage of fines less than 20%. The LRT tests were carried out on samples compacted at three moisture contents ( $w_{optm} - 2\%$ ,  $w_{optm}$  and  $w_{optm} + 2\%$ ) and at 95% and 100% of optimal dry density ( $\gamma_{doptm}$ ). Test results showed that the characteristic resilient Young's modulus ( $E_c$ ) of gravelly laterites soils depends on the compacted water content and the variation of the grains size distribution (sand ( $\phi < 2$  mm), motor ( $\phi < 0.5$  mm) and fines content ( $\phi < 0.063$  mm) obtained after (LRT). Materials with a high percent of fines ( $>20\%$ ), mortar and sand (Sindia and Lam-Lam) are more sensitive to variations in water content. The presence of water combined with the excess of fines leads

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to a decrease in modulus around 25% for Lam-Lam and 20.2% for Sindia. Materials containing a low percent of fines, mortar and sand (Badnogo and Dedougou) behave differently. And the resilient modulus increases about 225.67% for Badnogo and 312.24% for Dedougou with the rise of the water content for approximately unchanged the percentage of fines, mortar and sand. Granularity therefore has an indirect influence on the resilient modulus of the lateritic soils by controlling the effects of water on the entire system. Results of statistical analysis and coefficients of correlation (0.659 to 0.865) showed that the anisotropic Boyce's model is suitable to predict the volumetric ( $\varepsilon_v^*$ ) and deviatoric strain ( $\varepsilon_q^*$ ) with stress path ( $\Delta q/\Delta p$ ) of the lateritic soils. The predicted  $E_r$  resilient Young's modulus from anisotropic Boyce's model varies according to the evolution of the bulk stress ( $\theta = \sigma_1 + \sigma_2 + \sigma_3$ ). A correlation around 0.9 is obtained from the power law model.

### Keywords

Lateritic Soil, Cyclic Triaxial with Repeated Loading (LRT), Characteristic Resilient Young's Modulus, Anisotropic Boyce Model, Water Content, Grains Size Distribution

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## 1. Introduction

Any type of material in which a force is exerted undergoes deformations and has resistance limits. This rule is no exception for road materials, in particular for lateritic soils used in tropical areas such as Burkina Faso and Senegal in road construction. The design of pavements in tropical African countries is based on so called semi-empirical methods which mix empirical and rational approaches. These methods are based on the consideration of static loads (approximating traffic loads) and the assumption of a linear elastic behavior of materials described by Hooke's law [2]. But since the 1960, following several experimental and modeling works and, a new body of knowledge has been gathered on the behavior of unbound granular materials.

Previous studies (Boyce [3], Hornych [4], El Abd [5], and Gidel [6]) underlined the importance to study the resilient behavior (resilient modulus and permanent strain) of unbound granular materials (UGMs). They used Boyce's model to predict volumetric and deviatoric strain, Poisson ratio and the resilient modulus. Hornych *et al.* [7] found the limit of the Boyce's model and proposed the parameter  $\gamma$  to resolve the anisotropic matter.

More recent studies have shown that granular materials have a much more complex behavior [8]. To solve this problem, studies have been carried out in Senegal for the last two decades to discover the advanced parameters of granular materials. Fall [9], Ba [10], Samb [11], Dione [12], and Aidara [13] have contributed to deepen the knowledge on the advanced mechanical behavior of the road materials used in Senegal.

Fall [9] points out the importance of adequate characteristics for a good design of road structures. He highlights the advanced mechanical properties of laterites from Senegal through triaxial tests with monotonic and cyclic stress (by the method B with constant confining pressure loading (CCP) ( $\sigma_3$ ) and method A with variable confining pressure (VCP) ( $\sigma_3$ ) of the European standard).

Ba [10] and Samb [11] worked respectively on unbound gravel materials and gravel lateritic soils (unbound and cement-improved) from Senegal using the procedure Ia of NCHRP 1-37A (2004). Samb [11] and Dione [12] used the results of the above research on the resilient modulus for finite element modeling. Aïdara [13], carried out research for the determination of the complex and dynamic modulus of asphalt mixtures made with crushed gravel from Senegal by the LCMB method in Montreal, Canada.

This leads to describe the behavior of these materials as plastic and nonlinear elastic. In the road sector, the loads applied by traffic at a given point in the structure are rapidly varying with time with alternating charge/discharge periods. Omitting the stress rotation aspects, this mode of loading can be approximated in lab by LRT tests for which the material is both confined and submitted to an axial compressive and sinusoidal stress (Figure 1). For this study we used the SCHENCK (LRT) device from the former IFSTTAR in Nantes with the application of the European standard EN 13286-7: 2004, keeping the variable confining pressure (method A). The measured stress-strain results are interpreted for the resilient part by the application of the Boyce model, extended to axial anisotropy.

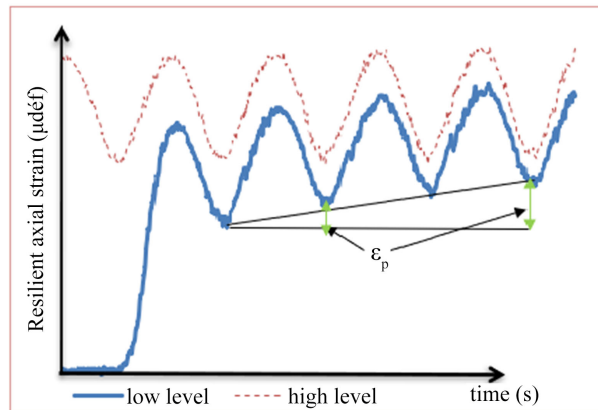
## 2. Physical and Mechanical Characteristics of Lateritic Materials

Gravel lateritic soils, very abundant in tropical and equatorial Africa, are widely used in building as well as road construction [14]. Gravel lateritic soils are a loose or indurated material, rich in iron or aluminum hydroxides, constituting soils to deep horizons, with an alteration profile. But the term laterite is controversial and can refer to different soils or parts of soils, according to authors. For Erhart *in* Kalenda [15], true laterites must have an armored horizon. For Hardy and Rodrigues *in* Fall [9], laterite is a product that must be rich in gibbsite. Mohr *in* Kalenda [15], for his part, considers that a laterite is a soil which is rich in indurated sesquioxides and that it is the ultimate state of a long evolution.

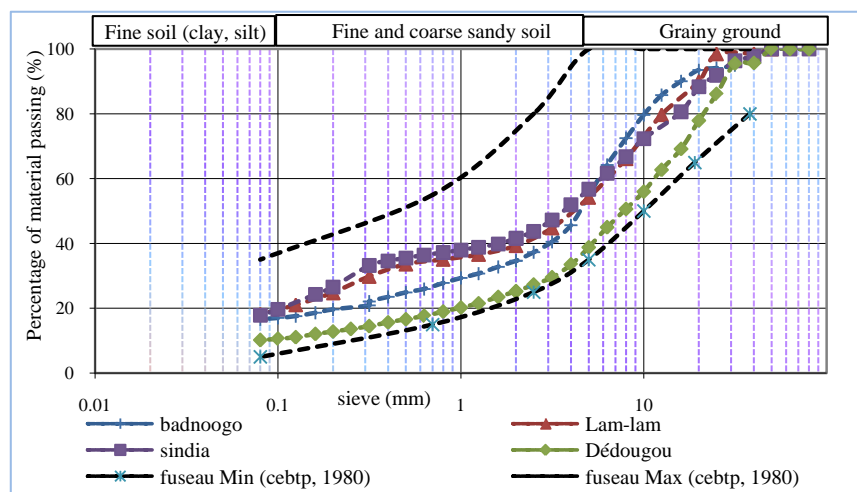
The study as a whole focused on the laterites of two (2) quarries in Senegal (Sindia and Lam-Lam) and two borrow sites in Burkina Faso (Badnogo and Dédougou) for further mechanical tests for the rheological behavior. Soil identification tests such as particle size analysis (NF EN 933-1), the determination of Atterberg limits (NF EN ISO 17892-12: 2004), the compactness test such as the Proctor test (NF P 94-093) and the California Bearing Ratio CBR test (NF P 94-078) were the subject of the first part of this study.

The summary of the material characterization tests is listed in Table 1 and Figure 2. It comes out that:

- The fine content ( $\phi < 0.08$  mm) is an average between 10% and 19% < 35%. Also, the percentage passing at 2 mm is an average between 25% and 42% < 70%;



**Figure 1.** Effects of the stress level applications and increased loading on development of the permanent strain ( $\epsilon_p$ ).



**Figure 2.** Particle size curve of the collected materials.

**Table 1.** Physical and mechanical characterization of lateritic soils samples.

Designation of borrow sites	Particle size Analysis				Materials classification	Parameters of the Atterberg limit	CBR compactness and lift parameters								
	Sand (%)	Fines (%)	Ratio				Water content	Dry density	Parameters of the Modified Proctor		CBR after 4 days of immersion				
			Cu	Cc					$\gamma^d_{max}$ (kN/m <sup>3</sup> )	$W_{opt}$ (%)		95% OPM			
Sindia	41.6	17.86	207	0.34	B5 GA	A2-4a (0)	GC - CL	29.85	9.2	2.7	4.7	2.76	19.7	9.66	54.8
Lam-Lam	39.3	18.19	130	0.28	B5 GA	A2-4a (0)	GC - CL	30.3	10.1	2.4	5.6	2.69	17.52	11.8	30.5
Dedougou	25.2	10.21	763	29.5	B4 GA	A2-4a (0)	GM - GC	27.5	7.2	3.2	4.3	2.82	22.5	8.05	65
Badnogo	34.6	16.63	129	11.6	B6 GA	A2-6 (0)	GC - CL	28	13	1.8	4.6	2.76	21.45	9.7	58

- The liquidity limit (WL) and the plasticity index (IP) are on average respectively 29% and 9.8%.  $5 < IP = 9.8 < 15$  where the materials have a low plastic state “Guide de Terrassement Routier” [16], also the optimum dry density ( $\gamma_{dopt}$ ) and the optimum water content ( $w_{opt}$ ) are on average 20 kN/m<sup>3</sup> and 9.8% respectively.

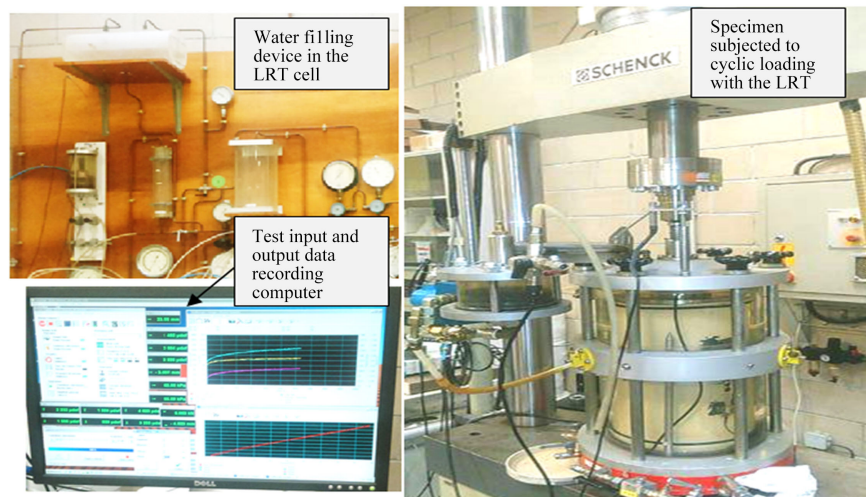
After the analysis of the material, it is observed that the samples, despite the diversity of their origin, are gravelly materials with little clay (lateritic gravel with little clay) and of class B4, B5 and B6 according to the book of “Guide de Terrassement Routier” [16]. We notice that materials with a high fine content (grading fraction with  $\% \phi < 80 \mu\text{m}$  is greater than 19%) have a low.

### 3. Material and Procedure for Determining the Resilient Modulus

The Resilient Modulus ( $M_r$ ) is an elastic modulus based on the recoverable strain under repeated loads [2]. Also, it defined as the unloading modulus after many cycles of repeated loading, is used in pavement engineering as an appropriate measure of stiffness for the layers in a pavement structure [9]. It determined as the ratio between the deviatoric stress ( $q = \sigma_1 - \sigma_3$ ) and the reversible axial strain ( $\varepsilon_r$ ). From the identification and characterization tests (particle size, Atterberg limit, Proctor, etc.), we noticed that the lateritic soil is somewhat similar to the road Untreated Graves Material (UGMs). This justifies the choice of this aforementioned standard. It should be noted that these tests in France began in the 1980s in laboratories. Pautre *et al.* in Hornych [4], Correia *et al.* in Hornych [4], Hornych *et al.* [7], Hornych [4] and Gidel [6] in Allou [17] have developed an apparatus for performing loading repeated triaxial (LRT) tests according to a standardized procedure. They simulated in the laboratory the conditions of loading in place by means of an LRT apparatus Gidel [6] and Allou [17].

#### 3.1. Preparation of Samples and LRT Apparatus before Testing (NF EN 13286-7)

Like road UGM, lateritic soils are natural granular materials without binder, which exhibit a highly non-linear elasto-plastic behavior, depending on the magnitude of the stresses applied and the number of loading cycles (Figure 1). The triaxial repeated loading test is widely used to study the mechanical behavior of these types of materials [5]. The LRT consists in subjecting a cylindrical specimen of untreated granular material to cyclic loadings, simulating the stresses existing in a pavement layer, and in measuring the axial and radial deformations of the specimen produced by these phase loads (Figure 3) [10]. The standard used is EN 13286-7: 2004 which defines two different tests procedures (method B with constant confining pressure loading (CCP) ( $\sigma_3$ ), stepwise method and method A with variable confining pressure (VCP) ( $\sigma_3$ ) the one we used.

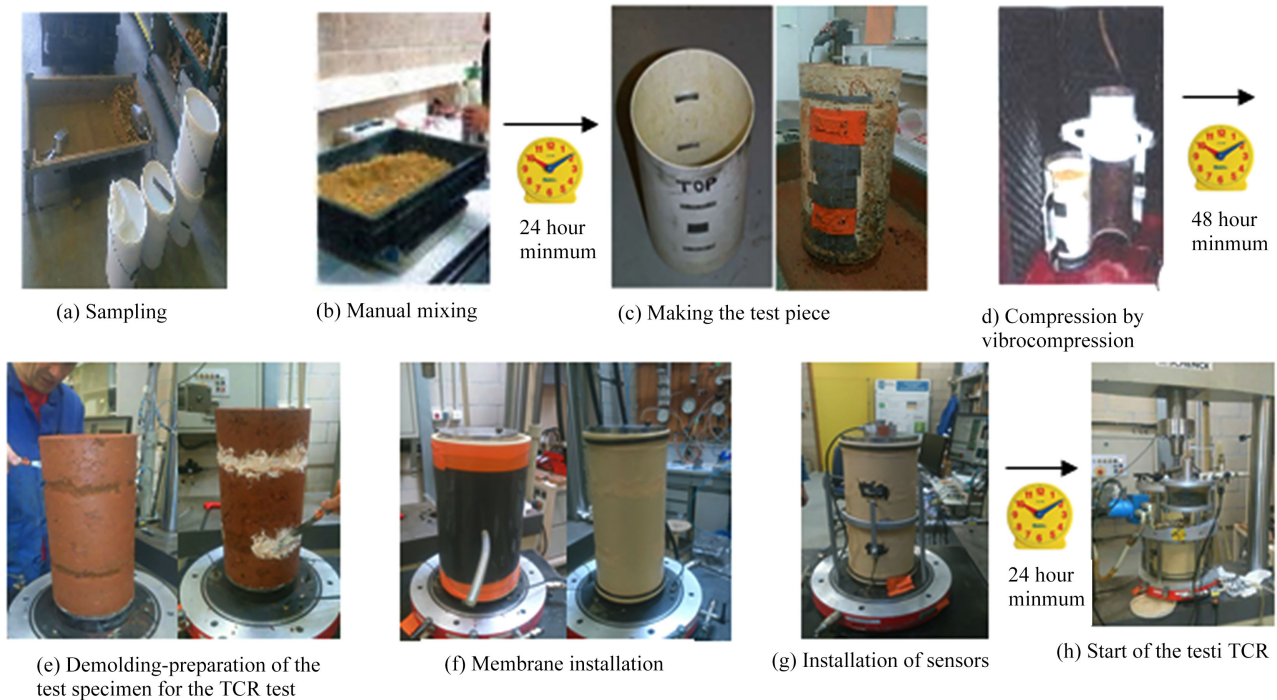


**Figure 3.** Overview of the SCHENCK triaxial device at IFSTTAR Nantes.

The test consists of placing a cylindrical specimen of the material compacted in advance (covered with a waterproof membrane) to be studied with a dimension of 160 mm in diameter and 320 mm in height in a cell filled with water and applying a large number of cycles to it. Loading is the result of a variable or constant isotropic pressure and a variable axial compressive force. Two displacement sensors (Hall effect sensor) positioned diametrically opposed in the two central quarters of the height of the specimen, measure the axial deformations. One ring-shaped sensor placed in the center of the surface of the specimen measures the radial strain (**Figure 4**). The supports for each axial Hall effect sensor are 2 studs fixed to the surface of the test piece and 160 mm apart. In addition to these sensors, there is an axial force sensor (20 kN) placed on the vertical axis of the specimen on the upper base, and a LVDT (Linear Variable Differential Transformers) sensor measuring the axial deformation and placed outside the cell.

These two loading systems make it possible to cyclically vary the axial force and the pressure in the cell, which makes it possible to carry out loadings according to different stress paths (several values of the  $\Delta q/\Delta p$  ratio).

The standard recommends three (3) types of compaction to prepare the sample (Manual, vibrating hammer and vibrocompression); our choice turned to Vibrocompression (NF EN13286-52) (**Figure 4**). The compacted sample is closed hermetically in the mold with adhesive tape and then stored in a room at 4°C in vertical position for a minimum of 48 hours. The sampling procedure until the start of the LRT test is shown schematically in (**Figure 4**). The LRT tests were carried out on samples compacted at three moisture contents ( $w_{opm} - 2\%$ ,  $w_{opm}$  and  $w_{opm} + 2\%$ ) and at 95% and 100% of optimal dry density ( $\gamma_{dopm}$ ). Depending on the characteristics of the materials chosen, we applied two (2) types of packaging with a high loading level of a deviatoric stress equal to 320 kPa; and a low loading level of a deviatoric stress equal to 160 kPa.



**Figure 4.** Test specimen manufacturing procedure before LRT test.

In order to properly identify the samples, we proceeded to a labeling system such as Bad/10.04/95.58 whose letters designate the site specimen, followed by the compaction water content and the compaction rate.

### 3.2. Definition and Procedure of the LRT Test (NF EN 13286-7)

The results obtained with this procedure can be used to determine values of the resilient modulus of elasticity of the material for different stress levels or parameters of nonlinear elastic models that can be used in the calculation methods of pavement design. These tests are carried out in two stages. In this procedure, a cyclic conditioning is first applied to the specimen, to stabilize the permanent deformations of the material and to obtain a resilient behavior (first phase).

Conditioning stabilizes the permanent deformations in order to study only the resilient behavior of the material in the second phase. It can be likened to the stresses to which the material is subjected during implementation and site traffic [5]. In this study for the first phase, the conditioning is carried out by applying a large number of cycles varying between 20,000 to 60,000 loading cycles at a frequency of 2 Hz, following a corresponding stress path ( $\Delta q/\Delta p = 2$ ) (Table 2) at the level of maximum stress applied during the test ( $\Delta p = 300$  kPa and  $\Delta q = 600$  kPa). Depending on the characteristics of the materials chosen, we applied two (2) types of packaging with a high loading level of a deviatoric stress equal to 320 kPa; and a low loading level of a deviatoric stress equal to 160 kPa.

In the second phase, the resilient behavior is then studied by applying successively, to the same sample, different stress paths ( $\Delta q/\Delta p$ ). This stress ratio is

taken equal to 0, 0.5, 1, 1.5 and 2 (Figure 5) and distributed over 19 loading sequences with three hundred (300) cycles per sequence, at the same frequency of 2 Hz (Table 3). The resilient Young's modulus is calculated as the mean of the last 5 cycles of each sequence from the recoverable axial strain and cyclic axial stress.

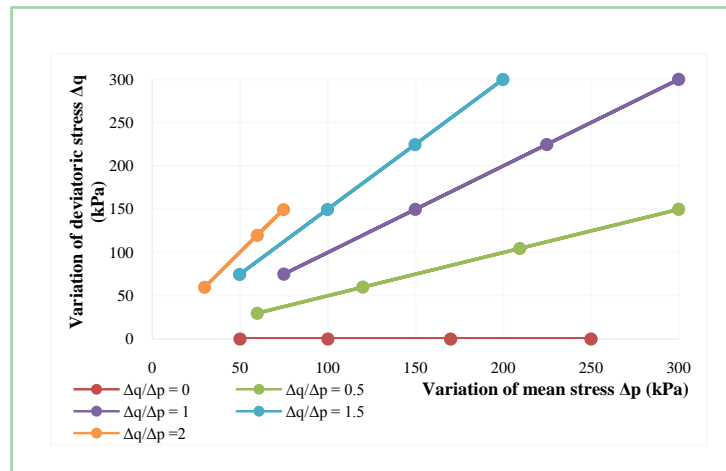


Figure 5. Different stress paths ( $\Delta q/\Delta p$ ).

Table 2. Summary of conditioning loading.

Designation	Labeling	Water content (%)	Density rate (%)	Dry density ( $G_s$ )	$\rho_{dopm}$ ( $g/cm^3$ )	Conditioning $\sigma/W$ (kPa/kN)/Number of cycles (2 Hz)			sequences number
						low	strong		
						160/3.216	235/4.7235	310/6.231	
Sindia ( $w_{opm} \% = 9.66$ )	Sin/11.7/95.71	11.7	95.71				$2.10^4$	$2.10^4$	
	Sin/12.10/95.75	12.1	95.75				$2.10^4$	$2.10^4$	
	Sin/9.90/95.80	9.9	95.8	2.76	1.97		$2.10^4$	$2.10^4$	19
	Sin/9.10/94.94	9.1	94.94				$2.10^4$	$2.10^4$	
	Sin/7.59/95.94	7.59	95.94				$2.10^4$	$2.10^4$	
Badnogo ( $w_{opm} \% = 9.7$ )	Bad/11.43/95.48	11.43	95.48				$2.10^4$		13
	Bad/10.04/95.58	10.04	95.58	2.76	2.45		$2.10^4$	$3.10^4$	
	Bad/7.57/95.32	7.57	95.32				$2.10^4$	$3.10^4$	19
	Bad/9.36/100.06	9.36	100.06				$2.10^4$	$2.10^4$	
Dedougou ( $w_{opm} \% = 8.05$ )	Ded/9.93/95.42	9.93	95.42				$2.10^4$	$2.10^4$	
	Ded/8.16/95.39	8.16	95.39	2.82	2.25		$2.10^4$	$2.10^4$	19
	Ded/6.23/95.52	6.23	95.52				$2.10^4$	$2.10^4$	
Lam-Lam ( $w_{opm} \% = 11.8$ )	Lam/14.56/95.21	14.56	95.21				$2.10^4$		15
	Lam/12.43/98.92	12.43	98.92	2.69	1.752		$3.10^4$	$3.10^4$	19
	Lam/13.92/96.51	13.92	96.51				$3.10^4$	$3.10^4$	19
	Lam/10.15/96.04	10.15	96.04				$3.10^4$	$3.10^4$	



**Table 3.** Summary of LRT test data.

$q/p$ ratio	sequence order	number of cycles	$\sigma_{3min}$	$\sigma_{3max}$	$P_{min}$	$P_{max}$	$Q_{min}$	$Q_{max}$	$W$
		2 Hz	kPa	kPa	kPa	kPa	kPa	kPa	kN
0	16	300	20	70	23.3	73.3	10	10	0.201
	17	300	20	120	23.3	123.3	10	10	0.201
	18	300	20	190	23.3	193.3	10	10	0.201
	19	300	20	270	23.3	273.3	10	10	0.201
0.5	1	300	20	70	23.3	83.3	10	40	0.804
	2	300	20	120	23.3	143.3	10	70	1.407
	3	300	20	195	23.3	233.3	10	115	2.3115
	4	300	20	270	23.3	323.3	10	160	3.216
1	5	300	20	70	23.3	98.3	10	85	1.7085
	6	300	20	120	23.3	173.3	10	160	3.216
	7	300	20	170	23.3	248.3	10	235	4.7235
	8	300	20	220	23.3	323.3	10	310	6.231
1.5	9	300	20	45	23.3	73.3	10	85	1.7085
	10	300	20	70	23.3	123.3	10	160	3.216
	11	300	20	95	23.3	173.3	10	235	4.7235
	12	300	20	120	23.3	223.3	10	310	6.231
2	13	300	20	30	23.3	53.3	10	70	1.407
	14	300	20	40	23.3	83.3	10	130	2.613
	15	300	20	45	23.3	98.3	10	160	3.216

Data are recorded by a computer connected directly to the LRT device using the TEMA-CONCEPT software for data processing. The axial and radial deformations must be measured with a measurement not exceeding the uncertainty ( $I$ ) defined by Equation (1)

$$I = 5 \times 10^{-3} \text{ mm} + 10^{-3} \times L \quad (1)$$

where  $L$  is the value of the measured displacement, in millimeter (mm). The measurements of all displacement sensors must be recorded separately [1]. After we are processing in order to bring out the desired results ( $\varepsilon_1^r$ ,  $\varepsilon_3^r$ ,  $\varepsilon_v$ ,  $\varepsilon_v$ ).

**Table 3** illustrates the sequences used as a function of the path  $q/p$ , the number of cycles by sequence the axial and radial strains obtained following the application of the deviatoric and average stresses. For each path  $q/p$  we notice that the stresses increase then decrease when the stress path changes.

### 3.3. Boyce Anisotropy Model

The desired objective is to be able to determine the characteristic resilient Young's modulus, the volumetric and shear deviation strains of the material and its variation as a function of several parameters such as the water content, the compactness. For this we will use Boyce's model to achieve these objectives [3]. The basic equation according to EN 13286-7: 2004 [1] is represented by Equa-

tion (2):

$$E_r = \frac{\sigma_1^r{}^2 + \sigma_1^r \sigma_3^r - 2\sigma_3^r{}^2}{\sigma_1^r \varepsilon_1^r + \sigma_3^r \varepsilon_1^r - \sigma_3^r \varepsilon_3^r} \quad (2)$$

$E_r$  (MPa) = Resilient Young's modulus,

$\sigma_1^r$  and  $\sigma_3^r$  are respectively axial and radial stresses;

$\varepsilon_1^r$  and  $\varepsilon_3^r$  are respectively the measured axial and radial strains.

Boyce's law results from a non-linear generalization of Hooke's law, based on a dependence of compressibility modulus  $K$  and shear modulus  $G$  as a function of stresses. Equations (3), (4) and (5) derive from Boyce's model [2] for the prediction of volumetric strain  $\varepsilon_v$  and deviatoric strain  $\varepsilon_q$  under imposed stress conditions. The volumetric strain ( $\varepsilon_v$ ) (<0 in contraction) and the deviatoric strain ( $\varepsilon_q$ ), using the sign of convention of the Soils Mechanics, are presented as below:

$$\varepsilon_v = \frac{1}{K_a} p_a^{(1-n)} p^n \left( 1 - \beta \left( \frac{q}{p} \right)^2 \right) \quad (3)$$

$$\varepsilon_q = \frac{1}{G_a} p_a^{(1-n)} p^n \left( \frac{q}{p} \right) \quad (4)$$

$$\beta = (1-n) \frac{K_a}{6G_a} \quad (5)$$

$K_a$ ,  $G_a$  are positive parameters (MPa),

$n$  is an exponent between 0 and 1;

$\beta$  is a parameter dimensionless, and characterizes the expansion of the material;

$p$  is the mean pressure (>0 in compression), and  $q$  is deviatoric stress;

$p_a$  is the atmospheric pressure.

The expressions of the resilient Young's modulus (6) and the Poisson's ratio (7) are deduced using the Equation (2) and the theory of elasticity:

$$E_r = \frac{9G_a \left( \frac{p}{p_a} \right)^{1-n}}{3 + \left( \frac{G_a}{K_a} \right) \left( 1 - \beta \left( \frac{q}{p} \right)^2 \right)} \quad (6)$$

$E_r$  (MPa) = The resilient Young's Modules,

$E_c$  (MPa) =  $E_r$  = The characteristic resilient Young's Modules (with  $p = 250$  kPa and  $q = 500$  kPa)

$$\nu = \frac{\frac{3}{2} - \left( \frac{G_a}{K_a} \right) \left( 1 - \beta \left( \frac{q}{p} \right)^2 \right)}{3 + \left( \frac{G_a}{K_a} \right) \left( 1 - \beta \left( \frac{q}{p} \right)^2 \right)} \quad (7)$$

$\nu$  = Poisson ratio,

$K_a$ ,  $G_a$ ,  $n$  and  $\beta$  are parameters of the model.

$p$  and  $q$  respectively mean average and deviatoric stress.

In order to take into account, the anisotropic nature of unbound materials, Hornych *et al.* [7] have modified Boyce's initial model by weighting the vertical stress  $\sigma_1$  by an anisotropy parameter  $\gamma$ . If the parameter  $\gamma$  is equal to 1, the material is isotropic. The material becomes more and more orthotropic when  $\gamma$  moves away from 1 and it is more rigid in the direction of the constraint affected by  $\gamma$  if it is less than 1 [18]. The expression of the model then becomes of (8) at (13):

$$\varepsilon_v^* = \frac{1}{K_a} \frac{P^{*n}}{p_a^{n-1}} \left[ 1 + \frac{(n-1)K_a}{6G_a} \left( \frac{q^*}{p^*} \right)^2 \right] \quad (8)$$

$$\varepsilon_q^* = \frac{1}{3G_a} \frac{P^{*n}}{p_a^{n-1}} \left( \frac{q^*}{p^*} \right) \quad (9)$$

$$p^* = \frac{\gamma\sigma_1 + 2\sigma_3}{3} \quad (10)$$

$$q^* = \gamma\sigma_1 - \sigma_3 \quad (11)$$

$$\varepsilon_v^* = \frac{\varepsilon_1}{\gamma} + 2\varepsilon_3 \quad (12)$$

$$\varepsilon_q^* = \frac{2}{3} \left( \frac{\varepsilon_1}{\gamma} - \varepsilon_3 \right) \quad (13)$$

$\sigma_1$  and  $\sigma_3$  are respectively the axial and radial stresses,

$\varepsilon_1$  and  $\varepsilon_3$  are respectively the resilient axial and isotropic (radial) strains.

According to Boyce's modified model [7], Equations (8) to (13) are used for the prediction of anisotropic volumetric strain  $\varepsilon_v^*$  and anisotropic deviatoric strain  $\varepsilon_q^*$ .

Equation (14) and (15) are obtained from the derivative of the potential energy:

$$\varepsilon_v^* = \frac{P^{*n}}{p_a^{n-1}} \left[ \frac{\gamma+2}{3K_a} + \frac{n-1}{18G_a} (\gamma+2) \left( \frac{q^*}{p^*} \right)^2 + \frac{\gamma-1}{3K_a} \frac{q^*}{p^*} \right] \quad (14)$$

$$\varepsilon_q^* = \frac{2}{3} \frac{P^{*n}}{p_a^{n-1}} \left[ \frac{\gamma-1}{3K_a} + \frac{n-1}{18G_a} (\gamma-1) \left( \frac{q^*}{p^*} \right)^2 + \frac{2\gamma+1}{6K_a} \frac{q^*}{p^*} \right] \quad (15)$$

#### 4. LRT Results and Interpretation

Following our LRT tests with variable confining pressure (method A), we deduce that:

- **Table 4** shows that:
  - Anisotropic Boyce's model gives the ratios correlations for the predicting of the volumetric and deviatoric strains which are between 0.659 to 0.865;

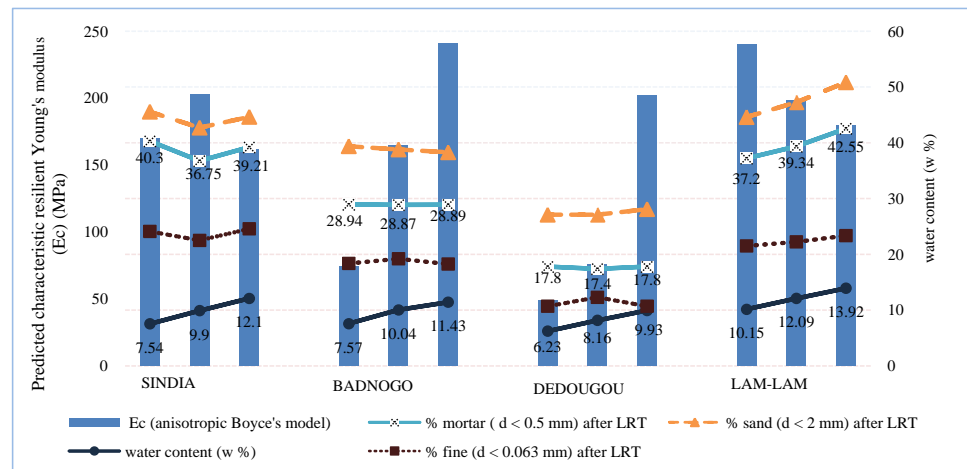
- The parameter  $n$  of the model is positive and between 0 and 1 which verifies the above condition;
- The parameter  $\gamma$  is positive and strictly less than 1, which justifies the rigidity of the materials and their anisotropic behavior;
- The characteristic resilient Young's modulus also increases when the density increases (Bad/10.04/95.58 and Bad/9.36/100.06; Lam/12.43/98.92 and Lam/13.92/96.51).
- The CEBTP model of correlation ( $E_c = MR = 5 \times CBR$ ) is not verified for these tests as shown in **Table 5**;
- The results show that the variation in particle size distribution ( $\phi = 2$  mm, 0.5 mm and 0.063 mm) and water content have a significant influence on the resilient modulus (**Figure 6**).

**Table 4.** Summary of the parameters of the Boyce's model taking into account the anisotropy of the grains.

Designation	Anisotropic Boyce's model parameters					$E_c$ (500/250) (MPa)	Ratio of determination		
	$K_a$ (MPa)	$G_a$ (MPa)	$n$	$\gamma$	Ratio of correlation		$\varepsilon_v^*$	$\varepsilon_q^*$	
SINDIA Wop = 9.66%	Sin/11.7/95.71	29.310	109.418	0.464	0.847	0.789	234	0.967	0.951
	Sin/12.10/95.75	36.370	66.859	0.475	0.512	0.795	162	0.993	0.941
	Sin/9.90/95.80	33.599	93.986	0.382	0.538	0.776	203	0.988	0.926
	Sin/9.10/94.94	37.741	86.618	0.429	0.411	0.807	179	0.987	0.951
	Sin/7.59/95.94	30.124	92.581	0.372	0.414	0.837	170	0.992	0.962
BADNOGO Wop = 9.7%	Bad/11.43/95.48	106.04	91.73	0.84	0.67	0.694	241	0.91	0.98
	Bad/10.04/95.58	42.283	61.174	0.498	0.552	0.806	165	0.968	0.972
	Bad/7.57/95.32	11.617	52.917	0.187	0.332	0.836	74	0.987	0.964
DEDUGOU Wop = 8.05%	Bad/9.36/100.06	17.710	94.381	0.135	0.709	0.715	190	0.973	0.885
	Ded/9.93/95.42	31.935	80.950	0.312	0.606	0.838	202	0.978	0.977
	Ded/8.16/95.39	10.416	64.257	0.141	0.342	0.841	76	0.983	0.970
LAM-LAM Wop = 11.8%	Ded/6.23/95.52	6.564	62.371	0.091	0.275	0.822	49	0.986	0.956
	Lam/14.56/95.21	22.085	61.548	0.454	0.400	0.659	114	0.918	0.852
	Lam/12.43/98.92	61.327	73.131	0.642	0.602	0.865	198	0.996	0.978
	Lam/13.92/96.51	59.619	66.749	0.699	0.640	0.835	180	0.991	0.973
	Lam/10.15/96.04	73.065	106.946	0.808	0.550	0.815	240	0.940	0.992

**Table 5.** Summary of evolution the  $E_c$  according to the CBR index after 92 hours of immersion to 95% of optimal dry density.

Designation	$E_c$ (250/500) (MPa)	CBR after 92 Hours of immersion to 95% of optimal dry density	CEBTP model MR/ CBR = 5
Sin/9.90/95.80	203	55	3.70
Bad/10.04/95.58	165	58	2.84
Ded/8.16/95.39	76	65	1.17
Lam/12.43/98.92	198	31	6.49

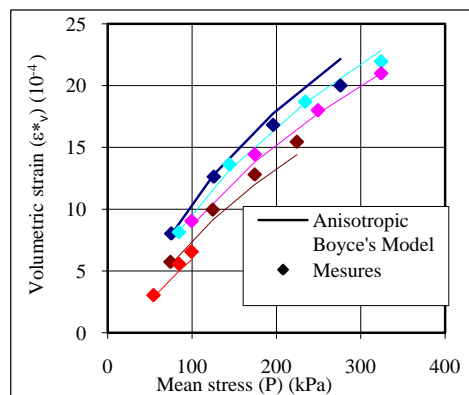


**Figure 6.** Evolution of the predicted characteristic resilient Young's modulus ( $E_c$ ) from anisotropic Boyce's model as a function of the water content and grain size distribution after LRT test.

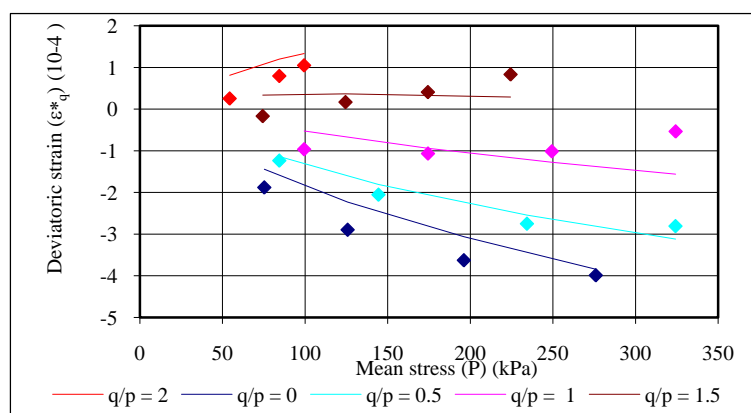
- ❖ For Lam-Lam, there is a drop of 25% of the resilient modulus when the water content increases from 10.2% to 13.9%, in addition to an increase in fines ( $\phi < 0.063$  mm) from 21.5% to 23%, mortar ( $\phi < 0.5$  mm) from 37.2% to 42.5% and sand ( $\phi < 2$  mm) from 44.6% to 50.84%;
- ❖ For Sindia there is an increase of 19.4% of the resilient modulus when the water content increases from 7.54% to 9.9%, accompanied by a decrease in fines ( $\phi < 0.063$  mm) from 24.1% to 22.5%, mortar ( $\phi < 0.5$  mm) from 40.3% to 36.7% and sand ( $\phi < 2$  mm) from 45.56% to 42.7%. Then there is a drop of 20.2% of the resilient modulus when the water content increases from 9.9% to 12.1%, followed by an increase in the content of fines ( $\phi < 0.063$  mm) from 22.5% to 24.57%, mortar ( $\phi < 0.5$  mm) from 36.75% to 39.21% and sands ( $\phi < 2$  mm) from 42.7% to 44.64%;
- ❖ Materials with a high percent of fines, mortar and sand (Sindia and Lam-Lam) are more sensitive to variations in water content. The presence of water combined with the excess of fines leads to a decrease in modulus;
- ❖ For Dedougou there is an increase of 312.24% of the resilient modulus when the water content increases from 6.2% to 9.9%, associated with a constant fines content ( $\phi < 0.063$  mm) of 10.7%, mortar ( $\phi < 0.5$  mm) of 17.8% and an increase in sand ( $\phi < 2$  mm) from 27.1% to 28.1%;
- ❖ For Badnogo there is an increase of 225.67% of the modulus when the water content increases from 7.6% to 11.4%, in addition to a slight decrease in fines ( $\phi < 0.063$  mm) from 18.37% to 18.26%, mortar ( $\phi < 0.5$  mm) from 28.94% to 28.89% and sand ( $\phi < 2$  mm) from 39.37% to 38.29%;
- ❖ Materials containing a low percent of fines, mortar and sand (Badnogo and Dedougou) behave differently. The resilient modulus increases with the rise of the water content for an approximately unchanged of the percent of fines, mortar and sand;
- ❖ Granularity therefore has an indirect influence on the resilient modulus of

the lateritic soils by controlling the effects of water on the entire system.

- **Figure 7** and **Figure 8** show the variation of the volumetric strain  $\varepsilon_v^*$  and deviatoric strain  $\varepsilon_q^*$  of the samples Sin/9.90/95.80 and Bad/10.04/95.58 as a function of the mean pressure  $p$ , is the increasing non-linearly independently of the applied stress paths. Furthermore, we notice that the deviatoric strains obtained at the stress paths  $q/p$  equal to 0, 0.5, 1 are negative and decrease as a function of  $p$ , while those at the  $q/p = 2$  is positive and increase according to  $p$ . In addition, when the ratio  $q/p$  is low, the volumetric strains are high while the deviatoric strain are low;
- **Figure 9** shows that the predicted resilient Young's modulus ( $E_r$ ) from the Boyce's model increases with the bulk stress ( $\theta = \sigma_1 + \sigma_2 + \sigma_3$ ). A correlation around 0.9 is obtained from the power law model;
- The standard NF EN-13286-7 [1] allows to classify the tested materials according to the characteristic resilient Young's modulus ( $E_c$ ). The results of  $E_c$  obtained about from 49 to 241 MPa therefore less than 250 MPa, so these materials are classified C3.

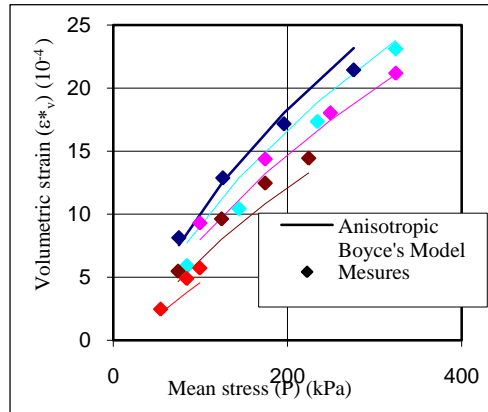


(a)

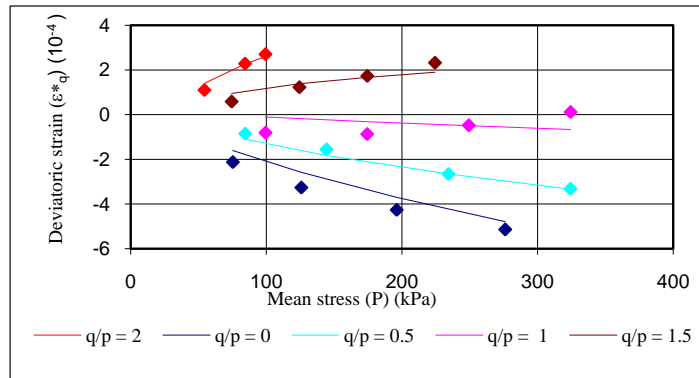


(b)

**Figure 7.** Evolution of the predicted from anisotropic Boyce's model of the sample Sin/9.90/95.80 as a function of the mean stress  $p$ : (a) of the volumetric strain ( $\varepsilon_v^*$ ), (b) of the deviatoric strain ( $\varepsilon_q^*$ ).

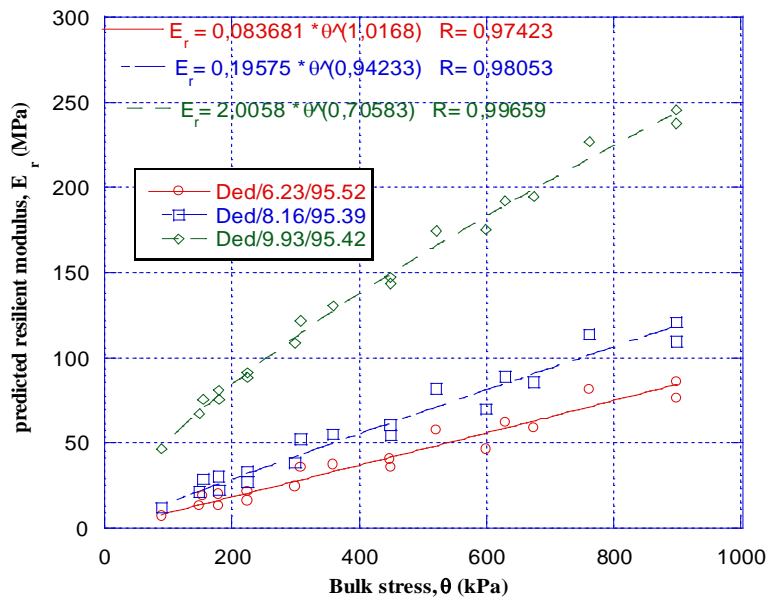


(a)

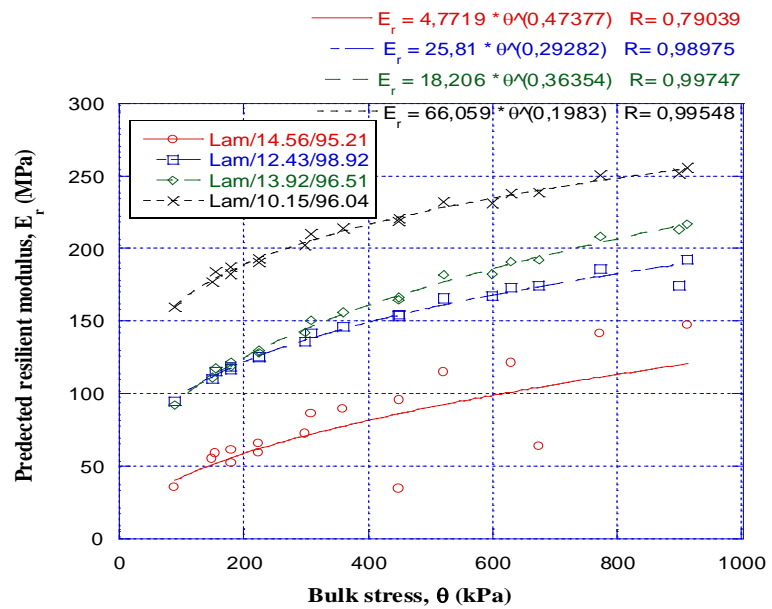


(b)

**Figure 8.** Evolution of the predicted from anisotropic Boyce's model of the sample Bad/10.04/95.58 as a function of the mean stress  $p$ : (a) of the volumetric strain ( $\epsilon_v^*$ ), (b) of the deviatoric strain ( $\epsilon_q^*$ ).



(a)



(b)

**Figure 9.** Evolution of the predicted resilient Young's modulus ( $E_r$ ) from Boyce's model, as a function of the bulk stress ( $\theta = \sigma_1 + \sigma_2 + \sigma_3$ ): (a) Ded/6.23/95.52, Ded/8.16/95.39 and Ded/9.93/95.42 samples; (b) Lam/14.56/95.21, Lam/12.43/98.92, Lam/13.92/96.51 and Lam/10.15/96.04 samples.

## 5. Conclusions

Four (4) gravel lateritic soils from Burkina Faso (Badnogo and Dedougou) and Senegal (Sindia, Lam-Lam) were the subject of this study. Despite the geographical diversity, the materials present approximately the same physical proprieties. They are of the gravelly materials with little clay (lateritic gravel with little clay) and of the class B4, B5 and B6 according to the book of "Guide de Terrassement Routier" [17]. The NF EN 13286-7 2004 standard was used to carry out these LRT tests at the University Gustave Eiffel (formerly "Institut Français des Sciences et Technologies des Transports de l'Aménagement et des Réseaux" (IFSTTAR)) in Nantes, France in the "Laboratoire Auscultation, Modélisation, Expérimentation des Infrastructures de Transport" (LAMES). We deduce that the mechanical behavior of gravelly laterites soils depends on the variation of the grains size distribution sand ( $\phi < 2$  mm), mortar ( $\phi < 0.5$  mm) and fines content ( $\phi < 0.063$  mm) obtained after the loading repeated of cyclic triaxial tests (LRT), and the water content.

Materials with a high percent of fines, mortar and sand (Sindia and Lam-Lam) are more sensitive to variations in water content. The presence of water combined with the excess of fines leads to a decrease in modulus around 25% for Lam-Lam and 20.2% for Sindia. So, materials containing a low percent of fines, mortar and sand (Badnogo and Dedougou) behave differently. The resilient modulus increases a range 225.67% for Badnogo and 312.24% for Dedougou



with the rise of the water content for almost constant of the percent of fines, mortar and sand. Granularity therefore has an indirect influence on the resilient modulus of the lateritic soils by controlling the effects of water on the entire system.

The modulus also increases when the density increases (Bad/10.04/95.58 and Bad/9.36/100.06; Lam/12.43/98.92 and Lam/13.92/96.51).

Results of statistical analysis and coefficients of correlation (0.659 to 0.865) showed that the anisotropic Boyce's model is suitable to predict the volumetric and deviatoric strain with stress path ( $\Delta q/\Delta p$ ) of the lateritic. The parameter  $\gamma$  is positive and strictly less than 1, which justifies the rigidity of the materials and their anisotropic behavior. These strains increase non-linearly as a function of the mean pressure independently of the applied stress paths.

The predicted resilient Young's modulus ( $E_r$ ) from the anisotropic Boyce model increases with the bulk stress ( $\theta = \sigma_1 + \sigma_2 + \sigma_3$ ). A good correlation around 0.9 is obtained from the power law model.

The standard NF EN-13286-7 [1] allows to classify the tested materials according to the characteristic resilient Young's modulus ( $E_c$ ). The results of  $E_c$  obtained are less than 250 MPa then the materials are class C3.

Many researchers have studied the behavior of these materials at macroscopic level, but understanding the true nature of their response at a microscopic, particulate level is a great challenge yet to be overcome. In perspective, mineralogical, fragmentability and degradability tests could help to better understand the behavior of lateritic soils under cyclic loading.

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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