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H_∞ Control for Externally Excited Building Structures with Active Mass Damper under Actuator Saturation

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Abstract

This paper investigates the application of active mass dampers to mitigate the vibrations of building structures subjected to unknown external excitations under controller saturation conditions. By utilizing an H_∞ control strategy, the optimal state feedback controller is derived by solving the linear matrix inequality problem for controller saturation. Case studies show that the proposed controller is capable of stabilizing the closed-loop system with good control performance and effectively suppressing vibrations in building structures under unknown external excitation. When compared to controllers that do not consider saturation, the proposed controller requires lower gain and results in reduced energy consumption. The research findings provide valuable insights for addressing real-world building structure control problems, contributing to both theoretical significance and practical applications.

Keywords

Structural Vibration, Active Mass Damper, H_∞ Control, Controller Saturation, Linear Matrix Inequality

1. Introduction

In recent years, the increasing number of high-rise buildings has greatly heightened the need for vibration damping control. High-rise buildings are susceptible to vibrations and pose risks of collapse when subjected to external forces such as earthquakes and strong winds. Hence, conducting research on vibration damping control for high-rise buildings has become of utmost importance. Over the past few decades, extensive research has been conducted on various techniques for vibration damping control in high-rise buildings. These techniques can be broadly categorized into two main groups: passive control and active control [1] [2] [3] [4] [5]. Passive control methods were initially employed to mitigate vibrations in building structures and were widely acknowledged for their advantages, such as independence from external energy, simple construction, easy maintenance, and low cost. However, passive control methods lack the capability to make real-time adjustments and optimize control performance in response to changes in the building system and environmental disturbances. Empirical evidence has shown that passive control is less effective in reducing structural vibrations in high-rise buildings and cannot sufficiently absorb vibration energy, subsequently failing to meet the requirements of practical applications [6] [7]. In contrast, active control entails the utilization of external energy in the damping devices to generate control forces that actively suppress structural vibrations in real-time. Active control provides benefits in terms of adaptability and control effectiveness [7] [8]. Common active control devices comprise active mass dampers, active tuned mass dampers, active tie systems, and active bracing systems [9]. Among these, the Active Mass Damper (AMD) has garnered significant popularity in high-rise buildings owing to its remarkable vibration damping performance and cost-effectiveness [10] [11] [12].

A variety of control algorithms have been employed in active mass dampers to mitigate vibrations in high-rise building structures [6] [13]. These control algorithms encompass various approaches, including model-based algorithms, data-based algorithms, and hybrid methods. The implementation of these algorithms aims to improve the effectiveness of vibration damping in high-rise buildings, reduce the influence of external excitations on the structural integrity, and create a safer and more comfortable environment for work and daily activities. H_∞ control presents notable advantages in the control domain. It obviates the necessity of prior knowledge about the system's structural model, operates effectively in both linear and nonlinear control scenarios, exhibits improved robustness, and does not mandate foreknowledge of system uncertainties. Consequently, H_∞ control enjoys widespread recognition as a dependable, flexible, and applicable control methodology for diverse practical systems [14].

The external excitation of high-rise building structures is often unknown. To address this challenge, H_∞ control is designed to regulate the system performance under worst-case external excitations. The fast response controller, developed using this approach, effectively improves the dynamic performance of the structural system, enabling it to meet the stringent requirements imposed by high-speed dynamic loads and emergency situations [15] [16] [17]. Consequently, H_∞ control theory has garnered significant attention in the field of civil engineering and holds immense potential for a wide range of applications. Active mass dampers were employed for vibration control in building structures subjected to seismic loading, and an H_∞ controller was designed accordingly [18]. A novel method was proposed for establishing vibration H_∞ control within a specific frequency range [19]. Furthermore, a robust H_∞ controller was designed by applying a constrained layer comprising damping and piezoelectric materials onto a flexible beam, and the control effectiveness of this approach was investigated [20]. To mitigate the dynamic response of reinforced concrete frame structures subjected to seismic acceleration, researchers introduced an H_∞ control method [21]. Furthermore, the study investigated the design of optimal H_∞ state feedback control, which took into account feedback gain constraints [22]. These studies not only laid a solid theoretical groundwork but also carried practical significance in the implementation of H_∞ control in civil engineering applications.

In actual high-rise building structural systems, the controller's output is often constrained. If the controller's output surpasses the acceptable range of the system, controller saturation may occur, resulting in system instability and potential damage [23] [24]. Therefore, it is crucial to consider structural vibration control in the presence of controller saturation. By doing so, the H_∞ control algorithm can ensure that the controller output remains within a reasonable range, aligns with the actual system behavior, and maintains control stability while improving vibration suppression. Additionally, this approach helps to protect the controller from overloading and reduces control energy consumption [25] [26]. In the field of controller design for building structures, various approaches have been proposed to tackle the challenges posed by saturation constraints. Notably, a gain scheduling controller design method [27] and an event-triggered algorithm [28] have been introduced to ensure optimal performance and stability for linear systems subject to input saturation. Furthermore, researchers have investigated the event-triggered tranquilization problem for discrete-time segmented affine systems with input saturation [29]. Moreover, adaptive neural network event-triggered controllers have demonstrated effectiveness in dealing with input saturation, unknown disturbances, and sensor failures [30]. However, relatively limited research has been conducted on controller saturation constraints in the control of building structures that incorporate active mass dampers [31] [32] [33] [34].

This study investigates the suppression of vibrations in a building structure subjected to unknown external excitations by utilizing an active mass damper (AMD) under controller saturation conditions. The objective is to provide an effective control strategy for real-time control of structural vibrations, thereby improving stability, safety, and comfort. The paper is structured as follows, and the contributions of this work can be summarized as follows:

1) The research employs an H_∞ control strategy and solves the linear matrix inequality (LMI) to derive the state-feedback H_∞ optimal control law in the presence of controller saturation. This approach effectively maintains system performance under worst-case disturbances, offering crucial theoretical support for practical implementations.

2) The paper specifically addresses the prevalent issue of controller saturation in real-world systems. Many physical systems, when subjected to excessively

large input signals, reach an upper limit known as saturation. By imposing input signal limitations within infinite paradigms, the researchers successfully mitigate controller saturation, ensuring system stability.

3) Notably, the study encompasses both theoretical and practical aspects. The obtained control law can be directly applied to real active mass damper systems, enabling the control of building structure vibrations. These practical implications introduce novel ideas and methods for vibration control in building structures.

In summary, this thesis presents innovative theoretical findings and provides practical guidelines. By employing the H_∞ control strategy, it adeptly addresses the challenge of controller saturation and successfully achieves vibration control in building structures. This contribution offers an effective solution to the field of vibration control. In Section 2, a dynamic model of the building structure with an active mass damper is established. In Section 3, an H_∞ control design method for the building structure is proposed under controller saturation, taking into account the constraints. In Section 4, an example analysis is conducted to verify the practical effectiveness of the proposed method, demonstrating its validity in addressing the actuator saturation problem. In Section 5, the main contributions and research results of the paper are summarized.

2. Structural Vibration Control Model with AMD

In the context of building vibration control, the top trolley moves under the drive of an electric motor, which generates a reaction force on the building structure. By adjusting the voltage feedback signal of the electric motor, the reaction force exerted on the building can be controlled to effectively suppress the vibrations in the structure.

The simplified model of an n-story building structure with an active mass damper (AMD) is shown in **Figure 1**. In this model, the top-level carriage is driven by an internal DC motor with a rotational inertia J_c. The relationship between the angular velocity $w_c(t)$ and the linear velocity $\dot{x}_c(t)$ is given by the following equation:

$$w_c(t) = \mu_1 \dot{x}_c(t) \tag{1}$$

The angular velocity of the trolley motor can be converted to linear velocity to accurately determine and control the motion of the trolley. μ_1 is a system parameter.

The trolley drive force F_c has a linear functional relationship with the trolley motor voltage $u(t)$ and the trolley linear speed $\dot{x}_c(t)$, as shown in the following expression:

$$F_c = \mu_2 u(t) + \mu_3 \dot{x}_c(t) \tag{2}$$

μ_2 and μ_3 represent system parameters.

By employing Lagrange's equations, the dynamical equations of the n-storey building system can be derived in the following manner:

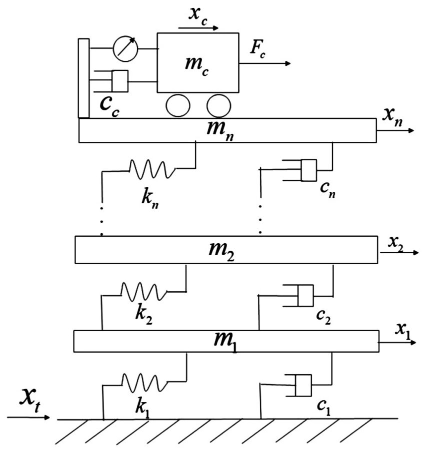

Figure 1. Simplified model of *n*-story building structure with AMD, where m_i, c_i, k_i and $x_i\left(i=1,2,\cdots,n\right)$ represent the mass, equivalent viscous damping coefficient, stiffness, and horizontal deflection, respectively, of the *i*th floor; m_c, c_c, F_c and x_c represent the mass, equivalent viscous damping coefficient, inertia force, and horizontal displacement of the active mass damper (AMD) installed on the roof of the building, relative to the top floor; and x_t is denotes the ground-level horizontal displacement, which represents the excitation applied to the building.

$$\begin{cases} m_1\ddot{x}_1(t)+(c_1+c_2)\dot{x}_1(t)-c_2\dot{x}_2(t)+(k_1+k_2)x_1(t)-k_2x_2(t)-k_1x_t(t)=0 \\ \vdots \\ m_c\ddot{x}_c(t)+(m_c+m_n)\ddot{x}_n(t)-c_n\dot{x}_{n-1}(t)+c_n\dot{x}_n(t)-k_nx_{n-1}(t)+k_nx_n(t)=0 \\ \left(m_c+J_c\mu_2^2\right)\ddot{x}_c^2(t)+m_c\ddot{x}_n(t)+(c_c-\mu_3)\dot{x}_c(t)=\mu_2u(t) \end{cases} \tag{3}$$

Define

$$\overline{x}(t)=\left[x_1(t),x_2(t),\cdots,x_n(t),x_c(t)\right] \tag{4}$$

The equation of state for the building structure under external excitation can be expressed concisely in matrix form as follows:

$$\overline{M}\ddot{\overline{x}}(t)+\overline{C}\dot{\overline{x}}(t)+\overline{K}\,\overline{x}(t)=\overline{B}_u u(t)+\overline{B}_w x_t(t) \tag{5}$$

where $\ddot{\overline{x}}(t)=\dfrac{\mathrm{d}^2\overline{x}(t)}{\mathrm{d}t^2},\dot{\overline{x}}=\dfrac{\mathrm{d}\overline{x}(t)}{\mathrm{d}t}$.

$$\overline{K}=\begin{bmatrix} k_2+k_1 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_3+k_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -k_n & 0 \\ 0 & 0 & \cdots & -k_n & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \overline{B}_w=\begin{bmatrix} k_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\overline{M}=\begin{bmatrix} m_1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & m_{n-1} & 0 & 0 \\ 0 & \cdots & 0 & m_c+m_n & m_c \\ 0 & \cdots & 0 & m_c & m_c+J_c\mu_2^2 \end{bmatrix}, \overline{B}_u=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mu_2 \end{bmatrix},$$

$$\bar{C}=\begin{bmatrix} c_1+c_2 & -c_2 & \cdots & 0 & 0 & 0 \\ -c_2 & c_3+c_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & c_n+c_{n-1} & -c_n & 0 \\ 0 & 0 & \cdots & -c_n & c_n & 0 \\ 0 & 0 & \cdots & 0 & 0 & c_c-\mu_3 \end{bmatrix}.$$

Since $\bar{M}$ is invertible, Equation (5) can be deformed as:

$$\ddot{\bar{x}}(t)=-\bar{M}^{-1}\bar{K}\bar{x}(t)-\bar{M}^{-1}\bar{C}\dot{\bar{x}}(t)+\bar{M}^{-1}\bar{B}_u u(t)+\bar{M}^{-1}\bar{B}_w x_t(t) \tag{6}$$

Define $A_1=-\bar{M}^{-1}\bar{K}$, $A_2=-\bar{M}^{-1}\bar{C}$, $B_u=\bar{M}^{-1}\bar{B}_u$, $B_w=\bar{M}^{-1}\bar{B}_w$, Equation (6) can be simplified as:

$$\ddot{\bar{x}}(t)=A_1\bar{x}(t)+A_2\dot{\bar{x}}(t)+B_u u(t)+B_w x_t(t) \tag{7}$$

Further define $x(t)=\left[\bar{x}(t),\dot{\bar{x}}(t)\right]^{\mathrm{T}}$, $w(t)=x_t(t)$, then the dynamics model of the final building structural system can be simplified as:

$$\dot{x}(t)=Ax(t)+B_2u(t)+B_1w(t) \tag{8}$$

where

$$A=\begin{bmatrix} O & E \\ -\bar{M}^{-1}\bar{K} & -\bar{M}^{-1}\bar{C} \end{bmatrix},\quad B_2=\begin{bmatrix} O \\ \bar{M}^{-1}\bar{B}_u \end{bmatrix},\quad B_1=\begin{bmatrix} O \\ \bar{M}^{-1}\bar{B}_w \end{bmatrix},$$

where O and E are denoted as the zero and unit matrices, respectively.

In practice, the control force of the controller is limited by a saturation constraint. These constraints are designed to prevent the controller from overload and effectively conserve input energy. To ensure the stability and reliability of the control system, the control inputs must satisfy the following constraints:

$$\|u(t)\|_\infty=\sqrt{\sup\left\{u^{\mathrm{T}}(t)u(t)\right\}}\le\mu \tag{9}$$

where μ is the maximum input value of the controller, serving as an imposed upper limit.

3. H_∞ Control Design under Controller Saturation

Under the external excitation, the control objective of the building is to minimize the relative displacement and velocity of each floor. This objective is achieved by formulating the control output equation as follows:

$$z(t)=C_1x(t) \tag{10}$$

where $z(t)$ represents the controlled output vector at time *t*, $x(t)$ represents the state vector at time *t*, and C_1 is the weighted coefficient matrix.

The equations governing the controlled system for the building structure, taking into account the saturation case of the controller, are as follows:

$$\begin{cases} \dot{x}(t)=Ax(t)+B_2u(t)+B_1w(t) \\ z(t)=C_1x(t) \\ \|u(t)\|_\infty\le\mu \end{cases} \tag{11}$$

where $x(t) \in R^n$ is the state vector of the building structure, including information such as displacements and velocities; $z(t) \in R^r$ is the output of the control response of the building structure; $w(t) \in R^q$ is the external perturbation, which mainly comes from the influence of factors such as earthquakes and random winds; and A, B_1, B_2, C_1 is the system parameter matrix, which describes the dynamics of the building structure.

In order to enhance the analysis and design process of controllers that satisfy system requirements, it is essential to introduce necessary definitions and underlying primitives.

Definition 1 Design a state feedback H_∞ controller $u = Kx$ such that the control system (11) is asymptotically stable and satisfies the H_∞ performance gain metric γ,

$$\left\|T_{\omega z}(s)\right\|_\infty = \sup_{w, \|w(t)\|_2 \neq 0} \frac{\|z(t)\|_2}{\|w(t)\|_2} < \gamma \tag{12}$$

where $\|z(t)\|_2 = \sqrt{\int_0^{+\infty} z^{\mathrm{T}}(t) z(t) \mathrm{d}t}$. Furthermore, by conducting a search for the optimal value of γ, the closed-loop control system's H_∞ optimal control design is obtained, and the optimal suppression of disturbances is realized.

Lemma 1 Schur Complement [13]: For a given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where S_{11} and S_{22} are symmetric matrices of appropriate dimension, and S_{12} is a matrix of appropriate dimension. Then the following three conditions are equivalent:

1) $S < 0$;
2) $S_{11} < 0$, $S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$;
3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0$.

In order to ensure control system stability and account for the bounded nature of the external disturbance $w(t)$, a state feedback-based H_∞ control method is used to achieve optimal damping. For the given system Equation (11), a method of H_∞ controller under controller saturation is proposed.

Theorem 1: Given the parameters γ, μ and the initial value $x(0)$, if there exist symmetric positive definite matrices X and matrices W of suitable dimension that satisfy the following matrix inequality:

$$\text{s.t.} \begin{bmatrix} AX + B_2 W + (AX + B_2 W)^{\mathrm{T}} & B_1 & (C_1 X)^{\mathrm{T}} \\ B_1^{\mathrm{T}} & -\gamma^2 I & 0 \\ C_1 X & 0 & -I \end{bmatrix} < 0, \tag{13}$$

$$X > 0$$

$$\begin{bmatrix} 1 & x(0)^{\mathrm{T}} \\ x(0) & X \end{bmatrix} \geq 0 \tag{14}$$

$$\begin{bmatrix} X & W^{\mathrm{T}} \\ W & \mu^2 I \end{bmatrix} \geq 0 \tag{15}$$

Then there exists a state-feedback H_∞ controller $u = Kx = WX^{-1}x$ and the closed-loop control system (11) satisfies the following conditions under controller saturation:

1) The closed-loop system (11) is asymptotically stable.

2) The system satisfies the performance index γ-H_∞.

3) The input saturation constraint $\|u(t)\|_\infty \le \mu$ is always satisfied.

Proof Define a Lyapunov functional as $V(x(t)) = x^{\mathrm{T}}(t)Px(t)$, where P is a symmetric positive definite matrix; $x^{\mathrm{T}}(t)Px(t) \le 1$. Taking the time derivative of $V(t)$ along the solution of (11) gives:

$$\dot{V}(x(t)) = \dot{x}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\dot{x}(t)$$
$$= \left[Ax(t) + B_2u(t) + B_1w(t)\right]^{\mathrm{T}} Px(t) + x^{\mathrm{T}}(t)P\left[Ax(t) + B_2u(t) + B_1w(t)\right]$$

and substituting $u = Kx$ into the above equation gives:

$$\begin{aligned}\dot{V}(x(t)) &= \dot{x}^{\mathrm{T}}(t)\left[(A+B_2K)^{\mathrm{T}}P + P(A+B_2K) + \frac{1}{\gamma^2}PB_1B_1^{\mathrm{T}}P + C_1^{\mathrm{T}}C_1\right]\dot{x}(t) \\ &\quad - z^{\mathrm{T}}(t)z(t) + \gamma^2 w^{\mathrm{T}}(t)w(t) - \left[\gamma w(t) - \frac{1}{\gamma}B_1^{\mathrm{T}}P\dot{x}(t)\right]^{\mathrm{T}}\left[\gamma w(t) - \frac{1}{\gamma}B_1^{\mathrm{T}}P\dot{x}(t)\right] \\ &\le -z^{\mathrm{T}}(t)z(t) + \gamma^2 w^{\mathrm{T}}(t)w(t)\end{aligned} \tag{16}$$

Integrating both sides of the above inequality from 0 to ∞, one has

$$V(x(\infty)) - V(x(0)) \le \int_0^{+\infty}\left(\gamma^2 w^{\mathrm{T}}(t)w(t) - z^{\mathrm{T}}(t)z(t)\right)\mathrm{d}t \tag{17}$$

The initial condition is known to be $x(0) = 0$, then $V(x(0)) = 0$, $V(x(\infty)) \ge 0$. Thus it is possible to obtain

$$\int_0^{+\infty}\left(z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t)\right)\mathrm{d}t < 0 \tag{18}$$

So the closed-loop control system (11) satisfies the γ-H_∞ performance specification.

If

$$(A+B_2K)^{\mathrm{T}}P + P(A+B_2K) + \frac{1}{\gamma^2}PB_1B_1^{\mathrm{T}}P + C_1^{\mathrm{T}}C_1 < 0 \tag{19}$$

$\dot{V}(x(t)) < 0$ can be obtained, and system (11) is asymptotically stable.

The inequality (19) is transformed into (20) by the Schur complement procedure.

$$\begin{bmatrix}(A+B_2K)^{\mathrm{T}}P + P(A+B_2K) + C_1^{\mathrm{T}}C_1 & PB_1 \\ B_1^{\mathrm{T}}P & \gamma^2 I\end{bmatrix} < 0 \tag{20}$$

the above equation is equivalent to

$$\begin{bmatrix}P(A+B_2K) + (A+B_2K)^{\mathrm{T}}P & PB_1 & C_1^{\mathrm{T}} \\ B_1^{\mathrm{T}}P & -\gamma^2 I & 0 \\ C_1 & 0 & -I\end{bmatrix} < 0 \tag{21}$$

Multiplying both side of the above inequality by block-diag $\{P^{-1}, I, I\}$ and where $X = P^{-1}$, and $W = KX = KP^{-1}$, The inequality (21) is transformed into (13) by the Schur complement procedure.

Since $x^{\mathrm{T}}(t)Px(t) \le 1$, then,

$$-1 + x^{\mathrm{T}}(t)Px(t) \le 0 \tag{22}$$

where $X = P^{-1}$, The inequality (22) is transformed into (14) by the Schur complement procedure.

The derivation of (15) is as follows: From $\|u(t)\|_{\infty} \le \mu$,

$$u(t)^{\mathrm{T}} u(t) = x^{\mathrm{T}}(t)K^{\mathrm{T}}Kx(t) \le \mu^2$$

Therefore,

$$\frac{1}{\mu^2} x^{\mathrm{T}}(t)K^{\mathrm{T}}Kx(t) \le 1 \tag{23}$$

Since

$$x^{\mathrm{T}}(t)X^{-1}x(t) < x^{\mathrm{T}}(0)X^{-1}x(0) \le 1 \tag{24}$$

where $t < 0$.

If

$$\frac{1}{\mu^2} x^{\mathrm{T}}(t)K^{\mathrm{T}}Kx(t) \le x^{\mathrm{T}}(t)X^{-1}x(t) \tag{25}$$

Then (23) is holds. Therefore, we have

$$x^{\mathrm{T}}(t)\left(\frac{1}{\mu^2}K^{\mathrm{T}}K - X^{-1}\right)x(t) \le 0 \tag{26}$$

The inequality (26) is transformed into (27) by the Schur complement procedure.

$$\begin{bmatrix} X^{-1} & K^{\mathrm{T}} \\ K & \mu^2 I \end{bmatrix} \ge 0 \tag{27}$$

By multiplying both side of the (27) by block-diag $\{X, I\}$ (15) is obtained, where $X = X^{\mathrm{T}}$; define $W = KX$.

This is the design of a state feedback H_∞ controller that takes controller saturation into account to effectively achieve optimal control of structural vibration and mitigate external disturbances. The following illustrative example is analyzed to validate the performance of the designed controller and ensure system stability under external excitation, while also meeting the performance requirements in the presence of controller saturation.

4. Case Study

The study focused on an in-depth numerical analysis of a two-story building structure as the subject of investigation. The purpose of this experiment was to gain a comprehensive understanding of the controller's performance under saturated conditions. To ensure the applicability and generalization of the results, simulations were conducted using the system parameters provided by Quanser.

The weighted coefficient matrix, C_1 is qualitatively selected according to the disturbance suppression performance index specific to high-rise buildings. Subsequently, utilizing the values of the system model parameters from **Table 1**, the coefficient matrix of the system (11) can be derived as follows:

$$A=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1470.5882 & 735.2941 & 0 & 0 & 0 & 0 \\ 335.4766 & -335.4766 & 0 & 0 & 0 & 5.9728 \\ -278.4885 & 278.4885 & 0 & 0 & 0 & -18.6534 \end{bmatrix}, B_2=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.9600 \\ 2.9981 \end{bmatrix},$$

$$C_1=\begin{bmatrix} 0.1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B_1=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 735.2941 \\ 0 \\ 0 \end{bmatrix}.$$

In practical applications, building systems are frequently subjected to diverse external excitations arising from the environment, loads, or other factors. To effectively evaluate the performance of building systems under such various external excitations, this study employs three distinct signal excitation methods: step signals, simple harmonic signals, and Gaussian white noise signals. Step signals are employed to simulate circumstances in which sudden changes occur within the system, resulting in significant alterations in the system's state variables over a short span of time. Conversely, simple harmonic signals are used to emulate periodic external disturbances experienced by the system, such as natural phenomena like earthquakes and wind vibrations. Gaussian white noise signals, on the other hand, are utilized to simulate random perturbations encountered by the system, including airflow fluctuations and seismic waves. These perturbations possess irregular characteristics and introduce an uncertain effect on the system's stability.

Table 1. Parameters of the building structural model.

Parameters	Numerical value	Parameters	Numerical value
m_1	0.68	m_c	0.65
m_2	1.38	c_c	3
c_1	0	k_c	0
c_2	0	μ_1	584
k_1	500	μ_2	1.7235
k_2	500	μ_3	−7.7236
J_c	3.9×10^{-7}		

The study also focuses on the reaction force exerted by the controller on the top floor of the building, as well as the pulling effect originating from the upper floor on the lower floor. The study concentrates on comprehending the stress-induced vibrational behavior observed by the top floor of the building when subjected to the three distinct signal excitation modes. The primary objective of this research endeavor is to provide a comprehensive assessment of the stability and dynamic performance exhibited by multi-story or high-rise building structural systems when exposed to various types of external disturbances. Such evaluations serve as a basis for optimizing and enhancing the design of building systems, effectively bolstering their resistance to disturbances within complex external environments.

The control input $\|u(t)\|_\infty \le \mu = 8000$ of the H_∞ controller is subject to constraints, and the optimal W, X, and their corresponding feedback gains are determined by minimizing performance metrics. This optimization task is accomplished using the Mincx solver available in the LMI toolbox.

$$K = WX^{-1} = 10^3 \times [-2.5935 \quad 2.6480 \quad -0.0000 \quad 0.0334 \quad 0.0576 \quad 0.0002]$$

The optimal performance is $\gamma = 0.1539$, and $\max_t \|u(t)\|_\infty = 313.4 \le \mu$.

Figure 2 depicts a comparison of building roof vibrations under step signal excitation with controlled and uncontrolled conditions. Without control, the roof exhibits intense oscillations from 0 to 5.3 seconds after the step signals input, characterized by significant overshooting and a longer regulation time of 3.3 seconds for stabilization. In contrast, employing the H_∞ control method with a saturated controller effectively mitigates roof vibrations $x_2(t)$, reducing the time to reach stabilization to 0.45 seconds and minimizing overshooting by 60.2%. This implies that H_∞ control under controller saturation ensures smoother response of $x_2(t)$ and faster attainment of equilibrium, thereby effectively limiting vibrations within the controllable range. Figure 3 shows the steady-state vibrations of the building structure under simple harmonic excitation. Without control, the roof amplitude closely follows the excitation frequency but is not effectively controlled. However, after implementing H_∞ control with the saturation controller, roof vibrations are reduced by 73.7%, effectively

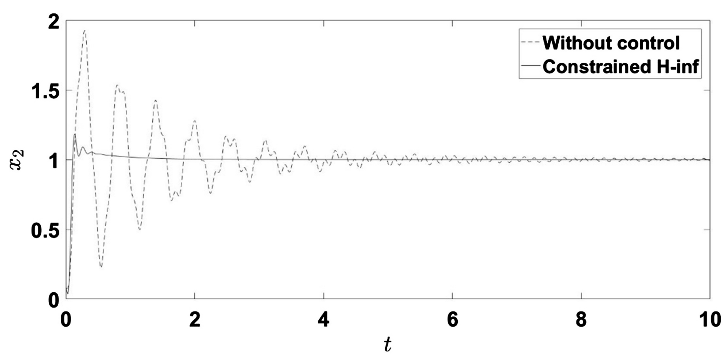

Figure 2. Image depicting the response of roof vibration under step excitation signal. $x_2(t)$ represents the horizontal displacement of the roof.

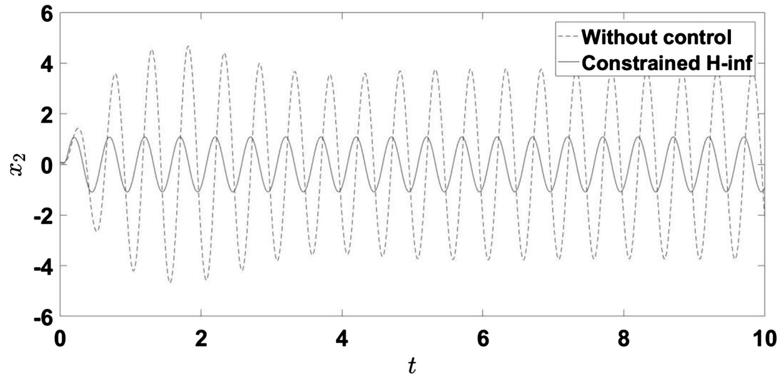

Figure 3. Image depicting the response of roof vibration under simple harmonic excitation. $x_2(t)$ represents the horizontal displacement of the roof.

suppressing the influence of the excitation signal on the building. **Figure 4** illustrates the $x_2(t)$ response curves of H_∞ controlled and uncontrolled under random signal excitation with the controller operating in a saturated state. The comparison shows that the building structure system is continuously affected by the disturbance and cannot reach a stable state. After adopting the control strategy, the displacement vibration $x_2(t)$ of the building structure is reduced by an average of 56.5%, accompanied by decreased fluctuation. This signifies that the adopted control measures significantly reduce the vibration effect of external excitation on the building structure and improve its dynamic performance, thereby establishing robust anti-disturbance capabilities.

From **Figure 3** and **Figure 4**, it can be found that the vibration amplitude of each floor is noticeably lower under the influence of control. This indicates that the active mass damper effectively absorbs and disperses external energy, thereby reducing the vibration of the building structure. In addition, the calculated control inputs consistently remain below the predetermined values for H_∞ control. This result indicates that the control inputs successfully achieve vibration damping objectives while adhering to the saturation constraints. Consequently, by considering the saturation characteristics of the controller, the proposed approach effectively reduces floor vibrations and enhances the structural resilience against vibrations. This preliminary validation of the input constraint strategy offers valuable insights for future research endeavors.

To comprehensively examine the impact of controller saturation characteristics on the effectiveness of the building system control, it is crucial to study the stress vibration of the building structure without considering the controller saturation characteristics To achieve this objective, a conventional H_∞ controller is designed by employing the method outlined in Equation (13) of Theorem 1. The Mincx solver from the LMI toolbox is employed to search for the optimal value of γ, thereby determining the feedback gain of the controller:

$$K_0 = W_0 X_0^{-1} = 10^5 \times [1.1071 \;\; 0.7649 \;\; -0.0002 \;\; 0.0140 \;\; -0.1235 \;\; 0.0407]$$

The optimal performance is $\gamma_0 = 0.1415$, and $\max_t \|u_0(t)\|_\infty = 6.821 \times 10^3$.

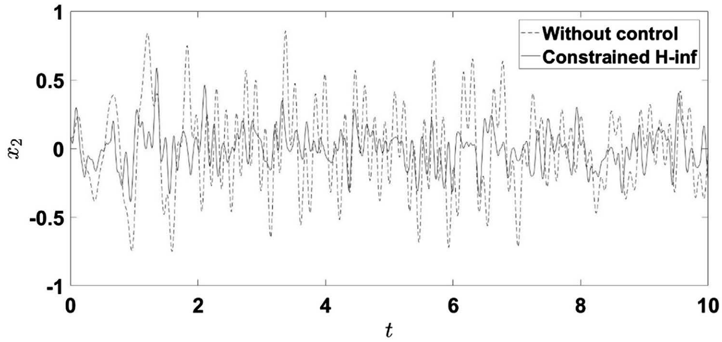

Figure 4. Image depicting the response of roof vibration under randomly perturbed excitation. $x_2(t)$ represents the horizontal displacement of the roof.

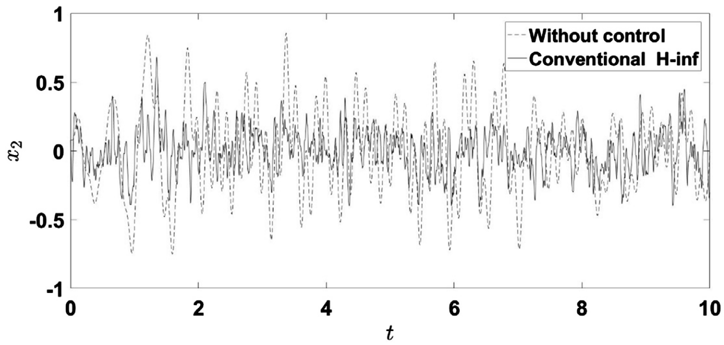

Figure 5. Image depicting the response of roof vibration under randomly perturbed excitation. $x_2(t)$ represents the horizontal displacement of the roof.

The calculations confirm that $K < K_0$, which aligns with the established understanding that smaller control gains tend to yield superior results. Consequently, when designing and implementing the controller proposed in Theorem 1, it is essential to account for the controller saturation problem in order to maximize the efficient utilization of input energy and achieve optimal control outcomes. This underscores the necessity of comprehensive consideration of controller saturation while designing controllers for vibration control, with the aim of attaining optimal control results. By comparing the vibration of the building structure $x_2(t)$ under the conventional control strategy demonstrated in **Figure 5** with the vibration under controller saturation in **Figure 4**, it can be observed that a significant reduction in structural vibrations. This reinforces the importance of fully addressing controller saturation during the design of vibration control systems, as it enables the realization of a more optimized control effect.

5. Conclusion Comments

This study implements an H_∞ control strategy with a active mass damper under controller saturation conditions to effectively mitigate vibrations in building structures, while ensuring adherence to control input constraints and meeting H_∞ performance requirements. The proposed strategy guarantees stability and optimal performance of the closed-loop system, while minimizing control energy consumption. Through example analysis, the feasibility and effectiveness of the proposed strategy are demonstrated. This control strategy provides an efficient vibration control method for real-world architectural structures. It aims to simultaneously avoid controller saturation and significantly reduce vibration amplitudes to enhance resilience against vibrations. Its application is crucial for improving the safety and reliability of important infrastructures such as large buildings and bridges. Future research can focus on complex structural control, multi-degree-of-freedom systems (e.g., high-rise buildings, large-span structures), and vibration and noise control. For example, investigating multi-modal vibration control and multi-input multi-output control strategies, as well as mitigating the interference of urban noise on structural vibrations. In summary, this research offers potential for vibration control in architectural structures and provides valuable directions for future research and application in important areas. It holds theoretical and practical significance in the field of architectural structural control, enhancing the understanding of system stability and robust control under controller saturation, and offering new insights and approaches for the optimization and control of architectural structures.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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