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Optimization of Coupled Periodic Antenna Using Genetic Algorithm with Floquet Modal Analysis and MoM-GEC

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Abstract

In this paper Genetic Algorithm has been integrated with Fouquet modal analysis to optimize radiation pattern of coupled periodic antenna. Floquet analysis is used with MoM-GEC (Moment-Generalized Equivalent Circuit) method to study a finite periodic array with uniform amplitude and linear phase distribution. This method is very advantageous for studying large antenna array since it considerably reduces the computation time and the number of operations. In this way, Genetic algorithm is introduced and combined with Floquet analysis to optimize the radiation pattern distribution of this coupled periodic antenna. The goal of the optimization is to provide a better radiation characteristic for the coupled periodic antenna with maximum side lobe level reduction.

Keywords

Periodic Antenna, Floquet Analysis, MoM-GEC Method, Genetic Algorithm

1. Introduction

Genetic algorithms calculation and optimization of the radiation pattern for periodic structure have recently been developed in multiple searches [1] [2] [3]. This paper presents a new optimization method based on Floquet modal analysis with Genetic algorithm to optimize the radiation of coupled periodic antenna [4]. This algorithm simulates biological evolutions, and it is based on the principle of genetic and natural selection. The coupling between the radiating elements increases with the dimension of the antenna arrays, making it one of the most important factors in the choice of the analysis method. This approach forces us to do a parametric study to provide an optimum radiation pattern for the coupled antenna. In this work, Floquet modal analysis is used and combined with MoM-GEC method. Floquet analysis [5] is used to reduce the formulation of the global periodic structure to one reference cell with periodic walls.

Moment MoM-GEC can be applied to this reference cell used to formulate the electric and the magnetic fields. This Floquet approach [6] [7] shows that the electromagnetic field distribution in periodic structures changes only by multiplication of a complex constant for a translation by one period in the global structure. The electromagnetic compilation of antenna arrays with an N pattern is then reduced to a calculation on one reference cell with periodic walls in a new modal base. The Floquet modal analysis introduces all possible Floquet states [8] [9] [10] and groups the coupling information of the overall structure.

This new method is applied to improve the formulation of high-density antennas. To obtain the optimized radiation pattern of coupled cell the Genetic algorithm must be adapted and applied with Floquet analysis.

This paper is organized as follows. In section two, the analytical details of the Floquet analysis formulation for periodic structure are described and 1-D antenna array example is given. In section three, the Genetic algorithm process is described and adapted to optimize radiation pattern based on Floquet phases and amplitudes. In section four, numerical tests are performed to assess the accuracy of proposed formulas. Finally in section five, conclusions have presented the results of this paper.

2. Problem Formulation: Periodic Antenna Array

This section presents the formulation of electromagnetic problem for the periodic antenna. A Floquet theory is proposed to reduce the periodic domain to a single cell with periodic walls. The Floquet approach announces that electromagnetic field distribution in periodic structure changes only by multiplication of complex constant for a translation by one period in the global structure. Then the electromagnetic compilation of N periodic antenna is reduced to a one reference cell with periodic wall in a new modal base. These periodic walls are an artificial wall, and they are implemented to group all phases coming from other cells. Floquet modal analysis is to bring back all spatial calculation to a new modal calculation. An electrical field is then formulated and solved through a MoM-GEC approach [11] [12] [13] in a spectral domain [14].

The structure under analysis is shown in **Figure 1**. The excitation is given by an E_0 voltage source [15] [16] placed in the middle of a metallic patch. The width and the length of patches are w and L. The spatial period along the x direction is d_x. The height of dielectric substance is h, and its relative permittivity εr is mounted on aground plane.

Periodic antenna arrays presentation using Floquet theory and MoM-GEC method. This structure is taken as finite in $\pm X$ and periodic with a period d_x. Floquet theorem can be used with this geometric periodicity, so the study of global structure is reduced to one cell with Floquet phases. The distribution of

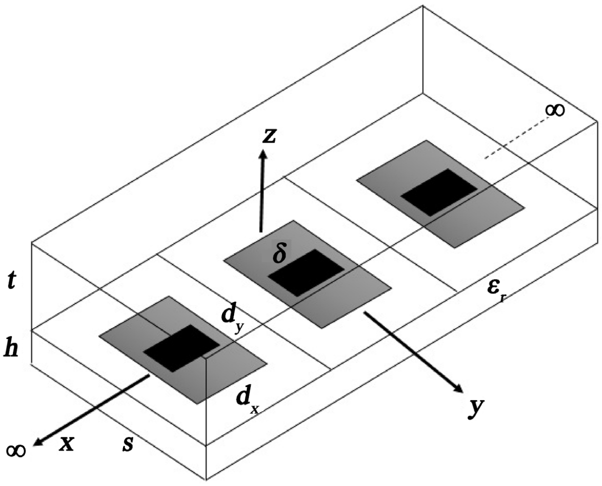

Figure 1. Periodic antenna arrays presentation using Floquet theory and MoM-GEC method.

the electric field differs only by one phase term compared to the adjacent cell. For details, see [17]. $\tilde{E}(x)$ represents an electric field reacting with this periodicity.

$$\begin{cases} \tilde{E}(x+d_x) = \exp(j\alpha d_x) * \tilde{E}(x) \\ \tilde{E}(x+2d_x) = \exp(j\alpha 2d_x) * \tilde{E}(x) \\ \tilde{E}(x+Nd_x) = \exp(j\alpha Nd_x) * \tilde{E}(x) \end{cases} \tag{1}$$

Each Floquet phase corresponds to a Floquet state, and the function $F_{\alpha m}$ characterizes all possible states.

$$F_{\alpha m} = \frac{1}{d_x}\exp(j\alpha x) * \exp\left(j\frac{2\pi mx}{d_x}\right) \tag{2}$$

where α and m correspond respectively to Floquet mode and spectral domain mode. The α values are in Brillouin domain:

$$\left[-\frac{\pi}{d_x}, \frac{\pi}{d_x}\right] \tag{3}$$

And for N discrete values of α, α_p are given by:

$$\alpha_p = \frac{2\pi p}{L} \tag{4}$$

where $L = N * d_x$ and: $-\frac{N}{2} \le p \le \frac{N}{2}$

The electric field of the central cell in spatial domain is $\tilde{E}_m$. The electric field $\tilde{E}_\alpha$ was associated in spectral domain, which models all waves emitted from other cells of periodic structure.

$$\tilde{E}_{m+1} = \tilde{E}_m \exp(j\alpha d_x) \tag{5}$$

Then

$$\tilde{E}_m = \frac{d_x}{2\pi}\int_{-\frac{\pi}{d_x}}^{\frac{\pi}{d_x}} \tilde{E}_\alpha \exp(j\alpha m d_x)\,\mathrm{d}\alpha \tag{6}$$

In the spectral domain, MoM-GEC technique [10] [18] [19] can be applied for this single cell with periodic walls to extract the electromagnetic parameter. The pertinent problem of the use an electric field integral equation can be solved by applying the GEC method. It can replace the integral equation by a simple equivalent circuit in the discontinuity surface and applies the laws of tension and current to extract the relation between electric and current field by using an admittance operator [20] [21]. The discontinuity surface contains metallic and dielectric parties. The equivalent circuit of the unit cell is shown in **Figure 2**.

The virtual electric field is defined on the metallic surface and is null on the dielectric part (E_{ea} is its dual). Similar examples are found in [22]. From this circuit, this system was deduced:

$$\begin{cases} \tilde{J}_{e\alpha} = \tilde{J}_{\alpha} \\ \tilde{E}_{e\alpha} = -\tilde{E}_{0\alpha} + \dfrac{1}{\hat{Y}_{\alpha}^{eq}} * \tilde{J}_{e\alpha} \end{cases} \tag{7}$$

The equivalent admittance [23] operator is:

$$\hat{Y}_{\alpha}^{eq} = \hat{Y}_{\alpha}^{upper} + \hat{Y}_{\alpha}^{down} \tag{8}$$

$\hat{Y}_{\alpha}^{upper}$ is the upper admittance operator of the infinite empty wave guide with periodic walls, and $\hat{Y}_{\alpha}^{down}$ is the down admittance operator of the short-circuited dielectric wave guide of height h with periodic walls.

$$\begin{aligned} \hat{Y}_{\alpha}^{upper} &= \sum_{mn} \left| f_{mn} \right\rangle y_{mn\alpha}^{upper} \left\langle f_{mn} \right| \\ \hat{Y}_{\alpha}^{down} &= \sum_{mn} \left| f_{mn} \right\rangle y_{mn\alpha}^{down} \left\langle f_{mn} \right| \end{aligned} \tag{9}$$

f_{mn} are the base propagation mode functions.

Next, the Galerkin method [24] was applied, where we project the excitation mode f_{mn} and the test function g_{pq} on the previous equation. The following system

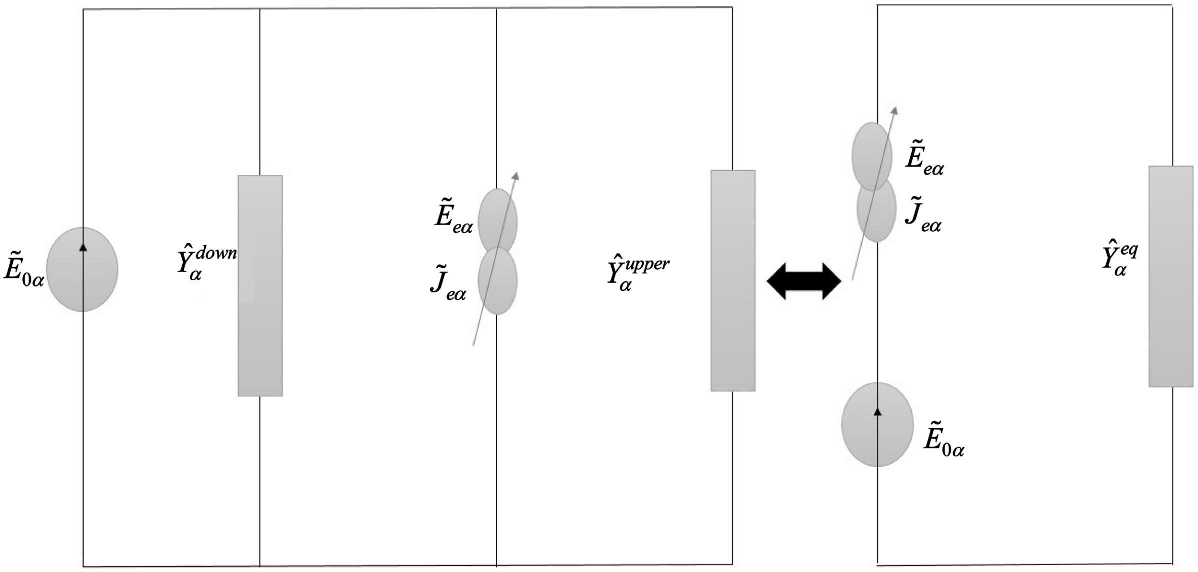

Figure 2. Equivalent circuit in spectral domain with MoM-CEM method of unit cell.

was deduced:

$$\begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} 0 & \langle f_0, g_2 \rangle & \cdots & \langle f_0, g_n \rangle \\ \langle g_1, f_0 \rangle & \langle g_1, Y_{eq\alpha}^{-1}, g_1 \rangle & & \\ \langle g_2, f_0 \rangle & & \ddots & \vdots \\ \langle g_3, f_0 \rangle & & \cdots & \langle g_n, Y_{eq\alpha}^{-1}, g_n \rangle \end{bmatrix} \begin{pmatrix} V_{0\alpha} \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{10}$$

The matrix form of the former equation can be developed as following:

$$\begin{pmatrix} I_{0\alpha} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & A^t \\ -A^t & B \end{pmatrix} \begin{pmatrix} V_{0\alpha} \\ X \end{pmatrix} \tag{11}$$

where A is the excitation vector and B is the coupling matrix. The test courant functions in metallic part are g_{pq}. The resolution of the previous system consequently helps to calculate the virtual electric field E_{ea} and the electric far field E_{Rad} of the coupled structure.

3. Genetic Algorithm Principles

The fact of system optimization function is to search some parameters to inhabit optimal result. The resolution of optimization problems is based necessarily on optimization algorithm. For example, genetic algorithm, gradient method, network method and quasi-Newton method [25] [26] [27] are used in various research. In this study, the Genetic algorithm has been selected. It simulates the genetic evolution. The change of the system parameters follows an evolution process based on the genetic rules that modify chromosomes. In the optimization context, the variables define each chromosomes gene. These chromosomes evolve with genetic law to an optimal chromosome. In the reproduction phase, the individuals are selected and then their structures are modified to generate new individuals for the next generation. The optimization process using genetic algorithm is clarified in **Figure 3**.

The initial population present a set of chromosomes that includes all the variables of this problem $X_1, \cdots, X_N$. The research area is defined between

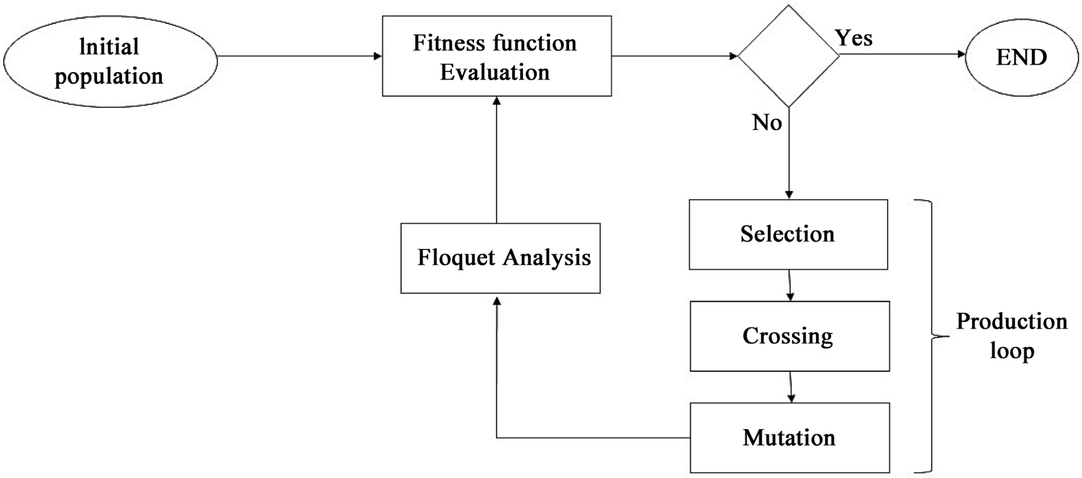

Figure 3. Optimization process of radiation pattern for coupled periodic antenna using Genetic algorithm.

$\left[X_{i\min}, X_{i\max}\right]$, $i \in \left[1, N\right]$, where X_i is the variable to optimize and N is the number of system parameter. The coding choice of chromosomes depends on the length of search area. In our case, the binary coding has been chosen. In the reproduction loop, selection function allows to choose the individuals on which ones apply the algorithm to create the next generation. Many selections method have been found such as proportional method, tournament method and ranking method.

The proportional selection is the most used method. The crossing is applied on two different individual and the result is a chromosome formed from his parent genes.

The crossing can be done in one or many points. The mutation is applied on one individual by the modification of one or many parent genes chosen randomly. The percentage of mutation is fixed, and a single child is provided. From generation to generation, the system will converge, and the population will involve towards the optimum. The optimization of pattern radiation [28] [29] form is done using Genetic algorithm and implemented over MATLAB software. The evolution of each pattern radiation is calculated through fitness function. This function represents the standard deviation between the antenna pattern and the desired template. The usual fitness function is the sum of absolute errors between the calculated values and the reference radiation pattern template. The expression of the fitness function is given by:

$$\text{Fitness} = \left| R_c\left(X_i, \theta\right) - R_{\text{Template}}\left(\theta\right) \right| \tag{12}$$

where N is the iteration number of genetic algorithm. The calculation of the radiation pattern for each iteration of the algorithm has been done based on the formulation of Floquet analysis. However, at each iteration, new parameters are produced by the Genetic algorithm. Finally, an evaluation of the function Fitness will be applied to validate the optimization result. To achieve good optimization, the fitness function tends to minimal value near to zero. This research of minimum will be effected by generating a sequence of vectors $X_1, X_2, \cdots, X_N$. In a convergent iterative process, an acceptable Fitness value can be reach after N iterations. At this point, the X_N value is used to generate a diagram that meets the requirements. To simulate periodic antenna array, it is necessary to consider a periodic structure with uniform spacing between elements and uniform excitation to apply Floquet theory reduction. So, its radiation pattern can be described by all Floquet mode in spectral domain.

In the context of optimization by the genetic algorithm, the parameters to be optimized are the variables which characterize a coupled periodic antenna. First, we start by encoding the system parameters. In our case the encoding binary type is used. The production mechanism [30] [31] of the initial population must be capable of producing non-homogeneous individuals of a population. In our case, the randomly model have been chosen. At this stage, we will identify the three phases of the Genetic algorithm. So, the tournament selection model have been chosen a, a crossover model at a single point characterized by a cross rate c and a mutation model with a mutation rate m. **Table 1** presents all the simulation parameters of the Genetic algorithm.

4. Result and Observations

4.1. Performance Method

In this section, the performance of our method combined with Genetic algorithm is evaluated. The computer code was implemented under MATLAB environment for measuring this performance. The proposed simulation is applied to a (4 * 1) elements array. The next step is to start studying digital complexity [32] for the analysis method. The numerical complexity of a mathematical problem is a measure of the number of resources required to solve that problem. In the framework of this study, we are interested in calculating the temporal complexity of the MoM-GEC method combined with the Floquet analysis. Time complexity is the time it takes to solve our electromagnetic problem. In computer science, time is measured by the number of elementary calculation steps, it is the number of operations used by the algorithm. The complexity depends on an integer parameter n which defines the input of the calculation method. The numerical complexity of an unspecified problem presents the preponderant term in the equation which defines the number of operations necessary to resolve of this problem. This preponderant term is the one that grows the fastest for large n. The $O(n)$ representation is used to define this complexity. The number of operations N_{MoM} required to analyze a planar antenna using the Moments method can be calculated using [26]:

$$N_{\text{MoM}} \simeq o\left(n^3\right) \tag{13}$$

With n is the number of discretization functions to describe the metallic part. In general, the MoM-GEC method is based on the calculation of the input impedance Z_{in} of the planar antenna structure.

However, the total time necessary to calculate this impedance Z_{in} is composed of:

- Time required for scalar product computation.
- Time required for multiplication/addition operations.
- Time required to invert a matrix of $(n*n)$ size.

For MoM-GEC method, the n parameter characterizes the number of test

Table 1. Genetic algorithm parameters to calculate radiation pattern of coupled periodic antenna.

Genetic algorithm parameter	Value
Number of parameters N	20
Population size	10
Stop condition	$N = 20$
Crossing rate c	0.1
Mutation rate m	0.1

functions which discretize the metallic part of the antenna. The number of operations $N_{\text{MoM-GEC}}$ can be expressed by:

$$N_{\text{MoM-GEC}} \simeq o\left(n^2\right) \tag{14}$$

Based on the two Equations (13) and (14), we can notice that for a large antenna structure, which require many test functions, $N_{\text{MoM-GEC}}$ is much lower than N_{MoM}. We can conclude that the MoM-GEC method is much more advantageous compared to the MoM method. Now we turn to the comparison of the MoM-GEC method with the Floquet analysis. We will show the contribution of Floquet's analysis in computation time. In the approach of the MoM-GEC method combined with the Floquet analysis, the formulation of the numerical complexity amounts to calculating the impedance $Z_{in\alpha}$ with the Floquet phases in the spectral domain. For a finite array antenna structure of size N_x, the number of Floquet phases is equal to the number of antenna elements. Indeed, the spatial calculation of an antenna of N_x elements requires $N_x * n^2$ elementary operations with the MoM-MCEG method.

The spectral formulation of the impedance $Z_{in\alpha}$ for a structure with N_x elements requires ($N_x * n^2$) elementary operations. However, using Equation (14), we can deduce the numerical complexity of the MoM-GEC method combined with the Floquet analysis for an antenna structure with N_x elements. The number of operations $N_{\text{MoM-GEC-Floquet}}$ is defined by [32]:

$$N_{\text{MoM-GEC-Floquet}} \simeq N_x o\left(n^2\right) \tag{15}$$

The numerical complexity was simulated as a function of the number of elements for the MoM-GEC and MoM-GEC-Floquet methods. **Figure 4** shows the evolution of the number of operations as a function of N_x for different analysis methods.

From this simulation, we can notice that for a large antenna structure NMoM-GEC-Floquet is much lower than $NMoM-MCEG. Therefore, the execution time of the MoM-GEC method combined with the Floquet analysis is obviously less than that of MoM-GEC. We can conclude that this Floquet analysis is very advantageous for studying large antenna structures since it considerably reduces the computation time and the number of operations. The results of the comparison in terms of calculation time and number of operations are shown in **Table 2**.

This table presents a comparison of the required time to calculate Z_{in} for a test functions number equal to 100. In the previous calculation, an i5 processor is used with 3.6 GHz speed. The measured computation time is approximately 2.5 ms for the MoM-GEC method and 0.0833 ms for the MoM-GEC method combined with the Floquet analysis. At this point the CPU time used by our method represent 3.33% of the CPU time used by the MoM-GEC method. This contribution in computing time will be exploited in the optimization process used by Genetic algorithm. Indeed, the integration of the Floquet analysis in the optimization process with the genetic algorithm allows us to save 96.66% of computation

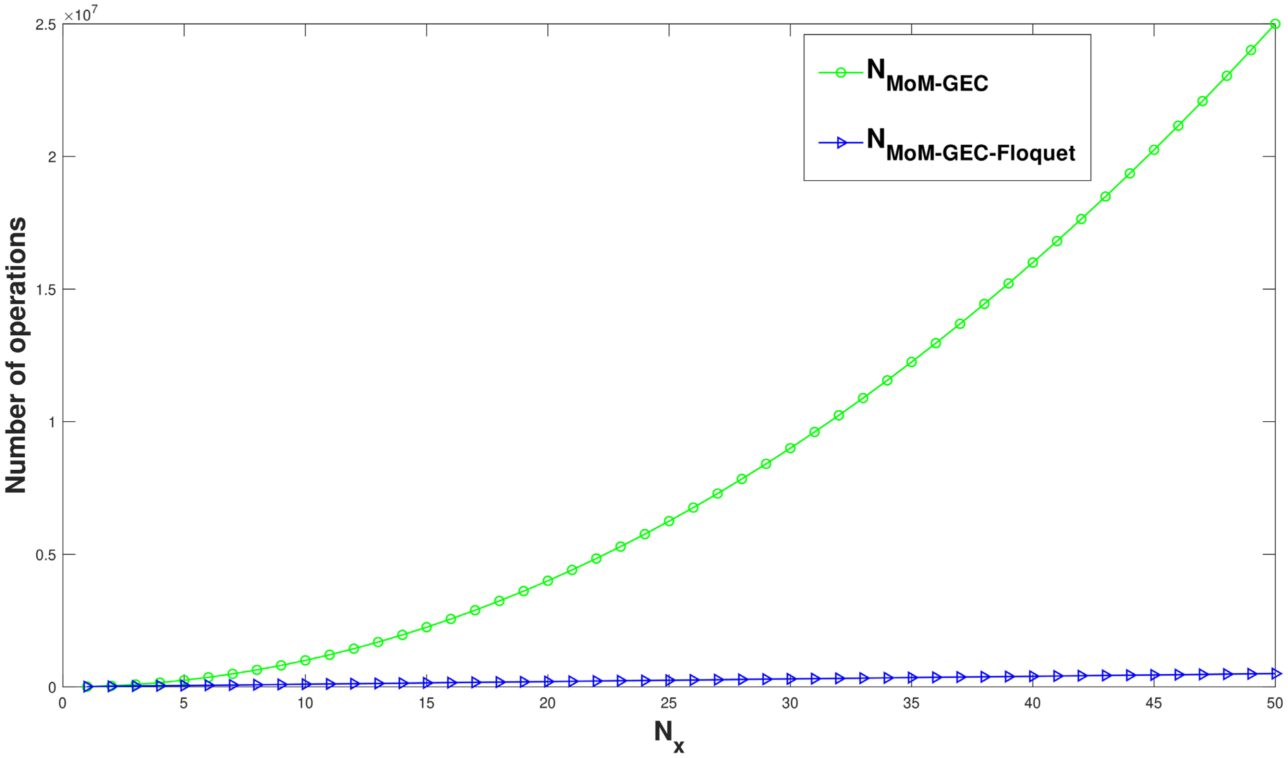

Figure 4. Evolution of the number of operations as a function of N_x.

Table 2. Comparison of the required time for Z_{in} calculation of periodic antenna (N_x = 30).

Method of analysis	Number of operations	Calculation time (ms)
MoM-GEC	9,000,000	2.5
MoM-GEC-Floquet	300,000	0.833

time for each iteration. This makes this method more useful and more beneficial in the context of optimizing antenna arrays.

4.2. Optimisation of the Periodic Structure Dimension

First, we will highlight the influence of structure periodicity d_x on the antenna radiation. The goal of Genetic algorithm optimization is to approach the antenna radiation towards a predefined radiation model.

The desired radiation pattern is specified from a template centered around the 0˚, having a maximum sidelobe level equal to −12 dB. Figure 5 shows the variation of the Fitness function as a function of the algorithm generations. The stopping condition for this simulation is the number of generations which is chosen at 20.

In this case, the optimization parameter is the periodicity d_x. The search space for this variable is the range [81,135] mm. The simulation result of the Fitness function shows that this function converges towards the value 0 and its optimal value is 0.72, which confirms the convergence of the Genetic algorithm. However,

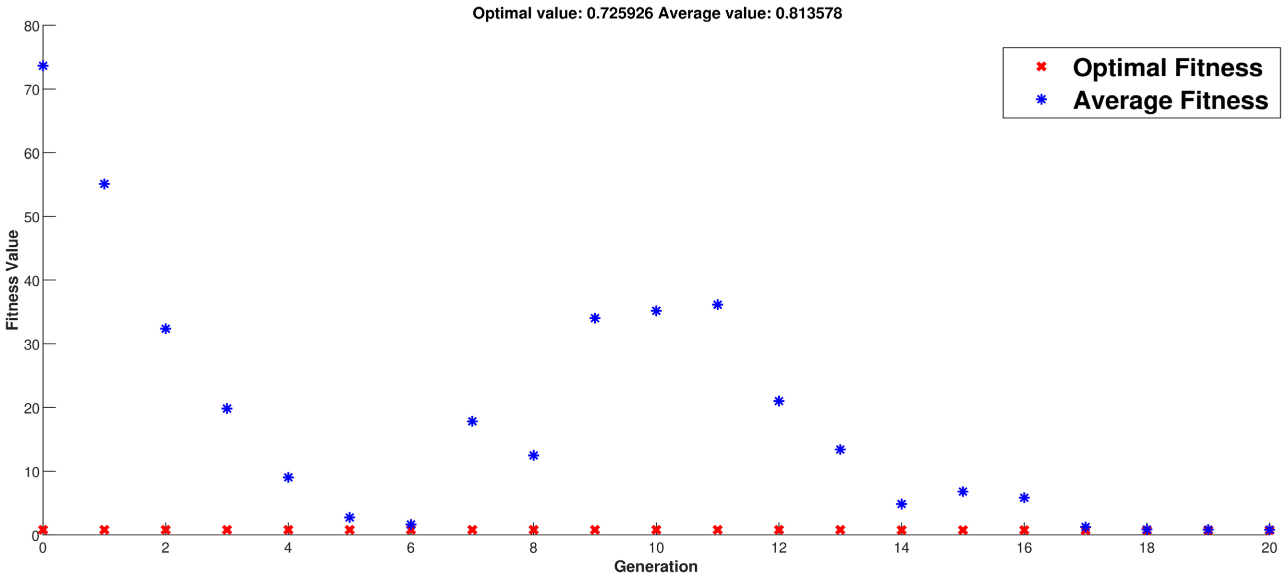

Figure 5. Evolution of the Fitness value as a function of the generations number (optimization of d_x).

it should take a periodicity value d_x equal to 120 mm to ensure radiation pattern convergence towards the chosen template.

4.3. Optimisation of the Excitation Source Dimension

After evaluating the effect of the periodicity on the antenna array radiation, in this part we are interested in measuring the influence of the metallic structure dimension on the radiation for the same periodic antenna. The optimization parameters are the length L and the width w of the metal part. To start the optimization process, the search space for each parameter was specified. However, the variable L belongs to the interval [26,46] mm and the variable w belongs to the interval [1,3] mm. The stopping criteria of the algorithm is set at 20 generations. **Figure 6** shows the variation of the Fitness function as a function of the algorithm generations. The result obtained shows that the Fitness function converges towards an optimal value of 16.93, which confirms the convergence of the Genetic algorithm in this study. We also notice that even at the convergence the genetic algorithm does not give a good Fitness value. we can deduce that these parameters L and w are not very important in the optimization process compared to the periodicity d_x. The optimal result of this simulation gives a value 45 mm for L parameter and a value 3 mm for w parameter.

4.4. Global Optimization

The two previous parts have approved the effectiveness of the periodicity and the metal part dimension on the optimization process for periodic antennas. In this part, a global optimization method is presented. This method all the parameters of the optimization algorithm. So, we will consider the periodicity of the structure and the dimension of the metallic part as optimization variables in the genetic algorithm.

Otherwise, the periodicity d_x, the metal part length L and the metal part width w are used in the optimization process to minimize the Fitness function. To start the optimization process, the search space for each parameter was specified. However, the variable L belongs to the interval [26,46] mm, the variable w belongs to the interval [1,3] mm and the variable d_x belongs to the interval [81,135] mm.

Figure 7 shows the variation of the Fitness function as a function of the generation number for the Genetic algorithm. The simulation result of the Fitness function approves its convergence, and its optimal value is 0.16, which validates the convergence of the Genetic algorithm. In the optimal case, the periodicity d_x

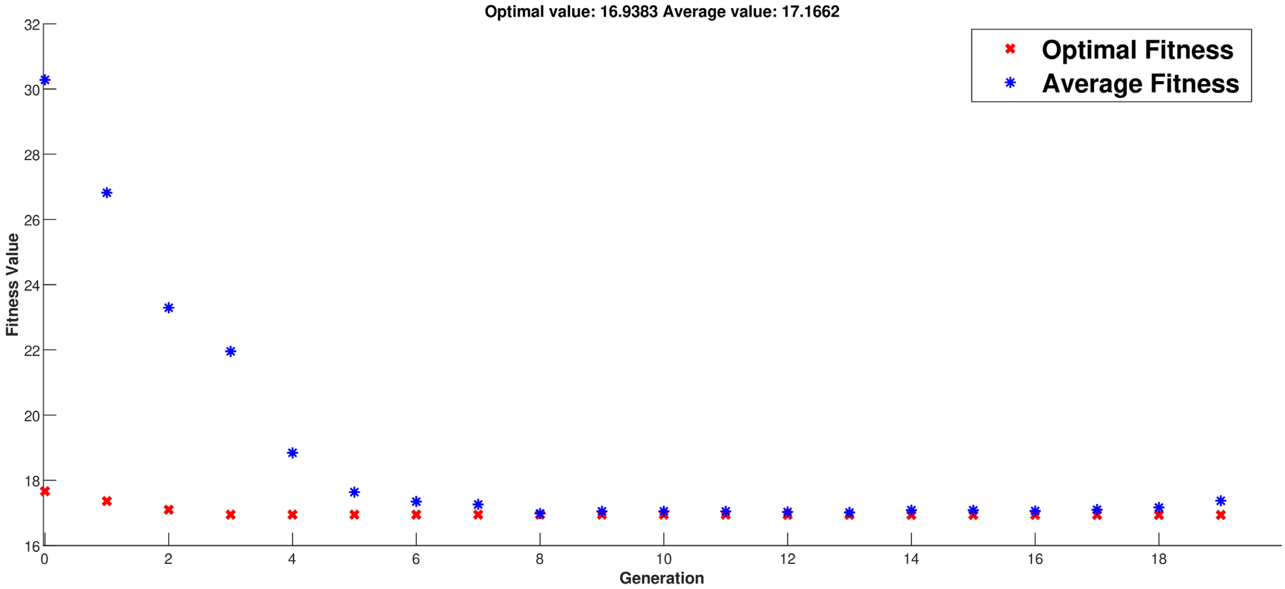

Figure 6. Evolution of the Fitness value as a function of the generations number (optimization of L et w).

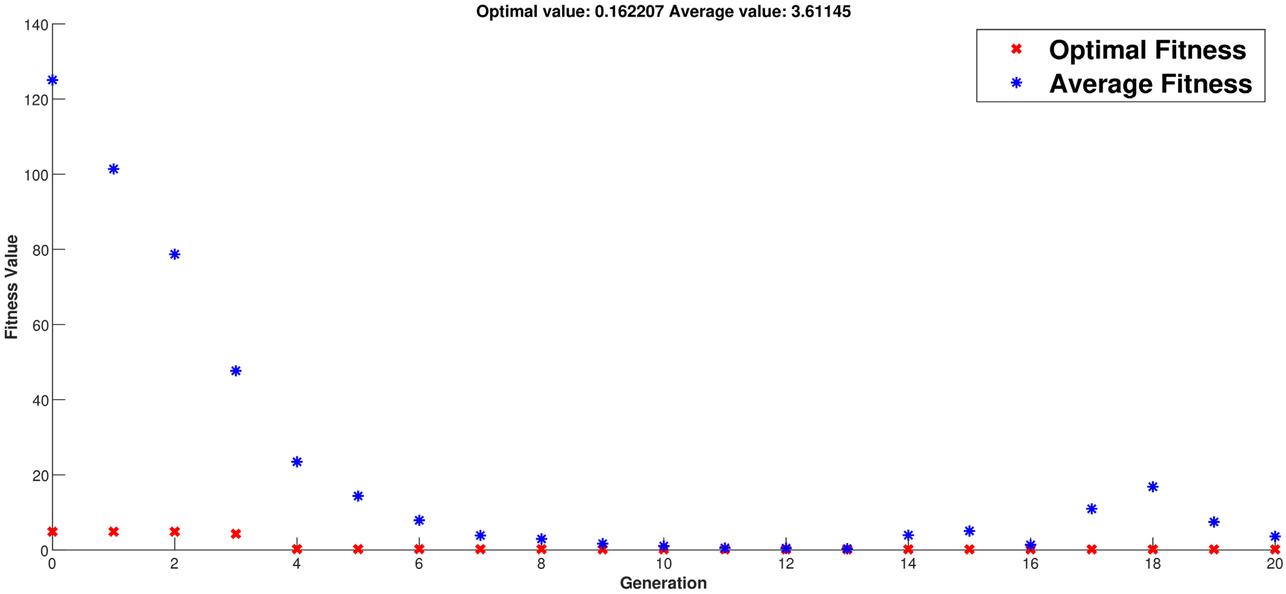

Figure 7. Evolution of the Fitness value as a function of the generations number (optimization of d_x, L et w).

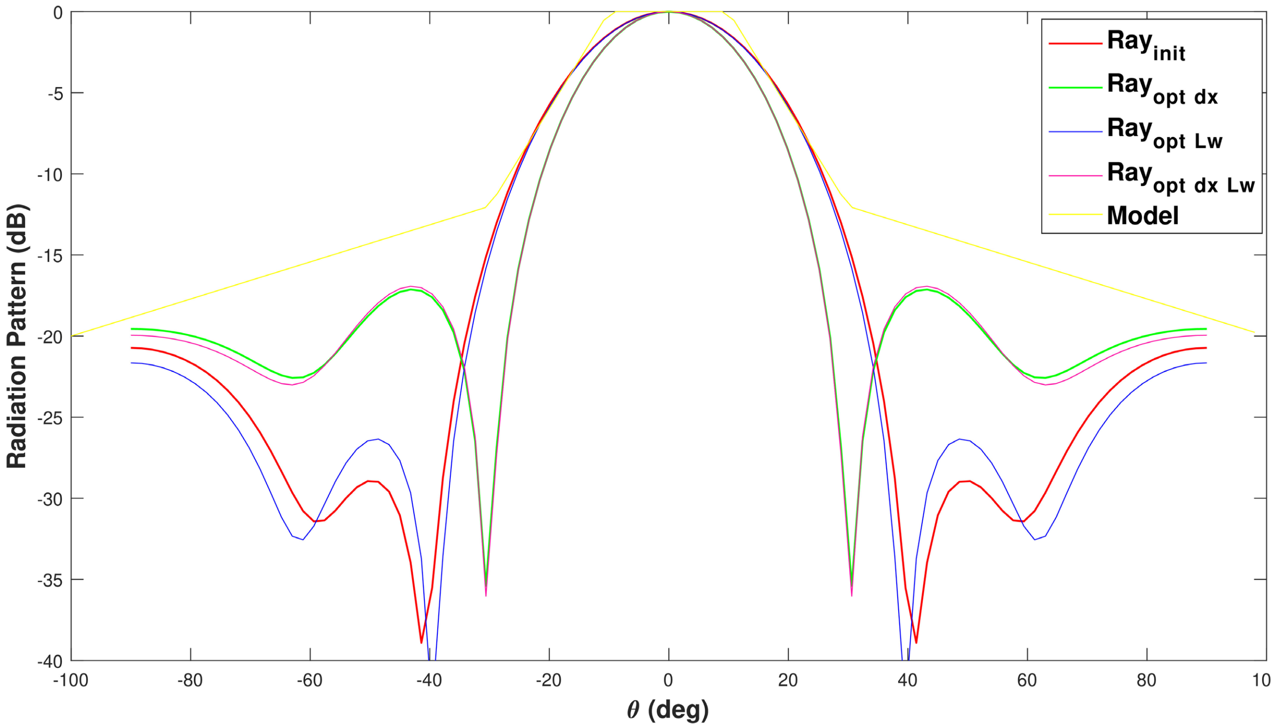

Figure 8. Evolution of the antenna radiation pattern for the three optimization cases.

is equal to 120 mm, the length of the metal part L is equal to 45 mm, and the width of the metal part w is equal to 3 mm. We now turn to the evaluation of the radiation pattern of the optimized structure. The superposition of all the radiation patterns for the three optimization cases is shown in **Figure 8**. This last figure shows a good convergence of the radiation pattern of the optimized structure towards the selected radiation model. We can also notice that better radiation characteristics have been obtained especially for the reduction level of the main lobe width. Finally, we can conclude that using the optimization by the genetic algorithm integrated in the Floquet analysis we will be able to determine the best parameters of a periodic antenna to benefit from a better radiation pattern.

5. Conclusion

In this contribution, we have presented a theoretical analysis of 1-D coupled periodic antennas with Folquet theory and MoM-GEC method. Then we have optimized the gain and the directivity of coupled periodic structure with Genetic algorithm. To enhance the radiation characteristics, we propose to choose the optimum parameters generated by the optimization algorithm (the periodicity d_x = 120 mm, the length of the metal part L = 45 mm, and the width of the metal part w = 3 mm). Floquet analysis with MoM-GEC is more useful and more beneficial and allows us to save 96.66% of computation time for each iteration of the optimization process. For a future work, we suggest applying different optimization techniques with Floquet analysis for a periodic structure to predict the optimal radiation pattern that can be adopted to decrease the side lobe level.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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