

Optimization of the Physical Aperture of the Parabolic Reflector Antenna

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Abstract

In fields like astronomy and radar technology, high-gain antennas are required for long-distance communication. Due to its relatively large gain, the use of parabolic antennas has become very popular over time, because they can easily achieve gains of above 30 dB at microwave and higher frequencies. Today, most systems' success depends on how well the antennas perform. These antennas are available in different types and sizes. Each antenna's effective area usually has less than the actual physical area of the antenna surface. This means that the unused area of the antenna is massive, and a waste. The aim of the research is to show that the actual physical aperture of a parabolic antenna can be reduced as much as possible to equal the effective area, as given by the antenna formula, thereby saving manufacturing costs, improve the aesthetics. In other words, the focus of this work is to experimentally show that reflector antenna can be made of smaller sizes but better performance. Measurements were taken from different positions from a parabolic antenna, the signal level measured and compared with signal levels for optimal performance.

Keywords

Parabolic, Antenna, Reception, Manufacturing, Cost

1. Introduction

Over the years, parabolic antenna usage has become very popular because of its extreme large gain at high frequencies, the gain limited by the physical size of the parabolic reflector. Today, the success of many systems depends on how functional the antennas used for them are. These antennas come in various types and sizes: wired antennas, aperture antennas, array antennas, reflector antennas, lens antennas, micro-strip antennas, etc. Antennas can be classified in terms of their

[1] [2] functional operations and uses [1] [2]. A simple reflector antenna consists of two components: a reflecting surface and a much smaller feed antenna at the reflector's focal point while a more complex reflector antenna involves a secondary reflector (a sub-reflector) at the focal point, which is illuminated by a primary feed [3] [4]. There are many factors that can affect the quantity and quality of signal of signal by a reflector antenna, as seen in different literature. The surface accuracy and electrical performance of the antenna in space are not considered in the work, as these constraints are assumed to be the same, and cancels out; according to Kang, J., Wang, W. *et al.* [2] [5], the thermal deformation of the reflector caused by temperature increase, has effect on the surface accuracy and electrical performance, it impairs performance.

Theory of the Parabolic Reflector Antenna

The reflector or antenna has two purposes, first they collect power in terms of electrical signals (scintillations) and second, they provide directionality, for propagation of electromagnetic signal [6]. Reflector antennas operate on the principles known long ago from the theory of geometrical optics (GO) [7] [8]. **Figure 1** shows an abstraction of the coordinate system of a parabolic antenna while **Figure 2** shows the cross section.

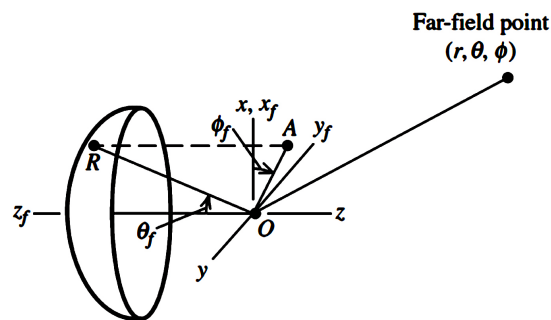


Figure 1. Shows the coordinate system of a parabolic.

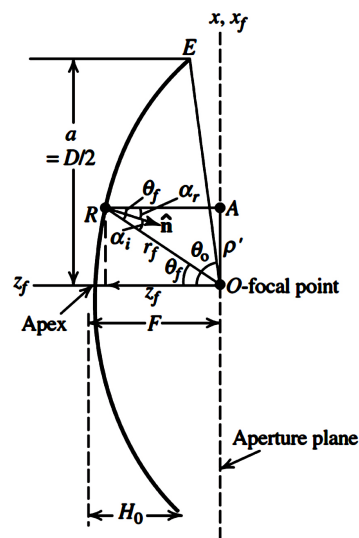


Figure 2. Cross-section of the reflector of parabolic antenna *xz*-plane.

The parabolic surface of the antenna in **Figure 1** is described by:

$$\rho'^2 = 4F(F - z_f), \rho' \ll a \tag{1}$$

Here, ρ' is the distance from a point A to the focal point O , where A is the projection of the point R on the reflector surface onto the axis-orthogonal plane (the aperture plane) at the focal point [9] [10]. For a given displacement: ρ' from the axis of the reflector, the point R on the reflector surface is a distance r_f away from the focal point O .

The position of R can be modeled by a pair of coordinates in either rectangular (ρ', z_f) or the polar (r_f, θ_f) . The distance r_f and the focal length F is related by:

$$r_f = \frac{2F}{1 + \cos \theta_f} = \frac{F}{\cos^2\left(\frac{\theta_f}{2}\right)} \tag{2}$$

The displacement P' from the axis of the reflector is given by:

$$\rho' = r_f \sin \theta_f = \frac{2F \sin \theta_f}{1 + \cos \theta_f} = 2F \tan \frac{\theta_f}{2} \tag{3}$$

A common parameter used in specifying a parabola is the F/D ratio. When F/D approaches infinity, the reflector becomes flat. Commonly used paraboloidal shapes are shown below in **Figure 3**. When $F/D = 0.25$, the focal point lies in the plane passing through the reflector's rim [10] [11].

Though the paraboloidal reflector is asymmetric, it is rotational symmetric.

To determine the angel from the feed-point to the reflector length, we use the relation.

$$\theta_0 = 2 \arctan \left[\frac{1}{4(F/D)} \right] \tag{4}$$

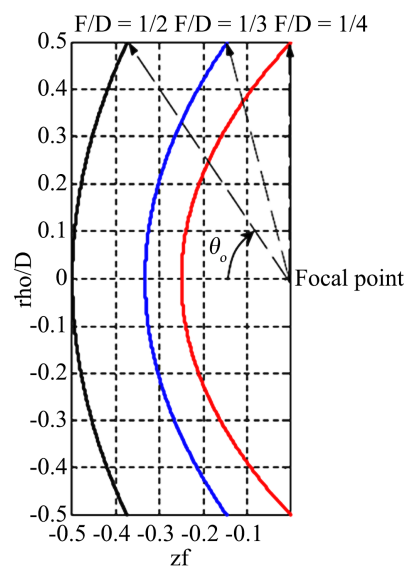


Figure 3. F/D ratio feed point angle.

For a reflector, the main design concern is matching the antenna feed pattern to the reflector, with a feed pattern of about -10 dB level in the direction of the rim, *i.e.* $F_f(\theta = \theta_0) = -10$ dB.

Given the height of the antenna H_0 and the diameter D , the focal distance F is given by

$$F = \frac{D^2}{16H_0} \quad (5)$$

To solve Equation (5), we substitute the following values into Equation (1): $\rho' = D/2$ & $z_f = F - H_0$.

Iterating different values for F/D into Equation (5), we come to the point where,

$$F/D = 1/4. \text{ Hence } H_0 = 1/4.$$

$$\Rightarrow H_0 = F \text{ i.e. the focal point is on the reflector's rim.}$$

In analyzing the electromagnetic distribution over the aperture, two methods are used [12] [13]:

- 1) Current distribution method, which makes use of Physical Optic (PO) approximation.
- 2) Aperture distribution method, which uses Geometric Optic ray tracing.

The current distribution method assumes that the incident field from the feed is known, and that it excites surface currents on the parabolic reflector's surface as

$$J_s = 2n \times H^i \quad (6)$$

By integrating the surface density over the surface of the dish, the far-field is obtained; it is assumed that the material used in the antenna is a perfectly conducting material. Using the principle of reciprocity of light, the incident rays from the feed horn is taken to be locally plan wave at the far-field [14] [15]. The field distribution at the aperture of the reflector antenna is necessary in order to calculate the far-field pattern and directivity, as the power density of the rays from the field decreases as the inverse-squared of the focal length, F , while the non-uniform aperture amplitude distribution varies as the inverse of the focal length (FL).

2. Research Methodology

The degree of functionality of a system depends on the different components, devices, methodologies and procedures that were carried out during the process of execution.

A schematic diagram of the various processes involved in the research is shown in **Figure 4**.

When the LNB collects radio waves from the dish, it sends out intermediate frequency (IF) to the decoder, the IF is then sent to the two-way splitter [16] [17]. A side of the splitter is connected to the television decoder and the other part is connected to the spectrum analyser. Measurements of the received signal

strength and behaviour of the system when it is functioning optimally is taken and recorded by the spectrum analyser [18] [19] (when it is functioning optimally, the reception on the Television is very clear).

The measurement of the spectrum analyser is also varied by moving the strut randomly until a clear reception is seen on the television; measurement for which the DSTV service becomes very clear is taken as the threshold value. The range of values for optimal service are noted, collected, and tabulated.

The surface area of the reflector antenna is reduced in stages by covering the dish with a serrated foam which disallows the reflection of signals to the feed, to control the electromagnetic energy received at the feed. This is achieved by moving the strut slowly, then recording the values of the received energy that are close to the threshold value, needed to get a good reception on the television. Different shapes of antenna area were used, as shown in **Figure 5**.

The results obtained are used to develop mathematical models.

$$\text{Antenna gain: } G = \frac{(P/S)_{\text{antenna}}}{(P/S)_{\text{isotropic}}}$$

where $(P/S)_{\text{antenna}}$ is the ratio of the intensity radiated (*i.e.* power per unit surface) by the antenna and $(P/S)_{\text{isotropic}}$ is the ratio of the intensity radiated (power per unit surface) by an isotropic antenna.

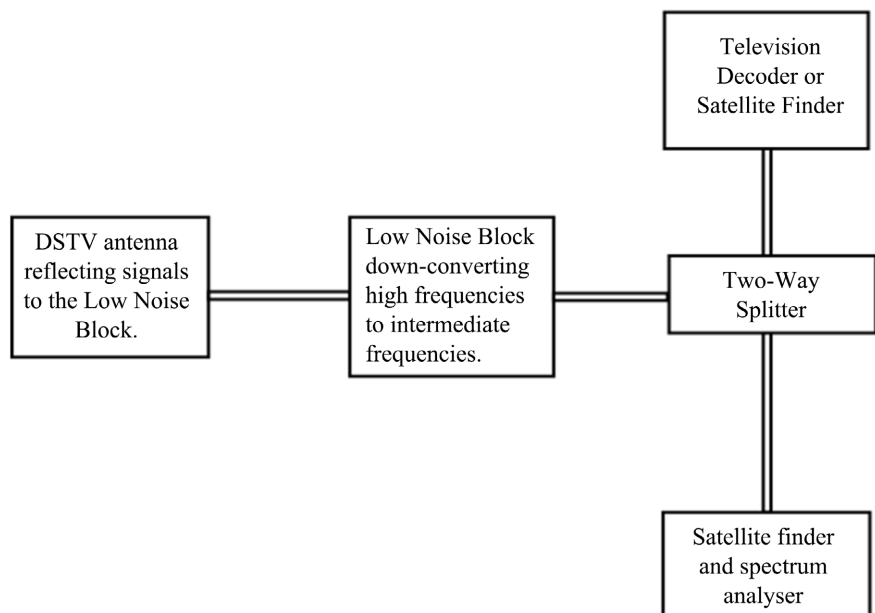


Figure 4. A schematic diagram of the steps involved in carrying out the research.

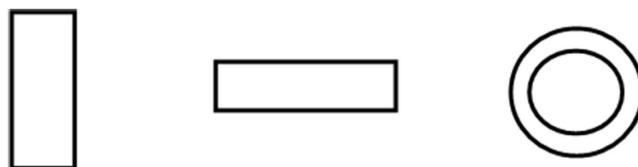


Figure 5. The different shapes the DSTV dish is reduced to.

Decibel Conversions:

- 1) 0 dB = 30 dBm = 1 Watt.
- 2) -30 dB = 0 dBm = 0.001 Watt.
- 3) 10 dB = 10 dBi = 7.85 dBd.

This work focuses on the parabolic reflector antenna with the DSTV dish used as the case study. From the principle of reciprocity, when a series of radio signals fall on a reflector whose shape is that of a parabola, the signals are reflected to the focal point of the reflector *i.e.* these signals become concentrated on a point and they all meet at this single point. On the other hand, if another source of energy (point source) is placed at the focal point (*i.e.* focus) of the parabolic reflector, parallel rays of signals are reflected. These reflected parallel rays are said to be “collimated” [20] [21]. The point source of the parabolic reflector antenna *i.e.* the receiver, is placed at the focal point or the focus of the parabolic reflector which is in front of the parabola. This arrangement is referred to as “front fed” as shown in **Figure 6**.

The parabolic reflector antenna functions similarly to a searchlight or flashlight reflector which directs the signals in a narrow beam or receives signals from one particular direction only.

3. Results and Analysis

Measurement Result

Summary Results

From the tables (**Tables 1-7**), it is observed that:

- 1) There’s a slightly linear relationship between the signal quality and the surface area of the parabolic antenna, *i.e.* the signal strength increases with increase in aperture surface area.
- 2) Kang, J., Wang, W., Zhang, S., Lian, S. and Bao, H. (2021) Thermal Distortion Compensation for Improving Electrical Performance of Reflector Antennas. *Open Journal of Applied Sciences*, 11, 523-540. doi: 10.4236/ojapps.2021.104037.
- 3) In order for television viewing to be made possible, the signal strength must be between 52% and 73%.

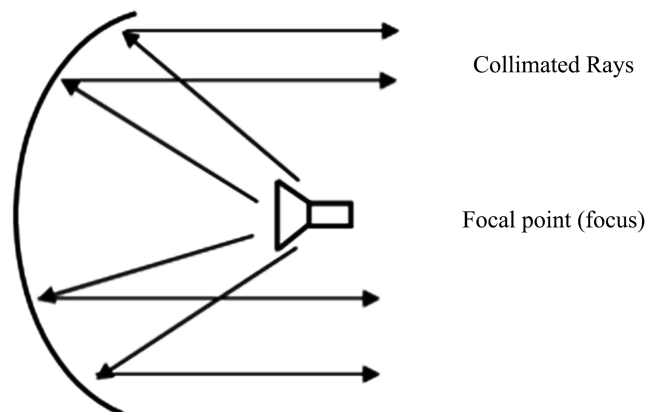


Figure 6. Working arrangement of the parabolic reflector antenna (front-fed) [22].

Table 1. When the right-hand side of the aperture was covered with the anechoic foam.

| HEIGHT (cm) | DIAMETER (cm) | AREA (cm ²) | STRENGTH (%) | SCALED AREA (cm ²) | NORMALIZED STRENGTH |
|-------------|---------------|-------------------------|--------------|--------------------------------|---------------------|
| 90 | 80.50 | 16156.47 | 73.00 | 4.03 | 0.73 |
| 90 | 79.00 | 15822.32 | 72.00 | 3.95 | 0.72 |
| 90 | 77.50 | 15489.9 | 71.00 | 3.87 | 0.71 |
| 90 | 76.00 | 15159.2 | 71.00 | 3.78 | 0.71 |
| 90 | 74.50 | 14830.19 | 68.00 | 3.70 | 0.68 |
| 90 | 73.00 | 14502.85 | 66.00 | 3.62 | 0.66 |
| 90 | 71.50 | 14177.14 | 67.00 | 3.54 | 0.67 |
| 90 | 70.00 | 13853.06 | 64.00 | 3.46 | 0.64 |
| 90 | 68.50 | 13530.56 | 62.00 | 3.38 | 0.62 |
| 90 | 67.0 | 13209.63 | 60.00 | 3.30 | 0.60 |

Table 2. When the left-hand side of the aperture was covered with the anechoic foam.

| HEIGHT (cm) | DIAMETER (cm) | AREA (cm ²) | STRENGTH (%) | SCALED AREA (cm ²) | NORMALIZED STRENGTH |
|-------------|---------------|-------------------------|--------------|--------------------------------|---------------------|
| 90.00 | 80.50 | 16156.47 | 72.00 | 4.03 | 0.72 |
| 90.00 | 79.50 | 15822.32 | 71.00 | 3.95 | 0.71 |
| 90.00 | 77.50 | 15489.9 | 70.00 | 3.87 | 0.70 |
| 90.00 | 76.00 | 15159.2 | 70.00 | 3.78 | 0.70 |
| 90.00 | 74.50 | 14830.19 | 69.00 | 3.70 | 0.69 |
| 90.00 | 73.00 | 14502.85 | 67.00 | 3.62 | 0.67 |
| 90.00 | 71.50 | 14177.14 | 66.00 | 3.54 | 0.66 |
| 90.00 | 70.00 | 13853.06 | 65.00 | 3.46 | 0.65 |
| 90.00 | 68.50 | 13530.56 | 64.00 | 3.38 | 0.64 |
| 90.00 | 67.00 | 13209.63 | 60.00 | 3.30 | 0.60 |

Table 3. When both the left and right sides of the aperture were covered with the anechoic foam.

| HEIGHT (cm) | DIAMETER (cm) | AREA (cm ²) | STRENGTH (%) | SCALED AREA (cm ²) | NORMALIZED STRENGTH |
|-------------|---------------|-------------------------|--------------|--------------------------------|---------------------|
| 90.00 | 79.00 | 15822.32 | 62.00 | 3.95 | 0.62 |
| 90.00 | 76.00 | 15159.2 | 62.00 | 3.78 | 0.62 |
| 90.00 | 73.00 | 14502.85 | 62.00 | 3.62 | 0.62 |
| 90.00 | 70.00 | 13853.06 | 61.00 | 3.46 | 0.61 |
| 90.00 | 67.00 | 13209.63 | 61.00 | 3.30 | 0.61 |
| 90.00 | 64.00 | 12572.37 | 61.00 | 3.14 | 0.61 |
| 90.00 | 61.00 | 11941.05 | 61.00 | 2.98 | 0.61 |
| 90.00 | 58.00 | 11315.47 | 60.00 | 2.82 | 0.60 |
| 90.00 | 55.00 | 10695.4 | 60.00 | 2.67 | 0.60 |
| 90.00 | 52.00 | 10080.61 | 60.00 | 2.52 | 0.60 |

Table 4. When the upper part was covered with the anechoic foam.

| HEIGHT (cm) | DIAMETER (cm) | AREA (cm ²) | STRENGTH (%) | SCALED AREA (cm ²) | NORMALIZED STRENGTH |
|-------------|---------------|-------------------------|--------------|--------------------------------|---------------------|
| 88.50 | 82.00 | 16249.71 | 72.00 | 4.06 | 0.72 |
| 87.00 | 82.00 | 16007.47 | 72.00 | 4.00 | 0.72 |
| 85.50 | 82.00 | 15765.68 | 70.00 | 3.94 | 0.70 |
| 84.00 | 82.00 | 15524.36 | 70.00 | 3.88 | 0.70 |
| 82.50 | 82.00 | 15283.54 | 68.00 | 3.82 | 0.68 |
| 81.00 | 82.00 | 15043.23 | 64.00 | 3.76 | 0.64 |
| 79.50 | 82.00 | 14803.46 | 64.00 | 3.70 | 0.64 |
| 78.00 | 82.00 | 14564.25 | 61.00 | 3.64 | 0.61 |
| 76.50 | 82.00 | 14325.64 | 61.00 | 3.58 | 0.61 |
| 75.00 | 82.00 | 14087.64 | 60.00 | 3.52 | 0.60 |

Table 5. When the lower part of the aperture is covered with the anechoic foam.

| HEIGHT (cm) | DIAMETER (cm) | AREA (cm ²) | STRENGTH (%) | SCALED AREA (cm ²) | NORMALIZED STRENGTH |
|-------------|---------------|-------------------------|--------------|--------------------------------|---------------------|
| 87.00 | 82.00 | 16007.47 | 72.00 | 4.00 | 0.72 |
| 84.00 | 82.00 | 15524.36 | 72.00 | 3.88 | 0.72 |
| 81.00 | 82.00 | 15043.23 | 72.00 | 3.76 | 0.72 |
| 78.00 | 82.00 | 14564.25 | 71.00 | 3.64 | 0.71 |
| 75.00 | 82.00 | 14087.64 | 71.00 | 3.52 | 0.71 |
| 72.00 | 82.00 | 13613.63 | 70.00 | 3.40 | 0.70 |
| 69.00 | 82.00 | 13142.48 | 68.00 | 3.28 | 0.68 |
| 66.00 | 82.00 | 12674.5 | 65.00 | 3.16 | 0.65 |
| 63.00 | 82.00 | 12210.04 | 64.00 | 3.05 | 0.64 |
| 60.00 | 82.00 | 11749.48 | 61.00 | 2.93 | 0.61 |

Table 6. When both the upper and lower parts of the dish were covered with the anechoic foam.

| HEIGHT (cm) | DIAMETER (cm) | AREA (cm ²) | STRENGTH (%) | SCALED AREA (cm ²) | NORMALIZED STRENGTH |
|-------------|---------------|-------------------------|--------------|--------------------------------|---------------------|
| 87.00 | 82.00 | 16007.47 | 63.00 | 4.00 | 0.63 |
| 84.00 | 82.00 | 15524.36 | 63.00 | 3.88 | 0.63 |
| 81.00 | 82.00 | 15043.23 | 62.00 | 3.76 | 0.62 |
| 78.00 | 82.00 | 14564.25 | 62.00 | 3.64 | 0.62 |
| 75.00 | 82.00 | 14087.64 | 61.00 | 3.52 | 0.61 |
| 72.00 | 82.00 | 13613.63 | 61.00 | 3.40 | 0.61 |
| 69.00 | 82.00 | 13142.48 | 61.00 | 3.28 | 0.61 |
| 66.00 | 82.00 | 12674.5 | 60.00 | 3.16 | 0.60 |
| 63.00 | 82.00 | 12210.04 | 60.00 | 3.05 | 0.60 |
| 60.00 | 82.00 | 11749.48 | 60.00 | 2.93 | 0.60 |

Table 7. When all four edges are covered with the anechoic foam.

| HEIGHT (cm) | DIAMETER (cm) | AREA (cm ²) | STRENGTH (%) | SCALED AREA (cm ²) | NORMALIZED STRENGTH |
|----------------|------------------|----------------------------|-----------------|-----------------------------------|------------------------|
| 88.00 | 80.00 | 15728.51 | 59.00 | 3.93 | 0.59 |
| 86.00 | 78.00 | 14982.73 | 58.00 | 3.74 | 0.58 |
| 84.00 | 76.00 | 14255.04 | 58.00 | 3.56 | 0.58 |
| 82.00 | 74.00 | 13545.44 | 58.00 | 3.38 | 0.58 |
| 80.00 | 72.00 | 12853.92 | 57.00 | 3.21 | 0.57 |
| 78.00 | 70.00 | 12180.49 | 57.00 | 3.04 | 0.57 |
| 76.00 | 68.00 | 11525.15 | 56.00 | 2.88 | 0.56 |
| 74.00 | 66.00 | 10887.9 | 56.00 | 2.72 | 0.56 |
| 72.00 | 64.00 | 10268.73 | 54.00 | 2.56 | 0.54 |
| 70.00 | 62.00 | 9667.65 | 52.00 | 2.41 | 0.52 |

4) From the tables, it was also observed that the minimum strength of the received signal must be received is about 52%. For any values below this, the decoder would be switched off.

5) From the tables, it was observed that as long as the aperture area was more than a certain percentage of the full aperture area, the decoder would be receiving sufficient power to enable clear TV viewing.

6) From the tables, it was observed that the range of values of the aperture area for which a sufficient signal was received falls between about 7800 - 16,160 cm² after a certain area of the antenna had been covered.

7) The surface area of the full-sized parabolic antenna was found to be about 16,492 cm². The minimum surface area of the antenna at which a signal was received was 9667 cm²; by expressing this as a percentage of the full aperture area, it was found to be about 58% of this value. This means that the size of the dish can be reduced by as much as 40% of the full size yet it will still perform its needed function of making television viewing possible.

8) Theoretically, the effective aperture of an antenna is about 70% of the total aperture. From the readings taken, it was shown that the true effective aperture of the parabolic reflector antenna is much less than this, it is about 58% of the total aperture.

9) The aim of this research work was to see if the actual area of the antenna can be made to be as close as possible to the effective area without affecting the signal reception. It has therefore been shown that the aperture size can definitely be made closer to the effective aperture. It can even be made smaller than the theoretical effective aperture without deterioration in the antenna's function.

4. Conclusion

Through the research, the manufacturing cost and purchasing cost of antennas

are reduced, the parabolic reflector antenna becomes very small and therefore easy to move about and various designs for the antenna are developed.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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