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Optimization of Market Behavior of a Transportation Company

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Abstract

The relevance of optimizing the interaction of participants in manufacture and transport systems is due to the growth of the scale of social production, the development of transnational companies and international trade at the present stage of market transformations, economic integration and development of transport corridors. In this paper we will optimize the market behavior of a transport enterprise in the manufacture and transportation chain. Objectives: To determine the best option for coordinating the actions of participants in the manufacture and transport chain and to optimize the mechanism for granting discounts on the tariff of the transport company. Research methods: economic and mathematical modeling, construction and mathematical analysis of models of manufacture and transport systems, optimization computer calculations, economic interpretation of the results. In this paper it will also be analyzed how full or partial integration of the participants of manufacture and transportation chains, such as coordination of actions, unification of interests can ensure not only a significant increase in the profits of each participant, but also an increase in production volumes and delivery of products to consumers with a corresponding decrease in prices. The paper also portrays how every participant in the manufacture and transportation chain can paradoxically benefit from the established mechanism that would provide discounts on the tariff of a transport company depending on the volume of cargo flow followed by principle the more cargo, the greater the discount. It will be shown how manufacturer benefits by reducing transportation costs, while the transport company earns less per container, but the outstripping growth in the number of these containers provides it with a higher profit. Consequently, the conditions for the feasibility of applying such a mechanism of tariff discounts will also be established.

Keywords

Economic and Mathematical Models, Manufacture and Transport Systems, Equilibrium, Tariff, Price, Competition, Integration, Discount, Optimum

1. Introduction

The problem of choosing the most effective schemes of interaction of participants in economic (including manufacture and transport) systems is especially important at the current stage of market transformation, economic integration and development of transport corridors in connection with growing social manufacture, multinational companies and international trade.

Serious interest in this issue is evidenced by appearance in recent years of a number of publications devoted to the synergetic effect in Campbell & Luchs, (1998), alliances in Dussage & Garnette (1999), mergers in Guajardo, Roennqvist, Flisberg, & Frisk (2018), Connecting to Compete 2014, Trade Logistics in the Global Economy (Arvis et al., 2014), a number of publication on supply chains, which has been published in recent years (Grazia Speranza, 2018; Zhang et al., 2021), in particular devoted to advantages of vertical integration of in the logistics system, however the relevant issues remain at a descriptive level (Bowersox & Closs, 1996; Shapiro, 2000; Geunes et al., 2002; Poirier, 2003; Voß & Woodruff, 2003; Stadtler & Kilger, 2004; Simchi-Levi et al., 2004; Shaelaie et al., 2018; Guajardo et al., 2018), including the creation (Shaelaie, Ranjbar, & Jamili, 2018; Waters, 2019); and monitoring (Simchi-Levi, Chen, & Bramel, 2004) of integrated logistics supply chains, transport systems construction (Geunes, Pardalos, & Romeijn, 2002).

However, the introduction to the subject topic in many sources is conducted mainly at the descriptive level, is often declarative in nature, is limited to the generalization of certain practical experience, the best case scenario with the simplest calculation formulas. The economic interests of individual participants in the systems are not taken into account, the task of balancing these interests is not set, so logistics systems are considered in a purely technological aspect, without taking into account the economic ones.

Therefore, manufacture and transport systems, which do not provide for the a priori unity of economic interests of the participants, can be considered a generalization of logistics. This approach, on the one hand, describes the reality much more adequately, on the other provides a wide spectrum of opportunities to establish patterns of effective management of such systems, in particular, through the construction and analysis of relevant economic and mathematical models.

The aim of this work is to establish the most effective scheme of interaction of supply chain participants, which includes the manufacturer, two related companies and the consumer market by analytical comparison of the equilibrium states of the various alternatives of cooperation and competition between the participants of the supply chain (Stadtler & Kilger, 2004).

Next, we optimize the tool for regulation of providing discounts on the tariff of the transport company and then setting the conditions for the feasibility of using this established tool.

2. Selection of the Best Competitive Option or Coordination Behavior of the Transport Enterprise in the Manufacture and Transport Chain

2.1. Competition of All Participants According to Cournot

For descriptive reasons we will consider the manufacture and transport supply chain, which includes the manufacturer, two related transport companies and the consumer market.

Such a chain—with the independence of the interests of all its participants—is shown in Figure 1, where B1—enterprise-manufacturer, T2 and T3—related transport enterprises, P—consumer market.

The manufacturer's costs for the manufacture of products in volume Q are described by a linear function $c_1 \cdot Q + d_1$, where d_1—constant, c_1—specific variable costs.

The costs of transport enterprises for the transportation of goods in the amount Q are expressed by similar functions, respectively, $c_2 \cdot Q + d_2$ and $c_3 \cdot Q + d_3$.

The demand function in the consumer market for simplicity is considered linear: $P = b - k \cdot Q$ (Figure 2).

The output that satisfies the condition:

$$A = b - c_1 - c_2 - c_3 > 0 \tag{1}$$

Alpha—the maximum possible market price should exceed the amount of specific variable costs in the manufacture and transport chain (otherwise the

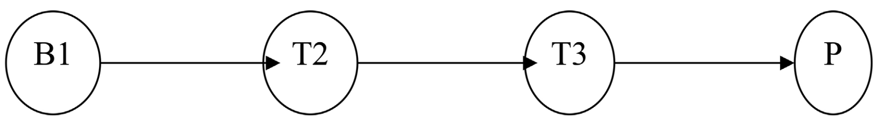

Figure 1. Manufacture and transport supply chain with the independence of all its members. Figure created by author.

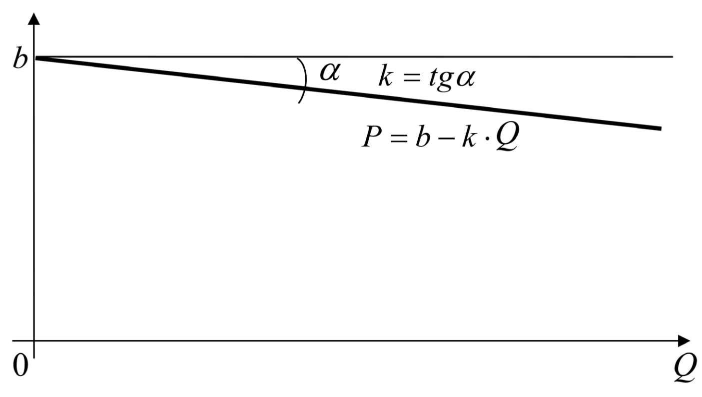

Figure 2. Dependence of the market price P on the volume of delivered products Q. Figure created by author.

manufacture and transportation of products makes no sense), i.e. the economic potential of the manufacture and transport chain Alpha should be positive.

With the independence of all participants in the system, each of them maximizes its profits (in fact, variable profits—excluding fixed costs that do not affect the optimization and will decrease)

$$\begin{aligned} F_1 &= P \cdot Q - c_1 \cdot Q - p_2 \cdot Q - p_3 \cdot Q \\ &= (b - k \cdot Q) \cdot Q - c_1 \cdot Q - p_2 \cdot Q - p_3 \cdot Q \to \max_{Q} \end{aligned} \tag{2}$$

$$F_2 = p_2 \cdot Q - c_2 \cdot Q \to \max_{p_2} \tag{3}$$

$$F_3 = p_3 \cdot Q - c_3 \cdot Q \to \max_{p_3} \tag{4}$$

The manufacturer's profit F_1 is maximized by the volume of production Q at these transport tariffs (per unit of transported or transhipped products) p_2 and p_3. The profit of the first transport enterprise F_2 is maximized according to its tariff p_2 (on which depends the volume of the produced and transported or transhipped goods Q at the given tariff of other transport enterprise p_3. The profit of the second transport enterprise F_3 is maximized according to its tariff p_3 at the given p_2.

We equate to zero the first derivative of the manufacturer's profit function

$$\mathrm{d}F_1 / \mathrm{d}Q = b - 2k \cdot Q - c_1 - p_2 - p_3 = 0,$$

hence the volume of manufacture

$$Q = (b - c_1 - p_2 - p_3)/(2k). \tag{5}$$

The second derivative $\mathrm{d}^2 F_1 / \mathrm{d}Q^2 = -2k < 0$ is negative, i.e. this Q achieves the maximum profit of the manufacturer.

Substituting this value of manufacture volume (5) in (3) and (4), we obtain

$$F_2 = (p_2 - c_2) \cdot (b - c_1 - p_2 - p_3)/(2k) \to \max_{p_2}, \tag{6}$$

$$F_3 = (p_3 - c_3) \cdot (b - c_1 - p_2 - p_3)/(2k) \to \max_{p_3}. \tag{7}$$

We equate to zero the corresponding first derivatives of the profit functions of transport enterprises

$$\mathrm{d}F_2 / \mathrm{d}p_2 = (b - c_1 + c_2 - 2p_2 - p_3)/(2k) = 0,$$

$$\mathrm{d}F_3 / \mathrm{d}p_3 = (b - c_1 + c_3 - 2p_3 - p_2)/(2k) = 0,$$

hence the transport tariffs

$$p_2 = (b - c_1 + c_2 - p_3)/2 \tag{8}$$

$$p_3 = (b - c_1 + c_3 - p_2)/2 \tag{9}$$

As the second derivatives $\mathrm{d}^2 F_2 / \mathrm{d}p_2^2 = \mathrm{d}^2 F_3 / \mathrm{d}p_3^2 = -1/k < 0$, under the such tariffs reach the maximum of the corresponding functions of profit of the transport enterprises.

Substituting the obtained expressions p_2 through p_3 (8) and p_3 through p_2 (9)

to each other, we determine the equilibrium according to Cournot (by analogy with a similar equilibrium in classical microeconomics) transport tariffs through the initial parameters of the manufacture and transport chain:

$$p_2 = \left(b + \left(c_2 - c_1\right) + \left(c_2 - c_3\right)\right)/3 = c_2 + A/3 \quad (10)$$

$$p_3 = \left(b + \left(c_3 - c_1\right) + \left(c_3 - c_2\right)\right)/3 = c_3 + A/3 \quad (11)$$

From (10) and (11) it is seen that the equilibrium Cournot tariff of this transport company with an increase per unit of economic potential of the system increases by 1/3, with a single increase in its own unit costs—increases by 2/3, but with a single increase in specific costs of other participants in the manufacture and transport chain—is reduced by 1/3.

The amount of equilibrium transport tariffs according to Cournot:

$$p_2 + p_3 = \left(2b + c_2 + c_3 - 2c_1\right)/3 = c_2 + c_3 + 2A/3, \quad (12)$$

substituting it in (5), we obtain the equilibrium value of the volume of produced and transported products

$$Q = \left(b - c_1 - c_2 - c_3\right)/\left(6k\right) = A/\left(6k\right), \quad (13)$$

$Q > 0$, as per the condition in (1).

Herewith the equilibrium profit of the manufacturer is a substitution (10), (11) and (13) in (2) and is equal to

$$F_1 = \left(b - c_1 - c_2 - c_3\right)^2/\left(36k\right) = A^2/\left(36k\right) \quad (14)$$

The equilibrium profit of each of the transport enterprises is substituted (10) and (11) in (6) and (7):

$$F_2 = F_3 = A^2/\left(18k\right). \quad (15)$$

The total profit of all independent participants in the manufacture and transport chain in equilibrium is

$$F_1 + F_2 + F_3 = 5A^2/\left(36k\right). \quad (16)$$

It is important to note that in contrast to the usual competitive market—in the manufacture and transport chain to any of its participants is unfavorable increase in unit costs of other enterprises. With increasing unit costs, the tariff of the transport company increases, with increasing unit costs of other participants - decreases (as noted above), but its traffic and profits decrease in both cases, and to the same extent (regardless of whose unit costs increased). This emphasizes the interconnectedness of the participants in the manufacture and transport chain - despite the independence of economic interests of enterprises.

2.2. Competition of All Participants According to Stackelberg

If now one of the transport companies (for example, T2), having learned in some way about the type of function (9) of the optimal response of another transport company (T3) to its tariff p_2, will act as a leader according to Stackelberg, it can substitute function (9) in function of its profit (6) and maximize it as a function of one of its variables p_2.

$$F_2 = (p_2 - c_2) \cdot \left(b - c_1 - p_2 - (b - c_1 + c_3 - p_2)/2\right)/(2k) \to \max_{p_2} . \quad (17)$$

Consequently

$$p_2 = c_2 + A/2 , \quad (18)$$

$$p_3 = c_3 + A/4 , \quad (19)$$

that is, the transport company, defines the leader according to Stackelberg, raises its tariff, and the transport company, defines as the follower according to Stackelberg, lowers it compared to the Cournot equilibrium tariffs (10) and (11). The sum of Stackelberg's equilibrium transport tariffs is

$$p_2 + p_3 = c_2 + c_3 + 3A/4 , \quad (20)$$

that is, it increases in comparison with (12), so the manufacturer loses from the intensification of competition from transport companies.

The consumer also loses, because the volume of products delivered to the market

$$Q = A/(8k) \quad (21)$$

decreases in comparison with (13).

Stackelberg's equilibrium producer's profit will be $F_1 = A^2/(64k)$ (much less than (14)), the profit of the transport company - the leader after Stackelberg will increase slightly (which was the meaning of his leadership) $F_2 = A^2/(16k)$, but due to a significant reduction in profits of the transport company, the follower according to Stackelberg's will be $F_3 = A^2/(32k)$, consequently, the total profit of all participants in the manufacture and transport system in the equilibrium of Stackelberg will be significantly reduced:

$$F_1 + F_2 + F_3 = 7A^2/(64k) . \quad (22)$$

2.3. Coordination of Actions of Transport Enterprises

Consider now the situation of integration (coordination of actions) of two adjacent transport companies (for example, combining the interests of the seaport and the railway or shipping company, Figure 3).

The description of the manufacturer by the objective function (2) and the optimal output (5) is preserved. Under the integration form the transport enterprises can be described by the united function of profit:

$$F_{23} = p_2 \cdot Q + p_3 \cdot Q - c_2 \cdot Q - c_3 \cdot Q \to \max_{p_2, p_3} . \quad (23)$$

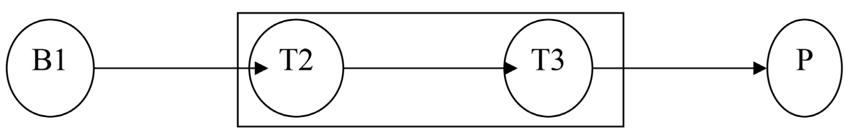

Figure 3. Manufacture and transport supply chain with the integration of transport companies, figure created by author.

Substituting for it the volume of transported goods Q from (5) we obtain

$$F_{23} = (p_2 + p_3 - c_2 - c_3) \cdot (b - c_1 - p_2 - p_3)/(2k) \to \max_{p_2, p_3}. \tag{24}$$

To maximize this profit function, we now equate to zero its partial derivatives

$$\partial F_{23}/\partial p_2 = \partial F_{23}/\partial p_3 = (b - c_1 + c_2 + c_3 - 2p_2 - 2p_3)/(2k) = 0,$$

hence the equilibrium end-to-end transport tariff

$$p_2 + p_3 = c_2 + c_3 + A/2. \tag{25}$$

Substituting (25) into (5), we obtain a new equilibrium volume of manufacture and transportation of products

$$Q = A/(4k). \tag{26}$$

Substituting (25) into (24), we obtain the total profit of transport enterprises in the new equilibrium state

$$F_{23} = A^2/(8k). \tag{27}$$

Substituting (25) and (26) in (2), we get the profit of the manufacturer

$$F_1 = A^2/(16k). \tag{28}$$

The total profit of all participants in the manufacture and transport chain is

$$F_1 + F_{23} = 3A^2/(16k). \tag{29}$$

Comparing (26) and (13), we see that due to the integration of transport companies (intermodal transport at a single rate) in the manufacture and transport chain, the equilibrium volume of production, transport and delivery to the consumer market increases 1.5 times (and in compared with the Stackelberg equilibrium (21)—2 times).

Thus from comparison (25) and (12) or (20) - the total transport tariff decreases accordingly.

With a single increase in specific transport costs, the equilibrium of the total transport tariff (12) increases by 1/3, and the equilibrium of the through transport rate (25)—by 1/2. With a single increase in unit production costs, the total tariff (12) is reduced by 2/3, and the through (25)—by 1/2. With a single increase in market potential, the total tariff (12) increases by 2/3, and the through (25)—by 1/2. Thus, the equilibrium end-to-end transport tariff is more sensitive to changes in actual transport costs and less sensitive to changes in external factors than the equilibrium total transport tariff.

In contrast to the tariff, the total profit of transport enterprises in their integration compared to the Cournot equilibrium (not to mention the Stackelberg equilibrium) increases slightly (1.125 times), which shows comparisons (27) and (25). Particularly significant—2.25 times—the profit of the manufacturer, as can be seen from comparison (28) and (14). The total profit of all participants, as shown by comparison (29) and (16), increases by 1.35 times.

Thus, the integration of transport companies in the manufacture and transport chain benefits not so much even themselves (their total profit increases by only 1125 times), as the cargo owner (producer profit growth of 2.25 times) and the consumer (increase in market output in 1.5 times, with a corresponding reduction in price).

2.4. Coordination of Actions of Manufacture and Transport Enterprises

Let us now consider the situation of combining the interests of the manufacturer with one of the transport companies - with an independent variable (**Figure 4**).

The total profit of such combined participants is expressed by the target function

$$F_{12} = (b - k \cdot Q) \cdot Q - c_1 \cdot Q - c_2 \cdot Q - p_3 \cdot Q \to \max_{Q} \tag{30}$$

(The services of the integrated transport company are now not paid at the tariff p_2, but costs for the cargo owner at prime cost c_2).

Equating to zero the first derivative of this function, we express the optimal volume of manufacture and transportation of products (similarly to (5), replacing p_2 with c_2)

$$Q = (b - c_1 - c_2 - p_3)/(2k). \tag{31}$$

Substitute (31) into the profit function of an independent transport company (4), we obtain

$$F_3 = (p_3 - c_3) \cdot (b - c_1 - c_2 - p_3)/(2k) \to \max_{p_3}. \tag{32}$$

Equate to zero the first derivative of (32)

$$\mathrm{d}F_3/\mathrm{d}p_3 = (b - c_1 + c_3 - 2p_3 - c_2)/(2k) = 0,$$

hence the equilibrium tariff of an independent transport company

$$p_3 = (b - c_1 + c_3 - c_2)/2 = c_3 + A/2. \tag{33}$$

Substituting (33) into (31), we obtain the equilibrium volume of manufacture and transportation of goods

$$Q = A/(4k), \tag{34}$$

which coincides with the equilibrium volume in the integration of transport enterprises (26), i.e. consumer gain is due to the fact of reducing the number of independent participants in the manufacture and transport chain from three to two and does not depend on how to achieve such a reduction.

The distribution of profits among the participants in the manufacture and

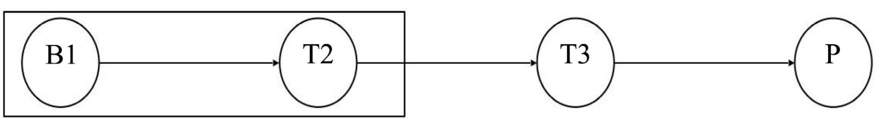

Figure 4. Manufacture and transport supply chain under the integration of the manufacturer with one of the transport companies. Figure created by author.

transport chain is significantly influenced by the nature of their integration. Substituting (33) into (32) we obtain

$$F_3 = A^2/(8k) = F_{23}, \tag{35}$$

that is, all the profits of the combined transport (27) now go to the transport company, which has retained its independence.

By substituting (33) and (34) in (30) we obtain

$$F_{12} = A^2/(16k), \tag{36}$$

that is, the profit of a manufacturer united with one of the transport enterprises coincides with the profit of an independent manufacturer (in the case of combined transport) (28) only now it has to be divided between the two.

Thus, when integrating a manufacturer with one of the transport enterprises of the manufacture and transport chain, the members of such an association lose, and the remaining independent transport enterprise wins. The total profit of all participants in the manufacture and transport chain and the volume of goods produced and delivered to the consumer market increase, respectively, 1.35 and 1.5 times, with each of these types of integration compared to the independence of all participants.

2.5. Coordination of Actions of All Participants of the Manufacture and Transport System

Let us now consider the situation of combining the interests of the manufacturer with one of the Let us finally investigate the situation of combining the interests of all participants in the manufacture and transport chain (**Figure 5**).

The combined profit function of integrated enterprises will look like

$$F_{123} = (b - k \cdot Q) \cdot Q - c_1 \cdot Q - c_2 \cdot Q - c_3 \cdot Q \to \max_Q, \tag{37}$$

The cost for manufacturing and all of products transportation is prime costs.

Equating to zero the first derivative, we obtain (similarly to (5), by replacing p_2 with c_2 and p_3 with c_3) the optimal volume of manufacture and transportation of products

$$Q = A/(2k). \tag{38}$$

Substituting (38) into (37) we obtain the optimal total profit of the entire manufacture and transport chain

$$F_{123} = A^2/(4k). \tag{39}$$

We see that with the full integration of all participants, the total profit of the manufacture and transport chain increases compared to partial integration by 1.33 times (divide (38) by (28)), and compared with the independence of all participants in the Cournot equilibrium by 1, 8 times (divide (38) by (16)).

Summary characteristics of options for competition or coordination of actions of participants in the manufacture and transport system are presented in **Table 1**.

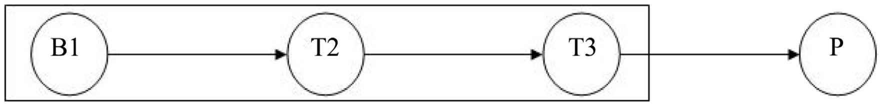

Figure 5. Manufacture and transport supply chain with the integration of all its participants. Figure created by author.

Table 1. Summary characteristics of options for competition or coordination of actions of participants in the manufacture and transport system. Table created by author.

Exponents	Independence of all participants		Integration transport enterprises	T2 integration with the manufacturer	Integration of all participants
	Cournot	Stackelberg (leader T2)			
p_2	$c_2 + A/3$	$c_2 + A/2$	$c_2 + c_3 + A/2$	c_2	c_2
p_3	$c_3 + A/3$	$c_3 + A/4$		$c_3 + A/2$	c_3
$p = p_2 + p_3$	$c_2 + c_3 + 2A/3$	$c_2 + c_3 + 3A/4$	$c_2 + c_3 + A/2$	$c_2 + c_3 + A/2$	$c_2 + c_3$
Q	$A/6k$	$A/8k$	$A/4k$	$A/4k$	$A/2k$
P	$b - A/6$	$b - A/8$	$b - A/4$	$b - A/4$	$b - A/2$
F_1	$A^2/36k$	$A^2/64k$	$A^2/16k$	$A^2/16k$	$A^2/4k$
F_2	$A^2/18k$	$A^2/16k$	$A^2/8k$		
F_3	$A^2/18k$	$A^2/32k$		$A^2/8k$	
$F_2 + F_3$	$A^2/9k$	$3A^2/32k$	$A^2/8k$		
$F_1 + F_2$	$A^2/12k$	$A^2/36k$		$A^2/16k$	
$F = F_1 + F_2 + F_3$	$5A^2/36k \approx 0.14\ A^2/k$	$7A^2/64k \approx 0.11\ A^2/k$	$3A^2/16k \approx 0.19\ A^2/k$	$3A^2/16k \approx 0.19\ A^2/k$	$A^2/4k = 0.25\ A^2/k$

Note that the growth of profits in the integration of participants - a very natural result (the reason the participants decided to unite). More interesting, revealing and even paradoxical is another established effect - the growth of manufacturing and transportation of products with the integration of participants in the manufacture and transport chain. After all, the growth of profits in integration (monopolization) is usually achieved through the reduction of manufacturing and the corresponding advanced price increase.

In the manufacture and transport chain with the full integration of participants, production volumes increase compared to partial integration 2 times (divide (38) by (34) or (26)), and compared to the independence of all participants 3 times at Cournot equilibrium (divide (38) by (13)) and 4 times at Stackelberg equilibrium (divide (38) by (21)).

Interestingly, all these numerical results of comparisons do not depend on specific quantitative values of the parameters of the production and transport system: the economic potential of the system and the elasticity of the consumer market, specific costs of producers and transporters and thus are of a fundamental pattern, due to the inherent features of a particular integration scheme.

Obtaining such qualitative results with the simplest linear functions of participants' costs and demand in the consumer market refutes possible assumptions about certain mathematical tricks and inadequate effects achieved through the use of specially selected functions of a special type.

From the above it can be concluded that the expressiveness of the figures obtained quite convincingly justifies the expediency of creating vertically integrated associations of manufacture and transportation enterprises, and the benefits are not only for the participants of manufacture and transportation chains, but also for consumers of products.

It is also worth noting that the established effect of reducing the equilibrium end-to-end transport tariff for intermodal transportation when transport enterprises are integrated compared to the sum of equilibrium tariffs of independent enterprises of related modes of transport, which also contributes to improving the efficiency of the manufacture and transport system as a whole.

3. Optimization of the Mechanism for Granting Discounts from the Tariff of a Transport Company

3.1. Theoretical Background

Define In addition to choosing the best option for competition or coordination in the manufacture and transport system, it is advisable for a transport enterprise to stimulate an increase in its cargo flow by providing discounts from its own tariff. Moreover, from our point of view, it is not the specific amount of the discount from the established tariff for a particular client that should be optimized, but the mechanism of granting such a discount itself - depending on the volume of services ordered by a given client of the transport enterprise (cargo owner).

Such problem formulations were considered, provided that the characteristics of the market to which the transported products are supplied are known. However, in reality, a transport company rarely has such information; it can only monitor the dependence of the received cargo flow on its own tariff. Therefore, it is of interest to study such a more realistic situation, to find the optimal values of the discount coefficient in it at different ratios of system parameters, and to find out the conditions under which the provision of a discount is beneficial for the transport company.

Let's introduce the following notation:

Q—cargo flow;

b—maximum possible cargo flow;

a—the elasticity of cargo flow at the tariff;

P—transportation tariff;

d—basic transportation tariff;

e—discount from the tariff with a single increase in cargo flow;

z—transportation cost.

Let us assume the simplest (linear) dependence of the cargo flow on the transportation tariff:

$$Q = b - a \cdot P > 0. \tag{40}$$

For simplicity and clarity we introduce a linear dependence of the discount from the basic tariff on the volume of cargo traffic (Figure 6):

$$P = d - e \cdot Q > z . \tag{41}$$

Then, $Q = b - aP = b - a(d - eQ) = b - ad + aeQ$

Hence, $Q = \frac{b - ad}{1 - ae}$

Then we obtain:

$$P = d - eQ = d - e\frac{b - ad}{1 - ae} = \frac{d - aed - be + aed}{1 - ae} = \frac{d - be}{1 - ae} .$$

Then the profit:

$$F = (P - z) \cdot Q = \left(\frac{d - be}{1 - ae} - z\right) \cdot \frac{b - ad}{1 - ae} = \frac{(b - ad)(d - be - z + aez)}{(1 - ae)^2} \to \max_e .$$

We equate to zero the first derivative of the profit function according to our control parameter—the discount factor e:

$$F_e' = \frac{b - ad}{(1 - ae)^3}\left(-b - abe - az + a^2ez + 2ad\right) = 0 ,$$

hence the optimal discount rate

$$e^* = \frac{2ad - b - az}{ab - a^2z} = \frac{a(d - z) - (b - ad)}{a(b - az)} . \tag{42}$$

Note that $e^* > 0$ for $a(d - z) > b - ad$, i.e. when the maximum possible increase in freight traffic due to discount $a(d - z)$ exceeds freight traffic at the base rate $Q_0 = b - ad$ (otherwise $e^* = 0$, i.e. discounts are not provided).

In other words, granting a discount is advantageous if the freight traffic at the base tariff d is less than half of the freight traffic at the tariff equal to the cost of z; if it is more than half (i.e. the elasticity of cargo flow at the tariff is not very high) - granting a discount is already unprofitable (Figure 7).

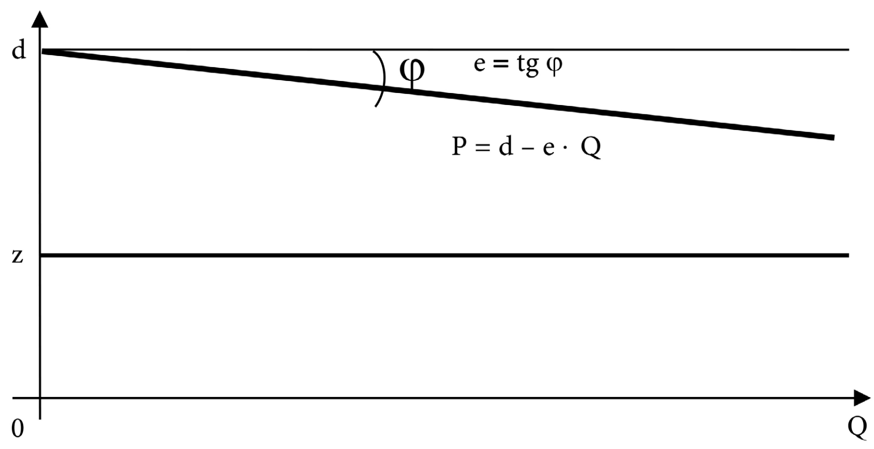

Figure 6. Dependence of the tariff and the size of the discount on the amount of freight traffic. Figure created by author.

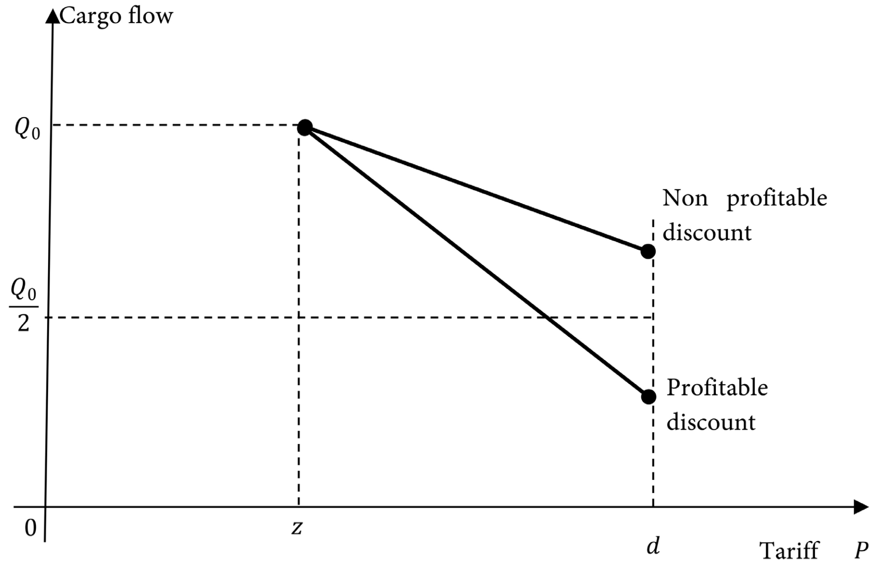

Figure 7. The ratio of system parameters for which the provision of a discount from the tariff is profitable or unprofitable. Figure created by author.

We show that the denominators in $Q=\frac{b-ad}{1-ae}$ and $P=\frac{d-bc}{1-ae}$ are positive at e^*.

$$ae^*=\frac{a(d-z)-(b-ad)}{b-az}<1 \text{ at } ad-az-b+ad<b-az,$$

$b-ad>0$ that is, if the freight flow at the base rate is positive, then $ae^*<1$, and for a given the optimal discount e^* is selected in the range $\left[0;\frac{1}{a}\right)$.

From the formula of the optimal discount factor

$$e^*=\frac{2ad-b-az}{ab-a^2z}=\frac{a(d-z)-(b-ad)}{a(b-az)}$$

it is immediately obvious that it increases with the increase of the basic tariff d (more basic tariff—more opportunities to provide a discount) and decreases with the increase of the maximum possible cargo flow b (when the cargo flow is so large—the discount is less relevant).

To clarify the atomic dependences e^* of z on a and, we find the corresponding derivatives:

$$\left(e^*\right)'_z=-\frac{2(b-ad)}{(b-ad)^2}<0$$

$$\left(e^*\right)'_a=\frac{2z(d-z)}{(b-az)^2}+\frac{1}{a^2}>0$$

that is, the optimal discount factor increases with the increase in the elasticity of freight traffic on the tariff (the discount becomes more influential on the traffic flow) and reduce the cost of overload (which expands the possibility of granting a discount).

In the case of even distribution of freight traffic between k customers, the optimal discount factor is increased by k times.

If the specific importance of the client i in the total cargo flow is α_i—the optimal discount factor for him will be $\frac{e^*}{\alpha_i}$.

3.2. Optimization Calculations

The constructed and analyzed economic-mathematical model of optimization of the mechanism of granting discounts on the tariff was implemented on a specific numerical example of the organization of container transportation from the port of Shanghai to Houston. The basic tariff for this transportation is d = 2100 US dollars/TEU on Hapag Lloyd website. Based on the processing of statistical data, a linear dependence of the demand for transportation on the tariff $Q = 1223 - 0.58 \cdot P$, i.e. $b = 1223$, $a = 0.58$ was constructed. Then, according to formula (42), we calculate the optimal discount coefficients for different possible values of transportation costs (1800, 1900, 2000, 2050 dollars per TEU) and the corresponding freight flows, tariffs, specific and total profits of the transport company (Table 2).

Table 2. Comparison of freight flows, tariffs, specific and total revenues at different costs of transportation in the absence or presence of discounts on the tariff. Table created by author.

Indicators	Options	1		2	
		Without discount	With discount	Without discount	With discount
Cost of transportation per 1 container, USD/TEU	z	1800		1900	
Discount coefficient, USD/TEU/TEU	e	0	1.64	0	1.60
Cargo flow, TEU	Q	5	89	5	63
Tarif USD/TEU	$P = d - e \cdot Q$	2100	1954	2100	2000
Specific profit (per 1 container), USD	$f = P - z$	300	154	200	100
Total profit, USD	$F = f \cdot Q$	1500	13,706	1000	6300
Indicators	**Options**	**3**		**4**	
		Without discount	With discount	Without discount	With discount
Cost of transportation per 1 container, USD/TEU	z	2000		2050	
Discount coefficient, USD/TEU/TEU	e	0	1.46	0	1.22
Cargo flow, TEU	Q	5	31	5	17
Tarif USD/TEU	$P = d - e \cdot Q$	2100	2055	2100	2080
Specific profit (per 1 container), USD	$f = P - z$	100	55	50	30
Total profit, USD	$F = f \cdot Q$	500	1705	250	510

We see that with a significant difference between the base rate and the cost (option 1) under the discount, the specific profit is almost halved (from \$300 to \$154 per TEU), but freight traffic due to the incentive effect of the discount mechanism increases 18 times, and total the profit increases 9 times.

With a smaller difference between the base tariff and the net price (option 2), the optimal discount ratio slightly decreases, the unit profit is halved from \$200 to \$100 per TEU, the cargo flow increases 13 times, and the total profit increases more than 6 times.

With an even smaller difference between the base tariff and the net price (options 3 and 4), the room for discounts is significantly reduced, the optimal discount ratios are reduced accordingly, the specific profit decreases from 100 to 55 and from 50 to 30 dollars per TEU, cargo flows increase by 6 and 3 times, and total profits increase by 3 and 2 times.

Thus, the optimal values of the discount coefficient have been established, depending on the system parameters, as well as the conditions when providing a discount is profitable or not. The discounts should not be too large so that the transport company does not start to incur losses, and not too small so that customers do not lose interest in the transportation services offered. Therefore, we find a certain "golden middle" in which the transport company, by reducing the specific profit on each transported container, increases the total profit due to the outstripping growth of cargo traffic.

4. Conclusion

After 1) It has been established that, unlike conventional economic systems, where monopolization leads to an increase in manufacturers' profits at the expense of consumers (reduction of manufacture volumes and outstripping price increases), in manufacture and transportation chains, full or partial integration (coordination of actions, unification of interests) of their participants ensures not only a significant increase in their profits, but also an increase in production volumes and delivery of products to consumers (with a corresponding decrease in prices), and even more so. It's hard to believe, but the math proves it.

2) The Cournot equilibrium as the basic (initial) state of the manufacture and transportation system defines the independence of the interests of all its participants. In an effort to increase its profit, a transport company can find out the decision-making mechanisms of other participants and use this information to optimize its actions, becoming a leader that is defined by Stackelberg competition (Stackelberg competition describes an oligopoly market model based on a non-cooperative strategic game where one firm (the "leader") moves first and decides how much to produce, while all other firms (the "followers") decide how much to produce afterwards). This leads to a significant increase in the leader's transportation tariff, a slight decrease in the tariff of the follower (another transportation company), while the leader's profit increases slightly (by 12.5%), the follower's and the producer's profits decrease almost twice, and the volume

of production decreases by a third. Thus, the intensification of competition among transport companies leads to very negative consequences for all participants, except for the leader.

3) If transport companies integrate (coordinate their actions, combine economic interests), they reduce their single (through) transport tariff compared to the sum of individual tariffs, while the profit of each of them increases by 12.5% (as the leader according to Stackelberg), the volume of production increases by half, and the producer's profit doubles, i.e., the integration of transport companies benefits not so much them as the producer and consumers of products.

4) It is extremely unprofitable for a transport company to integrate with a manufacturer if another transport company is independent, which will benefit significantly by solely receiving the profits of both transport companies.

5) The best option is the integration of all participants in the production and transportation system, with their total profit increasing by 2.25 times and the volume of production by 3 times, i.e. all participants in the system benefit at times, with consumers of products benefiting the most.

6) All these numerical results of comparisons of different variants of competition and integration do not depend on specific quantitative values of the parameters of the production and transport system: the economic potential of the system and the elasticity of the consumer market, the specific costs of the producer and transporters thus they are of a fundamental, legitimate, due to the inherent features of a particular integration scheme.

7) Obtaining such qualitative results with the simplest - linear - functions of costs of participants and demand in the consumer market refutes possible assumptions about certain mathematical tricks and inadequate effects achieved through the use of specially selected functions of a special kind.

8) The expressiveness of the obtained figures quite convincingly justifies the expediency of creating vertically integrated associations of production and transportation enterprises, and the profitability both from the point of view of participants in manufacture and transportation chains and from the point of view of consumers of products.

9) It should also be noted that the effect of reducing the equilibrium end-to-end transport tariff for intermodal transportation in the integration of transport enterprises compared to the sum of equilibrium tariffs of independent enterprises of related modes of transport is established.

10) A similar non-trivial effect, when all participants in the system paradoxically benefit (and not some at the expense of others, as is usual), was found when a mechanism was introduced to provide discounts on the tariff of a transport company depending on the volume of cargo flow (the more cargo, the greater the discount). In this case, the manufacturer benefits by reducing transportation costs, while the transport company earns less per container, but the outstripping growth in the number of these containers provides it with a higher profit.

11) The optimal values of the discount coefficient depending on the parame-

ters of the system, as well as the conditions when providing a discount is profitable or not, have been established. The discounts should not be too large so that the transport company does not start to incur losses, and not too small so as not to lose the motivation of customers to increase cargo flows.

12) The proposed mechanism for granting discounts on tariffs depending on the volume of orders can be applied not only to transport companies, but also to all service and trade enterprises.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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