Scientific Research Publishing

Open Journal of Applied Sciences, 2024, 14, 425-440
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

A Maintenance Service Improvement Approach Based on Queue Networks: Application

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How to cite this paper: Djami, A.B.N., Nzié, W., Ousman, B., Anyi, J.N. and Nguelcheu, U.N. (2024) A Maintenance Service Improvement Approach Based on Queue Networks: Application. *Open Journal of Applied Sciences*, **14**, 425-440.
https://doi.org/10.4236/ojapps.2024.142030

Received: September 7, 2023
Accepted: February 24, 2024
Published: February 27, 2024

Abstract

The quest to increase the performance of production systems that have become complex leads to the transfer to the maintenance function of the responsibility of guaranteeing the availability of such systems. Also, we will never stop saying that maintenance must integrate into all of the company's initiatives, to affirm its role, which is to ensure greater availability and sustainability of the means of production. The objective of this paper is to evaluate the reliability and availability of a system without knowing the distribution law of the operating times. Among the methods for evaluating dependability criteria (Fault Trees, Petri Nets, etc.), we are interested in queues that have the advantage of taking into account functional dependencies, thus allowing a quantified optimization of maintenance. Indeed, queues make it possible to model parallel or sequential processes, implementing operations taking place at the same time or one after the other, meeting the needs of modeling production systems. The main result of this paper is the study of the influence of availability on the reliability of a multi-state production system.

Keywords

Markov Chains, Queues, Availability, Reliability, Maintenance

1. Introduction

Queuing theory was developed to provide models for predicting the behavior of systems responding to random requests. The first problems studied concerned the congestion of telephone traffic. Erlang will find that a telephone system can be modeled by arrivals of fish customers and exponential uptime [1]. A queuing system could be represented by **Figure 1** and **Figure 2** [2].

The two queues schematized above are not the only types to be found in this area, but there are several. In a queuing process, if the client arrives and finds the server busy, it joins the queue associated with that server. At a certain point, the client is selected for service according to a rule known as a policy or service discipline. The expected service is then provided and the client exits the system.

In queue systems, a distinction is made between open systems where no restriction on the size of the queue is imposed; and closed systems where only a limited number of clients are allowed to stay in the queue.

In this paper, we are particularly interested in queuing networks, as they allow us to model and predict the behavior of the system in all its states. Thus, in any state of the system, reliability and availability can be assessed.

2. Markovian Queues

Markovian queues are those for which inter-arrivals and service times are exponential. Their Kendall notation will be of the form M/M/··· (M for Markovian...) [3]-[8].

2.1. The M/M/1 Queue

This queue is characterized by a Poissonian arrival with birth rate λ and an exponential service life with death rate μ [3] [4].

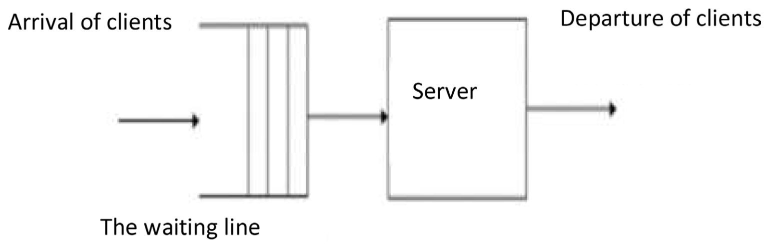

Figure 1. Single server system.

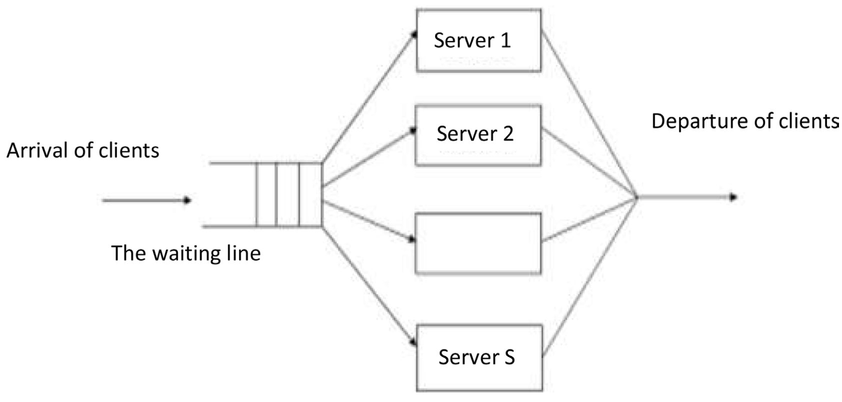

Figure 2. Multi server's system.

Let $\rho = \lambda/\mu$, the queue can be considered as a process of birth and death, for which we have the relation (1).

$$\lambda_n = \lambda, \quad \mu_n \begin{cases} \mu \text{ if } n \neq 0 \\ 0 \text{ if } n = 0 \end{cases} \tag{1}$$

The state probability (taking into account that $\rho \prec 1$ for there to be a steady state) is given by relation (2).

$$\begin{cases} \pi_n = \pi_0 \rho^n \\ \pi_0 = \dfrac{1}{\sum_{n=0}^{+\infty} \rho^n} = 1 - \rho \end{cases} \tag{2}$$

$$\text{So} \quad \pi_n = (1-\rho)\rho^n \tag{3}$$

All performance parameters are calculated for the case where the queue is stable $(\lambda \prec \mu,\ i.e\ \rho \prec 1)$ and for the steady state of the queue.

The illustrative diagram is given by Figure 3.

2.1.1. The Rate d

Here $d = \lambda$ because for all $n \geq 0$. Another way to look at it is to notice that the service is performed at a rate μ in every state where the system has at least one client [3] [4].

$$\begin{aligned} d &= Proba([\text{Queue not empty}])\mu \\ &= \sum_{n=1}^{+\infty} \pi_n \mu = [1 - \pi_0]\mu = \rho\mu = \lambda \end{aligned} \tag{4}$$

2.1.2. Server Utilization Rate

$$U = \sum_{n=1}^{+\infty} \pi_n = 1 - \pi_0 = \rho = \frac{\lambda}{\mu} \tag{5}$$

2.1.3. Average Number of Clients L

The average number of clients is calculated from the stationary probabilities of the following way:

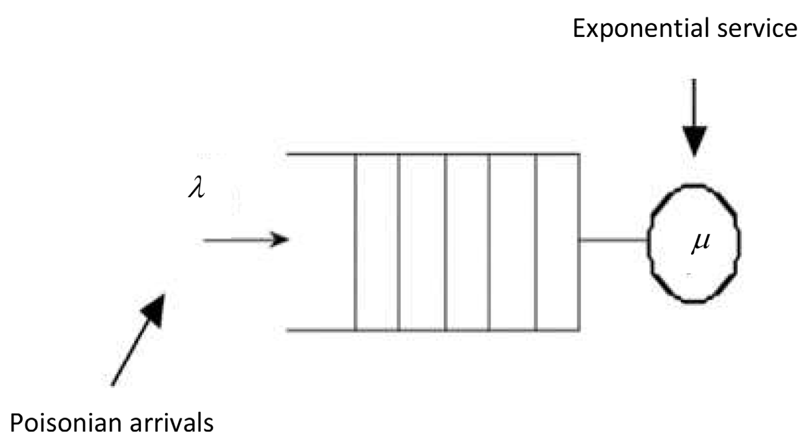

Figure 3. The M/M/1 queue [4].

$$L=\sum_{n=1}^{+\infty}n\pi_n=\sum_{n=1}^{+\infty}n(1-\rho)\rho^n$$
$$=\rho(1-\rho)\sum_{n=0}^{+\infty}(n+1)\rho^n$$
$$=\rho(1-\rho)\left(1+2\rho+3\rho^2+\cdots\right)$$
$$=\rho(1-\rho)\frac{d}{dp}\left(\rho+\rho^2+\rho^3+\cdots\right)$$
$$=\rho(1-\rho)\frac{d}{d\rho}\left(\frac{1}{1-\rho}-1\right)$$

$$\text{From where } L=\frac{\rho}{1-\rho} \tag{6}$$

2.1.4. Average Residence Time in System *T*

This parameter is obtained using Little's law, given by the relation (7).

$$T=\frac{L}{d}=\frac{1}{\mu(1-\rho)} \tag{7}$$

This formula can be decomposed according to relation (8).

$$T=\frac{1}{\mu}+\frac{\rho}{\mu(1-\rho)} \tag{8}$$

From the previous relationship, we deduce respectively the average time spent in the queue and the average number of customers in the queue by relationships (9) and (10).

$$T_q=\frac{\rho}{\mu(1-\rho)} \tag{9}$$

$$T=\frac{1}{\mu}+\frac{\rho}{\mu(1-\rho)} \tag{10}$$

2.2. The M/M/1/K Queue

Consider a single server system identical to the M/M/1 queue except that the capacity of the queue is finite. We therefore always have the following assumptions: the process of arriving customers in the queue is a Poisson process with rate λ and a customer's service time is an exponential random variable with rate μ. Let K be the capacity of the queue: this is the maximum number of clients that can be present in the system, either on standby or in service. When a customer arrives when there are already K customers present in the system, it is lost [3] [4].

Figure 4 shows an example of an M/M/1/K queue.

The process of birth and death modeling this type of queue is then defined by relation (11).

$$\lambda_n=\begin{cases}\lambda \text{ if } n\prec K\\ 0 \text{ if } n=K\end{cases},\quad \mu_n=\begin{cases}\mu \text{ if } n\neq 0\\ 0 \text{ if } n=0\end{cases} \tag{11}$$

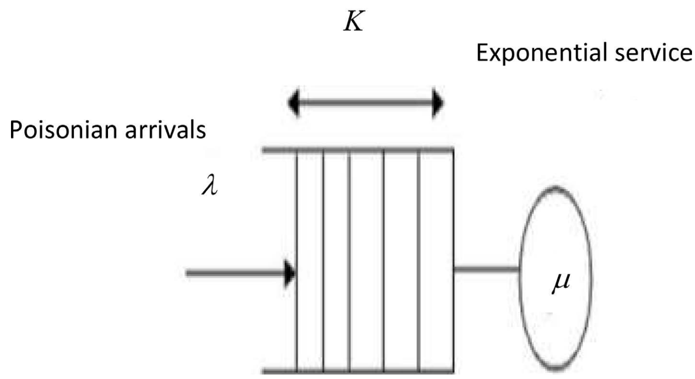

Figure 4. The M /M/1/K queue [9].

The integration of the recurring equation for performing calculations is given as follows:

$$\pi_n = \begin{cases} \pi_0 \rho^n \text{ if } n \le K \\ 0 \text{ if } n \succ K \end{cases} \tag{12}$$

$$\pi_0 = \begin{cases} \dfrac{1}{\sum_0^K \rho^n} = \dfrac{1-\rho}{1-\rho^{K+1}} \text{ if } \lambda \ne \mu \\ \dfrac{1}{K+1} \text{ if } \lambda = \mu \end{cases} \tag{13}$$

2.2.1. The Rate *d*

The system flow can be calculated in two equivalent ways: either by measuring the departure rate of clients leaving the server d_S, or by measuring the actual arrival rate of clients accepted into the d_e system. We would of course expect to obtain equality of these two rates.

The output rate of the server is equal to μ as soon as the queue is not empty:

$$\begin{aligned} d_S &= Proba\left(\left[\text{Queue not empty}\right]\right)\mu \\ &= \sum_{n=1}^{+\infty} \pi_n \mu = \left[1-\pi_0\right]\mu = \rho\mu = \frac{\rho-\rho^{K+1}}{1-\rho^{K+1}}\mu \end{aligned} \tag{14}$$

The effective input rate to the file is equal to λ from the moment a client arrives when the file is not full:

$$\begin{aligned} d_e &= Proba\left(\left[\text{Queue not full at arrival times}\right]\right)\lambda \\ &= \sum_{n=0}^{K-1} \pi_n \lambda = \left[1-\pi_0 K\right]\lambda = \frac{1-\rho^K}{1-\rho^{K+1}}\lambda \end{aligned} \tag{15}$$

Since $\rho = \dfrac{\lambda}{\mu}$, we have $d_e = d_S = d$, where d is the average throughput of the queue (entry or exit):

$$d = \frac{1-\rho^K}{1-\rho^{K+1}}\lambda \tag{16}$$

Note that, when K tends to infinity, we find the results of M/M/1, that is to say, provided that $\rho \prec 1$, which corresponds to the condition of stability of M/M/1.

2.2.2. *U(K)* Server Utilization Rate

$$U(K)=\sum_{n=1}^{K}\pi_n=1-\pi_0=\frac{\rho-\rho^{K+1}}{1-\rho^{K+1}}=\rho\frac{1-\rho^{K}}{1-\rho^{K+1}} \quad (17)$$

Thus, in the case of a queue with limited capacity, the utilization rate is no longer equal to ρ. Indeed, the rate of use is always equal to the ratio of the average input rate to the average service rate (Little's law): $U=\frac{d}{\mu}$.

Note that, as $K\to+\infty$, $U(K)\to\rho$ if $\rho\prec 1$, and towards one if $\rho\succ 1$ [10].

2.2.3. Average Number of Clients *L*

The average number of customers in the system is given by relation (18).

$$L=\frac{\rho}{1-\rho}\frac{1-(K+1)\rho^{K}+K\cdot\rho^{K+1}}{1-\rho^{K+1}} \quad (18)$$

2.2.4. Average Residence Time in System *T*

We calculate T by Little's formula, but λ_n not being constant, we must calculate $\bar{\lambda}$, that is:

$$\bar{\lambda}=\sum_{n=0}^{+\infty}P_n \quad \text{with} \quad P_n=\frac{1-\rho}{1-\rho^{K+1}}\rho^{n}$$

Hence: $$T=\frac{L}{\bar{\lambda}}=\frac{\rho}{1-\rho}\cdot\left(1-(K+1)\cdot\rho^{K}+K\cdot\rho^{K+1}\right) \quad (\rho\prec 1) \quad (19)$$

Table 1 illustrates the different symbols we will use in queues and their meaning.

Table 1. Symbols and meanings [2].

Symbols	Significations
P	System utilization rate
n	Average number of customers in the system (customers who are waiting and customers who are being served)
λ	Average customer arrival rate (failure rate)
μ	Average customer service throughput (repair rate)
T_q	Average waiting time the queue
T	Average waiting time in the system
P_0	Probability that there are zero units (customer) in the system
P_n	Probability that there are n units (clients) in the system
$\pi_n(t)$	Probability that the system is in state E_n at time t

3. Markov Chains

Markov chains—or State Space Method (MEE)—were developed in the 1950s for the reliability analysis of repairable systems [11] [12] [13] [14] [15].

This method consists of representing the operation of a system by a set of components that can be in a finite number of operating and fault states.

A graphic support (the state graph) makes it possible to visualize the different states of a system which are represented by circles and interconnected by oriented arcs which correspond to the transitions (breakdowns and repairs) between states. For an n-component system, if each component has two states (working and failing), the maximum number of states is 2^n. **Figure 5** shows an example of a Markovian model of a system.

To perform this analysis, you must first identify all the states of the system, classify them into operating states or failure states. Then, it is necessary to find out how to switch from one state to another during a malfunction or repair. With each transition from state E_i to state E_j, a transition rate L_{ij} is associated which is defined in such a way that $L_{ij} \cdot dt$ is equal to the probability of passing from E_i to E_j between two very close instants t and $t+dt$ knowing that we are at time t in E_i. Finally, the last step consists in calculating the probabilities of being in the different states during a system life period as well as of calculating the characteristics of dependability [12].

Using Markov graph modeling, time and stochastic dependencies are more widely taken into account than with classical methods.

In addition, they are easy to use [13].

4. Modeling of Availability and Reliability Using the Theory of Queues and Markov Chains

4.1. Definition of a Homogeneous Markov Chain over Time

We will assume that the system transits from state *i* to state *j* with a probability which depends only on states *i* and *j*. Such a system is said to be memoryless or even Markovian, which means that the probability does not depend on the states prior to *i* through which the system has passed during its history. Thus, the future state of the studied system depends only on its present state and not on its past states.

These numbers are arranged in the matrix $P = \left(P_{i,j}\right)_{(0 \le i, j \le n)}$. This matrix is stochastic because the (stochastic) vector in row *i* contains the probabilities of all

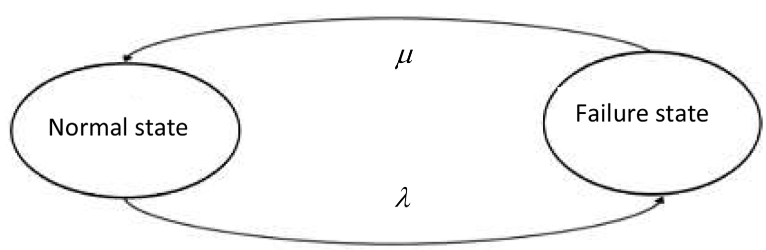

Figure 5. A Markovian model.

possible transitions starting from state i and therefore their sum is equal to one. The sequence of stochastic vectors $\pi(t)$ for $t = 0,1,2,\cdots$ satisfies the matrix recurrence formula given by Relation (20).

$$\begin{aligned}&\left(\pi_0(t+1), \pi_1(t+1), \cdots, \pi_n(t+1)\right)\\&= \left(\pi_0(t), \pi_1(t), \cdots, \pi_n(t)\right)\begin{pmatrix} P_{0,0} & P_{0,1} & \cdots & P_{0,n} \\ P_{1,0} & P_{1,1} & \cdots & P_{1,n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n,0} & P_{n,1} & \cdots & P_{n,n} \end{pmatrix}\end{aligned} \tag{20}$$

The more compact form is given by Equation (21).

$$\pi(t+1) = \pi(t)P \tag{21}$$

4.2. Definition of a Homogeneous Markov Chain over Time

The recurrence formula $\pi(t+1) = \pi(t)P$ applied at $t = 0,1,2,\cdots$, gives:

$$\begin{aligned}&\pi(1) = \pi(0)P\\&\pi(2) = \pi(1)P = \pi(0)P^2\\&\pi(3) = \pi(2)P = \pi(0)P^3\\&\vdots\end{aligned}$$

At any instant t (integer), the stochastic vector is therefore given by Equation (22).

$$\pi(t) = \pi(0)P^t \tag{22}$$

4.3. Stationary Distribution

If the eigenvalue 1 of the stochastic matrix P of a homogeneous Markov chain is simple and dominant (any other eigenvalue has a modulus strictly less than 1) then the sequence $\left(P^n\right)_{n\in N}$ converges to a strictly positive matrix P^∞ of the form given by expression (23), with $P_0 + P_1 + \cdots + P_n = 1$ [16].

$$P^\infty = \begin{pmatrix} P_0 & P_1 & \cdots & P_n \\ P_0 & P_1 & \cdots & P_n \\ \vdots & \vdots & \ddots & \vdots \\ P_0 & P_1 & \cdots & P_n \end{pmatrix} \tag{23}$$

Moreover, any sequence $(\pi_n)_{n\in N}$ defined by:
$\pi_0 = \left(P(X_0 = 0)\ P(X_0 = 1) \cdots P(X_0 = n)\right)$ and $\forall n \in N$, $\pi_{n+1} = \pi_n \times P$, $\pi_\infty = \left(P_0\ \ P_1\ \cdots\ P_n\right)$ converges to the vector π_∞ as defined by relation (24).

$$\pi_\infty = \left(P_0 \quad P_1 \quad P_2 \quad \cdots \quad P_n\right) \tag{24}$$

π_∞ is the unique distribution of probabilities verifying: $\pi \times P = \pi$.

4.4. Graphical Representation of Markov Chains

A homogeneous Markov chain with a set of states E can be represented by the directed graph G such that:

- its vertices are the states *E*,
- the arc from vertex *i* to vertex *j* exists if, $P_{i,j} \succ 0$,
- the evaluation of the arc $i \to j$ is given by the probability of transition $P_{i,j}$.

A Markov chain can then be seen as a random walk on Graph *G*: we draw randomly x_0 realization X_0 according to the π_0 law, then of x_0 we draw x_1 (realization of X_1) according to the transition probabilities of the arcs coming from x_0, etc [17].

5. Application

Either a production system, so the history of operating hours or failures during a year of operation reveals the times between failures (TBF) and the times to repair (TTR), given in Table 2.

We suppose that our production system can be found in three states: the state (E_0) where there is no failure with a probability of $1-\lambda$, the state (E_1) where there is a partial failure with a probability of $1-(\mu+\lambda)$ and the state (E_2) where we observe a total failure of the system with a probability of $1-\mu$. The probabilities of transitions between the different states are defined in the matrix of transitions given in relation (25).

$$P = \begin{pmatrix} 1-\lambda & P_0\lambda & \lambda(1-P_0) \\ P_1\mu & 1-(\mu+\lambda) & P_1\lambda \\ \mu(1-P_2) & \mu P_2 & 1-\mu \end{pmatrix} \quad (25)$$

We make the following working hypotheses:

- service disciplines first come, first served (FIFO),
- arrivals according to a Poisson process,

Table 2. History of operating hours (h).

Months of operating	TBF (h)	TTR (h)
January	510	150
February	680	32
March	312	23
April	540	40
May	618	57
June	514	42
July	613	40
August	223	38
September	146	-
October	233	21
November	424	5
December	620	26
Total	444	5433

- exponential service time,
- either the M/M/1 queue.

5.1. Graphical Representations of the Markov Chains of the Production System

From the previous matrix, we deduce the graph of the availability states, by Figure 6.

It is assumed that the production system is reliable if and only if the probabilities of repairs resulting from transitions from state (E_2) are zero (as long as we opt instead for palliative maintenance in this state).

Hence the reliability graph in Figure 7.

5.2. Parameters of Reliability and Maintainability of the Production System

The processing of data from the history of operating hours makes it possible to calculate the mean time between failure *(MTBF)*, the mean time to repair *(MTTR)*, the failure rate *(λ)* and the repair rate *(μ).*

$$MTBF = \frac{\sum TBF}{N} = 452.75\text{ h} \rightarrow \lambda = \frac{1}{MTBF} = 0.00221\text{ h}^{-1}$$

$$MTTR = \frac{\sum TTR}{N} = 37\text{ h} \rightarrow \mu = \frac{1}{MTTR} = 0.02702\text{ h}^{-1}$$

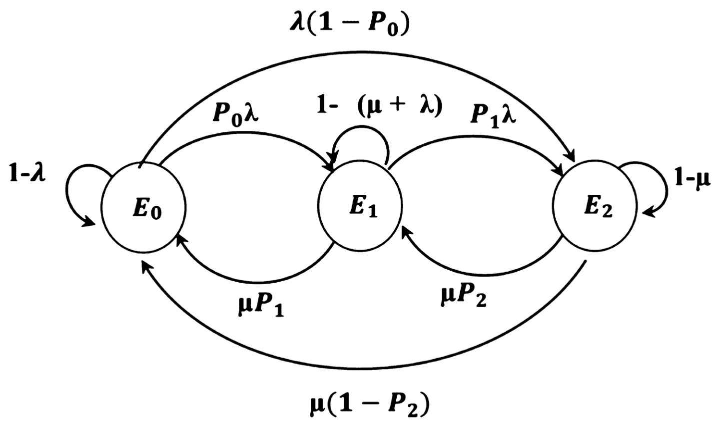

Figure 6. Production system availability status graph.

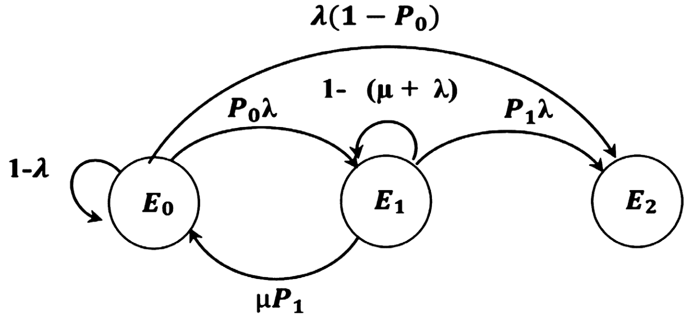

Figure 7. Production system reliability status graph.

5.3. Analysis of Production Chain Queue Networks

5.3.1. *U* Server Utilization Rate

By definition, the utilization rate is the probability that the server in the queue is busy.

From relation (5), $U = \rho = \frac{\lambda}{\mu} = 0.08179$.

5.3.2. Average Number of Clients L

From relation (6), $L = \frac{\rho}{1-\rho} = 0.08907$.

From relation 10, we deduce the average number of customers in the queue, *i.e.* $L_q = \frac{\rho^2}{1-\rho} = 0.00728$.

5.3.3. Average Residence Time *T*

This parameter is obtained using Little's law of relation (7).

$$T = \frac{1}{\mu(1-\rho)} = 40.30627$$

From relation (9), we deduce the average time spent in the queue, namely:

$$T_q = \frac{\rho}{\mu(1-\rho)} = 3.29664$$

5.3.4. System State Equation

In steady state, we have:

$$\begin{cases} P_0\lambda = \mu P_1 \\ P_1\lambda = \mu P_2 \end{cases} \Rightarrow \begin{cases} P_1 = \frac{\lambda}{\mu}P_0 = \rho P_0 \\ P_2 = \frac{\lambda}{\mu}P_1 = \rho P_1 = \rho^2 P_0 \end{cases} \Rightarrow P_n = \rho^n P_0 \left(n \in N\right)$$

We find the same general term of probability as that found in the literature [9].

Now, $P_0 = 1-\rho = 0.91821$, so $P_1 = 0.07510$ and $P_2 = 0.00614$.

We can see that $P_0 + P_1 + P_2 = 1$, so the production system can admit 0, 1 or 2 customers.

5.4. Production Availability Calculation

We have:

$$P = \begin{pmatrix} 1-\lambda & P_0\lambda & \lambda(1-P_0) \\ P_1\mu & 1-(\mu+\lambda) & P_1\lambda \\ \mu(1-P_2) & \mu P_2 & 1-\mu \end{pmatrix} = \begin{pmatrix} 0.99779 & 0.00202 & 0.00018 \\ 0.00202 & 0.97077 & 0.00016 \\ 0.02685 & 0.00016 & 0.97298 \end{pmatrix}$$

5.4.1. After Two Years of Operation

We set the following initial conditions: $(\pi_0, \pi_1, \pi_2)^{(1)} = (1,0,0)$.

The probability of the system in its states $((E_0),(E_1),(E_2))$ after two years of operation is given as follows:

$$\left(\pi_0,\pi_1,\pi_2\right)^{(2)}=(1,0,0)\begin{pmatrix}0.99779 & 0.00202 & 0.00018\\ 0.00202 & 0.97077 & 0.00016\\ 0.02685 & 0.00016 & 0.97298\end{pmatrix}$$
$$=(0.99779,0.00202,0.00018)$$

5.4.2. After Four Years of Operation

From the recurrence relation given by Equation (21), we deduce by conjecture that:

$$\left(\pi_0,\pi_1,\pi_2\right)^{(n)}=\left(\pi_0,\pi_1,\pi_2\right)^{(1)}P^{n-1}\left(n\in N\right)\tag{26}$$

So, after four years, the probability of the system in its states is given as follows:

$$\left(\pi_0,\pi_1,\pi_2\right)^{(4)}=\left(\pi_0,\pi_1,\pi_2\right)^{(1)}P^{3}$$
$$=(1,0,0)\begin{pmatrix}0.99779 & 0.00202 & 0.00018\\ 0.00202 & 0.97077 & 0.00016\\ 0.02685 & 0.00016 & 0.97298\end{pmatrix}^3$$
$$=(1,0,0)\begin{pmatrix}0.99341 & 0.00587 & 0.00052\\ 0.00588 & 0.91486 & 0.00045\\ 0.07821 & 0.00061 & 0.92112\end{pmatrix}$$
$$=(0.99340,0.00587,0.00052)$$

5.4.3. After Ten Years of Operation

$$\left(\pi_0,\pi_1,\pi_2\right)^{(10)}=\left(\pi_0,\pi_1,\pi_2\right)^{(1)}P^{9}$$
$$=(1,0,0)\begin{pmatrix}0.99779 & 0.00202 & 0.00018\\ 0.00202 & 0.97077 & 0.00016\\ 0.02685 & 0.00016 & 0.97298\end{pmatrix}^9$$
$$=(1,0,0)\begin{pmatrix}0.98048 & 0.00160 & 0.00143\\ 0.01616 & 0.76571 & 0.00114\\ 0.21512 & 0.00167 & 0.78165\end{pmatrix}$$
$$=(0.98048,0.00160,0.00143)$$
$$=(0.99340,0.00587,0.00052)$$

5.4.4. When the Production System Returns to Steady State

When $n\to+\infty$, Equation (26) becomes:

$$\left(\pi_0,\pi_1,\pi_2\right)^{+\infty}=\left(\pi_0,\pi_1,\pi_2\right)^{(1)}P^{+\infty}$$

However, according to the expression (23),

$$P^{\infty}=\begin{pmatrix}P_0 & P_1 & P_2\\ P_0 & P_1 & P_2\\ P_0 & P_1 & P_2\end{pmatrix}=\begin{pmatrix}0.91821 & 0.07510 & 0.00614\\ 0.91821 & 0.07510 & 0.00614\\ 0.91821 & 0.07510 & 0.00614\end{pmatrix}$$

$$\left(\pi_0,\pi_1,\pi_2\right)^{+\infty}=(1,0,0)\begin{pmatrix}0.91821 & 0.07510 & 0.00614\\ 0.91821 & 0.07510 & 0.00614\\ 0.91821 & 0.07510 & 0.00614\end{pmatrix}$$
$$=(0.91821,0.07510,0.00614)$$

We deduce that the production system reaches the operating limit (maximum number of years of operation) with an availability of 91.82%, when it is in state (E_0); with an availability of 7.51%, when it is in state (E_1) and with an availability of 0%, when it is in state (E_2).

On the other hand, we also note that during the life cycle of the production system, availability decreases in state (E_0). On the other hand, when it is in state (E_2), availability is zero, which I translate by the fact of its inactivity in this state.

5.5. Calculation of the Reliability of the Production System

According to the equations of state of the system, in a steady state, we have:

$$P_0 + P_1 + P_2 = 1 \Rightarrow P_0 + \rho P_0 + \rho^2 P_0 = 1$$

$$\Rightarrow P_0 = \frac{1}{1+\rho+\rho^2} = 0.91871; \quad P_1 = 0.07514 \quad \text{and} \quad P_2 = 0.00614.$$

In this case, the new matrix of system state transitions, in agreement with Figure 7 is given by:

$$P = \begin{pmatrix} 1-\lambda & P_0\lambda & \lambda(1-P_0) \\ P_1\mu & 1-(\mu+\lambda) & P_1\lambda \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0.99779 & 0.00203 & 0.00017 \\ 0.00203 & 0.97077 & 0.00016 \\ 0 & 0 & 0 \end{pmatrix}$$

5.5.1. After Two Years of Operation

We set the following initial conditions:

$$(\pi_0, \pi_1, \pi_2)^{(1)} = (1, 0, 0)$$

The probability of the system in its states after two years of operation is given as follows:

$$(\pi_0, \pi_1, \pi_2)^{(2)} = (1,0,0) \begin{pmatrix} 0.99779 & 0.00203 & 0.00017 \\ 0.00203 & 0.97077 & 0.00016 \\ 0 & 0 & 0 \end{pmatrix}$$
$$= (0.99779, 0.00203, 0.00017)$$

5.5.2. After Four Years of Operation

From relation 26, we deduce after four years that the probability of the system in its states is given as follows:

$$(\pi_0, \pi_1, \pi_2)^{(4)} = (\pi_0, \pi_1, \pi_2)^{(1)} P^3$$
$$= (1,0,0) \begin{pmatrix} 0.99779 & 0.00203 & 0.00017 \\ 0.00203 & 0.97077 & 0.00016 \\ 0 & 0 & 0 \end{pmatrix}^3$$
$$= (1,0,0) \begin{pmatrix} 0.99339 & 0.00590 & 0.00016 \\ 0.00590 & 0.91486 & 0.00015 \\ 0 & 0 & 0 \end{pmatrix}$$
$$= (0.99339, 0.00590, 0.00016)$$

5.5.3. After Ten Years of Operation

$$(\pi_0,\pi_1,\pi_2)^{(10)} = (\pi_0,\pi_1,\pi_2)^{(1)} P^9$$
$$= (1,0,0)\begin{pmatrix} 0.99779 & 0.00203 & 0.00017 \\ 0.00203 & 0.97077 & 0.00016 \\ 0 & 0 & 0 \end{pmatrix}^9$$
$$= (1,0,0)\begin{pmatrix} 0.98040 & 0.01612 & 0.00015 \\ 0.01612 & 0.76580 & 0.00012 \\ 0 & 0 & 0 \end{pmatrix}$$
$$= (0.98040, 0.01612, 0.00015)$$

5.5.4. When the Production System Returns to Steady State

We see that the last row of the matrix P is zero, therefore the matrix P is no longer stochastic, therefore the relation (23) is no longer applicable in a steady state.

On the other hand, in a steady state, we have: $\pi \times P = \pi$, because $\pi_{n+1} = \pi_n$, with $\pi = (\pi_0,\pi_1,\pi_2)$ and $\sum_{i=1}^{3} \pi_i = 1$.

So: $$\begin{cases} 0.00203\pi_0 + 0.97077\pi_1 = \pi_1 \\ 0.00017\pi_0 + 0.00016\pi_1 = \pi_2 \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases} \Rightarrow (\pi_0,\pi_1,\pi_2) = (0.93490, 0.06492, 0.00016).$$

The production system reaches the operating limit with a reliability of 93.49%, when it is in state (E_0); with 6.49% reliability, when in the (E_1) state and with 0% reliability, when in the (E_2) state.

5.6. Proposals for Improving the Reliability and Availability of the Production System

To have better availability and adequate reliability, the following operations must be carried out:

- Set up a group of executives and technicians specializing in the field of production chain operation;
- Develop a preventive maintenance program corresponding to the needs of the production line;
- Work in close collaboration with all other departments of the company;
- Ensure rigorous control of maintenance operations.

6. Conclusion

Having reached the end of the writing of this paper, whose general objective was to evaluate the availability and the reliability of the networks of Markov queues, we showed how to take advantage of the knowledge on the theory of the chains of Markov, in order to help in the analysis of waiting phenomena. We have presented an application on a production line where the arrival process is Poissonian. This approach allowed us to show the influence of availability on reliability. Based on the results obtained, proposals for improving maintenance were made.

Acknowledgements

The authors would like to thank the anonymous reviewers for their time and effort. Their constructive comments and helpful suggestions helped us to clarify the main paper's research contributions and improve its quality.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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