Open Journal of Applied Sciences, 2023, 13, 2273-2281
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

Synchronous Control of Complex Networks with Fuzzy Connections

Wei Chen, Yuanguang Zheng*

School of Mathematics and Information Sciences, Nanchang Hangkong University, Nanchang, China
Email: *zhengyg2718@163.com

How to cite this paper: Chen, W. and Zheng, Y.G. (2023) Synchronous Control of Complex Networks with Fuzzy Connections. *Open Journal of Applied Sciences*, **13**, 2273-2281.
https://doi.org/10.4236/ojapps.2023.1312177

Received: September 27, 2023
Accepted: December 5, 2023
Published: December 8, 2023

Abstract

This article is based on the T-S fuzzy control theory and investigates the synchronization control problem of complex networks with fuzzy connections. Firstly, the main stability equation of a complex network system is obtained, which can determine the stability of the synchronous manifold. Secondly, the main stable system is fuzzified, and based on fuzzy control theory, the control design of the fuzzified main stable system is carried out to obtain a coupling matrix that enables the complex network to achieve complete synchronization. The numerical analysis results indicate that the control method proposed in this paper can effectively achieve synchronization control of complex networks, while also controlling the transition time for the network to achieve synchronization.

Keywords

T-S Fuzzy Control, Synchronization, Complex Network

1. Introduction

Since Euler proposed and solved the "Seven Bridge Problem", complex networks have gradually entered the public's view. Afterwards, mathematicians Erdos and Renyi established random graph theory, which opened up systematic research on complex network theory. From 1998 to 1999, Watts and Strogatz, Barabási and Albert explained the small world [1] and scale-free characteristics of complex networks [2], respectively. Afterwards, more and more scholars began to pay attention to complex networks. In nature, complex networks can be seen everywhere, such as the World Wide Web [3], the Internet [4], the food web [5], the gene regulatory network [6], the cellular and metabolic network [7], the power grid [8], the road traffic network [9], various social relationship networks [10], and the climate network [11]. The study of complex networks has become a hot topic in today's society, and their connection with our lives is becoming increasingly close.

In complex networks, synchronization is a common dynamic behavior, which refers to the dynamic behavior of nodes in the network. Under different initial conditions, through the interaction between nodes, two or more nodes evolve independently over time, gradually approaching and ultimately reaching a relevant state. In the real world, synchronization phenomena can be seen everywhere [12], such as the synchronous flash of fireflies; the frogs in the pond will sing in unison. Due to the significant role and potential application prospects of synchronization in laser systems, superconducting materials, communication systems, and other fields, complex network synchronization has become a research hotspot in physics, biology, mathematics, and other fields. In the study of synchronization, people have found various forms of synchronization, including phase synchronization, cluster synchronization, amplitude synchronization, and so on. Among them, complete synchronization [13] refers to two or more identical dynamic systems, which, through the interaction between systems or the action of external forces, gradually approach the states of each system under different initial conditions, and ultimately reach an identical state.

The synchronization control of complex networks has always been a research hotspot and has attracted the attention of scholars. Synchronous control refers to the inability of a network to achieve synchronization through edge connections or coupling itself, requiring the addition of appropriate controllers and the use of system control theory and methods to promote network synchronization. So far, many scholars have devoted themselves to studying the synchronization control problem of complex networks. There are various control methods that can achieve synchronization of complex networks, such as traction control, pulse control, adaptive control, intermittent control, sampling control, etc. In order to fully utilize the dynamic characteristics of network synchronization in engineering practice, it is crucial to study the synchronization control of complex networks. So far, the development of efficient and practical control methods for achieving synchronous development of complex networks is still a topic worth studying.

With the rapid development of science and technology, the systems involved in practical engineering are becoming increasingly complex, making it difficult to establish accurate mathematical models, which makes it impossible to use traditional control methods for system control design. In this context, fuzzy control theory [14] has gradually developed. Fuzzy control is a control method based on fuzzy logic, which can handle systems that are difficult to accurately describe using traditional control methods, and then determine control rules through fuzzy reasoning, ultimately achieving control of the system. In 1973, Professor Zadeh LA [15] proposed the idea of transforming the language description of logical rules into relevant control rates, making it possible to make practical and human thinking oriented treatments for complex systems, laying the theoretical foundation for the early introduction of fuzzy controllers. In 1985, Takagi and Sugeno proposed a new fuzzy inference model called the T-S fuzzy model [16]. Among many fuzzy logic control methods, the T-S fuzzy model is widely adopted as a tool for the design and analysis of fuzzy control systems. The main idea of T-S fuzzy model is to construct a series of linear equations to represent each subsystem, and then connect these subsystems into a global model through membership functions. Therefore, this model can utilize existing linear system theory to analyze and synthesize a highly nonlinear dynamic system.

At present, the research on synchronous control of complex networks is of great significance for solving practical problems. However, it is difficult to establish accurate mathematical models of complex networks, and theoretically feasible control methods applied to real systems may result in significant deviations. Fuzzy controllers have the advantages of simple structure, strong robustness, insensitivity to parameter changes, and strong anti-interference ability, making them suitable for application in complex network synchronization control. Therefore, the combination of fuzzy control and complex network control methods has become one of the important directions for complex network synchronization control. Based on the above analysis, this article establishes a model of a complex network with fuzzy connections to achieve complete synchronization of each node. Study its specific synchronization characteristics through examples. The rest of this article is organized as follows: In Section 2, a complex network synchronization model with fuzzy connections is constructed. In Section 3, through instance analysis, it is determined whether this method can achieve complete synchronization in complex networks. In the final section, the research work of this article is summarized and related issues are discussed.

2. A Complex Network Model with Fuzzy Connections

2.1. General Continuous Time Coupled Network Model

Consider a continuous-time dissipative coupled network system consisting of N identical nodes, where the equation of state for the i-th node is:

$$\dot{X}_i = F\left(X_i(t)\right) + \sum_{j=1}^{N} H\left(X_j(t) - X_i(t)\right) \tag{2-1}$$

where $X_i = \left(x_{i1}, x_{i2}, \cdots, x_{in}\right)^{\mathrm{T}}$ it is the state variable of the i-th node $\left(i = 1, 2, \cdots, N\right)$, $F(\cdot) = \left(f_1(\cdot), f_2(\cdot), \cdots, f_n(\cdot)\right)^{\mathrm{T}} \in R^n$ it is a nonlinear vector field of a single node system, $H \in R^{n\times n}$ is the internal coupling matrix between the state variables of each node. For different internal coupling matrices H, the network has different synchronization capabilities. The purpose of control design is to find an appropriate internal coupling matrix H, so that the network system can achieve complete synchronization while meeting the given synchronization performance.

The system (2-1) can also be expressed in the following form:

$$\dot{X} = F\left(X_i(t)\right) + \sum_{j=1}^{N} a_{ij} H X_j(t) \tag{2-2}$$

Among the $a_{ij} = 1(i \neq j)$, $a_{ii} = -N + 1(i, j = 1, 2, \cdots, n)$, and satisfy the dissipation condition $\sum_j a_{ij} = 0$, $A = (a_{ij}) \in R^{n \times n}$ is the external coupling matrix of the network system, with a single eigenvalue $\lambda_1 = 0$ and an $N - 1$ multiple eigenvalues $\lambda_2 = -N$.

When each node system on the network starts from different initial values and evolves over a period of time, with $X_1(t) = X_2(t) = \cdots = X_n(t) = X(t)$, the network system (2-1) reaches complete synchronization. Due to the dissipative coupling condition, the synchronous state $X(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}} \in R^n$ must be the solution of a single isolated node, satisfying the equation:

$$\dot{X}(t) = F(X(t)) \tag{2-3}$$

In order to determine the stability of the synchronous epidemic $X_1(t) = X_2(t) = \cdots = X_n(t) = X(t)$, the following main stability equation needs to be investigated [17]:

$$\dot{\eta}(t) = DF(X(t))\eta(t) + \lambda_2 H \eta(t) \tag{2-4}$$

Among them, $\eta(t)$ represents the deviation in the cross-sectional direction of synchronous transmission, and $DF(X(t))$ is the Jacobi matrix of $F(X(t))$. When all eigenvalues corresponding to the main stable equation have negative real parts, the synchronous manifold is stable.

2.2. Continuous Time Fuzzy Coupled Network Model

To obtain the fuzzy coupling matrix H that enables the network to achieve complete synchronization, the main stability equation is transformed into the following fuzzy model in the form of $DF(X(t))$:

$$R_i: \text{IF } x_1(t) \text{ is } M_{i1},\ x_2(t) \text{ is } M_{i2},\ \cdots,\ x_p(t) \text{ is } M_{ip}$$

$$\text{THEN: } \dot{\eta}(t) = A_i \eta(t) + \lambda_2 G_i \eta(t) \quad i = 1, 2, \cdots, n$$

R_i represents the *i*-th fuzzy rule, $M_{i1}, M_{i2}, \cdots, M_{ip}$ are fuzzy sets, r is the number of fuzzy rules.

By applying single point fuzzification, product reasoning, and center weighted anti fuzzification reasoning methods, a global fuzzy system model can be obtained:

$$\dot{X}(t) = \sum_{i=1}^{r} h_i (A_i + \lambda_2 G_i) \eta(t) \tag{2-5}$$

Since $h_i(X(t)) = \omega_i(X(t)) / \sum_{i=1}^{r} \omega_i(X(t))$, and $\sum_{i=1}^{r} h_i(X(t)) = 1$, $\omega_i(X(t)) = \prod_{j=1}^{n} \omega_{ij}(x)$, where $\omega_{ij}(x)$ is the degree of the membership of $\omega_i(X(t))$, then in fuzzy systems (2-5), $DF(X(t))$ is fuzzified to $\sum_{i=1}^{r} h_i A_i$, and the internal coupling matrix H is fuzzified to $\sum_{i=1}^{r} h_i G_i$, where G_i is the matrix to be solved.

2.3. Fuzzy Coupling Matrix for Achieving Complete Synchronization

In order to obtain the fuzzy coupling matrix *H*, the fuzzy control theory is applied to the main stability Equation (2-5), and the following results can be obtained.

Theorem 1 Assuming that the number of triggering rules all t is less than or equal to s, where $1 < s \le r$, search for $X > 0, Y \ge 0$ and $M_i\left(1 = 1, \cdots, r\right)$ that:

$$-XA_i - A_i X + M_i^{\mathrm{T}} B_i^{\mathrm{T}} + B_i M_i - \left(s-1\right)Y > 0 \tag{2-6}$$

$$2Y - XA_i^{\mathrm{T}} - A_i X - XA_j^{\mathrm{T}} - -A_j X + M_j^{\mathrm{T}} B_i^{\mathrm{T}} + B_j M_i \ge 0 \tag{2-7}$$

$$i < j \quad \text{s.t.} \quad h_i \cap h_j \ne 0$$

where $X = P^{-1}$, $M_i = G_i X$, $Y = XQX$.

According to the above conditions, the feedback gain matrix G_i is solved, so that the internal coupling matrix $H\left(X\left(t\right)\right)$ can be solved.

3. Example Analysis

3.1. Fully Coupled Rossler System with Fuzzy Connection

Considering the fully coupled Rossler network system, the dynamic equation of the i-th node is:

$$\begin{cases} \dot{x}_i\left(t\right) = -\omega y_i\left(t\right) - z_i\left(t\right) \\ \dot{y}_i\left(t\right) = \omega x_i\left(t\right) + a y_i\left(t\right) \\ \dot{z}_i\left(t\right) = b + z_i\left(t\right)\left(x_i\left(t\right) - c\right) \end{cases} \tag{3-1}$$

Among them, $X_i\left(t\right) = \left(x_i\left(t\right), y_i\left(t\right), z_i\left(t\right)\right)^{\mathrm{T}}$ is the state variable of the i-th node system. Fix the Rossler system parameters of each node in the coupled network at $\omega = 1, a = b = 2, c = 5.7$, where the dynamics of a single node are chaotic.

The Jacobi matrix of $F\left(X\left(t\right)\right)$ in synchronous state $X\left(t\right)$ is:

$$DF\left(X\left(t\right)\right) = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ z\left(t\right) & 0 & x\left(t\right) \end{pmatrix}$$

Let $x\left(t\right) \in \left[-10 \ \ 10\right]$, $z\left(t\right) \in \left[-15 \ \ 15\right]$ denote the first rule of fuzzy systems $\left(i = 1,2,3,4\right)$, M_{11}, M_{12}, N_{11}, N_{12} are fuzzy sets, so we can represent the main stable system as the following fuzzy model:

$$R_i: \text{IF} \ \ x\left(t\right) \ \text{is} \ M_{i1} \ \text{and} \ z\left(t\right) \ \text{is} \ N_{i1}$$

$$\text{THEN} \quad \dot{X}\left(t\right) = A_i X\left(t\right) + G_i X\left(t\right)$$

where $X\left(t\right) = \left[x(t) \ \ y\left(t\right) \ \ z\left(t\right)\right]^{\mathrm{T}}$:

$$A_1 = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 15 & 0 & 10 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 15 & 0 & -10 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ -15 & 0 & 10 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ -15 & 0 & -10 \end{pmatrix}$$

$$M_{11}=\frac{x(t)+10}{20},\quad M_{12}=\frac{10-x(t)}{20},\quad N_{11}=\frac{15+z(t)}{30},\quad N_{12}=\frac{15-z(t)}{30}$$

Then, after gelatinization $DF(X(t))=\sum_{i=1}^{4}h_iA_i$, with $h_1=M_{11}(x(t))\cdot N_{11}(z(t))$, $h_2=M_{11}(x(t))\cdot N_{12}(z(t))$, $h_3=M_{12}(x(t))\cdot N_{11}(z(t))$, $h_4=M_{12}(x(t))\cdot N_{12}(z(t))$.

Here $s=4$, according to Theorem 1, the feedback gain matrix G_i can be obtained:

$$G_1=[8.4692\quad 2.0082\quad 4.5342],\ G_2=[8.4169\quad 1.9531\quad -5.4631]$$

$$G_3=[-6.6547\quad 1.8908\quad 4.6086],\ G_4=[-6.1522\quad 2.3480\quad -5.8842]$$

Thus, the coupling matrix is obtained as $H=\sum_{i=1}^{4}h_iG_i$.

In order to further verify the correctness of the obtained results, the total error is introduced:

$$E(t)=\sum_{i\neq j}\left|X_i(t)-X_j(t)\right|,\ i,j=1,\cdots,n$$

Figure 1 shows the time history of the total error. It can be seen from the graph that the total error $E(t)$ asymptotically stabilizes to zero in a relatively short period of time, indicating that the complex network has achieved complete synchronization.

3.2. Fully Coupled Lorenz System with Fuzzy Connection

Considering the fully coupled Lorenz network system, the dynamic equation of the *i*-th node is:

$$\begin{cases}\dot{x}_i(t)=a(y_i(t)-x_i(t))+c\sum_{j=1}^{N}a_{ij}x_j(t)\\ \dot{y}_i(t)=bx_i(t)-y_i(t)-x_i(t)z_i(t)\\ \dot{z}_i(t)=x_i(t)y_i(t)-cz_i(t)\end{cases}\tag{3-2}$$

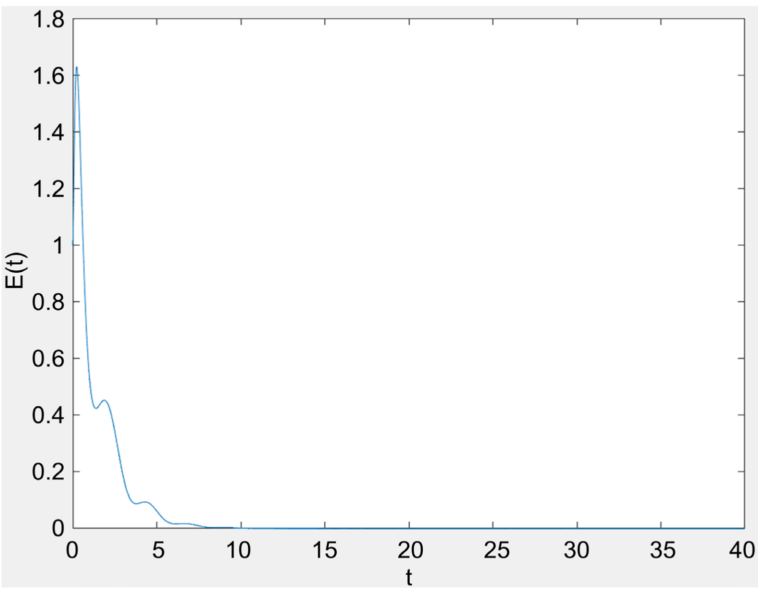

Figure 1. Time history of total error *E*(*t*).

Fix the Lorenz system parameters of each node in the coupled network at $a=10, b=28, z=\frac{8}{3}$, where the dynamics of a single node are chaotic.

The Jacobi matrix of $F\left(X(t)\right)$ in synchronous state $X(t)$ is:

$$DF\left(X(t)\right)=\begin{pmatrix} -10 & 10 & 0 \\ -z(t) & -1 & -x(t) \\ y(t) & x(t) & -\frac{8}{3} \end{pmatrix}$$

Let $x(t)\in[-10\ 10]$, $z(t)\in[-15\ 15]$ denote the first rule of fuzzy systems $(i=1,2,3,4)$, M_{11}, M_{12}, N_{11}, N_{12}, P_{11}, P_{12}, N_{12} are fuzzy sets, so we can represent the main stable system as the following fuzzy model:

$$R_i:\text{IF}\ \ x(t)\ \text{ is }\ M_{i1},\ \ y(t)\ \text{ is }\ N_{i1}\ \text{ and }\ z(t)\ \text{ is }\ P_{i1}$$

$$\text{THEN}\ \ \dot{X}(t)=A_iX(t)+G_iX(t)$$

where $X(t)=\left[x(t)\ \ y(t)\ \ z(t)\right]^{\mathrm{T}}$,

$$A_1=\begin{pmatrix} -10 & 10 & 0 \\ 10 & -1 & -30 \\ -20 & 30 & -\frac{8}{3} \end{pmatrix},\ A_2=\begin{pmatrix} -10 & 10 & 0 \\ -10 & -1 & -30 \\ -20 & 30 & -\frac{8}{3} \end{pmatrix},\ A_3=\begin{pmatrix} -10 & 10 & 0 \\ 10 & -1 & -30 \\ 20 & 30 & -\frac{8}{3} \end{pmatrix}$$

$$A_4=\begin{pmatrix} -10 & 10 & 0 \\ -10 & -1 & -30 \\ 20 & 30 & -\frac{8}{3} \end{pmatrix},\ A_5=\begin{pmatrix} -10 & 10 & 0 \\ 10 & -1 & 30 \\ -20 & -30 & -\frac{8}{3} \end{pmatrix},\ A_6=\begin{pmatrix} -10 & 10 & 0 \\ -10 & -1 & 30 \\ -20 & -30 & -\frac{8}{3} \end{pmatrix}$$

$$A_7=\begin{pmatrix} -10 & 10 & 0 \\ 10 & -1 & 30 \\ 20 & -30 & -\frac{8}{3} \end{pmatrix},\ A_8=\begin{pmatrix} -10 & 10 & 0 \\ -10 & -1 & 30 \\ 20 & -30 & -\frac{8}{3} \end{pmatrix}$$

$$M_{11}=\frac{x(t)+30}{60},\ M_{21}=\frac{30-x(t)}{60},\ N_{11}=\frac{y(t)+20}{40}$$

$$N_{11}=\frac{y(t)+20}{40},\ P_{11}=\frac{z(t)+10}{20},\ P_{21}=\frac{10-z(t)}{20}$$

Then, after gelatinization $DF\left(X(t)\right)=\sum_{i=1}^{8}h_iA_i$, with

$h_1=M_{11}\left(x(t)\right)\cdot N_{11}\left(y(t)\right)\cdot P_{11}\left(z(t)\right)$, $h_2=M_{11}\left(x(t)\right)\cdot N_{11}\left(y(t)\right)\cdot P_{12}\left(z(t)\right)$,
$h_3=M_{11}\left(x(t)\right)\cdot N_{12}\left(y(t)\right)\cdot P_{11}\left(z(t)\right)$, $h_4=M_{11}\left(x(t)\right)\cdot N_{12}\left(y(t)\right)\cdot P_{12}\left(z(t)\right)$,
$h_5=M_{12}\left(x(t)\right)\cdot N_{11}\left(y(t)\right)\cdot P_{11}\left(z(t)\right)$, $h_6=M_{12}\left(x(t)\right)\cdot N_{11}\left(y(t)\right)\cdot P_{12}\left(z(t)\right)$,
$h_7=M_{12}\left(x(t)\right)\cdot N_{12}\left(y(t)\right)\cdot P_{11}\left(z(t)\right)$, $h_8=M_{12}\left(x(t)\right)\cdot N_{12}\left(y(t)\right)\cdot P_{12}\left(z(t)\right)$.

Here $s=8$, according to Theorem 1, the feedback gain matrix G_i can be obtained:

$$G_1=\left[-52.8107\quad 53.3981\quad 57.8162\right],\ G_2=\left[69.3724\quad -68.1082\quad 58.5877\right]$$

$$G_3=\left[-52.8107\quad 53.3981\quad 287.7201\right],\ G_4=\left[68.8983\quad -67.6712\quad 287.4915\right]$$

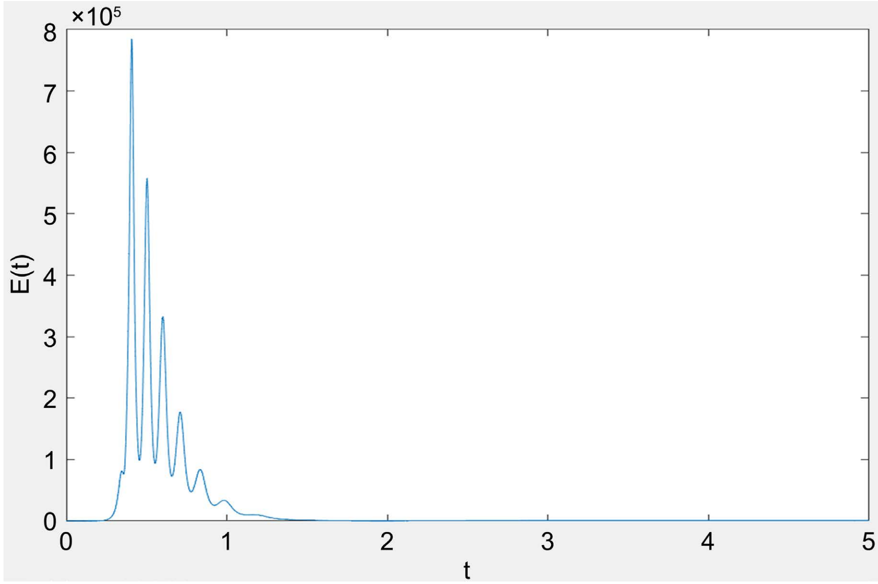

Figure 2. Time history of total error $E(t)$.

$$G_5 = [-52.0991 \quad 52.6932 \quad -287.5365], \quad G_6 = [70.1501 \quad -68.8164 \quad -287.7663]$$

$$G_7 = [-52.8107 \quad 53.3981 \quad 57.8162], \quad G_8 = [69.7762 \quad -68.7676 \quad -57.8620]$$

Thus, the coupling matrix is obtained as $H = \sum_{i=1}^{8} h_i G_i$.

In order to further verify the correctness of the obtained results, the total error is introduced:

$$E(t) = \sum_{i \neq j} \left| X_i(t) - X_j(t) \right|, \quad i, j = 1, \cdots, n$$

Figure 2 shows the time history of the total error. The graphical changes in this graph show that the complex network is synchronized.

4. Conclusion

This article is based on the T-S fuzzy control theory and first studies the synchronization control problem of complex networks with fuzzy connections. Example studies show that this method can not only achieve complete synchronization of complex networks, but also control the time to achieve synchronization. In practical application, this method has a simple structure, good control effect, and strong robustness. It will be widely applied in complex industrial fields, achieving greater economic and social benefits. This article only studies the synchronization problem of fully coupled complex networks using fuzzy control. Regarding complex networks with other structures and complex networks with time delays, it has not yet been addressed and deserves further research.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Watts, D.J. and Strogatz, S.H. (1998) Collective Dynamics of "Small-World" Networks. *Nature*, **393**, 440-443. https://doi.org/10.1038/30918

[2] Barabási, A.L. and Albet, R. (1999) Emergence of Scaling in Random Networks. *Science*, **286**, 509-513. https://doi.org/10.1126/science.286.5439.509

[3] Barabási, A.L., Albert, R. and Jeong, H. (2000) Scale-Free Characteristics of Random Networks: The Topology of the World-Wide Web. *Physica A*: *Statistical Mechanics & Its Applications*, **281**, 69-77. https://doi.org/10.1016/S0378-4371(00)00018-2

[4] Dorogovtsev, S.N. and Mendes, J. (2003) Evolution of Networks: From Biological Nets to the Internet and WWW. Oxford University Press, Oxford.

[5] Montoya, J.M. and Solé, R.V. (2002) Small World Patterns in Food Webs. *Journal of Theoretical Biology*, **214**, 405-412. https://doi.org/10.1006/jtbi.2001.2460

[6] Chesi, G. and Hung, Y.S. (2008) Stability Analysis of Uncertain Genetic Sum Regulatory Networks. *Automatica*, **44**, 2298-2305. https://doi.org/10.1016/j.automatica.2008.01.030

[7] Liang, J., Wang, Z. and Liu, X. (2008) Exponential Synchronization of Stochastic Delayed Discrete-Time Complex Networks. *Nonlinear Dynamics*, **53**, 153-165. https://doi.org/10.1007/s11071-007-9303-5

[8] Ravasz, E., Somera, L.A., Mongru, A.D., *et al.* (2002) Hierarchical Organization of Modularity in Metabolic Networks. *Science*, **297**, 1551-1556. https://doi.org/10.1126/science.1073374

[9] Crucitti, P., Latora, V. and Marchiori, M. (2004) A Topological Analysis of the Italian Electric Power Grid-Science Direct. *Physica A*: *Statistical Mechanics and Its Applications*, **338**, 92-97. https://doi.org/10.1016/j.physa.2004.02.029

[10] Lai, Y.C., Zhao, L., Ye, N., *et al.* (2005) Onset of Traffic Congestion in Complex Networks. *PHYSICAL REVIEW E*, **71**, Article ID: 026125. https://doi.org/10.1103/PhysRevE.71.026125

[11] Barabási, A.L., Jeong, H., Néda, Z., *et al.* (2002) Evolution of the Social Network of Scientific Collaborations. *Physica A*: *Statistical Mechanics & Its Applications*, **311**, 590-614. https://doi.org/10.1016/S0378-4371(02)00736-7

[12] Donges, J.F., Zou, Y., Marwan, N., *et al.* (2009) Complex Networks in Climate Dynamics. *The European Physical Journal Special Topics*, **174**, 157-179. https://doi.org/10.1140/epjst/e2009-01098-2

[13] Strogatz, S.H. (2001) Exploring Complex Networks. *Nature*, **410**, 268-276. https://doi.org/10.1038/35065725

[14] Shim, W. (2020) On the Gneric Complete Synchronization of the Disrete Kuramoto Model. *Kinetic and Related Models*, **13**, 979-1005. https://doi.org/10.3934/krm.2020034

[15] Tanaka, K. and Wang, H.O. (2001) Fuzzy Control Systems Design and Analysis. John Wiley & Sons, Inc., New York, 301.

[16] Takagi, T. and Sugeno, M. (1985) Fuzzy Identification of Systems and Its Applications to Modeling and Control. *IEEE Transactions on Systems Man & Cybernetics*, **15**, 116-132. https://doi.org/10.1109/TSMC.1985.6313399

[17] Zhang, Y.Y. and Zheng, Y.G. (2021) Synchronization of Transient Delay-Coupled Network. *Journal of Computer and Communications*, **9**, 25-27. https://doi.org/10.4236/jcc.2021.99002